Received September 11, 2019, accepted October 1, 2019, date of publication October 4, 2019, date of current version October 17, 2019.

*Digital Object Identifier 10.1109/ACCESS.2019.2945558*

# A PHD-Based Particle Filter for Detecting and Tracking Multiple Weak Targets

**ZHICHAO BAO, QIUXI JIANG, AND FANGZHENG LIU**

College of Electronic Engineering, National University of Defense Technology, Hefei 230037, China

Corresponding author: Zhichao Bao (baozhichao17@nudt.edu.cn)

**ABSTRACT** Joint detection and tracking weak target is a challenging problem whose complexity is intensified when there are multiple targets present at the same time. Some Probability Hypothesis Density (PHD) based track-before-detect (TBD) particle filters (PHD-TBD) are proposed to solve this issue; however, the performance is unsatisfactory especially when the number of targets is large because some assumptions in PHD are violated. We propose to modify the general PHD-TBD filter in two aspects to make the PHD processing available for TBD scenarios. First, the distribution of false alarms is approximated as the Poisson distribution through a threshold method, and then a clustering technique is proposed to solve the overestimation of the target number. A typical TBD scenario is used to test the effectiveness of the proposed method. Simulation results indicate that the proposed method outperforms the general method in terms of estimation accuracy and computational complexity.

**INDEX TERMS** Multitarget tracking, track-before-detect (TBD), particle filter, probability hypothesis density (PHD).

## I. INTRODUCTION

Joint detection and tracking a low signal-to-noise ratio (SNR) target, also referred to as weak, dim or stealthy target, is a thorny problem based on the traditional threshold detection method. If the detection threshold is set high, it is easy to lose weak targets, in reverse, a low detection threshold gives a high rate of false alarms. An alternative approach, referred to as track-before-detect (TBD) [1]–[3], is to supply the tracker with all of the raw sensor data and accumulate target information in successive observation data, which is proved effective in detecting and tracking weak targets. There are many algorithms that are proposed to realize the TBD approach. These algorithms can be mainly divided into two categories: the batch methods and the recursive methods. The batch methods, including the Hough Transform [4] and the dynamic programming algorithm [5]–[8], integrate target information by processing multiple scans of raw data at the same time, while the recursive methods update the target information recursively. Particle filter based approach [9]–[12] is a typical recursive method. As is studied in the survey [13], the batch methods require large computational resources and are inefficient, while particle filter based approach has been proven to be efficient in nonlinear and multitarget scenarios; thus, we will focus on the particle filter based approach in this paper.

In many practical scenarios, we have to monitor multiple targets simultaneously, i.e., jointly estimate the number of the targets and their states. The random finite set (RFS) theory [14], [15] has drawn wide attention and has been applied to many fields [16], [17]. It provides a systematic and rigorous procedure to solve the multitarget tracking problem. However, it involves multiple integrals and is computationally intractable. Probability Hypothesis Density (PHD) filter [18]–[20], which is the first-order statistical moment of the RFS, is developed to alleviate the computational intractability. The PHD filter operates on the single-target state space and avoids the combinatorial problems that arise from data association. These significant advantages make the PHD filter extremely attractive. The application of PHD filter has been extended to many fields. For example, the work in [21], [22] used a PHD filter to track a variable number of human groups in video, the work in [23] extended the PHD filter to accommodate nonstandard targets.

Thus, it is intuitively promising to combine the PHD and TBD approach (PHD-TBD) to solve the problem of joint detection and estimation of multiple weak targets. However, the PHD filter is designed for point measurements and cannot be applied to image observations directly. There are two assumptions that have to be satisfied when the PHD filter

The associate editor coordinating the review of this manuscript and approving it for publication was Tianhua Xu.

is used. First, no target generates more than one measurement, which cannot be guaranteed by general image observations. Some researchers solve this problem by limiting the response of target in only one resolution cell in the image observation [24], [25], but this method is demanding and makes the filter unavailable for nonstandard point observations. Second, false alarms have to be Poisson distribution, which is also violated in a typical TBD scenario. In [26], K. Punithakumar et al. proposed a sequence Monte Carlo (SMC) PHD-TBD approach for general image observations. Although the proposed method is shown to be a computationally efficient solution to the multitarget tracking problems with the varying number of targets, it assumes the false alarms to be a constant number, which is violated with the second assumption above, and the results need to be checked in more detail.

In the present work, to approximate the false alarms as Poisson distribution, we view the measurements whose intensities are above a predefined threshold as the measurements that generated by targets, while the other measurements as false alarms. As can be proved as follows, in this way, the distribution of false alarms can be approximated as Poisson distribution. To solve the overestimation of target number caused by one target generating multiple measurements, instead of limiting the response of target in one cell, we use a clustering technique to extract the multitarget state and the target number from all the particles after resampling. The rest of the paper is organized as follows. Section 2 constructs the multitarget state transition model and observation model based on RFS, Section 3 gives an overview on general PHD-TBD approach. In Section 4, we have made two significant changes to general PHD-TBD approach and proposed a new PHD-TBD method. In Section 5, some simulations are carried out and the conclusions are given in Section 6.

## II. MULTITARGET TBD RFS MODLE

### A. STATE OF THE RFS MODEL

The target state transition model describes the motion of the target and is one of the key factors that decide the performance of the tracking system. For simplicity of the representations, we use the capital $X$ to represent the multitarget state and the lowercase $x$ to represent the single target state. In multitarget system, the multitarget state at time $k$ can be naturally represented as a finite subset $X_k$. If there are $N(k)$ targets at time $k$ with state $x_{k,1}, \cdots, x_{k,N(k)}$, then,

$$X_k = \left\{x_{k,1}, \cdots, x_{k,t}, \cdots x_{k,N(k)}\right\} \tag{1}$$

is the multitarget state. The $t_{th}$ target state is represented as $x_{k,t}$, the single target state is represented as $x_k$ if without special indication. Each single target state contains the target position $(\mathrm{x}_k, \mathrm{y}_k)$ and velocity $(\dot{\mathrm{x}}_k, \dot{\mathrm{y}}_k)$ as

$$\mathrm{x}_k = \left[\mathrm{x}_k, \dot{\mathrm{x}}_k, \mathrm{y}_k, \dot{\mathrm{y}}_k\right]^T \tag{2}$$

For target motion, given multitarget state set $X_{k-1}$ at time $k-1$, each $x_{k-1} \in X_{k-1}$ survives at time $k$ with probability $e_{k|k-1}(x_{k-1})$ and its transition probability density from $x_{k-1}$ to $x_k$ is denoted as $f_{k|k-1}(x_k|x_{k-1})$. The surviving target motion is denoted as a RFS $S_{k|k-1}(x_{k-1})$, the RFS of target birth at time $k$ is denoted as $\Gamma_k$, and the RFS of targets spawning from the target with $x_{k-1}$ is represented by $B_{k|k-1}(x_{k-1})$, therefore the multitarget state $X_k$ is modelled by the union of RFSs as

$$X_k = \left[\bigcup_{x_{k-1}\in X_{k-1}} S_{k|k-1}(x_{k-1})\right] \cup \left[\bigcup_{x_{k-1}\in X_{k-1}} B_{k|k-1}(x_{k-1})\right] \cup \Gamma_k \tag{3}$$

### B. OBSERVATION OF THE RFS MODEL

In this paper, the observation is assumed to be a two-dimensional image consisting of $l \times m$ resolution cells. Each cell corresponds to a rectangular region of dimensions and the center of each ceil $(i,j)$ is defined to be at $(i\Delta_x, j\Delta_y)$ for $i = 1, \cdots, l$, $j = 1, \cdots, m$, the $\Delta_x$ and $\Delta_y$ represent the length of resolution cell in $x$-axis direction and $y$-axis direction respectively. The image data is the power intensity information in each resolution cell and can be expressed as

$$z_k = \{z_k^{(i,j)} : i = 1, \ldots, l, j = 1, \ldots, m\} \tag{4}$$

in which

$$z_k^{(i,j)} = \begin{cases} \sum_{t=1}^{N(k)} h_k^{(i,j)}(x_{k,t}) + v_k^{(i,j)} & H_1 : \text{if there are } N(k) \text{ targets} \\ v_k^{(i,j)} & H_0 : \text{if there is no target} \end{cases} \tag{5}$$

in which $v_k^{(i,j)}$ is the zero-mean white Gaussian noise with variance $\sigma^2$ and is assumed to be independent from cell to cell and from image to image, $h_k^{(i,j)}(x_{k,t})$ represents the degree of the $t_{th}$ target influence on adjacent image cells, and can be further expanded as

$$h_k^{(i,j)}(x_{k,t}) = \frac{\Delta_x \Delta_y I_0^t}{2\pi \Sigma^2} \exp\left\{-\frac{(i\Delta_x - \mathrm{x}_{k,t})^2 + (j\Delta_y - \mathrm{y}_{k,t})^2}{2\Sigma^2}\right\} \tag{6}$$

where $\Sigma$ stands for the amount of blurring introduced by the sensor, $\mathrm{x}_{k,t}$ and $\mathrm{y}_{k,t}$ represent the position of the $t_{th}$ target, $I_0^t$ is the $t_{th}$ target intensity and we assume the target intensity is constant. $H_0$ and $H_1$ are the hypotheses about the absence or presence of target, respectively.

As we assume the intensity information in each cell is independent of each other under the condition that the target state is given, the multitarget posterior probability density function (PDF) $p(z_k|X_k)$ can be expressed as the product of marginal PDFs

$$p(z_k|X_k, H) = \begin{cases} \prod_{i=1}^{l}\prod_{j=1}^{m} p_1\left(z_k^{(i,j)}|X_k\right) & \text{under } H_1 \\ \prod_{i=1}^{l}\prod_{j=1}^{m} p_0\left(z_k^{(i,j)}\right) & \text{under } H_0 \end{cases} \tag{7}$$

As the noise is assumed to be Gaussian distribution, the marginal PDFs can be further expressed as

$$p_0\left(z_k^{(i,j)}|H_0\right)=\mathcal{N}\left(z_k^{(i,j)};0,\sigma^2\right) \tag{8}$$

$$p_1\left(z_k^{(i,j)}|H_1\right)=\mathcal{N}\left(z_k^{(i,j)};\sum_{t=1}^{N(k)}h_k^{(i,j)}\left(x_{k,t}\right),\sigma^2\right) \tag{9}$$

where the Gaussian distribution is represented as "$\mathcal{N}$".

Considering that one single target only has a significant influence on the adjacent image cells, the posterior PDF of the single target $p\left(z_k|x_k\right)$ can be approximated as

$$p\left(z_k|x_k,H_1\right)\approx\prod_{i\in C_i(x_k)}\prod_{j\in C_j(x_k)}p_1\left(z_k^{(i,j)}|x_k\right)\prod_{i\notin C_i(x_k)}\prod_{j\notin C_j(x_k)}p_0\left(z_k^{(i,j)}\right) \tag{10}$$

where $C_i\left(x_k\right)$ and $C_j\left(x_k\right)$ are the sets of subscripts $i$ and $j$, respectively, corresponding to pixels affected by the target with state $x_k$.

## III. THE GENERAL PHD FILTER FOR TBD

In Section 2, we have modelled the multitarget TBD observations and collection of states as RFS, and a PHD-TBD approach can be formulated. As proposed in [18], the prediction and update equations of the PHD filter are presented as follows:

$$\begin{aligned}D_{k|k-1}\left(x|z_{1:k-1}\right)&=\int e_{k|k-1}\left(\varsigma\right)f_k\left(x|\varsigma\right)D_{k-1|k-1}\left(\varsigma|z_{1:k-1}\right)d\varsigma\\&\quad+\int b_{k|k-1}\left(x|\varsigma\right)D_{k-1|k-1}\left(\varsigma|z_{1:k-1}\right)d\varsigma+\gamma_k\left(x\right)\end{aligned} \tag{11}$$

$$\begin{aligned}D_{k|k}\left(x|z_{1:k}\right)&=\left[1-p_D\left(x\right)\right]D_{k|k-1}\left(x\right)\\&\quad+\sum_{z\in z_k}\frac{p_D\left(x\right)g_k\left(z|x\right)D_{k|k-1}\left(x\right)}{\kappa_k\left(z\right)+\int p_D\left(\varsigma\right)g_k\left(z|\varsigma\right)D_{k|k-1}\left(\varsigma\right)d\varsigma}\end{aligned} \tag{12}$$

where $D_{k|k}\left(x|z_{1:k}\right)$ is the PHD density whose integral $\int_S D_{k|k}\left(x|z_{1:k}\right)dx$ on any region $\mathcal{S}$ of state space is the cardinality

$$\hat{n}_k\left(\mathcal{S}\right)=\int\left|X\cap\mathcal{S}\right|p_k\left(X_k|z_{1:k}\right)\delta X \tag{13}$$

where the definition of set integral is

$$\int f\left(X\right)\delta X=\sum_{i=0}^{\infty}\frac{1}{i!}\int f\left(\{x_1,\cdots,x_i\}\right)dx_1\cdots dx_i \tag{14}$$

$b_{k|k-1}\left(x_k|x_{k-1}\right)$ and $\gamma_k\left(x_k\right)$ denote the intensity of $B_{k|k-1}\left(x_{k-1}\right)$ and $\Gamma_k$ at time $k$, $\kappa_k\left(z\right)=\lambda_k\cdot c\left(z\right)$ is the intensity of false alarms, $\lambda_k$ is the number of average false alarms, $c\left(z\right)$ is the distribution of false alarms; $z_{1:k}=\{z_1,\cdots,z_k\}$ is the time-sequence of observation sets, $z$ is one of the measurements in $z_k$, $p_D\left(x\right)$ is the state dependent probability of detection, $g_k\left(z|x\right)$ is the likelihood of the measurement that generated by targets. As there is no detection process before the update step, we assume that the observations contain all the target information; thus, $p_D\left(x\right)\equiv 1$. Therefore, the update equation (12) becomes

$$D_{k|k}\left(x|z_{1:k}\right)=\sum_{z\in z_k}\frac{g_k\left(z|x\right)D_{k|k-1}\left(x\right)}{\kappa_k\left(z\right)+\int g_k\left(z|\varsigma\right)D_{k|k-1}\left(\varsigma\right)d\varsigma} \tag{15}$$

Because the PHD propagation equations involve multiple integrals that have no computationally tractable closed-form expression, the sequential Monte Carlo (SMC) methods are used to approximate the PHD in [13]. Let the PHD density $D_{k-1|k-1}\left(x|z_{1:k-1}\right)$ at time $k-1$ be represented by a set of particles $\left\{w_{k-1}^{(r)},x_{k-1}^{(r)}\right\}_{r=1}^{L_{k-1}}$ where $w_{k-1}^{(r)}$ is the weight of the corresponding $r_{th}$ particle, $L_{k-1}$ is the total number of surviving particles, and thus the PHD density can be represented as

$$D_{k-1|k-1}\left(x|z_{1:k}\right)=\sum_{r=1}^{L_{k-1}}w_{k-1}^{(r)}\delta\left(x-x_{k-1}^{(r)}\right) \tag{16}$$

The predicted particles are generated by

$$x_{k|k-1}^{(r)}\sim\begin{cases}q_k\left(\cdot|x_{k-1}^{(r)}\right) & r=1,\cdots,L_{k-1}\\ v_k\left(\cdot\right) & r=L_{k-1}+1,\cdots,L_{k-1}+J_k\end{cases} \tag{17}$$

where $q_k\left(\cdot|x_{k-1}^{(r)}\right)$ and $v_k\left(\cdot\right)$ are the proposal density of the surviving particles and the new-born particles respectively, $J_k$ is the number of the new-born particles at time $k$.

The predicted PHD density can be denoted as

$$D_{k|k-1}\left(x|z_{1:k-1}\right)=\sum_{r=1}^{L_{k-1}+J_k}w_{k|k-1}^{(r)}\delta\left(x-x_{k|k-1}^{(r)}\right) \tag{18}$$

where

$$w_{k|k-1}^{(r)}=\begin{cases}\dfrac{e_{k|k-1}\left(x_{k-1}^{(r)}\right)f_k\left(x_{k|k-1}^{(r)}|x_{k-1}^{(r)}\right)+b_{k|k-1}\left(x_{k|k-1}^{(r)}|x_{k-1}^{(r)}\right)}{q_k\left(x_{k|k-1}^{(r)}|x_{k-1}^{(r)}\right)} & r=1,\cdots,L_{k-1}\\[2ex] \dfrac{\gamma_k\left(x_{k|k-1}^{(r)}\right)}{v_k\left(x_{k|k-1}^{(r)}\right)} & r=L_{k-1}+1,\cdots,L_{k-1}+J_k\end{cases} \tag{19}$$

The update PHD density can be represented as

$$D_{k|k}\left(x|z_{1:k}\right)=\sum_{r=1}^{L_{k-1}+J_k}w_k^{(r)}\delta\left(x-x_{k|k-1}^{(r)}\right) \tag{20}$$

where

$$w_k^{(r)}=\sum_{z\in z_k}\frac{g_k\left(z|x_{k|k-1}^{(r)}\right)w_{k|k-1}^{(r)}}{\kappa_k\left(z\right)+\sum_{r=1}^{L_{k-1}+J_k}g_k\left(z|x_{k|k-1}^{(r)}\right)w_{k|k-1}^{(r)}} \tag{21}$$

The target number is approximated as

$$\hat{n}_k=\sum_{r=1}^{L_{k-1}+J_k}w_k^{(r)} \tag{22}$$

According to the standard treatment of particle filter, we resample $\left\{w_k^{(r)}/\hat{n}_k,x_{k|k-1}^{(r)}\right\}_{r=1}^{L_{k-1}+J_k}$ to get the new particle set $\left\{w_k^{*(r)},x_k^{(r)}\right\}_{r=1}^{L_k}$.

## IV. THE PROPOSED PHD FILTER FOR TBD

### A. THE FALSE ALARMS

In the PHD assumptions, the distribution of false alarms is assumed to be the Poisson distribution and the number of false alarms is random. However, in the TBD scenario, the observation data provided by sensor is composed of a constant number, i.e., $l \times m$ measurements at a certain time. As a result, the general PHD-TBD algorithm needs to be modified. The measurements can be divided into two categories, namely measurements generated by targets and false alarms. When the number of measurements generated by targets is fixed for a period in the observing area, the remaining measurements are false alarms. Without considering computational complexity, each measurement in the observation is affected by the targets to a certain degree, i.e., there are no false alarms. In this way, we have to update the PHD filter using the entire $l \times m$ measurements, which is computationally intractable and unnecessary, because the targets only have a significant influence on the adjacent image cells. It is intuitive to assume that the measurement cells defined in (10) as the measurements generated by targets, and the other measurements as false alarms. Thus, the number of false alarms is equal to the number of total measurements minus the number of measurements generated by targets. When the number of measurements generated by targets is invariant, the number of false alarms is also invariant.

Assuming that there is no target in the observation. As the measurements are conditionally independent and obey Gaussian distribution, i.e.,

$$f\left(z_k^{(i,j)}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(z_k^{(i,j)} - \mu\right)^2}{2\sigma^2}\right) \tag{23}$$

To approximate the distribution of false alarms as the Poisson assumptions, we predefine a small threshold $\theta$ for the observation data, and the probability that one measurement exceeds this threshold can be calculated as

$$p\left(z_k^{(i,j)} \geq \theta\right) = \int_{\theta}^{+\infty} f\left(z_k^{(i,j)}\right) dz_k^{(i,j)} \geq p^* \tag{24}$$

As we assume there is no target, all the measurements that satisfy $z_k^{(i,j)} > \theta$ are regarded as false alarms and denoted as $T(z_k) = \left\{z_k^{(i,j)} | z_k^{(i,j)} > \theta\right\}$; the other measurements are discarded without further processing. Much care must be taken to choose the value of $\theta$ and a shrinkage method is proposed in [27] to find the optimal value. In this way, after introducing the threshold $\theta$ for each observation, the probability that one measurement exceeds this threshold is a constant, denoted as $p^*$; thus, the distribution of false alarms is a binomial distribution and its expectation is $Mp^*$. The binomial distribution can be approximated as Poisson distribution when $M$ is a large number. Thus, we can approximate the false alarms as the Poisson distribution and

$$\kappa_k(z) \approx \frac{Mp^*}{l \times \Delta_x \times m \times \Delta_y} \tag{25}$$

in which $p^* < 1$. As is often the case, we set the resolution $\Delta_x$, $\Delta_y$ equal one, and thus $\kappa_k(z) < 1$.

### B. THE OVERESTIMATE OF THE TARGET NUMBER

In the observation model of TBD scenario, one target can affect several adjacent cells, which violates the assumption in the PHD filter that no target generates more than one measurement. In TBD scenario, some measurements generated by the same target are used to update the PHD filter. However, the PHD filter assumes one measurement generated by the target is originated from one distinct target. Thus, the overestimation of the target number is inevitable. We will examine this problem theoretically first. Assuming that all the measurements generated by the targets are included in $T(z_k)$, the update equation for each particle can be rewritten as

$$w_k^{(r)} = \sum_{z \in T(z_k)} \frac{g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}}{\kappa_k(z) + \sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}} \tag{26}$$

the target number estimation becomes

$$\hat{n}_k = \sum_{z \in T(z_k)} \sum_{r=1}^{L_{k-1}+J_k} \times \frac{g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}}{\kappa_k(z) + \sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}} \tag{27}$$

As for one measurement $z \in T(z_k)$, its corresponding component in (27) is

$$\frac{\sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}}{\kappa_k(z) + \sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}} \tag{28}$$

As we generate the particles in the total observation space, there are always some particles that will scatter around the real target state and make the sum of weighted likelihoods

$$\sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)} \gg \kappa_k(z) \tag{29}$$

and thus the

$$\frac{\sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}}{\kappa_k(z) + \sum_{r=1}^{L_{k-1}+J_k} g_k\left(z|x_{k|k-1}^{(r)}\right) w_{k|k-1}^{(r)}} \approx 1 \tag{30}$$

and the estimation of target number can be approximated as

$$\hat{n}_k \approx |T(z_k)| \tag{31}$$

where the $|T(z_k)|$ represents the number of measurements in $T(z_k)$. The number of measurements in $T(z_k)$ is far more than the real target number as one target can generate several measurements. Therefore, the sum of particle weights cannot be used to estimate the target number in TBD scenario.

Although the overestimation of the target number is inevitable using the general PHD-TBD filter, it does not affect the estimation of the multitarget state. Then, the problem changes to extract the target number and the multitarget state from the particles after resampling at the same time. This turns out to be a typical clustering analysis problem and some mature algorithms are available. In this paper, we adopt the clustering algorithm in [28], which is proved effective and stable in cluster analysis. There may exist some other cluster algorithms, like the spectral clustering [29], which may have a better performance in cluster analysis, but the choice of the cluster algorithm is not the focus of this paper.

We view the estimation of the target number as a process of optimization. As in the clustering algorithm, we need to find a validity index to optimize the number of clusters. We choose to use the Davies-Bouldin Index (DBI) in [28] as the validity index.

Assume there are $\psi$ targets at time $k$, i.e., the particles after resampling will gather around $\psi$ clusters in the ideal situation. We denote the total cluster set as $C = \{C_1, \cdots C_\eta \cdots, C_\psi\}$, and define the average distance in each cluster as

$$avg\left(C_\eta\right) = \frac{2}{\left|C_\eta\right|\left(\left|C_\eta\right|-1\right)}\sum\nolimits_{1\le\alpha<\beta\le\left|C_\eta\right|} dist\left(x_k^{(\alpha)}, x_k^{(\beta)}\right) \tag{32}$$

where $\left|C_\eta\right|$ is the number of particles in the cluster $C_\eta$, $x_k^{(\alpha)}$ and $x_k^{(\beta)}$ are two different particles in $C_\eta$, $dist\left(x_k^{(\alpha)}, x_k^{(\beta)}\right) = \left\|x_k^{(\alpha)} - x_k^{(\beta)}\right\|_2$, is the Euler distance. Define the distance between two clusters as

$$d_{cen}\left(C_\varepsilon, C_\phi\right) = dist\left(\Omega_\varepsilon, \Omega_\phi\right) \tag{33}$$

where $\Omega_\varepsilon$ is the centre of $C_\varepsilon$, $\Omega_\phi$ is the centre of $C_\phi$. The definition of DBI is

$$DBI = \frac{1}{\psi}\sum_{\varepsilon=1}^{\psi}\max_{\varepsilon\neq\phi}\left(\frac{avg\left(C_\varepsilon\right)+avg\left(C_\phi\right)}{d_{cen}\left(\Omega_\varepsilon, \Omega_\phi\right)}\right) \tag{34}$$

Given the maximum target number $\psi_{\max}$, we just need to calculate the DBI successively by changing the value of $\psi$ from 1 to $\psi_{\max}$. The particular $\psi$ that minimizes DBI is the optimal estimation and is denoted as $\psi_{opt}$. The $\psi_{opt}$ is assumed to be the estimated target number, and the centrals of $\psi_{opt}$ clusters are the estimated target states.

## V. SIMULATIONS AND RESULTS

To test the effectiveness of the proposed PHD-TBD particle filter, we have designed a scenario that has a time-varying and unknown number of targets. A maximum of 5 targets are present at any time, and there are various target births/deaths throughout the scenario duration of $K = 50s$. The target motions are modelled by a nonlinear constant turn model, the target state has to be expanded to include the turn rate $\omega_k$, i.e., $\tilde{x}_k = \left[\, x_k^T \quad \omega_k \,\right]$, and

$$\begin{aligned} x_k &= F\left(\omega_{k-1}\right)x_{k-1} + G\varsigma_{k-1} \\ \omega_k &= \omega_{k-1} + \Lambda u_{k-1} \end{aligned} \tag{35}$$

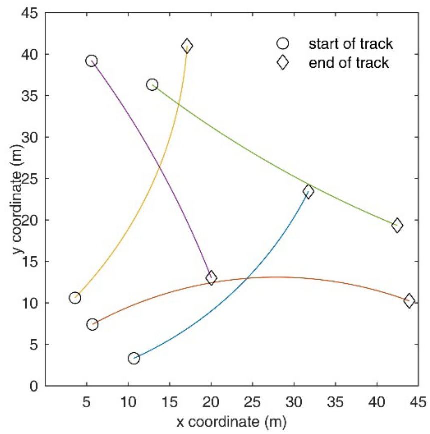


**FIGURE 1.** Ture tracks for 5 targets appearing and disappearing at different times.

where

$$F\left(\omega\right) = \begin{bmatrix} 1 & \frac{\sin\left(\omega\Lambda\right)}{\omega} & 0 & -\frac{1-\cos\left(\omega\Lambda\right)}{\omega} \\ 0 & \cos\left(\omega\Lambda\right) & 0 & \sin\left(\omega\Lambda\right) \\ 0 & \frac{1-\cos\left(\omega\Lambda\right)}{\omega} & 1 & \frac{\sin\left(\omega\Lambda\right)}{\omega} \\ 0 & \sin\left(\omega\Lambda\right) & 0 & \cos\left(\omega\Lambda\right) \end{bmatrix} \tag{36}$$

$$G = \begin{bmatrix} \Lambda^2/2 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda^2/2 & \Lambda \end{bmatrix}^T \tag{37}$$

$\Lambda$ is the sampling interval and let $\Lambda = 1s$, $G$ is the input matrix, the process noise $\varsigma_k$ is assumed to be $\varsigma_k \sim \mathcal{N}\left(\varsigma_k; 0, \sigma_\varsigma^2 I_2\right)$ where $I_2$ is an $2 \times 2$ identity matrix, $u_k$ is assumed to be $u_k \sim \mathcal{N}\left(u_k; 0, \sigma_u^2\right)$, and $\sigma_\varsigma = 0.01m/s^2$, $\sigma_u = \left(\pi/180\right) rad/s$.

The other parameter values are $\Delta_x = \Delta_y = 1, l = m = 45$, $\sigma = 1$, $\Sigma = 1$, $I_0 = 30$. The initial states of target are set as in Table 1.

**TABLE 1.** The initial states of target.

| Index | Initial state | Birth | Disappear |
|---|---|---|---|
| 1 | $[10 \quad 0.7 \quad 3 \quad 0.3 \quad \pi/180]^T$ | 1 | 40 |
| 2 | $[5 \quad 0.7 \quad 7 \quad 0.4 \quad -\pi/180]^T$ | 1 | 50 |
| 3 | $[3 \quad 0.6 \quad 10 \quad 0.6 \quad \pi/180]^T$ | 10 | 50 |
| 4 | $[5 \quad 0.6 \quad 40 \quad \text{-}0.8 \quad -\pi/360]^T$ | 10 | 40 |
| 5 | $[12 \quad 0.9 \quad 37 \quad \text{-}0.7 \quad \pi/360]^T$ | 20 | 50 |

As is modelled in Section 2, the observations can be generated using (4), (5) and (6). A typical observation is given in Fig. 2.

1000 particles are used to maintain per expected target's track, $J_k = 3000$ new-born particles are used to explore the target space. We generate the positions of new-born particles by uniformly sampling from $T\left(z_k\right)$, the velocities of the birth particles are generated from the uniform distributions, i.e., $\dot{x}_k \sim U\left(-1, 1\right), \dot{y}_k \sim U\left(-1, 1\right)$, the turning rate of

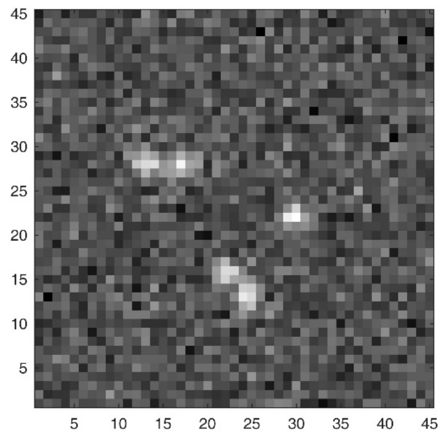


**FIGURE 2.** **An image observation of 5 targets at time 30.**

the new-born particles are generate from uniform distribution, i.e., $\omega_k \sim U\left(-\pi/90, \pi/90\right)$.

Several times of the position distribution of the particles after resampling and the targets are given in Fig. 3.

As is shown in Fig. 3, the particles gather around the target position over different times, which means that the PHD filter works well. Then we just need to extract the target number and target states from the particle clusters. The general PHD-TBD filter, the proposed PHD-TBD, and the ideal PHD-TBD filter that assume the true target number at different times is already known are used to estimate the multitarget state and the target number. The results are shown in Fig. 4 and Fig. 5.

Fig. 4 and Fig. 5 should be explained together. As is shown in the two figures above, the general PHD-TBD filter overestimates the target number, which is shown in Fig. 4 as many target states overlap around the real target states, and shown in Fig. 5 as the estimated target number seriously deviates from the truth. The performance of the proposed PHD-TBD filter improves greatly, which is shown in Fig. 4 as the state overlap phenomenon reduces greatly, and shown in Fig.5 as the estimated target states are in accordance with the true values. The ideal PHD-TBD filter is designed for comparison, which is the upper bound that a PHD-TBD filter can reach.

The optimal subpattern assignment (OSPA) [30] is a mathematically consistent yet intuitively meaningful way to jointly capture differences in cardinality and individual elements between two finite sets. To confirm these single run results, 100 Monte Carlo tests are performed and averaged. Fig. 6 shows the estimation errors in terms of the Monte Carlo averaged OSPA distance (for $c = 100m, p = 1$) for the general, the proposed and the ideal PHD-TBD filter. These results confirm that the proposed PHD-TBD filter performs accurately and consistently, though the performance is not ideal at several times, is still better than the general PHD-TBD filter. The estimation error of the general PHD-TBD filter remains at a relatively high level because of the overestimation of the target number.

To study the effect of target intensity on the performance of the filters, we assume the multiple targets have the same intensity which is based on various choices of the SNR

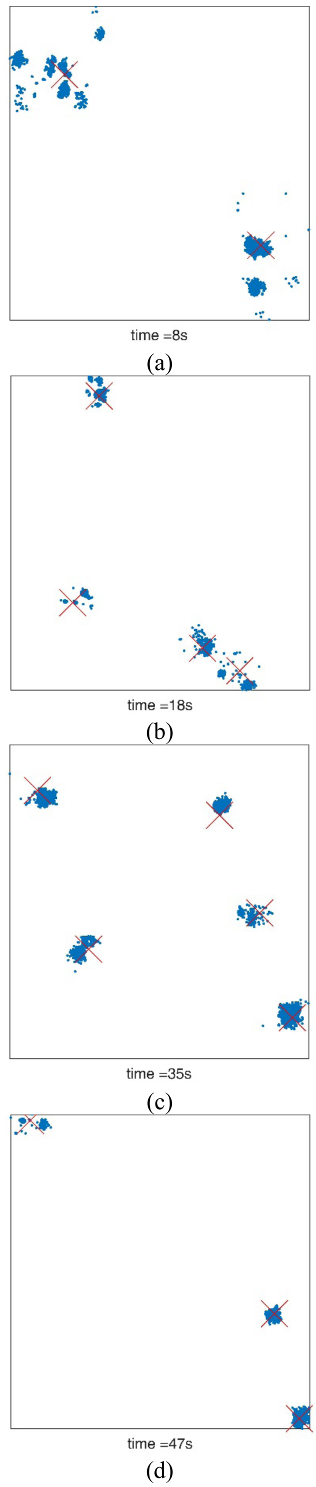


**FIGURE 3.** **The position distribution of particles and targets over several times. One particle is denoted as a little dot and the targets are represented as 'x'. (a) At time 8; (b) At time 18; (c) At time 35; (d) At time 47.**

given by

$$SNR = 10\log\left(\frac{I_0 \Delta x \Delta y / 2\pi\Sigma^2}{\sigma}\right)^2 \tag{38}$$

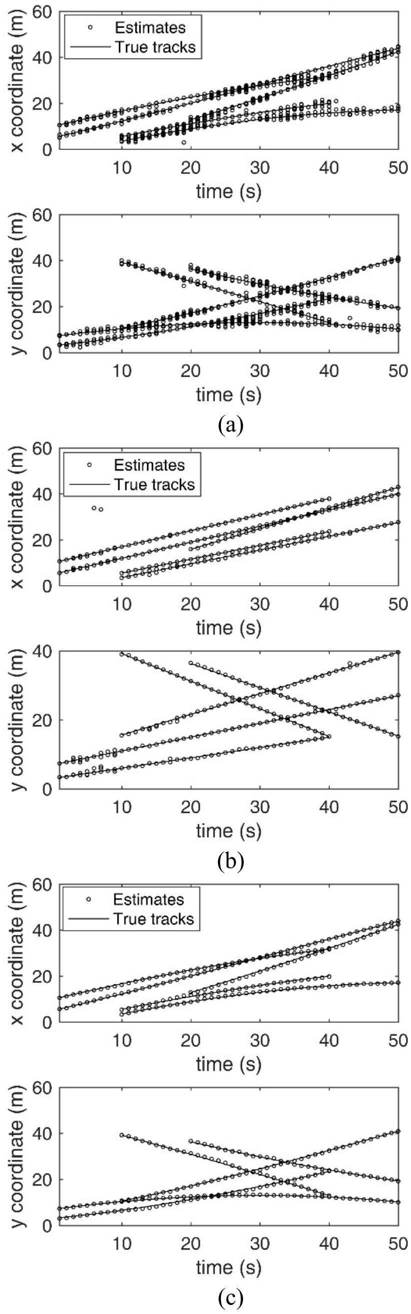


**FIGURE 4.** The multitarget states estimation given by the general PHD-TBD filter, the proposed PHD-TBD filter, and the ideal PHD-TBD filter. (a) The general PHD-TBD filter; (b) The proposed PHD-TBD filter; (c) The ideal PHD-TBD filter.

We test the performance over 100 Monte Carlo runs for 3, 6, 9, 12 dB for the proposed, the general and the idea PHD-TBD filter. The result is shown in Fig. 7.

Fig. 7 shows that the performance of the filters all gets better with higher target intensity. The proposed PHD-TBD filter outperforms the general PHD-TBD filter significantly with various SNR settings.

The computational demands of the general PHD-TBD and the proposed PHD-TBD filter are assessed by benchmarking their processing times on a generic PC with 2.5 GHz CPU and

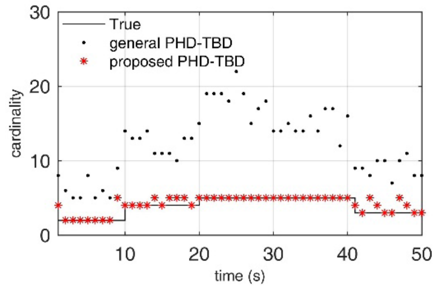


**FIGURE 5.** The estimation of target number.

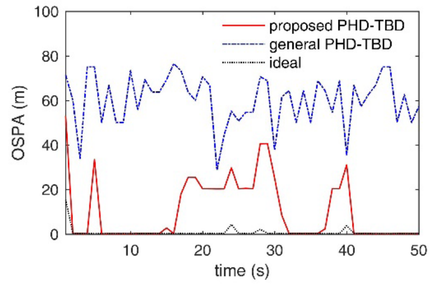


**FIGURE 6.** Monte Carlo averaged OSPA miss distance versus time with $c = 100m$ and $p = 1$ for the general, the proposed and the ideal PHD-TBD filter.

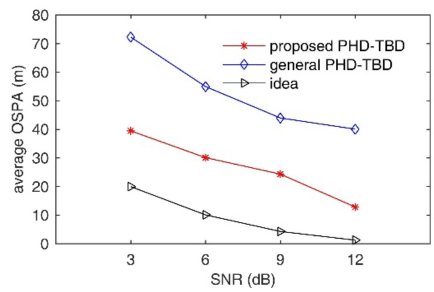


**FIGURE 7.** Monte Carlo averaged OSPA miss distance versus SNR with $c = 100m$ and $p = 1$ for the general, the proposed and the ideal PHD-TBD filter.

4.0 GB RAM in the Matlab environment. Though the proposed PHD-TBD filter has to find optimal target number and it seems to have increased the computation demand superficially. In fact, as the proposed method gives a much better estimation of the target number, the computation demand for multitarget state extraction decreases greatly. The average single running time for the general PHD-TBD filter is 15.47s, while the single running time for the proposed PHD-TBD filter is 9.69s, which shows that the proposed method is more efficient.

## VI. CONCLUSION

In this paper, we have modified the TBD observations to make it available for PHD filter and proposed a new PHD-TBD approach. A threshold method is used to adapt the distribution of false alarms to Poisson distribution, and a clustering technique is used to solve the problem of overestimation of the target number. Simulation results indicate that the proposed method is superior to the general PHD-TBD method in both estimation accuracy and running efficiency. The ideal PHD-TBD filter which assumes the target number is known at different times is also set for comparison. It indicates that if the target number has been better estimated, i.e., using a better clustering technique, the performance can be improved further, and such work will be carefully considered in the future study.

## ACKNOWLEDGMENT

The authors would like to thank professor Ba-Ngu Vo for sharing the PHD related Matlab codes on his blog.

## REFERENCES

[1] S. Buzzi, M. Lops, and L. Venturino, "Track-before-detect procedures for early detection of moving target from airborne radars," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 3, pp. 937–954, Jul. 2005.

[2] X. Deng, Y. Pi, M. Morelande, and B. Moran, "Track-before-detect procedures for low pulse repetition frequency surveillance radars," *IET Radar, Sonar Navigat.*, vol. 5, no. 1, pp. 65–73, Sep. 2011.

[3] I. S. Reed, R. M. Gagliardi, and L. B. Stotts, "A recursive moving-target-indication algorithm for optical image sequences," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 26, no. 3, pp. 434–440, May 1990.

[4] B. D. Carlson, E. D. Evans, and S. I. Wilson, "Search radar detection and track with the Hough transform. III. Detection performance with binary integration," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 30, no. 1, pp. 102–108, Jan. 1994.

[5] L. A. Johnston and V. Krishnamurthy, "Performance analysis of a dynamic programming track before detect algorithm," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 38, no. 3, pp. 228–242, Jul. 2002.

[6] D. Zheng, S. Wang, and X. Qin, "A dynamic programming track-before-detect algorithm based on local linearization for non-Gaussian clutter background," *Chin. J. Electron.*, vol. 25, no. 3, pp. 583–590, Aug. 2016.

[7] H. Wang, J. Yi, and X. Wan, "Greedy algorithm-based track-before-detect in radar systems," *IEEE Sensors J.*, vol. 18, no. 17, pp. 7158–7165, Sep. 2018.

[8] W. Yi, M. R. Morelande, L. Kong, and J. Yang, "An efficient multi-frame track-before-detect algorithm for multi-target tracking," *IEEE J. Sel. Topics Signal Process.*, vol. 7, no. 3, pp. 421–434, Apr. 2013.

[9] D. J. Salmond and H. Birch, "A particle filter for track-before-detect," in *Proc. Amer. Control Conf.*, Arlington, VA, USA, Jun. 2001, pp. 3755–3760.

[10] M. G. Rutten, B. Ristic, and N. J. Gordon, "A comparison of particle filters for recursive track-before-detect," in *Proc. Int. Conf. Inf. Fusion*, Philadelphia, PA, USA, Jul. 2005, pp. 169–175.

[11] H.-T. Su, T.-P. Wu, H.-W. Liu, and Z. Bao, "Rao–Blackwellised particle filter based track-before-detect algorithm," *IET Signal Process.*, vol. 2, no. 2, pp. 169–176, Jun. 2008.

[12] S. Ebenezer and S. Papandreou-Suppappola, "Generalized recursive track-before-detect with proposal partitioning for tracking varying number of multiple targets in low SNR," *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2819–2834, Jun. 2016.

[13] M. Hadzagic, H. Michalska, and E. Lefebvre, "Track-before detect methods in tracking low-observable targets: A survey," *Sensors Transducers Mag.*, vol. 54, no. 1, pp. 374–380, Aug. 2005.

[14] B.-T. Vo, B.-N. Vo, and H. G. Hoang, "An efficient implementation of the generalized labeled multi-Bernoulli filter," *IEEE Trans. Signal Process.*, vol. 65, no. 8, pp. 1975–1987, Apr. 2017.

[15] B.-N. Vo, B.-T. Vo, and D. Phung, "Labeled random finite sets and the Bayes multi-target tracking filter," *IEEE Trans. Signal Process.*, vol. 62, no. 24, pp. 6554–6567, Dec. 2014.

[16] Z. Sutton, P. Willett, and Y. Bar-Shalom, "Tracking multiple evolving threats with cluttered surveillance observations," in *Proc. IEEE Sensor Array Multichannel Signal Process. Workshop*, Sheffield, U.K., Jul. 2018, pp. 21–25.

[17] R. Hoseinnezhad, B.-N. Vo, B.-T. Vo, and D. Suter, "Visual tracking of numerous targets via multi-Bernoulli filtering of image data," *Pattern Recognit.*, vol. 45, no. 10, pp. 3625–3635, Oct. 2012.

[18] R. P. S. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 4, pp. 1152–1178, Oct. 2003.

[19] R. Mahler, "PHD filters of higher order in target number," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 4, pp. 1523–1543, Oct. 2007.

[20] B.-N. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo methods for multitarget filtering with random finite sets," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 4, pp. 1224–1245, Oct. 2005.

[21] Y.-D. Wang, J.-K. Wu, A. A. Kassim, and W.-M. Huang, "Tracking a variable number of human groups in video using probability hypothesis density," in *Proc. Int. Conf. Pattern Recognit.*, Hong Kong, Aug. 2006, pp. 1127–1130.

[22] P. Feng, W. Wang, S. M. Naqvi, and J. Chambers, "Adaptive retrodiction particle PHD filter for multiple human tracking," *IEEE Signal Process. Lett.*, vol. 23, no. 11, pp. 1592–1596, Nov. 2016.

[23] R. Mahler, "PHD filters for nonstandard targets, I: Extended targets," in *Proc. Int. Conf. Inf. Fusion*, Seattle, WA, USA, Jul. 2009, pp. 915–921.

[24] X. Luo, C. Yang, R. Chen, and Z. Shi, "Improved SMC-PHD filter for multi-target track-before-detect," in *Proc. IEEE 84th Veh. Technol. Conf.*, Montreal, QC, Canada, Sep. 2016, pp. 1–6.

[25] B. K. Habtemariam, R. Tharmarasa, and T. Kirubarajan, "PHD filter based track-before-detect for MIMO radars," *Signal Process.*, vol. 92, no. 3, pp. 667–678 Jul. 2012.

[26] K. Punithakumar, T. Kirubarajan, and A. Sinha, "A sequential Monte Carlo probability hypothesis density algorithm for multitarget track-before-detect," *Proc. SPIE*, vol. 5913, pp. 59131S-1–59131S-8, Sep. 2005.

[27] H. Tong, H. Zhang, H. Meng, and X. Wang, "A shrinkage probability hypothesis density filter for multitarget tracking," *EURASIP J. Adv. Signal Process.*, vol. 2011, pp. 116–129, Dec. 2011.

[28] D. L. Davies and D. W. Bouldin, "A cluster separation measure," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. PAMI-1, no. 2, pp. 224–227, Apr. 1979.

[29] T. Xiang and S. Gong, "Spectral clustering with eigenvector selection," *Pattern Recognit.*, vol. 41, no. 3, pp. 1012–1029, Mar. 2008.

[30] D. Schuhmacher, B.-T. Vo, and B.-N. Vo, "A consistent metric for performance evaluation of multi-object filters," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3447–3457, Aug. 2008.

**ZHICHAO BAO** was born in 1991. He received the B.S. and M.S. degrees from the Electronic Engineering Institute, Hefei, Anhui, China, in 2014 and 2016, respectively. He is currently pursuing the Ph.D. degree with the National University of Defense Technology, Hefei. His current research interest mainly focuses on radar target tracking.

**QIUXI JIANG** was born in 1960. He graduated from Xidian University, Xi'an, Shanxi, China. He is currently a University Professor and a Doctoral Supervisor with the National University of Defense Technology. From 2010 to 2015, he has authored two books: *Network Radar Countermeasure Systems* (National Defence Industry Press, 2010), *Introduction to Innovative Engineering* (National Defence Industry Press, 2014). His current research interests include signal and data processing, and radar countermeasure technology.

**FANGZHENG LIU** was born in 1983. He received the B.S., M.S., and Ph.D. degrees from the Electronic Engineering Institute, Hefei, Anhui, China, in 2006, 2009, and 2012, respectively. He is currently a Lecturer with the National University of Defense Technology, Hefei. His current research interest mainly focuses on signal and information processing.