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Single Machine Job Sequencing With a Restricted Common Due Window

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ABSTRACT This article deals with the problem of sequencing N jobs on a single machine with a restrictive common due window. The objective is to minimize the total weighted earliness-tardiness penalties, which conform to just-in-time (JIT) manufacturing. A novel backtracking simulated annealing (BSA) algorithm with a backtracking mechanism and an effective coding scheme is proposed herein to solve this problem. The performance of the proposed BSA algorithm is compared with that of the best available algorithm and the simulated annealing (SA) algorithm using four benchmark problem sets. The computational results reveal that the backtracking mechanism can improve the performance of the SA algorithm and make the proposed BSA algorithm outperform the state-of-the-art algorithm. The proposed BSA algorithm is sufficiently efficient to satisfy the real-world scheduling requirements of the JIT manufacturing system.

INDEX TERMS Scheduling, single machine, common due window, simulated annealing, backtracking.

I. INTRODUCTION

The single-machine scheduling problem (SMSP) is one of the most studied manufacturing systems owing to its practicability [1]. In recent decades, many investigations of various SMSPs have examined the dispatching rules [2], [3] and efficient heuristic algorithms [4]–[7], while applying various criteria. An increasing number of manufacturers are paying attention to just-in-time (JIT) production modes as they are confronted with numerous challenges, including increased product customization, short product life cycle and shortened time to market. In practice, meeting the specified due dates of jobs is critical for JIT production [8], [9]. Sidney [10] studied an SMSP with the objective of minimizing the total tardiness-earliness penalties for all jobs with target starting times and corresponding due dates. Gens and Levner [11] focused on minimizing the penalties of delayed jobs in an SMSP, and proposed a fast algorithm for approximating a tight bound on delay penalties. While these studies on SMSPs allowed different due dates for different jobs, Kanet [12] concentrated on a special case of the SMSP with a common due date. The

goal was to minimize the mean deviation of job completion times from the common due date. Raghavachari [13] determined the common due date in an SMSP, and sequenced jobs with minimization of the weighted mean absolute deviation of job completion times. He showed that the optimal job sequence is V-shaped. A V-shaped job sequence indicates a subset of jobs that are sorted in order of non-increasing job processing times, while the remaining jobs, are sorted in order of non-decreasing job processing times. Krieger and Raghavachari [14] further revealed that the optimal schedule that minimized the sum of the penalties (early or late) for all jobs is V-shaped when all jobs have the same penalty function. Owing to its effectiveness, the V-shaped sequence has become a useful property for efficiently finding optimal or near-optimal schedules in SMSPs with variable or constant common due dates [15]–[17].

SMSPs with a common due date (CDD) have been proved to be NP-complete [18], [19]. A review of the literature by Gordon, et al. [20] included a comprehensive discussion of the computational difficulty of solving such SMSPs. In light of this difficulty of computation using exact methods (such as by the branch-and-bound algorithm [21]), heuristic algorithms have become popular owing to their tractability and

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efficiency, as revealed by a study on the ant colony optimization algorithm [22]. Although various SMSPs have been extensively studied, most relevant research concerns SMSPs with a CDD [23], [24]. However, the assumption of the single due date may be unrealistic in some real-world operations.

The common due window (CDW), which has various applications in JIT manufacturing, chemical processing, project scheduling, and information technology [25], is a generalization of the CDD. In this type of scheduling problem, jobs completed within a specified time interval are not penalized, while jobs completed before or after the time interval are penalized accordingly [26]. There are many use cases and fields of application regarding CDD. For example, if customers order some goods from a supplier, they generally agree to accept small deviations from a fixed delivery date due to unavoidable uncertainty such as ships waiting for embarkation in a harbor. If a bundle of goods is transported in bulk delivery, the arrival and departure times of a truck may be specified by the customers, during which time the goods must be dispatched. Furthermore, the agreement of due windows typically occurs in problems associated with tour planning [27].

In recent years, many researchers have focused on the study of SMSPs with CDWs. The most famous and interesting studies in the literature were selected and discussed as follows. Anger *et al.* [28] first introduced the concept of a common due window (CDW) to the SMSP. They revealed that the SMSP with the objective of minimizing the number of early and tardy jobs can be solved in polynomial time. Thereafter, Kramer and Lee [29] considered variable and fixed CDWs in SMSPs, and solved the related problems using a polynomial-time algorithm and a pseudo-polynomial-time algorithm, respectively. Liman and Ramaswamy [30] considered a restricted common due window (RCDW) and unrestricted common due window (UCDW) in SMSPs in which the weighted sum of earliness penalties and the weighted number of tardy jobs is minimized. A CDW problem is called restrictive if the range of the CDW influences the optimal sequence of jobs; it is called unrestrictive if the left border of the range exceeds the sum of the processing times of all jobs, or if the range of the CDW is a decision variable [27]. Liman and Ramaswamy [30] proved that the UCDW case is NP-complete and presented dynamic programming algorithms to find the optimal schedule. Ventura and Weng [31] concentrated on the RCDW in an SMSP in which the mean absolute deviation of job completion times is minimized. They presented a Lagrangian relaxation procedure and two efficient heuristics for obtaining lower bounds of the optimal solutions. Yoo and Martin-Vega [32] investigated an RCDW in SMSP with the objective of minimizing the total earliness and tardiness penalties. They showed that the problem and its similar problem with release dates can be solved in polynomial-time by a modified Moore's Algorithm [33]. Yeung *et al.* [34] further proved that the SMSP with an RCDW problem, and the objective of minimizing the total weighted earliness, tardiness and flow time penalties,

becomes NP-hard. Biskup and Feldmann [27] investigated an RCDW in SMSP and the objective of minimizing the total weighted earliness and tardiness penalties. They supported the claim of Azizoglu and Webster [35] that an optimal solution to the SMSP with an RCDW exists in which jobs that are completed early or late exhibit the well-known V-shaped property. The orders of early and late jobs follow the dispatching rules of the weighted longest processing time (WLPT) and weighted shortest processing time (WSPT), respectively. Biskup and Feldmann [27] presented eight types of possible optimal sequences. Since the problem is NP-hard, they proposed a greedy heuristic (GH) algorithm to find initial feasible solutions, and improved the process using three meta-heuristics: the evolutionary strategy (ES), simulated annealing (SA) and threshold accepting (TA) algorithms. To demonstrate the efficiency of the GH algorithm, 250 benchmark test problems were used. However, they seem to have failed to consider that, with respect to the eight types of possible optimal sequences, the processing of the first job may straddle the boundaries of the RCDW.

On the other hand, a number of recent studies have considered the SMSP with a UCDW problem. For example, Li [36] investigated three different variants of the SMSP with a UCDW problem and batch deliveries. The objective was to minimize the total cost. He proposed polynomial-time solution procedures for the corresponding problems with significantly lower computational complexities than those of known algorithms in the literature. Liu *et al.* [37] considered the SMSP with a UCDW problem involving convex resource-dependent processing times. The objective was to minimize the total resource consumption cost under the constraint of a given schedule cost. They showed that the problem is polynomially solvable. Zhao *et al.* [38] investigated an SMSP with a UCDW, time-dependent processing times, and a controllable rate-modifying activity. The objective was to minimize the sum of earliness, tardiness, due-window-related costs and resource-related costs. They proposed a polynomial solution for the problem under consideration. Liu *et al.* [39] studied four SMSPs with a UCDW problem, where the processing time of the job was affected by the learning and positional effects. They proved that all the presented problems are polynomially solvable. Zhang *et al.* [40] studied the SMSP with a UCDW problem, linear decreasing processing times and maintenance activities, which are two common and important factors in scheduling practice. They proposed some optimality properties for the CDW assignment problem, and formulated them to obtain a polynomial time algorithm. Mor [41] extended the classical method of minmax CDD assignment and single-agent SMSPs to a setting involving two competing agents and a multi-agent setting. Furthermore, he generalized the problems to the SMSP with a UCDW problem and introduced efficient polynomial time solutions for all studied problems. Yin [42] investigated an SMSP with a UCDW and job-dependent learning effect, and showed that it can be solved in polynomial time. Wang and Li [43] dealt with four bi-criteria

TABLE 1. The computational complexity and solution algorithms of existing research.

CDW Type	Authors	Complexity	Algorithms
RCDW	Anger, et al. [28]	P	$O(n \log n)$
	Kramer and Lee [29]	P	$O(n \log n)$
	Liman and Ramaswamy [30]	P	$O(n^2 d \sum p_j)$
	Biskup and Feldmann [27]	NP-hard	Heuristic
	Ventura and Weng [31]	NP-hard	Heuristic
	Yoo and Martin-Vega [32]	P	Heuristic
	Yeung et al. [34]	P	Dynamic Programming
	Azizoglu and Webster [35]	NP-hard	Branch-and-Bound
UCDW	Mor [26]	P	$O(n), O(n^3)$
	Liman and Ramaswamy [30]	NP-hard	$O(n^2 D \sum p_j)$
	Li [36]	P	$O(n^4), O(n^5)$
	Liu et al. [37]	P	$O(n \log n)$
	Zhao et al. [38]	P	$O(n^4)$
	Liu et al. [39]	P	$O(n^3), O(n \log n)$
	Zhang et al. [40]	P	$O(n \log n)$
	Mor [41]	P	$O(\max\{n^A, n^B\}), O((m^A + m^B)n^k + (m^B \log m^B) + (m^A)^2)$
	Yin [42]	P	$O(n^3)$
	Wang and Li [43]	P, NP-hard	$O(n)$, Mixed Integer Linear Programming
	Wang et al. [44]	P	$O(n \log n), O(n^3)$

SMSPs with a UCDW problem and resource-dependent processing times, in which the resource amounts assigned to the jobs can be either discrete or continuous. The authors proposed pseudo-polynomial-time algorithms and an optimal algorithm, which can help practitioners addressing corresponding problems faced in their specific environments. At the same year, Mor [26] studied two extensions of min-max SMSP with a UCDW problem. The first problem is to minimize the maximum scheduling cost subject to maximal resource consumption; the second one is to minimize the resource consumption subject to an upper bound on the scheduling measure. It was proved that both considered problems are polynomially solvable. Wang *et al.* [44] dealt with an SMSP with a UCDW problem, in which the objective was to minimize the total position-dependent weighted cost. A polynomial time solution algorithm was provided for the corresponding problem.

The computational complexity and solution algorithms of existing research for SMSPs with RCDW and UCDW are summarized in Table 1. Generally, it can be seen in Table 1 that there are many pseudo-polynomial-time algorithms for the polynomially solvable problems, but only a few meta-heuristic algorithms for the NP-hard problems. For further detailed discussion on SMSCDWAPs, the reader is referred to the recent survey article by Janiak *et al.* [25].

Motivated by the excellent research of Biskup and Feldmann [27], this study focuses on the SMSP with an RCDW in which the total weighted earliness-tardiness penalties are minimized. A novel backtracking simulated annealing (BSA)

algorithm, which uses a backtracking mechanism to escape from local optima sequences and an effective coding scheme to find possible optimal sequences and waiting times, is proposed herein. Twelve types of possible optimal sequences are presented in calculating the total weighted earliness-tardiness penalties. The performance of the proposed BSA algorithm is demonstrated by comparing its computational results with those obtained using the state-of-the-art ES algorithm [27] and the simulated annealing (SA) algorithm in solving four sets of benchmark problems. The remainder of this paper is organized as follows. The following section defines the considered SMSP. Section 3 discusses 12 types of possible optimal sequences and the formulae for the corresponding objective functions. Section 4 describes in detail the proposed BSA algorithm. Section 5 presents the computational experiments and results obtained using four benchmark problem sets. Finally, Section 6 draws conclusions and offers suggestions regarding directions for future research.

II. PROBLEM STATEMENT, DEFINITIONS AND NOTATION

The SMSP with an RCDW in this work is described as follows. The following notations are used.

Notations:

- j job index
- α_j unit penalty associated with the earliness of job J_j
- β_j unit penalty associated with the tardiness of job J_j
- p_j processing time of job J_j
- C_j completion time of job J_j

- d_E earliest due date
- d_T latest due date
- C_E earliest possible completion time of all jobs
- h_E given parameters that determine d_E
- h_T given parameters that determine d_T
- E_j earliness of job J_j
- T_j tardiness of job J_j

Consider a set of N jobs $\mathbf{J} = \{J_j | j = 1, \dots, N\}$ to be processed on a single machine with an RCDW. The objective is to determine the sequence of all jobs that minimizes the total weighted earliness-tardiness penalties. By applying the three-field classification scheme of Graham et al. [45], the addressed SMSP can be expressed as the triplet $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$, where E_j and T_j are the earliness and tardiness of job J_j ($j = 1, 2, \dots, N$), respectively, and α_j and β_j are the unit penalties (penalty weights) associated with the earliness and tardiness of job J_j , respectively. Let $p_j, j = 1, \dots, N$, be the processing time of job J_j , and $C_j, j = 1, \dots, N$, be the completion time of job J_j . For the RCDW, let d_E and d_T represent the earliest (left boundary) and latest (right boundary) due dates, respectively. With reference to Feldmann and Biskup [46], the size and position of the RCDW, based on d_E and d_T , are predetermined as:

$$d_E = \lfloor h_E \cdot C_E \rfloor = \left\lfloor h_E \cdot \sum_{j=1}^N p_j \right\rfloor \quad (1)$$

$$d_T = \lfloor h_T \cdot C_E \rfloor = \left\lfloor h_T \cdot \sum_{j=1}^N p_j \right\rfloor \quad (2)$$

where C_E is the earliest possible completion time of all jobs, and h_E and h_T are the given parameters that determine d_E and d_T , respectively.

Throughout the paper, parameters h_E and h_T satisfy the inequality $0 < h_E < h_T < 1$ such that $(d_T - d_E) < C_E$. Additionally, the latest due date satisfies $d_T \geq \min_{j=1, \dots, N} p_j$; otherwise, an optimal sequence can be easily obtained by sequencing all jobs in order of non-decreasing p_j/β_j . The critical assumptions made in the $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$ problem herein are described as follows.

- All jobs are independent of each other and processed consecutively on one machine.
- The first job in a production sequence may be processed after the beginning of the scheduling horizon, which is at time zero.
- The machine can only process a job once and must process all jobs without any interruption from the beginning of the processing of the first job to the completion of the last job.
- The setup time of the machine is negligible.
- No job is interrupted and no machine breaks down.
- The size and position of the RCDW are predetermined and fixed.
- The RCDW is smaller than the makespan of the N jobs.

- The latest due date (right boundary) of the RCDW is after the earliest possible completion of any one job.

III. TWELVE TYPES OF POSSIBLE OPTIMAL SEQUENCES

With respect to the $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$ problem, Biskup and Feldmann [27] discussed eight types of possible optimal sequences. The orders of early and tardy jobs follow the WLPT and WSPT rules, respectively (and so exhibit V-shaped property). For ease of explanation, let S_j ($j = 1, 2, \dots, N$) be the starting time of job J_j ; $\mathbf{E} = \{J_j | C_j \leq d_E\}$, $\mathbf{W} = \{J_j | S_j \geq d_E, C_j \leq d_T\}$, and $\mathbf{T} = \{J_j | S_j \geq d_T\}$ denote the sets of non-straddling jobs with starting and completion times before, within and after the RCDW, respectively. Then, an optimal sequence exhibits the following well-known properties [35].

Property 1: In an optimal sequence, jobs must be in a V-shaped arrangement, meaning that jobs in set \mathbf{E} (or \mathbf{T}) are ordered by non-increasing (or non-decreasing) ratio p_j/α_j (or p_j/β_j).

Property 2: An optimal sequence exists in which either the job in the first position begins at time zero or one job is completed at d_E or d_T .

Property 1 implies that one or two straddling jobs may be present in the optimal sequence, and Property 2 means that an optimal sequence may exist in which all jobs have production waiting times. Given these two properties, twelve types of possible optimal sequences are provided, presented in Fig. 1. Therein, J_E (with $S_j < d_E \vee d_E < C_j \leq d_T$), J_T (with $d_E \leq S_j < d_T \vee C_j > d_T$) and J_B (with $S_j < d_E \vee C_j > d_T$) represent left-straddling, right-straddling and double-straddling individual jobs, with starting and completion times that straddle the RCDW boundaries d_E, d_T and both d_E and d_T , respectively. Note that the twelve cases are established under the following assumptions:

- $d_E \geq \min\{p_1, p_2, \dots, p_N\}$; otherwise, the problem is trivial. The optimal solution is ordering the jobs according to non-decreasing ratios p_j/β_j and starting the first job at time zero.
- $d_T - d_E < \sum_{j=1, \dots, N} p_j$; otherwise, the problem becomes trivial.

Case 1 ($\mathbf{W} = \phi$) involves production waiting time and right-straddling job J_T . Case 2 ($\mathbf{E} = \mathbf{W} = \phi$) involves left-straddling job J_E and right-straddling job J_T . Case 3 ($\mathbf{E} = \mathbf{W} = \phi$) involves double-straddling job J_B . Cases 4 and 5 involve production waiting times, and Case 4 involves left-straddling job J_E . In Case 6, all jobs are non-straddling jobs. Case 7 involves right-straddling job J_T with production waiting time. In Cases 8 to 12, production begins at time zero; Cases 8 and 9 involve the left-straddling job J_E and right-straddling job J_T , respectively. Cases 10 and 11 ($\mathbf{E} = \phi$) involve both a left-straddling job J_E and a right-straddling job J_T . Case 12 ($\mathbf{W} = \phi$) involves the double-straddling job J_B . In more detail, the first, seventh and ninth cases are characterized by the existence of one job completed exactly in d_E ; the fourth and eighth cases are characterized by the

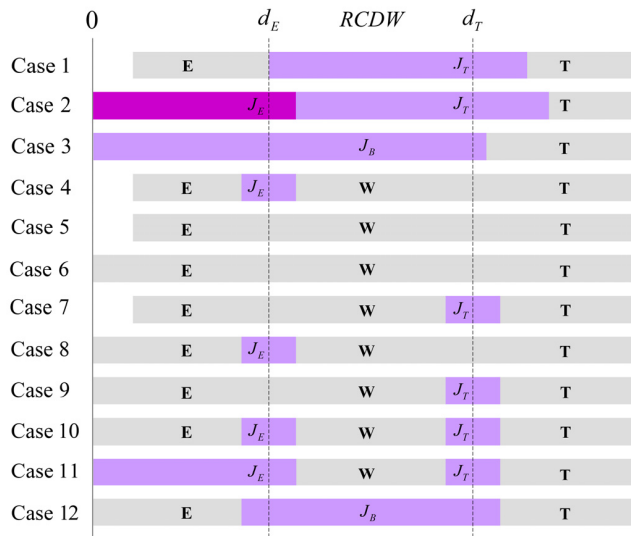


FIGURE 1. Twelve types of possible optimal sequences.

existence of one job completed exactly in d_T ; the fifth and sixth cases are characterized by the existence of one job completed exactly in d_E and another job completed exactly in d_T . In the remaining five cases: cases 2, 3, 10, 11 and 12, at least one straddling job occurs. In the third and twelfth cases a double-straddling job occurs. These straddling jobs are stressed by shading. In cases 1, 3, 4, 7, 8, 9 and 12 only one straddling job emerges and in cases 2, 10 and 11 two straddling jobs occur. Moreover, in the first, fourth, fifth, and seventh cases, their first job of set **E** starts later than time point zero. To simulate the leading idle time of these cases, set **E** is moved slightly to the right in Fig. 1. It is noted that for an optimal solution the existence of two straddling jobs or a double-straddling job are inconsistent with leading idle time (Property 2). Otherwise, the total weighted earliness-tardiness penalties could be decreased by moving all jobs to the left or to the right. Additionally, as seen in cases 1, 2, 3, 10, 11 and 12, sets **E** and **W** can be empty, but we have to mention that set **T** cannot be empty, as an empty set **T** contravenes the assumption that the CDW is restrictive.

These twelve types provide a more complete and accurate perspective on all possible optimal sequences associated with various straddling jobs and production waiting times. To facilitate the proposed BSA algorithm to evaluate possible candidate solutions, these cases are classified into six groups (**G1-G6**); each is associated with a formula for the total weighted earliness-tardiness penalties, as follows.

G1. Case 1: $\sum_{j \in \mathbf{E}} \alpha_j (d_E - C_j) + \beta (J_T) [p(J_T) + d_E - d_T] + \sum_{j \in \mathbf{T}} \beta_j [p(J_T) + d_E - d_T + p_j]$

G2. Case 2: $\beta (J_T) [p(J_E) + p(J_T) - d_T] + \sum_{j \in \mathbf{T}} \beta_j [p(J_E) + p(J_T) - d_T + p_j]$

G3. Case 3: $\beta (J_B) [p(J_B) - d_T] + \sum_{j \in \mathbf{T}} \beta_j [p(J_B) - d_T + p_j]$.

G4. Cases 4 to 6: $\sum_{j \in \mathbf{E}} \alpha_j (d_E - C_j) + \sum_{j \in \mathbf{T}} \beta_j p_j$.

G5. Cases 7 to 11: $\sum_{j \in \mathbf{E}} \alpha_j (d_E - C_j) + \beta (J_T) [p(J_T) + d_E - d_T] + \sum_{j \in \mathbf{T}} \beta_j [p(J_T) + d_E - d_T + p_j]$

G6. Case 12: $\sum_{j \in \mathbf{E}} \alpha_j (d_E - C_j) + \beta (J_B) T (J_B) + \sum_{j \in \mathbf{T}} \beta_j [T (J_B) + p_j]$
where $T (J_B) = \sum_{j \in \mathbf{E}} p_j + p (J_B) - d_T$.

IV. PROPOSED BSA ALGORITHM

This work develops a novel SA-based heuristic, called backtracking simulated annealing (BSA), to solve the $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$ problem. The SA algorithm is a well-known local search-based meta-heuristic that can escape from the local optima by accepting, with small probability, worse solutions during the search process. This famous algorithm has been successfully used to solve many hard combinatorial optimization problems, such as neural net [47], benchmark functions [48], image restoration problem [49], 0-1 Knapsack Problem [50], and quadratic assignment problem [51]. An SA algorithm typically begins with a randomly generated initial solution. Then, at each iteration, it finds a solution in the neighborhood of the current solution. If the new solution is better than the current solution, it replaces the latter with the former and the search process resumes from the new current solution. It also allows a worse neighborhood solution to replace the current solution, with a small probability, so that the procedure can escape local optima at which it may otherwise become trapped. The proposed BSA algorithm applies a backtracking mechanism to escape from the local optima sequences and an effective coding scheme to search for possible optimal sequences and the waiting time for the $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$.

The following subsections describe the solution representation and coding procedures, the neighborhood solutions, the parameters used in the proposed BSA algorithm, and the procedure of its implementation.

A. SOLUTION REPRESENTATION AND CODING PROCEDURE

In this study, a solution is coded using a non-negative integer value to represent the waiting time, LT ($0 \leq LT \leq d_E$), and n integers to specify an ordered list of n jobs. Given a waiting time and an ordered list, the corresponding solution Π is coded using the following two steps. In the first step, the completion time $C_{[j]}$ ($j = 1, \dots, N$) of the j^{th} job in an ordered list is calculated as follows:

$$C_{[1]} = LT + p_{[1]} \quad (3)$$

$$C_{[j+1]} = C_{[j]} + p_{[j+1]} \quad (4)$$

Based on its completion time, each job is identified as being a member of one of the three sets of non-straddling job (**E**, **W**, and **T**) or one of the three straddling jobs (J_E , J_T and J_B). In the second step, the jobs in **E** are re-arranged in order of non-increasing ratio p_j/α_j , while the jobs in **T** are re-arranged in order of non-decreasing ratio p_j/β_j . Since the value of the objective function must be computed frequently in the search process, the method for quickly sorting

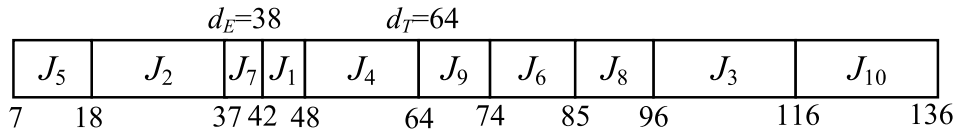


FIGURE 2. The Gantt chart of the optimal solution.

Begin

1. Input $T_0, T_f, I_{iter}, \alpha, \eta_{\min}, B_{non-improving}$ and problem instance;
 2. Generate initial solution Π by RGH heuristic
 3. $T \leftarrow T_0; B_{non} \leftarrow 0; \Pi_{Best} \leftarrow \Pi; \eta_r = \eta_{\min}$ ($r=1, 2, 3$)
 4. **while** ($T > T_f$) **do**
 5. for ($i=0; i \leq I_{iter}; i++$)
 6. Generate $\rho \leftarrow random()$;
 7. Let LT be the leading idle time of solution Π ;
 8. $\eta_T = \eta_1 + \eta_2 + \eta_3$;
 9. **if** ($\rho < \eta_1 / \eta_T$ and $LT < d_E$) { $LT \leftarrow LT+1; r_index \leftarrow 1$;}
 10. **else if** ($\rho > (\eta_1 + \eta_2) / \eta_T$ and $LT > 0$) { $LT \leftarrow LT-1; r_index \leftarrow 2$;}
 11. **else** { $r_index \leftarrow 3$;}
 12. Randomly choose two jobs in different set (E, W, T) using new LT and swap them to obtain new solution Π_{new} ;
 13. $\Delta E \leftarrow obj(\Pi_{new}) - obj(\Pi)$;
 14. **if** ($\Delta < 0$) **then** $\eta_{r_index} \leftarrow \eta_{r_index} + 1$;
 15. **else if** ($\Delta > 0$) **then** $\eta_{r_index} \leftarrow \eta_{r_index} - 1$;
 16. **else** $\eta_{r_index} \leftarrow \eta_{r_index} - 0.1$;
 17. **if** ($\eta_{r_index} < \eta_{\min}$) **then** { $\eta_{r_index} \leftarrow \eta_{\min}$;}
 18. **if** ($\Delta \leq 0$) **then** $\Pi = \Pi_{new}$;
 19. **else** {
 20. Generate $\gamma \leftarrow random()$;
 21. **if** ($\gamma < e^{-\Delta E/T}$) **then** { $\Pi \leftarrow \Pi_{new}$;}
 22. }
 23. **if** ($obj(\Pi) < obj(\Pi_{Best})$) **then** { $\Pi \leftarrow \Pi_{Best}; B_{non} \leftarrow 0$;}
 24. **else** { $B_{non} \leftarrow B_{non} + 1$;}
 25. **if** ($B_{non} = I_{iter} \times B_{non-improving}$) **then** { $B_{non} \leftarrow 0; \Pi \leftarrow \Pi_{Best}$;}
 26. **end for**
 27. $T \leftarrow \alpha T$;
 28. **end while**
- End**

FIGURE 3. Pseudo-code of the proposed BSA algorithm.

jobs in a V-shape that was proposed by Lin, et al. [52] is used. This method is performed using two pre-established lookup tables to quickly determine the sequences of jobs in \mathbf{E} and \mathbf{T} . Procedures and an example of its use were presented by Lin, et al. [52]. After the jobs in \mathbf{E} and \mathbf{T} are sorted in a V-shape, the ordered list in the solution is coded as $(\mathbf{E}, J_E, \mathbf{W}(\text{or } J_B), J_T, \mathbf{T})$. Notably, if J_B exists, then $\mathbf{W} = \phi$.

The coding procedure is demonstrated by applying it to a random generated instance (see Table 2) with ten-job,

$d_E = 38$ and $d_T = 64$. Given a waiting time of seven and a permutation list (5, 2, 7, 1, 4, 3, 9, 10, 6, 8), from Eqs. (3) and (4), $C_5 = 7 + 11 = 18, C_2 = 18 + 19 = 37, C_7 = 37 + 5 = 42, C_1 = 42 + 6 = 48, C_4 = 48 + 16 = 64, C_3 = 64 + 20 = 84, C_9 = 84 + 10 = 94, C_{10} = 94 + 20 = 114, C_6 = 114 + 11 = 125, C_8 = 125 + 11 = 136$. Therefore, $\mathbf{E} = \{J_5, J_2\}, J_E = J_7, J_B = \Phi, J_T = \Phi, \mathbf{W} = \{J_1, J_4\}, \mathbf{T} = \{J_3, J_9, J_{10}, J_6, J_8\}$, and the right straddling or double-straddling job does not exist. Finally, by sorting the jobs in \mathbf{E} and \mathbf{T} in a V-shape, the ordered list in the solution

TABLE 2. Data for a random generated instance.

	1	2	3	4	5	6	7	8	9	10
p_j	6	19	20	16	11	11	5	11	10	20
α_j	5	8	5	8	3	6	9	7	10	5
β_j	9	12	1	15	12	1	13	1	2	1

is recoded as (5, 2, 7, 1, 4, 9, 6, 8, 3, 10), and the solution is then coded as $\Pi = (7|5, 2, 7, 1, 4, 9, 6, 8, 3, 10)$. The Gantt chart of this solution is shown in Fig. 2. This solution is indeed the optimal solution of the above problem instance.

B. NEIGHBORHOOD

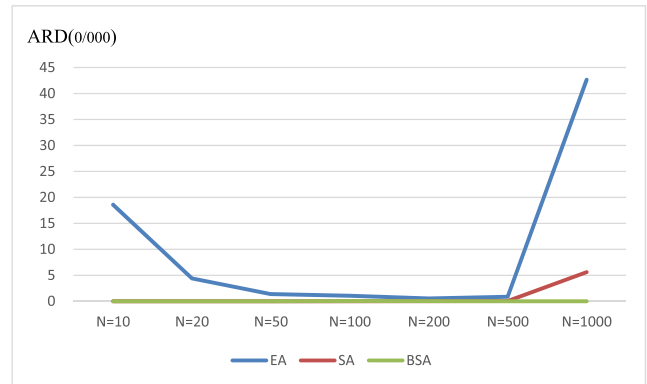
The change operator of the LT and the job swap operator are used to generate the solution from the neighborhood of the current solution Π . The set of solutions in the neighborhood of the current solution Π is denoted as $\mathcal{N}(\Pi)$. In each iteration, the LT is determined by applying one of three rules: increase one (R_1), reduce one (R_2), and keep the same (R_3) the current LT , according to the formula $prob_r = \eta_r / \sum_{l=1}^3 \eta_l$, ($r = 1, 2, 3$), where $prob_r$ is the probability of choosing rule R_r and η_r is the fitness value of rule R_r , which is auto-tuned in each iteration according to the following criteria:

- (1) If the current solution is improved and updated to a new obtained solution that is generated by applying a selected rule (R_r), then set $\eta_r =: \eta_r + 1$;
- (2) Otherwise, if the new obtained solution that is generated by applying a selected rule (R_r) is worse than the current solution, then set $\eta_r =: \eta_r - 1$. If $\eta_r < \eta_{\min}$, and then $\eta_r =: \eta_{\min}$, where η_{\min} is the minimal allowed value of η_r ($r = 1, 2, 3$).
- (3) Otherwise, $\eta_r =: \eta_r - 0.1$.

Notably, in the application of the three LT updating rules, the possible range of LT ($0 \leq LT \leq d_E$) must be considered. If the LT is zero or greater than d_E , the R_2 and R_1 cannot be applied, respectively. For example, if current LT is 7, new LT will be 8, 6, and 7 for R_1 , R_2 , and R_3 rule, respectively. After the LT is changed, a new feasible solution Π_{new} is generated from $\mathcal{N}(\Pi)$ by randomly choosing and swapping the i^{th} and the j^{th} positions of jobs in Π . Notably, if the selected jobs are in the same set \mathbf{E} , \mathbf{W} , or \mathbf{T} , the sequence of jobs in Π cannot be improved; therefore, two jobs may not be selected from a single set. For example, if $\Pi = (7|5, 2, 7, 1, 4, 9, 6, 8, 3, 10)$, then $\mathbf{E} = \{J_5, J_2\}$, $J_E = J_7$, $J_B = \Phi$, $J_T = \Phi$, $\mathbf{W} = \{J_1, J_4\}$, $\mathbf{T} = \{J_3, J_9, J_{10}, J_6, J_8\}$. Swap J_9 and J_{10} will not change the objective function value of Π because the V-shape property is applied; therefore, two jobs which are not in the set can be swapped.

C. BSA PROCEDURES

The pseudo-code of the proposed BSA algorithm is shown in Fig. 3. Let T_0 and T_f represent the initial and final temperatures, respectively; I_{iter} denotes the total number of

**FIGURE 4.** The average ARDs of all compared algorithms.

iterations that the perturbation should repeat at a certain temperature; α indicates the control coefficient of the cooling schedule; η_{\min} represents the minimal value of η_r ($r = 1, 2, 3$), where η_r is the fitness value of choosing rule R_r ; $B_{non-improving}$ stands for the cumulative number of consecutive temperature reductions. If the best value of the objective function is not improved by $B_{non-improving}$ consecutive temperature reductions, then the incumbent solution will be backtracked to the current best solution. The detailed procedures of the proposed BSA algorithm to be used to solve the $1|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$ problem are described as follows.

Initially, the current temperature T is set to T_0 and an initial solution Π is obtained using the revised greedy heuristic (RGH) [53]. The value of the objective function of Π is denoted as $obj(\Pi)$. The current best solution Π_{best} is set to Π , and $obj(\Pi_{best})$ is initialized as $obj(\Pi)$.

At each iteration, a neighborhood solution Π_{new} with waiting time is generated from $\mathcal{N}(\Pi)$, and its objective function value is evaluated. If $obj(\Pi_{new})$ is not worse than $obj(\Pi)$, then Π_{new} replaces Π as the incumbent solution. Otherwise, Π_{new} is accepted as the incumbent solution with a small probability. This probability is typically calculated using the Boltzmann function. More specifically, let $\Delta E = obj(\Pi_{new}) - obj(\Pi)$; then the probability of replacing Π with a worse neighborhood solution Π_{new} is $e^{(-\Delta E/T)}$. Such a replacement is implemented by randomly generating a number $0 < r < 1$ and replacing Π with Π_{new} when $r < e^{(-\Delta E/T)}$.

The current temperature T decreases after I_{iter} iterations at the current temperature, according to the formula $T =: \alpha T$, $0 < \alpha < 1$. If Π_{best} is not improved in $B_{non-improving}$, then the backtracking mechanism is implemented by setting the current solution Π to Π_{best} . The searching procedure terminates when the current temperature is lower than the final temperature T_f , and the best solution is then output.

V. COMPUTATIONAL EXPERIMENTS AND RESULTS

The performance of the proposed BSA algorithm is compared with those of SA (BSA without a backtracking mechanism) and EA [27], which is the best available algorithm published in the literature. All of the BSA, SA, and EA algorithms

TABLE 3. ARDs of the compared algorithms for each size of test problem.

Problem Set	N	ES		SA		BSA	
		ARD	Time*	ARD	Time	ARD	Time
I	10	3.019	0.010	0.000	0.046	0.000	0.101
	20	0.000	0.047	0.000	0.140	0.000	0.185
	50	1.230	0.435	0.000	0.622	0.000	0.662
	100	0.415	2.219	0.010	2.031	0.053	2.065
	200	0.233	13.328	0.012	7.398	0.029	7.457
	Average		0.979	-	0.004	-	0.016
II	500	1.459	173.8	0.034	43.6	0.003	44.0
	1000	74.079	1302.5	0.021	169.1	0.001	169.4
	Average		37.769	-	0.027	-	0.002
III	10	34.165	0.011	0.000	0.045	0.000	0.100
	20	8.764	0.053	0.000	0.140	0.000	0.186
	50	1.513	0.455	0.000	0.641	0.000	0.686
	100	1.689	2.404	0.000	2.156	0.000	2.205
	200	0.842	14.276	0.011	8.009	0.009	8.012
	Average		34.165	-	0.000	-	0.000
IV	500	0.292	186.9	0.037	46.6	0.003	47.1
	1000	11.197	1422.3	11.197	180.3	0.000	181.3
	Average		5.744	-	5.617	-	0.002
Total Average		9.921	-	0.809	-	0.007	-

*: CPU time in seconds.

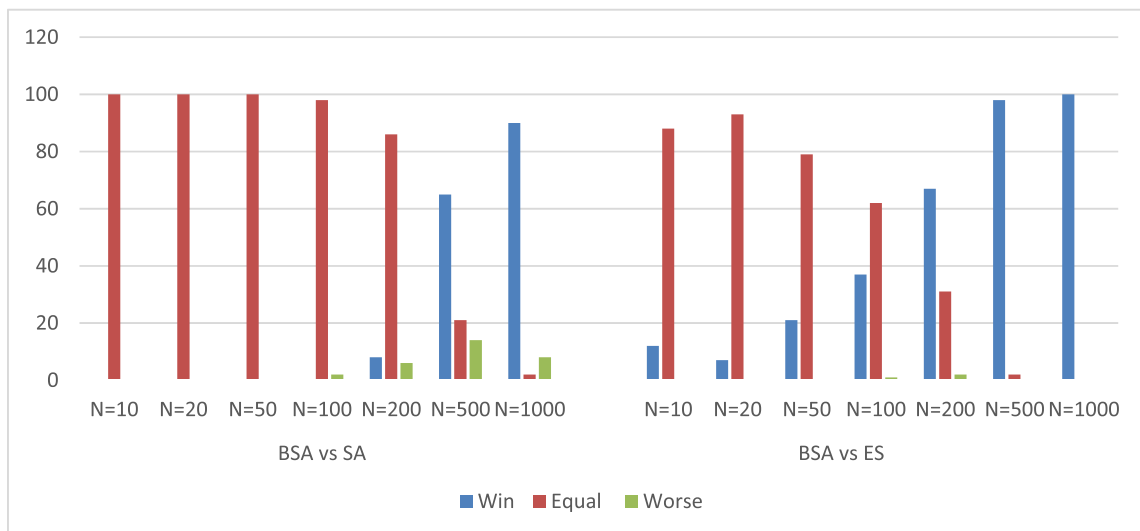


FIGURE 5. The number of solutions obtained by BSA is better, equal to, or worse than those obtained by SA and EA.

utilize an initial solution that is obtained using RGH [53]. The proposed BSA algorithm was coded using C language and executed on a personal computer with an Intel Core 2 i7-920

2.67 GHz CPU and 4 GB of RAM. EA was re-coded and run on the same computer; the computational times were then compared. The following subsection describes the

TABLE 4. Paired *t*-tests on ARDs of the compared algorithms for each size of test problem.

BSA vs. SA	Problem Set	<i>N</i>	Mean	S-Dev	<i>t</i> -value	DF	<i>P</i> -value	Significant
	I	10	0.000	0.000	-	49	-	No
		20	0.000	0.000	-	49	-	No
		50	0.000	0.000	-	49	-	No
		100	-0.043	0.215	1.764	49	0.08397	No
		200	-0.017	0.122	1.432	49	0.15845	No
	II	500	0.031	0.062	3.761	49	0.00045	Yes
		1000	0.020	0.024	6.070	49	0.00000	Yes
	III	10	0.000	0.000	-	49	-	No
		20	0.000	0.000	-	49	-	No
		50	0.000	0.000	-	49	-	No
100		0.000	0.000	-	49	-	No	
200		0.002	0.038	0.956	49	0.34399	No	
IV	500	0.033	0.056	4.468	49	0.00005	Yes	
	1000	11.197	32.536	2.707	49	0.00932	Yes	
BSA vs. ES	Problem Set	<i>N</i>	Mean	S-Dev	<i>t</i> -value	DF	<i>P</i> -value	Significant
	I	10	3.019	0.000	1.423	49	0.16111	No
		20	0.000	0.000	-	49	-	No
		50	1.230	0.000	2.624	49	0.01155	Yes
		100	0.355	0.233	2.786	49	0.00758	Yes
		200	0.204	0.086	3.949	49	0.00025	Yes
	II	500	1.456	2.052	5.216	49	0.00000	Yes
		1000	74.078	35.976	14.816	49	0.00000	Yes
	III	10	34.165	0.000	3.451	49	0.00116	Yes
		20	0.000	0.000	2.537	49	0.01443	Yes
		50	1.513	0.000	2.954	49	0.00481	Yes
100		1.656	0.011	3.934	49	0.00026	Yes	
200		0.833	0.038	6.396	49	0.00000	Yes	
IV	500	0.288	0.285	7.357	49	0.00000	Yes	
	1000	11.197	32.536	2.645	49	0.01096	Yes	

benchmark problems, parameter value determination, and computational results.

A. BENCHMARK PROBLEMS

To evaluate the performance of the proposed BSA algorithm in solving the $1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problem, four problem sets (I, II, III, and IV) extended from the well-known benchmark problems [54] are used. The Problem set I comprises the benchmark problems with the number of jobs $N = \{10, 20, 50, 100, 200\}$, and possible combinations of RCDW parameters $(h_E, h_T) = \{(0.1, 0.2), (0.1, 0.3), (0.2, 0.5), (0.3, 0.4), (0.3, 0.5)\}$. Ten benchmark problems are generated for each combination

yielding a total of 250 benchmark problems in the problem set I. Notably, problem set I was also used in tests by Biskup and Feldmann [53] and Ying *et al.* [54]. The experimental design of problem set II is the same as that of problem set I, but with $N = \{500, 1000\}$. Therefore, problem set II comprised 100 benchmark problems. The experimental designs of problem sets III and IV are the same as those of problem sets I and II, respectively, except that $(h_E, h_T) = \{(0.4, 0.5), (0.4, 0.6), (0.5, 0.6), (0.5, 0.7), (0.6, 0.7)\}$. As a result, a total of 700 benchmark problems are considered.

The performance of the compared algorithms is evaluated using the average relative deviation (ARD), defined as

TABLE 5. The number and percentage of solutions obtained by the BSA are better, equal to, or worse than those obtained by SA and ES algorithms.

Problem Set	N	BSA vs. SA			BSA vs. ES								
		Better	Equal	Worse	Better	Equal	Worse						
I	10	0	0.00%	50	100.00%	0	0.00%	1	2.00%	49	98.00%	0	0.00%
	20	0	0.00%	50	100.00%	0	0.00%	0	0.00%	50	100.00%	0	0.00%
	50	0	0.00%	50	100.00%	0	0.00%	9	18.00%	41	82.00%	0	0.00%
	100	0	0.00%	48	96.00%	2	4.00%	14	28.00%	35	70.00%	1	2.00%
	200	4	8.00%	42	84.00%	4	8.00%	28	56.00%	21	42.00%	1	2.00%
Sub total		4	1.60%	240	96.00%	6	2.40%	52	20.80%	196	78.40%	2	0.80%
II	500	29	58.00%	13	26.00%	8	16.00%	50	100.00%	0	0.00%	0	0.00%
	1000	40	80.00%	2	4.00%	8	16.00%	50	100.00%	0	0.00%	0	0.00%
	Sub total		69	69.00%	15	15.00%	16	16.00%	100	100.00%	0	0.00%	0
III	10	0	0.00%	50	100.00%	0	0.00%	11	22.00%	39	78.00%	0	0.00%
	20	0	0.00%	50	100.00%	0	0.00%	7	14.00%	43	86.00%	0	0.00%
	50	0	0.00%	50	100.00%	0	0.00%	12	24.00%	38	76.00%	0	0.00%
	100	0	0.00%	50	100.00%	0	0.00%	23	46.00%	27	54.00%	0	0.00%
	200	4	8.00%	44	88.00%	2	4.00%	39	78.00%	10	20.00%	1	2.00%
Sub total		4	1.60%	244	97.60%	2	0.80%	92	36.80%	157	62.80%	1	0.40%
IV	500	36	72.00%	8	16.00%	6	12.00%	48	96.00%	2	4.00%	0	0.00%
	1000	50	100.00%	0	0.00%	0	0.00%	50	100.00%	0	0.00%	0	0.00%
	Sub total		86	86.00%	8	8.00%	6	6.00%	98	98.00%	2	2.00%	0
Total Average		163	23.29%	507	72.43%	30	4.29%	342	48.86%	355	50.71%	3	0.43%

follows:

$$ARD = \frac{\left[\sum_{i=1}^n \frac{obj_i^h - obj_i^{best}}{obj_i^{best}} \right]}{n} \times 10,000\text{‰}$$

Here, obj_i^h is the value of the objective function in the i^{th} benchmark problem that was obtained by algorithm h ; obj_i^{best} is the best value of the objective function in the i^{th} benchmark problem that was obtained by any of the compared algorithms, n is the number of benchmark problems under consideration, and ‰ is a per ten thousand sign.

B. PARAMETER VALUE DETERMINATION

Since all of the relevant parameters may influence the performance of the proposed BSA algorithm, extensive computational testing was carried out to evaluate them. In the preliminary tests, the following combinations of parameter values were used in 16 benchmark problems that were randomly selected from the four sets thereof, and each problem was solved by three independent applications of the proposed BSA algorithm: $I_{iter} \in \{500, 1000, 1500, 2000\}$; $B_{non-improving} \in \{5, 10, 15, 20\}$; $\eta_{min} \in \{10, 20, 30\}$; $T_0 \in \{3, 5, 10, 15, 20\}$; $\alpha \in \{0.96, 0.97, 0.98, 0.99\}$, and $T_F \in \{0.01, 0.05, 0.10, 0.15, 0.20\}$. The test results showed that the best performance of the BSA algorithm was achieved within a reasonable computation time using $T_0 = 10$, $T_F = 0.1$, $I_{iter} = 1000$,

$B_{non-improving} = 5$, $\eta_{min} = 10$, and $\alpha = 0.98$. Accordingly, these parameter values were used in the experiments.

C. COMPUTATIONAL RESULTS

Table 3 presents the ARD and average running time (in seconds) to solve a problem of each size. The total average ARD for all 700 instances that was obtained using the proposed BSA algorithm is 0.007‰, whereas the corresponding values that were obtained by SA and ES are 0.809‰ and 9.921‰, respectively. Obviously, the proposed BSA algorithm outperforms the state-of-the-art ES algorithm and the traditional SA heuristic in solving the $|RCDW| \sum (\alpha_j E_j + \beta_j T_j)$ problem. The computational times of the BSA and SA algorithms are almost equal because they apply the same termination condition. In contrast, the computational times of the BSA and SA algorithms are much shorter than that of ES when $N \geq 200$, indicating that the encoding scheme of BSA and SA is more efficient than that of ES. As shown in Fig. 4, compared with ES, the proposed BSA can provide smaller ARDs for all different job numbers. Fig. 4 shows that SA and BSA can provide almost the same ARDs when the number of jobs is smaller than 500.

To verify the effectiveness of the proposed BSA algorithm, paired t -tests are performed on the ARD obtained using this algorithm to compare it with those of ES and SA algorithms. Table 4 reveals that the proposed BSA algorithm significantly outperforms the SA and ES algorithms for $N = 500$ and 1000 with problem sets II and IV at the 95% confidence level.

TABLE 6. The best known solutions of benchmark problem sets I and II.

K	(h_E, h_T)	Problem data set I					Problem set II	
		$N = 10$	$N = 20$	$N = 50$	$N = 100$	$N = 200$	$N = 500$	$N = 1000$
1	(0.1, 0.2)	1896	4089	39461	139568*	474405	2812324	13514502
	(0.1, 0.3)	1330	2713	28225	95211	324463	1915938	9315188
	(0.2, 0.5)	540	1162	12754	39487	136751	810403	3914623
	(0.3, 0.4)	919	2294	21110	72110	246659	1476558	6948451
	(0.3, 0.5)	587	1559	13971	45826	158116	944399	4404641
2	(0.1, 0.2)	947	8251	29043	120484	517302	3229169	11733558
	(0.1, 0.3)	539	5950	20133	82026	353001	2212062	8024367
	(0.2, 0.5)	191	2770	8468	35290	145421	935690	3368177
	(0.3, 0.4)	432	4482	15150	62421	267687	1679279	6098871
	(0.3, 0.5)	265	2923	9428	39705	169542	1076044	3882473
3	(0.1, 0.2)	1488	5881	33180	124317	466667	2950841	11383782
	(0.1, 0.3)	1012	4067	23020	86236	319357	2000406	7695608
	(0.2, 0.5)	398	1675	9962	38174	134610	834686	3140985
	(0.3, 0.4)	760	3035	17508	67046	244991	1534635	5778518
	(0.3, 0.5)	462	1998	11388	44048	156984	980797	3633073
4	(0.1, 0.2)	2128	8977	25856	122901	564830	3075510	11201090
	(0.1, 0.3)	1576	6609	17544	84112	396584	2075252	7590369
	(0.2, 0.5)	712	3113	7373	35498	176837	841660	3148904
	(0.3, 0.4)	1162	4830	13609	65057	304632	1567957	5800727
	(0.3, 0.5)	740	3210	8418	41584	200659	988910	3672325
5	(0.1, 0.2)	1150	4028	31456	119115	488829	2983984	11857765
	(0.1, 0.3)	755	2850	21689	82398	333482	2048254	8023851
	(0.2, 0.5)	284	1192	8947	34860	139842	858055	3287539
	(0.3, 0.4)	542	2112	15747	60963	254510	1534573	6087728
	(0.3, 0.5)	339	1341	9956	38781	162120	971873	3830152
6	(0.1, 0.2)	1479	6306	33452	133545	458004	2654972	11067696
	(0.1, 0.3)	1023	4247	23261	89563	311824	1775879	7484909
	(0.2, 0.5)	439	1557	10221	35146	126398	708038	3097750
	(0.3, 0.4)	779	3042	17392	65472	233366	1328187	5707324
	(0.3, 0.5)	500	1778	11178	40858	146243	827793	3627473
7	(0.1, 0.2)	2093	10204	42234	129849	428095	3027551	12684832
	(0.1, 0.3)	1521	7492	29274	90963	285328	2069491	8686112
	(0.2, 0.5)	717	3573	12000	39336	115477	873675	3651121
	(0.3, 0.4)	1190	5722	20696	67795	220289	1582907	6592156
	(0.3, 0.5)	809	3846	12935	43737	139043	1014404	4205935
8	(0.1, 0.2)	1644	3742	42218	153965	474446	2976788	11686538
	(0.1, 0.3)	1287	2519	28403	106517	320676	1998263	7940720
	(0.2, 0.5)	670	990	11154	44963	129912	815282	3279303
	(0.3, 0.4)	952	1801	20965	80269	235477	1506415	6031322
	(0.3, 0.5)	680	1069	12913	51414	146955	947726	3794560
9	(0.1, 0.2)	1466	3317	33222	111474	508275	3224615	11144396
	(0.1, 0.3)	1121	2342	23840	75152	350341	2205768	7539465
	(0.2, 0.5)	492	1056	10968	31266	147166	922679	3136842
	(0.3, 0.4)	772	1767	17972	57102	263243	1658772	5795388
	(0.3, 0.5)	513	1187	11935	36319	168041	1059669	3689098
10	(0.1, 0.2)	1835	4673	31492	112799	514927	2977821	11822252
	(0.1, 0.3)	1384	3266	22036	78670	358435	2031659	8028517
	(0.2, 0.5)	691	1355	9653	34068	157881	848389	3293589
	(0.3, 0.4)	1047	2419	16510	59922	273834	1539930	6052594
	(0.3, 0.5)	717	1474	10597	38639	178442	974261	3813645

* Bold values indicate the best known solutions found by the BSA algorithm.

TABLE 7. The best known solutions of benchmark problem sets III and IV.

K	(h_E, h_T)	Problem set III					Problem set IV	
		$N = 10$	$N = 20$	$N = 50$	$N = 100$	$N = 200$	$N = 500$	$N = 1000$
1	(0.4, 0.5)	693	2115	16159	56462	195486	1188996	5298116
	(0.4, 0.6)	470	1478	10660	37147	129847	794983	3390555
	(0.5, 0.6)	632	2115	13458	51549	181331	1120240	4582742
	(0.5, 0.7)	462	1507	9492	36399	127891	786787	3175075
	(0.6, 0.7)	613	2148	13103	51398	181331	1120976	4539980
2	(0.4, 0.5)	408	3202	11321	47469	210182	1326189	4792029
	(0.4, 0.6)	265	2054	7240	31478	136800	878713	3125881
	(0.5, 0.6)	408	2529	9958	42562	189590	1227048	4340121
	(0.5, 0.7)	265	1675	6613	30468	132771	864513	3040446
	(0.6, 0.7)	408	2219	9670	42544	189489	1226290	4339326
3	(0.4, 0.5)	597	2590	13685	54382	197114	1239010	4547306
	(0.4, 0.6)	402	1783	8862	36553	131818	822586	2966739
	(0.5, 0.6)	554	2539	12000	49716	182261	1165025	4180534
	(0.5, 0.7)	402	1753	8379	35478	128261	815161	2901968
	(0.6, 0.7)	571	2501	11964	49716	181828	1165026	4179117
4	(0.4, 0.5)	808	3406	10651	52257	242258	1241531	4611941
	(0.4, 0.6)	545	2184	6842	34891	163792	821906	3042188
	(0.5, 0.6)	664	2585	9707	48432	219420	1165534	4292135
	(0.5, 0.7)	474	1664	6683	33703	158107	816389	3001860
	(0.6, 0.7)	620	2207	9630	48078	218734	1165534	4292137
5	(0.4, 0.5)	414	1660	11820	46227	201387	1177569	4810733
	(0.4, 0.6)	283	1124	7721	29439	132217	756670	3136870
	(0.5, 0.6)	377	1570	10525	39576	184939	1037437	4450742
	(0.5, 0.7)	269	1078	7463	26525	129898	722465	3086022
	(0.6, 0.7)	373	1542	10502	38437	184939	1035553	4452216
6	(0.4, 0.5)	600	2227	13004	50347	181802	1046776	4577050
	(0.4, 0.6)	417	1418	8121	32258	116811	691409	3045836
	(0.5, 0.6)	575	2075	10342	44289	165372	988565	4297410
	(0.5, 0.7)	397	1430	6696	30557	114213	687317	3011451
	(0.6, 0.7)	545	2114	9545	43879	165372	989247	4296003
7	(0.4, 0.5)	983	4228	15039	51898	180276	1259415	5184062
	(0.4, 0.6)	688	2755	9586	34124	119709	833425	3392192
	(0.5, 0.6)	866	3307	12799	45306	172017	1162321	4675651
	(0.5, 0.7)	609	2189	8895	31823	119341	817557	3272245
	(0.6, 0.7)	812	2885	12585	44608	172017	1161582	4663784
8	(0.4, 0.5)	735	1244	16283	62674	179248	1181993	4705887
	(0.4, 0.6)	448	780	10701	41454	113806	768471	3045284
	(0.5, 0.6)	494	1102	15228	58020	157327	1080104	4276411
	(0.5, 0.7)	318	739	10582	41074	108709	752411	2969460
	(0.6, 0.7)	421	1096	15145	58184	156738	1080817	4274879
9	(0.4, 0.5)	571	1465	13592	45447	205130	1304375	4673809
	(0.4, 0.6)	335	1041	8964	29422	133219	861164	3093889
	(0.5, 0.6)	413	1428	11129	41001	182230	1198181	4366051
	(0.5, 0.7)	278	1038	7443	28315	127845	842986	3049762
	(0.6, 0.7)	394	1428	10203	40962	181889	1197429	4366051
10	(0.4, 0.5)	763	1804	12409	47525	216838	1201709	4738093
	(0.4, 0.6)	507	1095	7800	31158	143958	776891	3074400
	(0.5, 0.6)	584	1499	10176	43228	193926	1072017	4322155
	(0.5, 0.7)	387	872	7024	30141	136437	747205	3012394
	(0.6, 0.7)	507	1289	10090	43086	192192	1070428	4322154

* Bold values indicate the best known solutions found by the BSA algorithm.

However, the BSA algorithm is not statistically better than the SA algorithm for $N = 10, 20, 50, 100$ and 200 with problem sets I and III, perhaps because the SA algorithm also performs

well when it is applied to small and medium-sized problems. Nevertheless, the BAS is indeed statistically better than ES, even when N is 50 .

The computational results are analyzed with a focus on the number of solutions obtained by the proposed BSA algorithm; they are better, equal to, or worse than those obtained by the SA and EA algorithms. As shown in Table 5, the proposed BSA algorithm is better than, equal to, and worse than the SA algorithm in 163 out of 700, 507 out of 700, and 30 out of 700 benchmark problems, respectively. The proposed BSA algorithm is better than, equal to, and worse than the ES algorithm in 342 out of 700, 355 out of 700, and 3 out of 770 benchmark problems, respectively. In the benchmark problems with $N = 500$ and 1000, 67% and 90%, respectively, the solutions obtained using the proposed BSA algorithm are better than those obtained using the SA algorithm. In the test instances with $N = 200, 500$ and 1000, 67%, 98% and 100%, respectively, of the solutions obtained using the proposed BSA algorithm are better than those obtained using the EA algorithm. The analytical results reveal that BSA outperforms SA in most benchmark problems with $N \geq 500$, whereas BSA outperforms ES in most of benchmark problems with $N \geq 200$. As shown in Fig. 5, compared with ES, the proposed BSA can provide much better solutions when the number of jobs increases. Fig. 5 shows that compared with SA, the BSA can provide better solutions when the number of jobs is equal to and larger than 200.

To provide a benchmark for future research, Appendix Table 6 presents the best known solutions of the 250 and 100 benchmark problems in problem sets I and II, respectively, while Appendix Table 7 presents the best known solutions of the 250 and 100 benchmark problems in problems sets III and IV, respectively.

VI. CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

This paper concerns the $1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problem, which is not only theoretically but also practically interesting. We present a complete perspective on all possible optimal sequences associated with various straddling jobs and production waiting times. An effective and efficient BSA algorithm, which includes a backtracking mechanism and an effective coding scheme, is proposed to solve the above problem. Computational experiments that involve extensive benchmark test instances demonstrate that the proposed backtracking mechanism can improve the performance of the SA algorithm and make the proposed BSA algorithm significantly outperform the best available algorithm published in the literature. This research contributes by providing useful optimization approaches to the $1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problem. Since few algorithms are currently available for solving this strongly \mathcal{NP} -complete problem, the presented approaches can help practitioners solve real-world $1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problems with respect to the JIT manufacturing system.

Many interesting related topics warrant further investigation. First, the problem herein should be extended to include various plausible objectives. Second, more effective and efficient meta-heuristics for solving the

$1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problem warrant further exploration. Third, more research is needed to develop exact methods for solving the $1|RCDW|\sum(\alpha_j E_j + \beta_j T_j)$ problem. Fourth, further investigations of problem variants with additional realistic constraints, such as sequence-dependent setup times and release times, would support a rich body of future studies. Fifth, the SMSP with an RCDW in which a bi-objective function value is minimized, would be an interesting target of research. Finally, future research could consider other production systems (such as flow-shop and job-shop) that involve an RCDW.

APPENDIX

See Tables 6 and 7.

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