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Filtering Design of Switched Systems With Markovian Jump Parameters and Lipschitz Nonlinearity via Fuzzy Approach

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ABSTRACT In this paper, the design of H_∞ filtering is addressed when nonlinear Markovian switching systems are modeled as a class of switched systems based on T-S fuzzy technique. Different from the previous research, the transition probabilities are time-variant and not known exactly in the Markovian switching systems. The addressed plants are represented by the fuzzy switched systems with global Lipschitz nonlinearities and random noises depending on states as well as sensor nonlinearities. And the influences of data packet losses, sensor saturation and output logarithmic quantization are taken into consideration at the same time. The stability of the systems and the desired H_∞ performance are obtained via the fuzzy Lyapunov functional method. The solutions of H_∞ filtering are derived by the application of the cone complementarity linearisation (CCL) procedure. Finally, the validity of the suggested design technique is showed via a simulation example.

INDEX TERMS Hybrid systems, switched systems, Markovian jump systems, T-S fuzzy systems, H_∞ filtering, Lipschitz nonlinearity.

I. INTRODUCTION

In the fields of signal processing [1], [2], for estimating unavailable state variables from noisy measurements [3], H_∞ filtering is considered to be the most important of fashionable methods via certain measured output. When compared with the famous Kalman filtering, it is not essential to consider the statistical characteristic of disturbances, but only the bounded energy of disturbances is supposed. A appropriate design is the main objective of H_∞ filtering, which not only ensures that the minimization of infinite norm of estimation error systems satisfies a provided attenuation level of the disturbance [4], [5], but that the filtering error system [6]–[10] is stable. The above researches on filtering problems present a uniform skeleton frame and theoretical basis for mathematical modeling as well as stability analysis in our work.

As is well known to us, compared with linear systems, in practical systems more complexities and difficulties are produced by severe nonlinearities when the physical plants are analyzed and modeled [11]. It is worth pointing out that many researchers apply the powerful T-S fuzzy method to address nonlinearities when they deal with nonlinear systems to achieve any specific accuracy that users want [12], [13]. In the T-S fuzzy switched systems [14] with stochastic perturbation, the issue in terms of the dissipativity is solved in [15]. Based on a multiple discontinuous Lyapunov function, the tighter bound on mode-dependent average dwell time is presented by the stability condition of the T-S fuzzy switched system in [16]. Moreover, in order to get better performance for T-S fuzzy switched systems, based on the values of membership functions a switched controller of the systems is introduced in [17]. Nevertheless, the complications of the above designs are caused by introducing many relaxation condition, which inevitably brings about a cumbersome computational demand and some conservativeness. It is necessary to point out that factors, such as decision of rule type, selection of rule number, and so on lead to great troubles in achieving the goal that the nonlinear systems are fitted completely by the fuzzy model in practice. The mentioned

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factors always inevitably lead to the vast majority of model errors. The remained nonlinearity which has not be processed by the T-S method in the system, which is the main cause of error formation. And we suppose that the nonlinearity satisfies the condition of global Lipschitz in this paper. Since the perspective of extensive practical application, it is of great significance to further research the analysis and synthesis issue of fuzzy systems with nonlinearities, which partially motivates our works.

On the other hand, dynamic engineering systems may suffer unexpected abrupt variations which are frequently caused by maintenances or failures of the components, environmental disturbances and so on in practical problems. Markovian switching systems, also known as Markovian jump systems (MJSs), have been introduced as powerful and appropriate tool to represent such complex situations [18], [19]. Particularly, on account of the small gain theorem, fuzzy MJSs with time varying delays [20] are concerned in [21], in which the transition probabilities are subject to uncertain dropout rate. For the T-S fuzzy Markovian switching stochastic systems with probabilistic time varying delays, the control problem based on passivity and the one of reliable mixed H_∞ are considered in [22]. It is worth mentioning that in most of the aforementioned studies on MJSs, the considered transition probabilities of MJSs are completely known or time-invariant. However, in general, it is supposed that the jump time is exponentially distributed in MJSs, which restricts the utilizations of MJSs. Meantime, in some sense, the considered transition probabilities in MJSs are constants such that the gained results depending on it are conservative. The networked system that can be considered as MJSs is a typical example, in which packet dropouts [23], signal quantization [24] and sensor nonlinearities [25] are in such system. The fact that in different periods signal quantization and packet dropouts are different causes that the transition rates vary and are uncertain through the whole working region. The above factors leads to time-varying transition probabilities. Different from the MJSs, a time variant matrix of transition probabilities is the greatest feature of nonhomogeneous MJSs (NMJSs). Therefore, in [26], a few novel methods are farther enhanced for NMJSs. It is a bold attempt that the extension of the fuzzy control strategy for NMJSs is bright and reasonable in this paper, which motivates us for the works.

Recently, the problem of filter design with the occurrence of gain variations [27] as well as the quantization [28] have been handled by the fuzzy inference method. The research of filtering for switched systems [15] and MJSs [29] has been extensively developed. For the nonlinear NMJSs, based on mode-dependent, Yin et al. have addressed the $l_2 - l_\infty$ filtering problem by some presented slack matrices via T-S fuzzy method [30]. Based on T-S fuzzy technique when nonlinear MJSs are modeled as a class of switched systems with performance constraint, the problem of reliable filter design is addressed [31]. However, note that the aforementioned studies are obtained under the condition of linear sensors. Besides, in the realistic implementation, remarkable limitations on all-round aspects of sensor performance often appear unfortunately due to external noises, temperatures, sensor aging and so on. And that brings out the nonlinear characteristic of sensors [25] and the distortion of transmitted signals. What's more, the measurements of sensors require new techniques when the signals are sent. In particular, based fuzzy approach the problem of H_∞ filtering for NMJSs with nonlinear sensor has not yet been adequately researched, which stimulates us for this study.

It is essential to estimate the information of real situations in the practical NMJSs for getting the almost truthful signals by the filtering. In general, the proposed NMJSs have much broader applications than the conventional MJSs. The fact is that the conventional conservative MJSs may be considered to be a special case of the NMJSs. In practical engineering systems the phenomenon of nonlinearity is frequently encountered, which is a origin of performance deterioration and instability in some sense. The appearance of nonlinearities which exist in the model or sensors extremely complicates the filter synthesis and the stability analysis of the systems. And then, how to apply the technique to get the filter gains is a key to ensure that the conservativeness is reduced. Based on the above observations, the research of the mentioned problems is of both theoretical and practical significance, and worthy of further research. But as far as the writer knows, in the existing literature, no research has been done on the filter problems for fuzzy NMJSs under the case of model nonlinearity and sensor nonlinearity coexisting.

With those motivations, this paper pays attention to the problems of H_∞ filtering for a kind of nonhomogeneous fuzzy Markovian switching systems in which Lipschitz nonlinearities and sensor nonlinearities exist simultaneously. The main contributions of this paper are as follows.

1. The influences of random packet dropouts, quantization and sensor nonlinearities are taken into consideration simultaneously. In the above circumstances, when the transition probabilities are time-variant and partially known simultaneously in the MJSs, the H_∞ filter problems for fuzzy NMJSs under the case of Lipschitz nonlinearity and sensor nonlinearity coexisting is the core objective of this research.

2. In our work, a key characteristic is that we employ the conception of nonhomogeneous fuzzy MJSs with global Lipschitz nonlinearities. In particular, in the case where the transition probabilities are partially known and time-varying, we study the systems in which Lipschitz nonlinearities and sensor nonlinearities exist simultaneously. The solutions to the H_∞ filtering problem are derived by the application of the CCL procedure.

Notation: In this work, the employed notation is relatively standard. Suppose a complete probability space (Ω, F, Pr) in which Pr, F and Ω denotes the probability measure defined over F, $\sigma-$ algebra of events and the sample space, respectively. The expectation of α is represented by $E\{\alpha\}$. The conditional expectation of α on β is indicated by $E\{\alpha/\beta\}$.

0 and I stand for the zero matrix and the identity matrix with suitable dimensions. The notation $P > 0$ (≥ 0) means that the matrix is positive definite (semi-definite) according to real symmetric structure. The norm of conventional Euclidean is defined by $\|\cdot\|$. In this paper, the suitable dimensions of matrices are assumed.

II. PROBLEM FORMULATION

Firstly, the following discrete-time T-S fuzzy switching systems with Markov jump parameters are considered. In practice, it is impossible for us to get a very large number of rules of the fuzzy model to completely fit the nonlinear system. We make the systems possess Lipschitz nonlinearity and state-dependent disturbances on probability space (Ω, F, Pr).

A. T-S FUZZY MJSS MODEL

The i-th rule of fuzzy MJSs:

Rule i : if $o_1(k)$ is $\overline{\lambda}_{i1}$ and $\cdots$ and $o_\ell(k)$ is $\overline{\lambda}_{i\ell}$ then

$$\begin{cases} x(k+1) = A_i(r_k)x(k) + B_i(r_k)v(k) + f(r_k, x_k) \\ +[E_i(r_k)x(k) + G_i(r_k)v(k)]w(k), \\ y(k) = C_i(r_k)x(k), \\ y_\phi(k) = \phi(y(k)) + D_i(r_k)v(k), \\ z(k) = L_i(r_k)x(k), \end{cases} \tag{1}$$

where $o(k) = [o_1(k) \cdots o_\ell(k)]^T$ are known premise variables. $\overline{\lambda}_{ij}$ is the fuzzy set which is applied with the $i-th$ model rule and $j-th$ premise variable component; $x(k) \in R^{n_x \times 1}$ is the state; $y(k) \in R^{n_y \times 1}$ is the measured output; $\phi(\cdot)$ denotes the nonlinear saturation function; $z(k) \in R^{n_z \times 1}$ is the estimated output; $w(k) \in R^{1\times 1}$ is a standard one-dimensional random process on a probability space (Ω, F, Pr). The $w(k)$ sequence satisfies $E\{w(k)\} = 0$, $E\{w(k)^2\} = 1$, $E\{w(k_1)w(k_2)\} = 0$ for $k_1 \neq k_2$. $A_i(r_k)$, $B_i(r_k)$, $C_i(r_k)$, $D_i(r_k)$, $L_i(r_k)$, $E_i(r_k)$, $G_i(r_k)$ are given with appropriate dimensions system matrices. $(i = 1, 2, \cdots, \lambda)$, the scalar λ is the number of rules. $f(r_k, x_k) \in R^{n_f \times 1}$ is a function of real nonlinear vector, and which satisfies the following condition of global Lipschitz:

$$\begin{cases} \|f(r_k, x_k)\| \leq \|Jx(k)\|, \\ \|f(r_k, x_k) - f(r_k, \hat{x}_k)\| \leq \|J(x(k) - \hat{x}(k))\|, \end{cases} \tag{2}$$

where $x(k)$ and $\hat{x}(k)$ are arbitrary two state vector, and J is given. In system (1), $v(k) \in l_2[0, \infty)$ is external disturbance and $v(k) \in R^{n_v \times 1}$, where $l_2[0, \infty)$ is the space of nonanticipatory square-summable stochastic process $\bar{\varphi}(\cdot) = (\bar{\varphi}(k))_{k\in N}$ on N with respect to $F(k)_{k\in N} \subset F$, and $\bar{\varphi}(\cdot)$ satisfies the following condition

$$\|\bar{\varphi}\|_2^2 = E\left\{\sum_{k=0}^{\infty} \|\bar{\varphi}(k)\|^2\right\} = \sum_{k=0}^{\infty} E\left\{\|\bar{\varphi}(k)\|^2\right\} < \infty. \tag{3}$$

The final fuzzy system is inferred as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^{\lambda} h_i[A_i(r_k)x(k) + B_i(r_k)v(k) + f(r_k, x_k) \\ +[E_i(r_k)x(k) + G_i(r_k)v(k)]w(k)], \\ y(k) = \sum_{i=1}^{\lambda} h_i C_i(r_k)x(k), \\ y_\phi(k) = \sum_{i=1}^{\lambda} h_i[\phi(C_i(r_k)x(k)) + D_i(r_k)v(k)], \\ z(k) = \sum_{i=1}^{\lambda} h_i L_i(r_k)x(k), \end{cases} \tag{4}$$

where consider for all k: $h_i(o(k)) = \bar{\omega}_i(o(k))/\sum_{i=1}^{\lambda} \bar{\omega}_i(o(k))$, $\bar{\omega}_i(o(k)) = \prod_{j=1}^{\ell} \bar{\lambda}_{ij}(o_j(k))$, $\bar{\omega}_i(o(k)) \geq 0$, $\sum_{i=1}^{\lambda} \bar{\omega}_i(o(k)) > 0$, $\sum_{i=1}^{\lambda} h_i(o(k)) = 1$, $h_i(o(k)) \geq 0$, $i = 1, 2, \cdots, \lambda$. In the discussion, we write $h_i = h_i(o(k))$, $x_k = x(k)$, $y_k = y(k)$, $\hat{x}_k = \hat{x}(k)$ and so on for brevity.

$\{r_k, k \geq 0\}$ is defined to denote the nonhomogeneous Markov chain which takes values in the space $\Im = \{1, 2, \cdots, \omega\}$. The matrix of transition probability is represented by $\Lambda(k) = \{\pi_{mn}(k)\}, m, n \in \Im$. From mode m at time k to mode n at time $k+1$, the transition probability is denoted by $\pi_{mn}(k) = \Pr(r_{k+1} = n \,|r_k = m)$, and $\pi_{mn}(k) \geq 0, \forall m, n \in \Im$, $\sum_{n=1}^{\omega} \pi_{mn}(k) = 1$. In the system (4), $\Lambda(k) = \{\pi_{mn}(k)\}$ is the time-variant matrix and it is proposed as a polytope as follows: $P_\Lambda = \{\Lambda(k) = \sum_{\tau=1}^{\kappa} \zeta_\tau(k)\Lambda^{(\tau)}$, $\sum_{\tau=1}^{\kappa} \zeta_\tau(k) = 1$, $0 \leq \zeta_\tau(k) \leq 1\}$, where the vertices of P_Λ are denoted by $\Lambda^{(\tau)}$, $(\tau = 1, 2, \cdots \kappa)$, and κ is the number of the chosen vertices. $\Lambda^{(\tau)}$ includes some partially unknown or uncertain elements. We consider

$$\begin{aligned} \Im_k^m &= \left\{\varphi_t^m(k) = \sum_{\tau=1}^{\kappa} \zeta_\tau(k)\pi_{mn}^{(\tau)} \text{ is know}\right\} \\ \Im_{uk}^m &= \left\{\bar{\varphi}_t^m(k) = \sum_{\tau=1}^{\kappa} \zeta_\tau(k)\pi_{mn}^{(\tau)} \text{ is unknow}\right\} \end{aligned} \tag{5}$$

where $\forall 1 \leq t \leq \omega$, $\pi_{mn}^{(\tau)} \in \Lambda^{(\tau)}$, $\forall m, n \in \Im$.

Remark 1: It is necessary to point out that factors, such as decision of rule type, selection of rule number, and so on lead to great troubles in achieving the goal that the actual nonlinear systems are fitted completely through the fuzzy model in practice. In this paper, the fuzzy systems with nonlinearities are presented.

B. SENSOR SATURATION, QUANTIZATION AND UNRELIABLE COMMUNICATION LINKS

Let $\phi(\cdot) \in \left[K_1 \; K_2\right]$ for some given diagonal matrices $K_1 \geq 0$ and $K_2 \geq 0$ with $K_2 \geq K_1$. $\phi(\cdot)$ satisfies the following condition

$$(\phi(y_k) - K_1 y_k)^T (\phi(y_k) - K_2 y_k) \le 0, \forall y_k \in R^{n_y \times 1}. \quad (6)$$

We decompose $\phi(y_k)$ into a nonlinear part and a linear one

$$\phi(y_k) = \phi_s(y_k) + K_1 y_k, \quad (7)$$

where $\phi_s(y_k) \in \$_s$, the set $\$_s$ is defined as

$$\$_s = \left\{ \phi_s : {\phi_s}^T (y_k)(\phi_s(y_k) - K y_k) \le 0 \right\}, \quad K = K_2 - K_1. \quad (8)$$

From Fig. 1, we can know that the signals are quantized in the environment of network under the unreliable links of communication. Before the signal is conveyed in the digital channel, the output $y_\phi(k)$ are quantized in the circumstances. The system (4) is considered to subordinate to logarithmic quantizer $q_m(\cdot) = q_{r_k}(\cdot)$ which is characterized by

$$q_m(\cdot) = \left[q_m^{(1)}(\cdot)\ q_m^{(2)}(\cdot)\ \ldots\ q_m^{(n_y)}(\cdot) \right]^T, m \in \Im, \quad (9)$$

where $q_m^{(n)}(\cdot)$ is assumed to be symmetric $q_m^{(n)}(y_\phi^{(n)}(k)) = -q_m^{(n)}(-(y_\phi^{(n)}(k)), (n = 1, \ldots, n_y)$. For $m \in \Im$, the set of quantification levels of $q_m^{(n)}(\cdot)$ is represented by $\Upsilon_n = \left\{ \pm\eta_\hbar^{(m,n)} \middle| \eta_\hbar^{(m,n)} = (\rho^{(m,n)})^\hbar \cdot \eta_{(0)}^{(m,n)}, \hbar = \pm 1, \pm 2, \ldots \right\} \cup \left\{ \eta_{(0)}^{(m,n)} \right\} \cup \{0\}, 0 < \rho^{(m,n)} < 1, \left\{ \eta_{(0)}^{(m,n)} \right\} > 0$, where $\rho^{(m,n)}$ denotes the quantizer density of the subquantizer $q_m^{(n)}(\cdot)$ and $\eta_{(0)}^{(m,n)}$ is the initial value for the subquantizer $q_m^{(n)}(\cdot)$. The quantizer $q_m^{(n)}(\cdot)$ is represented as follows:

$$q_m^{(n)}(y_\phi^{(n)}(k)) = \begin{cases} \eta_\hbar^{(m,n)}, & \dfrac{\eta_\hbar^{(m,n)}}{1+\delta^{(m,n)}} < y_\phi^{(n)}(k) < \dfrac{\eta_\hbar^{(m,n)}}{1-\delta^{(m,n)}} \\ 0, & y_\phi^{(n)}(k) = 0 \\ -q_m^{(n)}(-y_\phi^{(n)}(k)), & y_\phi^{(n)}(k) < 0, \end{cases} \quad (10)$$

where $\delta^{(m,n)} = (1 - \rho^{(m,n)})/(1 + \rho^{(m,n)})$ is the parameter of the quantizer. Based on [24], the logarithmic quantizer (10) may be described by

$$q_m(y_\phi(k)) = (I_{n_y} + \Delta_{(m,n_y)}) y_\phi(k), \quad (11)$$

where $\Delta_{(m,n_y)} = diag\left\{ \delta^{(m,1)}, \cdots, \delta^{(m,n_y)} \right\}, 0 < \Delta_{(m,n_y)} < I_{n_y}$.

From Fig. 1, it can be seen that data losses occur randomly. Based on the application of stochastic technique, the phenomenon of data losses is denoted as follows

$$y_f(k) = \alpha(k) q_m(y_\phi(k)) = \alpha(k)(I_{n_y} + \Delta_{(m,n_y)}) y_\phi(k). \quad (12)$$

The $\alpha(k)$ is applied to denote the data dropout where $\alpha(k)$ satisfies Bernoulli random distribution. Consider $\alpha(k)$ as follows

$$\begin{cases} \Pr\{\alpha(k) = 1\} & = E\{\alpha(k)\} = \bar{\alpha}, \\ \Pr\{\alpha(k) = 0\} & = 1 - E\{\alpha(k)\} = 1 - \bar{\alpha}, \\ Var\{\alpha(k)\} = E\left\{ (\alpha(k) - \bar{\alpha})^2 \right\} = (1 - \bar{\alpha})\bar{\alpha} = \alpha_1^2, \end{cases} \quad (13)$$

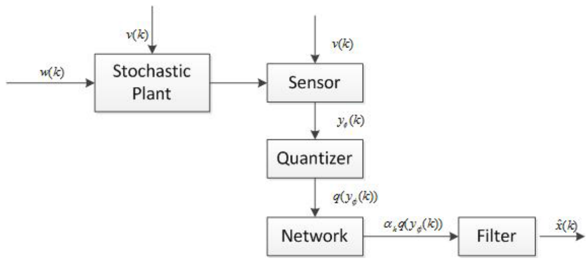

FIGURE 1. Plant flow chart.

where $\bar{\alpha} \in [0, 1]$ is constant. According to the (7), one can obtain

$$y_f(k) = \alpha_k (I_{n_y} + \Delta_{(m,n_y)}) \sum_{i=1}^{\lambda} h_i[(\phi_s(C_i(r_k)x_k) + K_1 C_i(r_k)x_k + D_i(r_k)v_k)]. \quad (14)$$

C. THE PLANT FORM OF FILTER

We apply the output measurements to design the filter for the system (4). The following non-linear filter of full order is selected.

Rule i : if $o_1(k)$ is $\overline{\lambda}_{i1}$ and $\cdots$ and $o_\ell(k)$ is $\overline{\lambda}_{i\ell}$ then

$$\begin{cases} \hat{x}(k+1) = A_{fi}(r_k)\hat{x}(k) + B_{fi}(r_k)y_f(k) + f(r_k, \hat{x}_k), \\ \hat{z}(k) = L_{fi}(r_k)\hat{x}(k), \hat{x}(k) = 0, \end{cases} \quad (15)$$

where $\hat{x}(k) \in R^{n_{\hat{x}} \times 1}$ and $\hat{z}(k) \in R^{n_{\hat{z}} \times 1}$ denotes, respectively, the state of the filter, the estimated output of the filter. $A_{fi}(r_k)$, $B_{fi}(r_k)$, $L_{fi}(r_k)$ are the gains matrices to be determined. Then the fuzzy filter is

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{\lambda} h_i[A_{fi}(r_k)\hat{x}(k) \\ + B_{fi}(r_k)y_f(k) + f(r_k, \hat{x}_k)], \\ \hat{z}(k) = \sum_{i=1}^{\lambda} h_i L_{fi}(r_k)\hat{x}(k), \hat{x}(k) = 0. \end{cases} \quad (16)$$

Define the following error variables $e_x(k) = x(k) - \hat{x}(k)$, $f_e(r_k, e_x) = f(r_k, x_k) - f(r_k, \hat{x}_k)$, $e(k) = \left[x_k^T\ e_x^T(k) \right]^T$, $e_z(k) = z(k) - \hat{z}(k)$. Subtracting (16) from (4), one obtain

$$\begin{cases} e_x(k+1) = \sum_{i=1}^{\lambda} h_i \sum_{j=1}^{\lambda} h_j [A_{fj}(r_k)e_x(k) + [A_i(r_k) \\ - A_{fj}(r_k) - \bar{\alpha}B_{fj}(r_k)(I_{n_y} + \Delta_{(m,n_y)})K_1 C_j(r_k)]x_k \\ - \bar{\alpha}B_{fj}(r_k)(I_{n_y} + \Delta_{(m,n_y)})\phi_s(y_k) \\ + [B_i(r_k) - \bar{\alpha}B_{fj}(r_k)(I_{n_y} + \Delta_{(m,n_y)})D_j(r_k)]v_k \\ + (E_i(r_k)x_k + G_i(r_k)v_k)w(k) - (\alpha_k - \bar{\alpha})B_{fj}(r_k)(I_{n_y} \\ + \Delta_{(m,n_y)})[K_1 C_j(r_k)x_k + \phi_s(y_k) \\ + D_j(r_k)v_k] + f_e(r_k, e_x)]. \end{cases} \quad (17)$$

Combining (17) and (4), the filtering error dynamics system is obtain as follows

$$\begin{cases} e(k+1) = \sum_{i=1}^{\lambda} h_i \sum_{j=1}^{\lambda} h_j\{[\bar{A}_{1ij} - \Delta\bar{A}_{1j}]e(k) - [\bar{H}_{1j} \\ +\Delta\bar{H}_{1j}]\phi_s(y_k) + [\bar{B}_{1ij} - \Delta\bar{B}_{1j}]v_k \\ +(\alpha_k - \bar{\alpha}) \times [(\bar{A}_{2j} - \Delta\bar{A}_{2j})e(k) \\ +(\bar{H}_{2j} + \Delta\bar{H}_{2j})\phi_s(y_k) \\ +(\bar{B}_{2j} + \Delta\bar{B}_{2j})v_k] + (\bar{E}_i e(k) + \bar{G}_i v_k)w_k + F(k, e_k)\}, \\ e_z(k) = \sum_{i=1}^{\lambda} h_i \sum_{j=1}^{\lambda} h_j \bar{L}_{ij} e(k), \end{cases} \tag{18}$$

where

$$\bar{A}_{1ij} = \begin{bmatrix} A_i(r_k) & 0 \\ \bar{\bar{A}}_{ij}(r_k) & A_{fj}(r_k) \end{bmatrix},$$
$$\bar{\bar{A}}_{ij}(r_k) = A_i(r_k) - A_{fj}(r_k) - \bar{\alpha}B_{fj}(r_k)K_1C_j(r_k),$$
$$\bar{A}_{2j} = \begin{bmatrix} 0 & 0 \\ -B_{fj}(r_k)K_1C_j(r_k) & 0 \end{bmatrix},$$
$$\bar{B}_{1ij} = \begin{bmatrix} B_i(r_k) \\ B_i(r_k) - \bar{\alpha}B_{fj}(r_k)D_j(r_k) \end{bmatrix},$$
$$\bar{B}_{2j} = \begin{bmatrix} 0 \\ -B_{fj}(r_k)D_j(r_k) \end{bmatrix},$$
$$\bar{H}_{1j} = \begin{bmatrix} 0 \\ \bar{\alpha}B_{fj}(r_k) \end{bmatrix}, \bar{H}_{2j} = \begin{bmatrix} 0 \\ -B_{fj}(r_k) \end{bmatrix},$$
$$\Delta\bar{A}_{1j} = \begin{bmatrix} 0 & 0 \\ \bar{\alpha}B_{fj}(r_k)\Delta_{(m,n_y)}K_1C_j(r_k) & 0 \end{bmatrix},$$
$$\Delta\bar{A}_{2j} = \begin{bmatrix} 0 & 0 \\ B_{fj}(r_k)\Delta_{(m,n_y)}K_1C_j(r_k) & 0 \end{bmatrix},$$
$$\Delta\bar{B}_{1j} = \begin{bmatrix} 0 \\ \bar{\alpha}B_{fj}(r_k)\Delta_{(m,n_y)}D_j(r_k) \end{bmatrix},$$
$$\Delta\bar{B}_{2j} = \begin{bmatrix} 0 \\ -B_{fj}(r_k)\Delta_{(m,n_y)}D_j(r_k) \end{bmatrix},$$
$$\Delta\bar{H}_{1j} = \begin{bmatrix} 0 \\ \bar{\alpha}B_{fj}(r_k)\Delta_{(m,n_y)} \end{bmatrix},$$
$$\Delta\bar{H}_{2j} = \begin{bmatrix} 0 \\ -B_{fj}(r_k)\Delta_{(m,n_y)} \end{bmatrix},$$
$$\bar{E}_i = \begin{bmatrix} E_i(r_k) & 0 \\ E_i(r_k) & 0 \end{bmatrix}, \quad \bar{G}_i = \begin{bmatrix} G_i(r_k) \\ G_i(r_k) \end{bmatrix},$$
$$\bar{L}_{ij} = [L_i(r_k) - L_{fj}(r_k) \; L_{fj}(r_k)],$$
$$F(k, e_k) = \begin{bmatrix} f(r_k, x_k) \\ f_e(r_k, e_x) \end{bmatrix}.$$

Remark 2: The premise variables and jump modes are considered to be available in the previous design. The filter in (16) is regarded as the mode-dependent one as in [26] and [30]. Generally, compared with the mode-dependent method, as a result of the neglecting of the obtained premise variables and jump modes, more conservativeness are brought out in the mode-independent method.

D. DEFINITION AND LEMMA

Definition 1 ([32]): The system (18) with $v(t) \equiv 0$ is considered to be stochastically stable, if any initial condition $e(0) \in R^n$, $r_0 \in \Im$ holds and there exists a scalar $W > 0$ such that the following expression holds

$$E\left\{\sum_{k=0}^{\infty} ||e(k)||^2 \,|_{e(0),r_0}\right\} < W \cdot E||e(0)||^2. \tag{19}$$

Definition 2 ([31]): For a given constant $\gamma > 0$, under zero initial condition $v(k) \equiv 0$, the system (18) is considered to be stochastically stable with an H_∞ performance γ, then for all nonzero $v(k) \in l_2[0, \infty)$ the following condition holds

$$E\left\{\sum_{k=0}^{\infty} ||e_z(k)||^2\right\} \le \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2. \tag{20}$$

Lemma1 (Yakubovich [33]): Let $T_i \in R^{n_T \times n_T}$, $(i = 0, 1 \cdots p)$ be symmetric matrices. If the following condition on T_i, $(i = 0, 1 \cdots p)$, $\hat{G}_0 = \vartheta^T T_0 \vartheta > 0, \forall \vartheta \neq 0$ $s.t.$ $\hat{G}_i = \vartheta^T T_i \vartheta \ge 0$, $(i = 0, 1 \cdots p)$ holds and there exists scalars $g_i \ge 0$, $(i = 0, 1 \cdots p)$ such that one yield

$$\hat{G}_0 - \sum_{i=1}^{p} g_i \hat{G}_i > 0. \tag{21}$$

Lemma2 ([32]): Suppose M, N and T are real matrices with appropriate dimensions and $T^T T \le I$, then for any scalar $\varepsilon > 0$, one can have

$$MTN + N^T T^T M^T \le \varepsilon^{-1} M M^T + \varepsilon N^T N. \tag{22}$$

Lemma3 ([32]): If the following conditions are founded

$$M_{ii} < 0, \quad i = 1, 2, \cdots \lambda. \tag{23}$$
$$\frac{1}{\lambda - 1} M_{ii} + \frac{1}{2}(M_{il} + M_{li}) < 0, \quad i \neq l, i, l = 1, 2, \cdots \lambda. \tag{24}$$

Then we have the following inequality

$$\sum_{i=1}^{\lambda} \sum_{l=1}^{\lambda} h_i h_l M_{il} < 0. \tag{25}$$

III. MAIN RESULTS

Theorem 1: For a supposed disturbance attenuation level $\gamma > 0$, the system (18) is stochastically stable and the filter gains of the system (15) are solvable if there exists positive definite matrices $P_l(m)$, $m \in \Im$,$(l = 1, \ldots, \lambda)$ and scalars $g_1 > 0$, $g_2 > 0$ to fulfill the following inequalities

$$\hat{A}_{ij}^T (\sum_{n=1}^{\omega} \pi_{mn}{}^{(\tau)} \hat{P}_l(n)) \hat{A}_{ij} + Q_{ij} < 0, \tag{26}$$
$$\Gamma_{ijl\tau}^1(m) = \begin{bmatrix} -\wp_l^{-1} & \aleph_t^m \hat{A}_{ij} \\ * & \prod_t^m Q_{ij} \end{bmatrix} < 0, \quad n \in \Im_k, \tag{27}$$
$$\Gamma_{ijl\tau}^2(m) = \begin{bmatrix} -\hat{P}_l^{-1}(n) & \hat{A}_{ij} \\ * & Q_{ij} \end{bmatrix} < 0, \quad n \in \Im_{uk}, \tag{28}$$

where

$$
\begin{aligned}
-\wp_l^{-1} &= diag\left\{-\hat{P}_l^{-1}(\varphi_1^m)\cdots -\hat{P}_l^{-1}(\varphi_t^m)\right\},\\
\aleph_k^m &= [\sqrt{\pi^{(\tau)}_{m\varphi_1^m}}I \quad \cdots \quad \sqrt{\pi^{(\tau)}_{m\varphi_t^m}}I],\\
\tilde{P}_l(n) &= \sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n),\\
\prod_t^m &= \sum\nolimits_{n\in\Im_k^m}\pi_{mn}{}^{(\tau)},\\
\hat{A}_{ij} &= \begin{bmatrix}\tilde{A}_{1ij} & \tilde{B}_{1ij} & -\tilde{H}_{1j} & I\\ \tilde{A}_{2j} & \tilde{B}_{2j} & \tilde{H}_{2j} & 0\\ \bar{E}_i & \bar{G}_i & 0 & 0\end{bmatrix},\\
\hat{P}_l(n) &= \begin{bmatrix}P_l(n) & 0 & 0\\ 0 & \alpha_1^2P_l(n) & 0\\ 0 & 0 & P_l(n)\end{bmatrix},\\
Q_{ij} &= \begin{bmatrix}-P_i+\bar{g}\bar{J}^T\bar{J}+\bar{L}_{ij}^T\bar{L}_{ij} & 0 & \bar{C}_j^TK^T & 0\\ * & -\gamma^2 & 0 & 0\\ * & * & -2I & 0\\ * & * & * & -\bar{g}I\end{bmatrix},\\
\bar{J} &= diag\left\{J\ J\right\},\\
\bar{g} &= diag\left\{g_1\ g_1\ g_2\ g_2\right\},\\
\tilde{A}_{1ij} &= \bar{A}_{1ij}-\Delta\bar{A}_{1j}, \tilde{A}_{2j}=\bar{A}_{2j}-\Delta\bar{A}_{2j},\\
\tilde{B}_{1ij} &= \bar{B}_{1ij}-\Delta\bar{B}_{1j}, \tilde{B}_{2j}=\bar{B}_{2j}+\Delta\bar{B}_{2j},\\
\tilde{H}_{1j} &= \bar{H}_{1j}+\Delta\bar{H}_{1j}, \tilde{H}_{2j}=\bar{H}_{2j}+\Delta\bar{H}_{2j},\\
\bar{C}_j &= \left[C_j(m) \quad 0\right].
\end{aligned}
$$

Proof 1: Defined $v(k)\equiv 0$, the stochastic stability of the filter error system (18) is investigated. We suppose $r_k = m$ at time instant k and the following fuzzy Lyapunov function is selected for system (18). $V(k,m)=e^T(k)\left[\sum_{i=1}^{\lambda}h_iP_i(m)\right]e(k)$, where $P_i(m)>0$, supposing $h_l^+ = h_l^+(o(k+1))$, $\zeta_\tau(k)=\zeta_\tau$, one can have

$$
\begin{aligned}
&\Delta V(k,r_k)\\
&= E\left\{V(e(k+1),r_{k+1})|e_k,r_k\right\}-V(e(k),r_k)\\
&= [\sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}h_ih_j(\tilde{A}_{1ij}e_k-\tilde{H}_{1j}\phi_s(y_k)+F_k)]^T\\
&\quad\times[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)\\
&\quad\times[\sum_{a=1}^{\lambda}\sum_{o=1}^{\lambda}h_ah_o(\tilde{A}_{1ao}e_k-\tilde{H}_{1o}\phi_s(y_k)+F_k)]\\
&\quad-e_k^T\left[\sum_{i=1}^{\lambda}h_iP_i(m)\right]e_k\\
&\quad+e_k^T[\sum_{i=1}^{\lambda}h_i\bar{E}_i]^T[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)]\\
&\quad\times[\sum_{a=1}^{\lambda}h_a\bar{E}_a]e_k+\bar{\alpha}_1^2[\sum_{j=1}^{\lambda}h_j(\tilde{A}_{2j}e_k+\tilde{H}_{2j}\phi_s(y_k))]^T\\
&\quad\times[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)]\\
&\quad\times[\sum_{o=1}^{\lambda}h_o(\tilde{A}_{2o}e_k+\tilde{H}_{2o}\phi_s(y_k))].
\end{aligned}\tag{29}
$$

Due to the saturation function $\phi_s(y_k)$, from (8) one have $-2\phi_s^T(y_k)\phi_s(y_k)+2\phi_s^T(y_k)Ky_k>0$, namely, $-2\phi_s^T(y_k)\phi_s(y_k)+2\phi_s^T(y_k)K\sum_{j=1}^{\lambda}\bar{C}_je(k)>0$.

From (29), one have

$$
\begin{aligned}
\Delta V &= \sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\sum_{a=1}^{\lambda}\sum_{o=1}^{\lambda}h_ih_jh_ah_o\{[\tilde{A}_{1ij}e_k-\tilde{H}_{1j}\phi_s(y_k)+F_k]^T\\
&\quad\times[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)]\\
&\quad\times[\tilde{A}_{1ao}e_k-\tilde{H}_{1o}\phi_s(y_k)+F_k]\\
&\quad+(\bar{E}_ie_k)^T[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)]\bar{E}_ae_k\\
&\quad+\bar{\alpha}_1^2[\tilde{A}_{2ij}e_k+\tilde{H}_{2j}\phi_s(y_k)]^T\\
&\quad\times[\sum_{l=1}^{\lambda}h_l^+\sum_{\tau=1}^{\kappa}\zeta_\tau\sum_{n=1}^{\omega}\pi_{mn}{}^{(\tau)}P_l(n)]\\
&\quad\times[\tilde{A}_{2ao}e_k+\tilde{H}_{2o}\phi_s(y_k)]\\
&\quad-2\phi_s^T(y_k)\phi_s(y_k)+e^T(k)\bar{C}_j^TK^T\phi_s(y_k)\\
&\quad+\phi_s^T(y_k)K\bar{C}_je(k)\}-e_k^T\left[\sum_{i=1}^{\lambda}h_iP_i(m)\right]e_k\\
&\le \xi^T(k)[\sum_{l=1}^{\lambda}h_l^+\sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\sum_{\tau=1}^{\kappa}h_ih_j\zeta_\tau\Pi]\xi(k),
\end{aligned}\tag{30}
$$

where

$$
\xi(k)=\left[e^T(k),\phi_s^T(y_k),F_k^T\right]^T,
$$

$$
\Pi=\begin{bmatrix}\tilde{A}_{1ij} & -\tilde{H}_{1j} & I\\ \tilde{A}_{2j} & \tilde{H}_{2j} & 0\\ \bar{E}_i & 0 & 0\end{bmatrix}^T\begin{bmatrix}\tilde{P}_l(n) & & \\ & \alpha_1^2\tilde{P}_l(n) & \\ & & \tilde{P}_l(n)\end{bmatrix}\times\begin{bmatrix}\tilde{A}_{1ij} & -\tilde{H}_{1j} & I\\ \tilde{A}_{2j} & \tilde{H}_{2j} & 0\\ \bar{E}_i & 0 & 0\end{bmatrix}+\begin{bmatrix}-P_i & \bar{C}_j^TK^T & 0\\ K\bar{C}_j & -2I & 0\\ 0 & 0 & 0\end{bmatrix}.
$$

From the Lipschitz nonlinearity $F(k,e_k)=\begin{bmatrix}f(k,x_k)\\ f_e(k,e_x)\end{bmatrix}$, on the other side, from (2) one have

$$
f^T(k,x_k)f(k,x_k)-x_k^TJ^TJx_k=\xi_k^T\psi_1\xi_k\le 0,\tag{31}
$$

where $\psi_1=diag\{-J^TJ\ 0\ 0\ I\ 0\}$, and

$$
f_e^T(k,e_x)f_e(k,e_x)-e_x^TJ^TJe_x=\xi_k^T\psi_2\xi_k\le 0,\tag{32}
$$

where $\psi_2 = diag\{0\ -J^TJ\ 0\ 0\ I\}$. In light of Lemma 1, if there exists matrices $P_i(m)$, $P_l(n)$ and scalars $g_1 > 0$, $g_2 > 0$ under the constraints (31)-(32), one have

$$\tilde{\Pi} = \Pi - g_1\psi_1 - g_2\psi_2 < 0. \quad (33)$$

Clearly, (33) is equivalent to $\Delta V(k, r_k) < 0$ and under $v(k) \equiv 0$ the filtering error system (18) is stochastically stable. On the other hand, from (33), based on Schur complement, one can have

$$\begin{bmatrix} \nabla_{11} & \nabla_{12} \\ * & \nabla_{22} \end{bmatrix} < 0, \quad (34)$$

where

$$\nabla_{11} = \begin{bmatrix} -\tilde{P}_l^{-1}(n) & 0 & 0 \\ * & -\frac{1}{\alpha_1^2}\tilde{P}_l^{-1}(n) & 0 \\ * & * & -\tilde{P}_l^{-1}(n) \end{bmatrix},$$

$$\nabla_{12} = \begin{bmatrix} \tilde{A}_{1ij} & -\tilde{H}_{1j} & I \\ \tilde{A}_{2j} & \tilde{H}_{2j} & 0 \\ \bar{E}_i & 0 & 0 \end{bmatrix},$$

$$\nabla_{22} = \begin{bmatrix} -P_i(m) + \bar{g}\bar{J}^T\bar{J} & \bar{C}_j^TK^T & 0 \\ * & -2I & 0 \\ * & * & -\bar{g}I \end{bmatrix}.$$

It is obvious that the condition (26) can imply (34). From (33), one can obtain that there exists a small scalar $\alpha_0 > 0$ such that $\tilde{\Pi} < diag\{-\alpha_0 I\ 0\ 0\}$ holds, which implies $E\{\Delta V(k, r_k)\} < -\alpha_0\|e(k)\|^2$ and $E\left\{\sum_{k=0}^{\infty} ||e(k)||^2\right\} \leq \frac{1}{\alpha_0}E\{V(0)\} \leq W \cdot E\{\|e(0)\|^2\}$. Let $W = \frac{1}{\alpha_0}[(\hat{\lambda}_{\max}(\sum_{i=1}^{\lambda} h_i P_i(m))]$ hold. Therefore, in light of Definition1, the system (18) is stochastically stable. In the following section, the objective is that the H_∞ performance is introduced and for all nonzero $v(k) \in l_{E_2}[0, \infty)$ the filtering system (18) satisfies Definition 2. The index of H_∞ performance is as follows $\hat{J}(k) = E\left\{e_z^T(k)e_z(k)\left|_{\Theta(k)}, r_k\right.\right\} - \gamma^2 v^T(k)v(k) + E\left\{V(k+1)\left|_{\Theta(k)}, r_k\right.\right\} - V(k)$, then, by the same mentioned method we can obtain

$$\hat{J}(k) \leq \Theta^T(k)[\sum_{l=1}^{\lambda} h_l^+ \sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\sum_{\tau=1}^{\kappa} h_i h_j \zeta_\tau \Xi]\Theta(k), \quad (35)$$

where

$$\Theta(k) = \left[e^T(k), v^T(k), \phi_s^T(y_k), F_k^T\right]^T,$$

$$\Xi = \begin{bmatrix} \tilde{A}_{1ij} & \tilde{B}_{1ij} & -\tilde{H}_{1j} & I \\ \tilde{A}_{2j} & \tilde{B}_{2j} & \tilde{H}_{2j} & 0 \\ \bar{E}_i & \bar{G}_i & 0 & 0 \end{bmatrix}^T \times \begin{bmatrix} \tilde{P}_l(n) & 0 & 0 \\ 0 & \alpha_1^2\tilde{P}_l(n) & 0 \\ 0 & 0 & \tilde{P}_l(n) \end{bmatrix} \times \begin{bmatrix} \tilde{A}_{1ij} & \tilde{B}_{1ij} & -\tilde{H}_{1j} & I \\ \tilde{A}_{2j} & \tilde{B}_{2j} & \tilde{H}_{2j} & 0 \\ \bar{E}_i & \bar{G}_i & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -P_i(m) + L_{ij}^TL_{ij} & 0 & \bar{C}_j^TK^T & 0 \\ * & -\gamma^2 & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & 0 \end{bmatrix}.$$

From (31) and (32), in light of Lemma 1, if there exists scalars $g_1 > 0$, $g_2 > 0$, obviously, we have $\Xi < 0$ if and only if the following expression holds:

$$\tilde{\Xi} = \Xi - g_1\tilde{\psi}_1 - g_2\tilde{\psi}_2 < 0, \quad (36)$$

where

$$\tilde{\psi}_1 = diag\{-J^TJ \quad 0 \quad 0 \quad 0 \quad I \quad 0\},$$
$$\tilde{\psi}_2 = diag\{0 \quad -J^TJ \quad 0 \quad 0 \quad 0 \quad I\}.$$

In fact, from Schur complement, the condition $\tilde{\Xi} < 0$ can be equivalent to (26). From (26), one can have

$$\begin{cases} \hat{A}_{ij}^T[\sum_{n=1}^{\omega} \pi_{mn}{}^{(\tau)}\hat{P}_l(n)]\hat{A}_{ij} + Q_{ij} \\ = \hat{A}_{ij}^T[\sum_{n\in\Im_k^m} \pi_{mn}{}^{(\tau)}\hat{P}_l(n)]\hat{A}_{ij} + \sum_{n\in\Im_k^m} \pi_{mn}{}^{(\tau)}Q_{ij} \\ +\hat{A}_{ij}^T[\sum_{n\in\Im_{uk}^m} \pi_{mn}{}^{(\tau)}\hat{P}_l(n)]\hat{A}_{ij} + \sum_{n\in\Im_{uk}^m} \pi_{mn}{}^{(\tau)}Q_{ij}. \end{cases} \quad (37)$$

From Schur complement, for each $n \in \Im_k^m$, pre- and post multiplying by $diag\left\{-\hat{P}_l^{-1}(\varphi_1^m) \cdots -\hat{P}_l^{-1}(\varphi_t^m)\ I\right\}$, one can have

$$\begin{bmatrix} -\hat{P}_l^{-1}(\varphi_1^m) & \cdots & 0 & \sqrt{\pi_{m\varphi_1^m}^{(\tau)}}\hat{A}_{ij} \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & -\hat{P}_l^{-1}(\varphi_t^m) & \sqrt{\pi_{m\varphi_t^m}^{(\tau)}}\hat{A}_{ij} \\ * & * & * & \prod_t^m Q_{ij} \end{bmatrix}, \quad (38)$$

$$\sum_{n\in\Im_{uk}^m} \pi_{mn}{}^{(\tau)}\begin{bmatrix} -\hat{P}_l^{-1}(n) & \hat{A}_{ij} \\ * & Q_{ij} \end{bmatrix}. \quad (39)$$

According to (27)-(28), we can obtain $\hat{J}(k) \leq 0$ and

$$E\left\{\sum_{k=0}^{\infty} ||e_z(k)||^2\right\} \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2.$$

The proof is finished.

Remark 3: Note that on the above proof of Theorem 1, the matrix inequalities are applied to supply conveniences of mathematical derivation. At the same time, it will lead to more conservativeness. One feasible method as in [34] is to present a constant gain matrix in order to decrease the conservativeness. Without loss of generality, in this paper the fuzzy Lyapunov function is utilized to tackle it.

From the Theorem 1, it is difficult to look for the gains of the filter due to the uncertainties. Based on Lemma 2, we can obtain the following theorem.

Theorem 2: For a supposed disturbance attenuation level $\gamma > 0$, the system (18) is stochastically stable and the filter gains $A_{fj}(m)$, $B_{fj}(m)$ and $L_{fj}(m)$ $(j = 1, \ldots, \lambda\ )$ of system (15) are solvable if there exists scalars $\varepsilon_q > 0$, $(q = 0, 1, 2, \cdots, 3t)$ and positive definite matrices $P_l(m)$,

$m \in \Im$, $(l = 1, \ldots, \lambda)$ such that the following inequalities hold:

$$\Gamma^1_{ijl\tau}(m) = \begin{bmatrix} -\wp_j^{-1} & \aleph_k^m \bar{A}_{ij} & \varrho_{13} & 0 \\ * & \varrho_{22} & 0 & \varrho_{24} \\ * & * & \varrho_{33} & 0 \\ * & * & * & \varrho_{44} \end{bmatrix} < 0, \quad n \in \Im_k, \tag{40}$$

$$\Gamma^2_{ijl\tau}(m) = \begin{bmatrix} -\hat{P}_l^{-1}(n) & \bar{A}_{ij} & Y_{13} & 0 \\ * & Y_{22} & 0 & Y_{24} \\ * & * & Y_{33} & 0 \\ * & * & * & Y_{44} \end{bmatrix} < 0, \quad n \in \Im_{uk}, \tag{41}$$

where

$$\bar{A}_{ij} = \begin{bmatrix} \bar{A}_{1ij} & \bar{B}_{1ij} & -\bar{H}_{1j} & I \\ \bar{A}_{2j} & \bar{B}_{2j} & \bar{H}_{2j} & 0 \\ \bar{E}_i & \bar{G}_i & 0 & 0 \end{bmatrix},$$

$$\varrho_{13} = diag\left\{E_0K_j(m)\ E_0K_j(m)\ 0 \cdots E_0K_j(m)\ E_0K_j(m)\ 0\right\},$$

$$\varrho_{22} = \prod_t^m \cdot \begin{bmatrix} \tilde{\varrho} & 0 & \bar{C}_j^T K^T & 0 \\ * & -\gamma^2 & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & -\bar{g}I \end{bmatrix},$$

$$\tilde{\varrho} = -P_i(m) + \bar{g}\bar{J}^T\bar{J} + \frac{\varepsilon_0}{\prod_t^m}\Phi_{4j}^T(m)\Phi_{4j}(m),$$

$$\varrho_{24} = \begin{bmatrix} \sqrt{\frac{\Pi_k^m}{\varepsilon_0}}\bar{L}_{ij}^T(m) & 0 & 0 & 0 \\ \Phi_{5j}^T(m) & 0 & 0 & 0 \\ E_0^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\varrho_{44} = diag\left\{-\varepsilon_0^{-1}I\ -\varepsilon_0^{-1}I\ -\varepsilon_0^{-1}I\ -\varepsilon_0^{-1}I\right\},$$

$$\varepsilon_0 = \bar{\alpha}^2(\varepsilon_1 + \varepsilon_4 + \cdots \varepsilon_{3t-2}) + (\varepsilon_2 + \varepsilon_5 + \cdots \varepsilon_{3t-1}),$$

$$\varrho_{33} = diag\left\{-\varepsilon_1(\pi^{(\tau)}_{m\varphi_1^m})^{-1}I\ -\varepsilon_2(\pi^{(\tau)}_{m\varphi_1^m})^{-1}I\ -I \right.$$
$$\left. \cdots -\varepsilon_{3t-2}(\pi^{(\tau)}_{m\varphi_t^m})^{-1}I\ -\varepsilon_{3t-1}(\pi^{(\tau)}_{m\varphi_t^m})^{-1}I\ -I\right\},$$

$$Y_{13} = diag\left\{E_0K_j(m)\ E_0K_j(m)\ 0\right\},$$

$$Y_{24} = \begin{bmatrix} \sqrt{\frac{1}{\varepsilon'}}\bar{L}_{ij}^T(m) & 0 & 0 & 0 \\ \Phi_{5j}^T(m) & 0 & 0 & 0 \\ E_0^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Y_{33} = diag\left\{-\varepsilon_1 I\ -\varepsilon_2 I\ -I\right\},$$

$$Y_{44} = diag\left\{-\varepsilon'^{-1}I\ -\varepsilon'^{-1}I\ -\varepsilon'^{-1}I\ -\varepsilon'^{-1}I\right\},$$

$$Y_{22} = \begin{bmatrix} \tilde{Y} & 0 & \bar{C}_j^T K^T & 0 \\ * & -\gamma^2 & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & -\bar{g}I \end{bmatrix},$$

$$\tilde{Y} = -P_i(m) + \bar{g}\bar{J}^T\bar{J} + \varepsilon'\Phi_{4j}^T(m)\Phi_{4j}(m),$$

$$\varepsilon' = \bar{\alpha}^2\varepsilon_1 + \varepsilon_2.$$

Proof 2: Let

$$\begin{aligned}
\bar{A}_{1ij} &= \Lambda_i(m) + E_0K_j(m)\Phi_{1j}(m), \\
\bar{A}_{2j} &= E_0K_j(m)\Phi_{2j}(m), \\
\bar{B}_{1ij} &= R_i(m) + E_0K_j(m)\bar{\alpha}\Phi_{3j}(m), \\
\bar{B}_{2j} &= E_0K_j(m)\Phi_{3j}(m), \\
\bar{H}_{1j} &= E_0K_j(m)(\bar{\alpha})E1_0, \\
\bar{H}_{2j} &= E_0K_j(m)(-E1_0), \\
\Delta\bar{A}_{1j} &= E_0K_j(m)\Delta_{(m,n_y)}\bar{\alpha}\Phi_{4j}(m), \\
\Delta\bar{A}_{2j} &= E_0K_j(m)\Delta_{(m,n_y)}\Phi_{4j}(m), \\
\Delta\bar{B}_{1j} &= E_0K_j(m)\Delta_{(m,n_y)}\bar{\alpha}\Phi_{5j}(m), \\
\Delta\bar{B}_{2j} &= E_0K_j(m)\Delta_{(m,n_y)}(-\Phi_{5j}(m)), \\
\Delta\bar{H}_{1j} &= E_0K_j(m)\Delta_{(m,n_y)}\bar{\alpha}E1_0, \\
\Delta\bar{H}_{2j} &= E_0K_j(m)\Delta_{(m,n_y)}(-E1_0),
\end{aligned}$$

$$\Lambda_i(m) = \begin{bmatrix} A_i(m) & 0 \\ A_i(m) & 0 \end{bmatrix}, \quad \Phi_{1j}(m) = \begin{bmatrix} -I & I \\ -\bar{\alpha}K_1C_j(m) & 0 \end{bmatrix},$$

$$\Phi_{2j}(m) = \begin{bmatrix} 0 & 0 \\ -K_1C_j(m) & 0 \end{bmatrix}, \quad E_0 = \begin{bmatrix} 0 \\ I \end{bmatrix}, \; E1_0 = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix},$$

$$\Phi_{3j}(m) = \begin{bmatrix} 0 \\ -D_j(m) \end{bmatrix}, \quad \Phi_{4j}(m) = \begin{bmatrix} 0 & 0 \\ K_1C_j(m) & 0 \end{bmatrix},$$

$$\Phi_{5j}(m) = \begin{bmatrix} 0 \\ D_j(m) \end{bmatrix}, \quad R_i(m) = \begin{bmatrix} B_i(m) \\ B_i(m) \end{bmatrix},$$

$$K_j(m) = \begin{bmatrix} A_{fj}(m) & B_{fj}(m) \end{bmatrix},$$

$$\bar{L}_{ij}(m) = \begin{bmatrix} L_i(m) & 0 \end{bmatrix} + L_{fj}(m)\begin{bmatrix} -I & I \end{bmatrix}.$$

It is seen that $\Gamma^1_{ijl\tau}(m) < 0$ in (27) is equivalent to

$$\Gamma^1_{ijl\tau}(m) = \bar{\Gamma}^1_{ijl\tau}(m) + \Delta\bar{\Gamma}^1_{ijl\tau}(m) < 0, \tag{42}$$

in which

$$\bar{\Gamma}^1_{ijl\tau}(m) = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{22} \end{bmatrix},$$

$$\Upsilon_{11} = diag\left\{\bar{\Upsilon}_{11} \cdots \bar{\Upsilon}_{tt}\right\},$$

$$\bar{\Upsilon}_{11} = diag\left\{-P_l^{-1}(\varphi_1^m)\ -P_l^{-1}(\varphi_1^m)\ -P_l^{-1}(\varphi_1^m)\right\},$$

$$\bar{\Upsilon}_{tt} = diag\left\{-P_l^{-1}(\varphi_t^m)\ -P_l^{-1}(\varphi_t^m)\ -P_l^{-1}(\varphi_t^m)\right\},$$

$$\Upsilon_{12} = \begin{bmatrix} \hat{\Upsilon}_1 \\ \vdots \\ \hat{\Upsilon}_t \end{bmatrix}, \quad \hat{\Upsilon}_1 = \sqrt{\pi^{(\tau)}_{m\varphi_1^m}}\bar{A}_{ij}, \; \hat{\Upsilon}_t = \sqrt{\pi^{(\tau)}_{m\varphi_t^m}}\bar{A}_{ij},$$

$$\Upsilon_{22} = \prod_v^m \cdot \begin{bmatrix} \breve{\Upsilon} & 0 & \bar{C}_j^T K^T & 0 \\ & -\gamma^2 & 0 & 0 \\ & * & -2I & 0 \\ & * & * & -\bar{g}I \end{bmatrix},$$

$$\breve{\Upsilon} = -P_i(m) + \bar{g}\bar{J}^T\bar{J} + L_{ij}^T(m)L_{ij}(m).$$

We rewrite the formula $\Gamma^1_{ijl\tau}(m)$ as follows:

$$\begin{aligned}
\Gamma^1_{ijl\tau}(m) &= \bar{\Gamma}^1_{ijl\tau}(m) + \Delta\bar{\Gamma}^1_{ijl\tau}(m) \\
&= \bar{\Gamma}^1_{ijl\tau}(m) + \Delta\bar{\Gamma}_1(m) + \Delta\bar{\Gamma}_2(m) + \Delta\bar{\Gamma}_3(m) \\
&\quad \cdots + \Delta\bar{\Gamma}_{3t-2}(m) + \Delta\bar{\Gamma}_{3t-1}(m) + \Delta\bar{\Gamma}_{3t}(m).
\end{aligned}$$

Let

$$\begin{aligned}
\eta_1 &= \Big[\sqrt{\pi^{(\tau)}_{m\varphi^m_1}}(E_0K_j(m))^T \quad 0 \quad 0 \\
&\quad \cdots 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\Big]^T, \\
\mu_1 &= \big[0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \\
&\quad -\bar{\alpha}\Phi_{4j}(m) \quad -\bar{\alpha}\Phi_{5j}(m) \quad -\bar{\alpha}E_0 \quad 0\big], \\
\Delta\bar{\Gamma}_1(m) &= \eta_1\Delta_{(m,n_y)}\mu_1 + {\mu_1}^T{\Delta^T}_{(m,n_y)}{\eta_1}^T, \\
\eta_2 &= \Big[0 \quad \sqrt{\pi^{(\tau)}_{m\varphi^m_1}}(E_0K_j(m))^T \quad 0 \\
&\quad \cdots 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\Big]^T, \\
\mu_2 &= \big[0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \\
&\quad -\Phi_{4j}(m) \quad -\Phi_{5j}(m) \quad -E_0 \quad 0\big], \\
\Delta\bar{\Gamma}_2(m) &= \eta_2\Delta_{(m,n_y)}\mu_2 + {\mu_2}^T{\Delta^T}_{(m,n_y)}{\eta_2}^T, \\
\Delta\hat{\Gamma}_3(m) &= 0, \\
&\vdots \\
\eta_{3t-2} &= \Big[0 \quad 0 \quad 0 \quad \cdots \\
&\quad \sqrt{\pi^{(\tau)}_{m\varphi^m_1}}(E_0K_j(m))^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\Big]^T, \\
\mu_{3t-2} &= \big[0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \\
&\quad -\bar{\alpha}\Phi_{4j}(m) \quad -\bar{\alpha}\Phi_{5j}(m) \quad -\bar{\alpha}E_0 \quad 0\big], \\
\Delta\bar{\Gamma}_{3t-2}(m) &= \eta_{3t-2}\Delta_{(m,n_y)}\mu_{3t-2} + {\mu_{3t-2}}^T{\Delta^T}_{(m,n_y)}{\eta_{3t-2}}^T, \\
\eta_{3t-1} &= \Big[0 \quad 0 \quad 0 \\
&\quad \cdots 0 \quad \sqrt{\pi^{(\tau)}_{m\varphi^m_1}}(E_0K_j(m))^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\Big]^T, \\
\mu_{3t-1} &= \big[0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0 \quad 0 \\
&\quad -\Phi_{4j}(m) \quad -\Phi_{5j}(m) \quad -E_0 \quad 0\big], \\
\Delta\bar{\Gamma}_{3t-1}(m) &= \eta_{3t-1}\Delta_{(m,n_y)}\mu_{3t-1} + {\mu_{3t-1}}^T{\Delta^T}_{(m,n_y)}{\eta_{3t-1}}^T, \\
\Delta\bar{\Gamma}_{3t}(m) &= 0, \\
&\vdots
\end{aligned}$$

Then, using Lemma 2, we have

$$\begin{aligned}
&\bar{\Gamma}^1_{ijl\tau}(m) + \varepsilon_1{\mu_1}^T\mu_1 + {\varepsilon_1}^{-1}\eta_1{\eta_1}^T \\
&\quad + \varepsilon_2{\mu_2}^T\mu_2 + {\varepsilon_2}^{-1}\eta_2{\eta_2}^T + \cdots \\
&\quad + \varepsilon_{3v-2}{\mu_{3v-2}}^T\mu_{3v-2} + {\varepsilon_{3v-2}}^{-1}\eta_{3v-2}{\eta_{3v-2}}^T \\
&\quad + \varepsilon_{3v-1}{\mu_{3v-1}}^T\mu_{3v-1} + {\varepsilon_{3v-1}}^{-1}\eta_{3v-1}{\eta_{3v-1}}^T,
\end{aligned}$$

which yields

$$\Gamma^1_{ijl\tau}(m) < \bar{\Gamma}^1_{ijl\tau}(m) + Z < 0, \tag{43}$$

where

$$\begin{aligned}
Z &= \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix}, \\
Z_1 &= diag\Big\{\varepsilon_1^{-1}(\pi^{(\tau)}_{m\varphi^m_1})(E_0K_j(m))(E_0K_j(m))^T \\
&\quad \times \varepsilon_2^{-1}(\pi^{(\tau)}_{m\varphi^m_1})(E_0K_j(m))(E_0K_j(m))^T \; 0 \\
&\quad \cdots \varepsilon_{3v-2}^{-1}(\pi^{(\tau)}_{m\varphi^m_t})(E_0K_j(m))(E_0K_j(m))^T \\
&\quad \times \varepsilon_{3v-2}^{-1}(\pi^{(\tau)}_{m\varphi^m_t})(E_0K_j(m))(E_0K_j(m))^T \; 0\Big\},
\end{aligned}$$

$$Z_2 = \varepsilon_0 \begin{bmatrix} \Phi^T_{4j}(m) \\ \Phi^T_{5j}(m) \\ E^T_0 \\ 0 \end{bmatrix} \big[\Phi_{4j}(m) \quad \Phi_{5j}(m) \quad E_0 \quad 0\big].$$

In order to get the gain $L_{fj}(m)$, the position of $\varepsilon_0\Phi^T_4(m)\Phi_4(m)$ and the one of $\Pi^m_k\bar{L}^T_{ij}(m)\bar{L}_{ij}(m)$ are reversed. We can obtain (27) by Schur complement for each $n \in \Im^m_k$, and in the same way one have (28) for each $n \in \Im^m_{uk}$. The proof is finished.

From the Theorem 2, it is difficult to find the gains of the filter due to the conservativeness. We use the CCL algorithm to tackle it. In terms of Lemma 3, one can have the following theorem.

Theorem 3: For a supposed disturbance attenuation level $\gamma > 0$, the system (18) is stochastically stable and the filter gains $A_{fj}(m)$, $B_{fj}(m)$ and $L_{fj}(m)$ $(j = 1, \ldots, \lambda)$ of system (15) are solvable if there exists scalars $\varepsilon_q > 0$, $(q = 0, 1, 2, \cdots, 3t)$ and positive definite matrices $P_l(m)$, $L_l(m)$, $m \in \Im$, $(l = 1, \ldots, \lambda)$ such that the following inequalities hold:

$$\Gamma^1_{iil\tau}(m) < 0, \quad n \in \Im_k, \tag{44}$$

$$\frac{1}{\lambda-1}\Gamma^1_{iil\tau}(m) + \frac{1}{2}(\Gamma^1_{ijl\tau}(m) + \Gamma^1_{jil\tau}(m)) < 0, \quad i \neq j \tag{45}$$

$$\Gamma^2_{iil\tau}(m) < 0, \quad n \in \Im_{uk}, \tag{46}$$

$$\frac{1}{\lambda-1}\Gamma^2_{iil\tau}(m) + \frac{1}{2}(\Gamma^2_{ijl\tau}(m) + \Gamma^2_{jil\tau}(m)) < 0, \quad i \neq j \tag{47}$$

$$P_l(m)L_l(m) = I. \tag{48}$$

Proof 3: In terms of Lemma 3, if the matrix inequalities (44)-(48) hold. Then one can have the following inequality

$$\sum_{l=1}^{\lambda} h^+_l \sum_{i=1}^{\lambda}\sum_{j=1}^{\lambda}\sum_{\tau=1}^{\kappa} h_ih_j\zeta_\tau(\Gamma^1_{ijl\tau}(m) + \Gamma^2_{ijl\tau}(m)) < 0. \tag{49}$$

The proof is completed.

We introduce the basic notion of the CCL algorithm. If $\bar{n}$ dimensional $P_l(m) > 0$, $L_l(m) > 0$, $m \in \Im$, $(l = 1, \ldots, \lambda)$ are solutions for the condition of LMI:

$$\begin{bmatrix} P_l(m) & I \\ I & L_l(m) \end{bmatrix} \geq 0, \quad \forall m \in \Im, \tag{50}$$

then, $tr(\sum_m P_l(m)L_l(m)) \geq \bar{n}$, furthermore, if and only if $P_l(m)L_l(m) = I$,

$$tr(\sum\nolimits_m P_l(m)L_l(m)) = \bar{n}. \tag{51}$$

In this paper, the quantized H_∞ filter design problem is as follows:

$$mintr(\sum\nolimits_{l,m} P_l(m)L_l(m)), \tag{52}$$

subject to (44)-(47) and (50). Then the conclusions in Theorem 3 are handled if there exists solutions based on $mintr(\sum_{l,m} P_l(m)L_l(m)) = \lambda\bar{n}$ subjecting to (44)-(47) and (51). The algorithm in Table 1 is proposed to solve the above problem by us.

TABLE 1. Filter design algorithm.

Programme Algorithm
Step 1: Set $k=0$. Seek a feasible set $(P_l{}^0(m), L_l{}^0(m), A^0_{fi}(m), B^0_{fi}(m), L^0_{fi}(m))$ to satisfy (44)-(47) and (50).
Step 2: Solve the following issue $\min tr(\sum_{l,m}(P_l(m)L_l^k(m)+P_l^k(m)L_l(m)))$ s.t. (44)-(47) and (50).
Step 3: The achieved variables $(P_l(m), L_l(m), A_{fi}(m), B_{fi}(m), L_{fi}(m))$ are substituted into the inequality (40) and (41). If the inequality (40) and (41) are hold, with $\left\|tr(\sum_{l,m}P_l(m)L_l(m))-\lambda\bar{n}\right\|<\bar{\delta}$ for any sufficiently small scalar $\bar{\delta}>0$, then obtain the feasible solutions $(P_l(m), L_l(m), A_{fi}(m), B_{fi}(m), L_{fi}(m))$. EXIT.
Step 4: If $k>\bar{N}$, where $\bar{N}$ is the allowed maximum number of iterations , EXIT.
Step 5: Set $k=k+1$, $(P_l{}^k(m), L_l{}^k(m), A^k_{fi}(m), B^k_{fi}(m), L^k_{fi}(m))=(P_l(m), L_l(m), A_{fi}(m), B_{fi}(m), L_{fi}(m))$ and go to Step 2.

Remark 4: Based on convex optimization algorithm, it is too hard to get a minimum because of nonlinear matrix inequality (48). However, by the application of the CCL algorithm [26], the sequential optimization problem formulated from the non-convex feasibility problem (48) can be solved. In the previous algorithm, note that, an iteration method is applied to tackle the minimization problem.

IV. ILLUSTRATIVE EXAMPLE

In the section, an example of a Pulse-Width-Modulation (PWM)-driven boost converter is applied to illustrate the validity of the proposed design method of filter. Considering the PWM-driven boost converter as described in [35], in each switched period T the $s(t)$ switches once at most and it is controlled by the PWM device. $e_s(t)$ is the source voltage, R is the load resistance, C is the capacitance and L is the inductance. This kind of power converter can be modeled as switched system in recent years. The problems of their corresponding stabilization have been researched too. By introducing variables $\tilde{\tau}={}^{t}/_{T}$, $L_1={}^{L}/_{T}$, $C_1={}^{C}/_{T}$, and $e_c(t)\le\widehat{E}$. The differential equations of the model are as follows:

$$\begin{cases}\dot{i}_L(\tilde{\tau})=-(1-s(\tilde{\tau}))\dfrac{1}{L_1}e_c(\tilde{\tau})+s(\tilde{\tau})\dfrac{1}{L_1}e_s(\tilde{\tau})\\ \dot{e}_c(\tilde{\tau})=-\dfrac{1}{RC_1}e_c(\tilde{\tau})+(1-s(\tilde{\tau}))\dfrac{1}{C_1}i_L(\tilde{\tau})\end{cases}\tag{53}$$

Then, from (53) we can obtain that three modes are in the continuous-time MJSs,

$$\dot{x}=A(r_{\tilde{\tau}})x+f(r_{\tilde{\tau}},x_{\tilde{\tau}}),\ r_{\tilde{\tau}}\in\{1,2,3\}\tag{54}$$

where,

$$x=[i_L,e_c]^T,$$
$$A\{1\}=\begin{bmatrix}0&0\\0&-{}^{T}/_{RC}\end{bmatrix},\quad f(1,x_{\tilde{\tau}})=\begin{bmatrix}{}^{T\widehat{E}}/_{L}\\0\end{bmatrix},$$
$$A\{2\}=\begin{bmatrix}0&{}^{T}/_{L}\\-{}^{T}/_{C}&-{}^{T}/_{RC}\end{bmatrix},\quad f(2,x_{\tilde{\tau}})=0,$$
$$A\{3\}=\begin{bmatrix}0&{}^{T}/_{2L}\\-{}^{T}/_{2C}&-{}^{T}/_{RC}\end{bmatrix},\quad f(3,x_{\tilde{\tau}})=\begin{bmatrix}{}^{T\widehat{E}}/_{2L}\\0\end{bmatrix}.$$

And as in [33] and [35],we assume the control matrices to be $B_1=B_2=B_3=\begin{bmatrix}0.5&0\end{bmatrix}^T$, and set the sampling time to be $T_s=0.5$, $C=1$, $L=1$, $R=1$, $\widehat{E}=1$. Respectively, T taking values 1 and 0.5, by the normalization and discretization technique the matrixes of parameters for the system (1) are listed as follows:

Plant Rule 1 : if $x_1(k)$ is $h_1(x_1(k))$ then

$$\begin{cases}x(k+1)=A_1(r_k)x(k)+B_1(r_k)v(k)+f(r_k,x_k)\\ +[E_1(r_k)x(k)+G_1(r_k)v(k)]w(k),\\ y(k)=C_1(r_k)x(k),\\ y_\phi(k)=\phi(y(k))+D_1(r_k)v(k),\\ z(k)=L_1(r_k)x(k),\end{cases}\tag{55}$$

Plant Rule 2 : if $x_1(k)$ is $h_2(x_1(k))$ then

$$\begin{cases}x(k+1)=A_2(r_k)x(k)+B_2(r_k)v(k)+f(r_k,x_k)\\ +[E_2(r_k)x(k)+G_2(r_k)v(k)]w(k),\\ y(k)=C_2(r_k)x(k),\\ y_\phi(k)=\phi(y(k))+D_2(r_k)v(k),\\ z(k)=L_2(r_k)x(k),\end{cases}\tag{56}$$

Remark 5: Shown as stated in [35], the considered boost converter is with only two nominal switched modes. We give a dummy couple $A\{3\}, f(3,x_{\tilde{\tau}})$ such that the states are steered to the selected initial values $x(0)=[2,-1]^T$. The couple $A\{3\}$, $f(3,x_{\tilde{\tau}})$ can be obtained by solving on $A(r_{\tilde{\tau}})x(0)+f(r_{\tilde{\tau}},x_{\tilde{\tau}})=0$. The obtained dummy couple has also a physical interpretation in this case: the states approximate the sliding surface of the system with infinite switched rate in the neighborhood of its initial values $x(0)$.

Mode1:

$$A_1\{1\}=\begin{bmatrix}1.0000&0\\0&0.6065\end{bmatrix},\quad B_1\{1\}=\begin{bmatrix}0.2500\\0.0\end{bmatrix},$$
$$A_2\{1\}=\begin{bmatrix}1.0000&0\\0&0.7788\end{bmatrix},\quad B_2\{1\}=\begin{bmatrix}0.2500\\0.00\end{bmatrix},$$
$$E_1\{1\}=\begin{bmatrix}0.5&0.1\\-0.1&-0.1\end{bmatrix},\quad E_2\{1\}=\begin{bmatrix}0.4&-0.2\\-0.1&-0.2\end{bmatrix},$$
$$L_1\{1\}=\begin{bmatrix}0.04&-0.07\\0.05&0.01\\0.06&-0.03\end{bmatrix},$$

$$L_2\{1\} = \begin{bmatrix} 0.01 & -0.02 \\ 0.03 & -0.02 \\ 0.01 & -0.04 \end{bmatrix},$$
$$C_1\{1\} = \begin{bmatrix} -0.34 & -0.5 \end{bmatrix}, \quad C_2\{1\} = \begin{bmatrix} -0.047 & -0.23 \end{bmatrix},$$
$$D_1\{1\} = 0.1, \quad D_2\{1\} = 0.23,$$
$$G_1\{1\} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \quad G_2\{1\} = \begin{bmatrix} 0.05 \\ -0.2 \end{bmatrix}.$$

Mode2:

$$A_1\{2\} = \begin{bmatrix} 0.8956 & 0.3773 \\ -0.3773 & 0.5182 \end{bmatrix}, \quad B_1\{2\} = \begin{bmatrix} 0.2409 \\ -0.0522 \end{bmatrix},$$
$$A_2\{2\} = \begin{bmatrix} 0.9713 & 0.2189 \\ -0.2189 & 0.7524 \end{bmatrix}, \quad B_2\{2\} = \begin{bmatrix} 0.2476 \\ -0.0287 \end{bmatrix},$$

Mode3:

$$A_1\{3\} = \begin{bmatrix} 0.9735 & 0.1947 \\ -0.1947 & 0.5841 \end{bmatrix}, \quad B_1\{3\} = \begin{bmatrix} 0.2477 \\ -0.0265 \end{bmatrix},$$
$$A_2\{3\} = \begin{bmatrix} 0.9928 & 0.1103 \\ -0.1103 & 0.7722 \end{bmatrix}, \quad B_2\{3\} = \begin{bmatrix} 0.2494 \\ -0.0144 \end{bmatrix},$$

Else

$$E_1\{1\} = E_1\{2\} = E_1\{3\}, \quad E_2\{1\} = E_2\{2\} = E_2\{3\},$$
$$C_1\{1\} = C_1\{2\} = C_1\{3\}, \quad C_2\{1\} = C_2\{2\} = C_2\{3\},$$
$$D_1\{1\} = D_1\{2\} = D_1\{3\}, \quad D_2\{1\} = D_2\{2\} = D_1\{3\},$$
$$G_1\{1\} = G_1\{2\} = G_1\{3\}, \quad G_2\{1\} = G_2\{2\} = G_2\{3\},$$
$$L_1\{1\} = L_1\{2\} = L_1\{3\}, \quad L_2\{1\} = L_2\{2\} = L_2\{3\},$$
$$J = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Membership functions for rule 1, 2 are given as follows:

$$h_1(x_1(k)) = \begin{cases} 1 & x_1(k) \le -1 \\ 0.5 - 0.5x_1(k) & -1 \le x_1(k) \le 1 \\ 0 & \text{else}, \end{cases}$$
$$h_2(x_1(k)) = 1\text{-}h_1(x_1(k)).$$

In order to obtain the stochastically stable closed-loop system (18) with H_∞ performance attenuation level, the design of the filter gain is our goal in (15). Firstly, we propose the design of filter. As opened up before our eyes in Fig. 1, the signals of the filter and ones of the model are always projected into piecewise-constant signals before transmissions. The logarithmic quantizer (11) makes the signal $y(k)$ quantize. Then, the relationship between matrices $P_l(m)$, $L_l(m)$, $m \in \Im$, $(l = 1, \ldots, \lambda)$ and $A_{fj}(m)$, $B_{fj}(m)$, $L_{fj}(m)$ $(j = 1, \ldots, \lambda)$ is explored by the application of the algorithm which is in terms of the CCL procedure. The flow chart in Fig. 2. is achieved such that the experimental section starts with these details. $\rho^{(1,1)} = 0.6667$, $\rho^{(1,2)} = 0.7391$, $\rho^{(1,3)} = 0.6$ and $\eta^{(1,1)}_{(0)} = \eta^{(1,2)}_{(0)} = \eta^{(1,3)}_{(0)} = 0.0001$ are the selected quantizer densities. It can be calculated that $\delta^{(1,1)} = 0.4$, $\delta^{(1,2)} = 0.5$ and $\delta^{(1,3)} = 0.25$ hold. We apply the CCL algorithm and the LMIs of theorem 3 when $\bar{\alpha} = 0.8$,

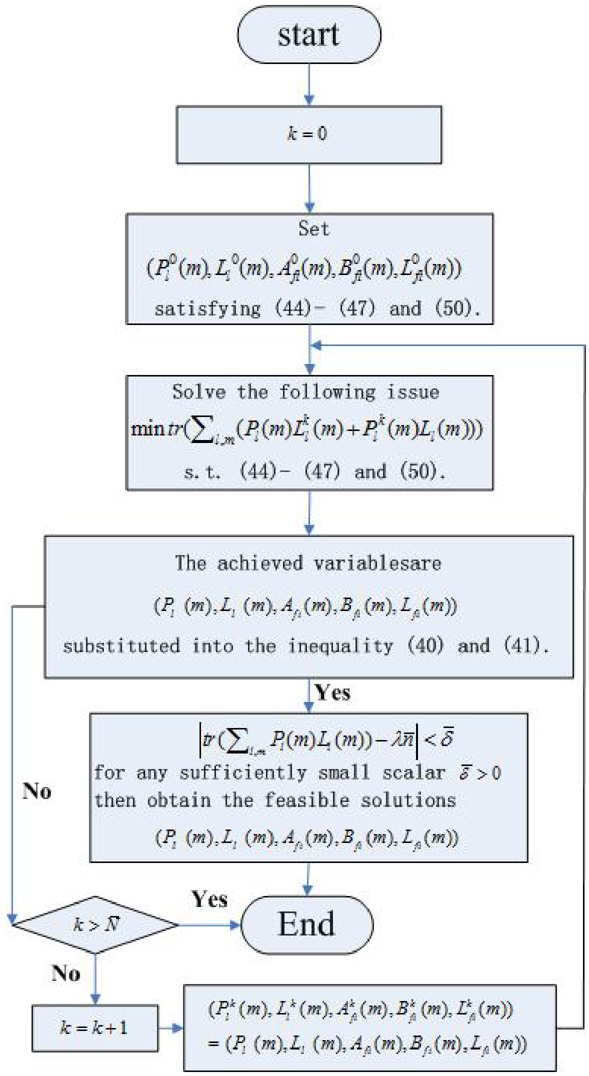

FIGURE 2. Algorithm flowchart.

$K_1 = 0.6$, $K_2 = 0.8$, $g_1 = 0.05$, $g_2 = 0.08$, $\varepsilon_1 = 0.002$, $\varepsilon_2 = 0.001$. The gains of filter are listed below

Filter1:

$$A_{f1}\{1\} = \begin{bmatrix} 0.0023 & 0.0004 \\ -0.0055 & 0.0108 \end{bmatrix},$$
$$A_{f2}\{1\} = \begin{bmatrix} 0.0025 & 0.0004 \\ -0.0016 & 0.0200 \end{bmatrix},$$
$$B_{f1}\{1\} = 1.0 * e^{-3} * \begin{bmatrix} -0.0017 \\ -0.0195 \end{bmatrix},$$
$$B_{f2}\{1\} = \begin{bmatrix} -0.0002 \\ -0.0072 \end{bmatrix},$$
$$L_{f1}\{1\} = \begin{bmatrix} 0.0241 & -0.1210 \\ 0.0310 & 0.0293 \\ 0.0367 & -0.0441 \end{bmatrix},$$
$$L_{f2}\{1\} = \begin{bmatrix} 0.0046 & -0.0480 \\ 0.0167 & -0.0513 \\ 0.0027 & -0.0989 \end{bmatrix}.$$

Filter2:

$$A_{f1}\{2\} = \begin{bmatrix} 0.0002 & -0.0001 \\ -0.0113 & 0.0096 \end{bmatrix},$$
$$A_{f2}\{2\} = \begin{bmatrix} 0.0009 & -0.0014 \\ -0.0178 & 0.0276 \end{bmatrix},$$
$$B_{f1}\{2\} = 1.0 * e^{-3} * \begin{bmatrix} 0.0192 \\ -0.8267 \end{bmatrix},$$
$$B_{f2}\{2\} = 1.0 * e^{-3} * \begin{bmatrix} -0.0016 \\ -0.3539 \end{bmatrix},$$
$$L_{f1}\{2\} = \begin{bmatrix} 0.1032 & 0.0124 \\ 0.0474 & 0.0238 \\ 0.0919 & 0.0244 \end{bmatrix},$$
$$L_{f2}\{2\} = \begin{bmatrix} 0.0419 & 0.0066 \\ 0.0672 & 0.0065 \\ 0.0695 & 0.0121 \end{bmatrix}.$$

Filter3:

$$A_{f1}\{3\} = \begin{bmatrix} 0.0002 & -0.0006 \\ 0.0113 & -0.0124 \end{bmatrix},$$
$$A_{f2}\{3\} = \begin{bmatrix} -0.0004 & 0.0007 \\ 0.0134 & -0.0304 \end{bmatrix},$$
$$B_{f1}\{3\} = \begin{bmatrix} 0.0010 \\ 0.0134 \end{bmatrix},$$
$$B_{f2}\{3\} = \begin{bmatrix} 0.0003 \\ 0.0050 \end{bmatrix},$$
$$L_{f1}\{3\} = \begin{bmatrix} 0.0152 & 0.0253 \\ 0.0328 & 0.0024 \\ 0.0343 & 0.0168 \end{bmatrix},$$
$$L_{f2}\{3\} = \begin{bmatrix} -0.0119 & 0.0079 \\ -0.0057 & 0.0221 \\ -0.0232 & 0.0130 \end{bmatrix}.$$

Let $\zeta_\tau(k) = h_i(x_1(k))$. The matrices of transition probabilities are listed as follows

$$\Lambda^{(1)} = \begin{bmatrix} 0.3 & ? & ? \\ ? & 0.75 & ? \\ ? & ? & 0.2 \end{bmatrix}, \quad \Lambda^{(2)} = \begin{bmatrix} 0.4 & ? & ? \\ ? & ? & 0.45 \\ ? & 0.6 & ? \end{bmatrix},$$
$$\Lambda^{(3)} = \begin{bmatrix} ? & ? & 0.15 \\ ? & 0.75 & ? \\ ? & ? & 0.55 \end{bmatrix}, \quad \Lambda^{(4)} = \begin{bmatrix} 0.3 & ? & ? \\ ? & ? & 0.45 \\ ? & 0.6 & ? \end{bmatrix},$$

where, ? represents the unknown element.

Fig. 3 shows that the missing of random data packet is described. The external disturbance is given as $v(k) = 1/(1+0.05 * k^2)$. The sensor nonlinearities are $\phi(y_k) = (K_1 + K_2)(y_k)/2 + (K_2 - K_1)sin(y_k)/2$. Moreover, we assume that $x(0) = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} -2 & 1 \end{bmatrix}^T$, $z(0) = \begin{bmatrix} -0.5 & 1 & 0 \end{bmatrix}^T$ and $\hat{z}(0) = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix}^T$ are the initial value of the model state, the initial value of the filter state, the initial value of the estimated output and the initial value of the filter estimated output, respectively. Under the switching signal, the output $q_m(y_\phi(k))$ of the quantized signals, the saturation signals $y_\phi(k)$ and the output $y(k)$ of the model are displayed in Fig. 4.

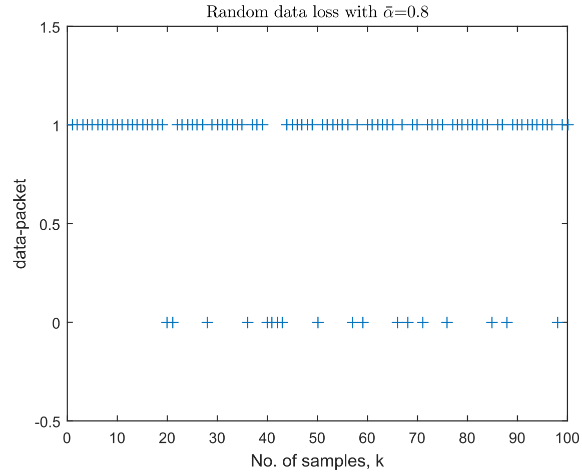

FIGURE 3. The missing of random data packet.

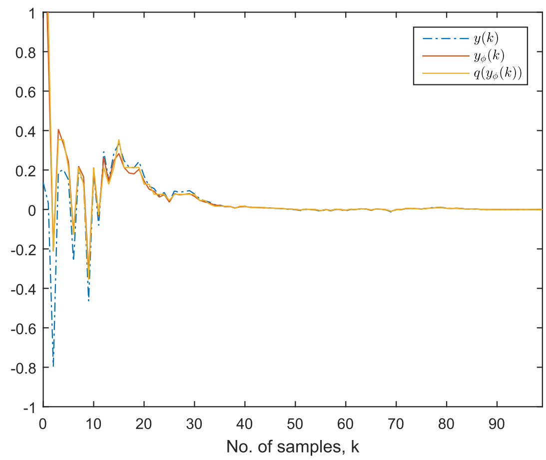

FIGURE 4. The output $y(k)$, saturation and quantized signals.

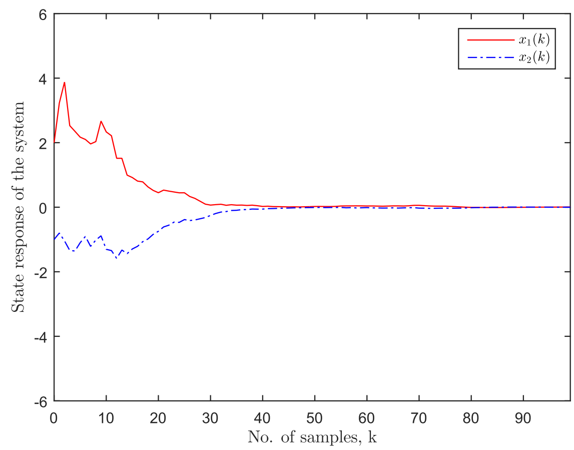

FIGURE 5. The state $x_1(k)$ and $x_2(k)$.

It may be known that based on the mode jumping sequence the mode-dependent quantization is revised well. It can be seen from Fig. 5, Fig. 6 that the states $x(k)$ of the plant may be estimated by the filter states $\hat{x}(k)$, which further indicates the merits of the presented control approach. Under traditional filtering schemes [36], the state variables $x(k)$ are

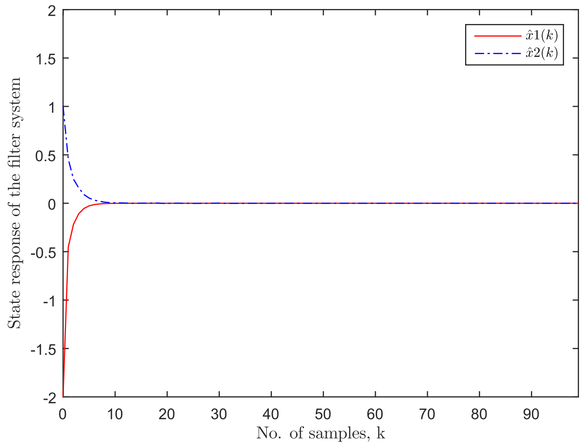

FIGURE 6. The filter state $\hat{x}_1(k)$ and $\hat{x}_2(k)$.

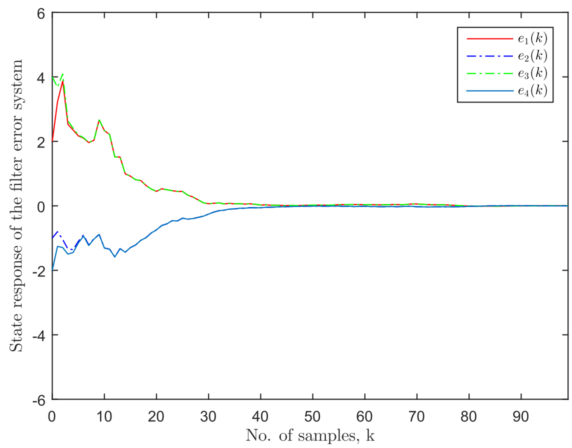

FIGURE 7. The state signals of the error system.

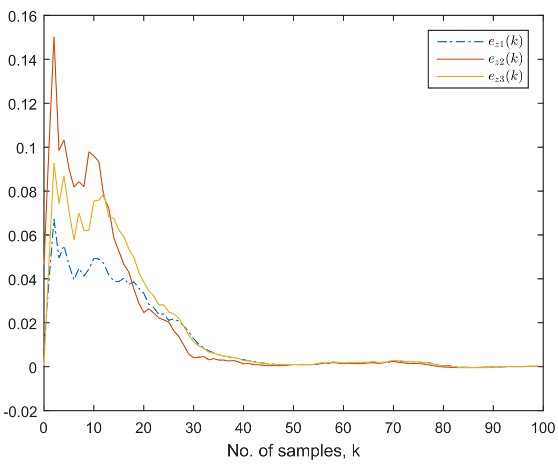

FIGURE 8. Error estimation.

shown in Figs. 9. It is obvious that the trajectories of $x(k)$ have not converged to zero in a short period of time, which represents the traditional filtering is not effective. Finally, the state responses of the filtering error dynamics system with H_∞ performance are presented in Fig. 7. The output errors between the estimated outputs of the plant and the ones of the filter are expressed in Fig. 8. From Fig. 8, it can be found that the amplitudes of the output errors are smaller and denser,

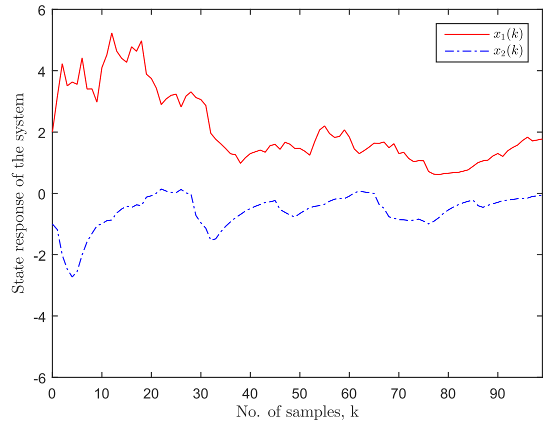

FIGURE 9. The state signals of the traditional scheme.

which further means that a better performance exists in the error system.

Remark 6: Also, we employ the traditional filtering method such as those in [36] to solve the filtering problem for Markovian jump systems, where the transition probabilities which are time variant and partly unknown has not been considered there. The designed filter is not effective. A limitation of our method is that computation time is long. Further investigation which involves the tuning of the parameters and speeding up the calculations is to be done. Despite these difficulties, it is proved that the obtained solution for the considered application is efficient.

V. CONCLUSION

In this paper, we have dealt with the H_∞ filter problems for the nonhomogeneous MJSs with global Lipschitz nonlinearities and random noises depending on states. Particularly in a network environment, the transition probabilities described as convex polyhedron are time variant and partly unknown. From the model to filter we considered simultaneously the effects of sensor nonlinearities, data packet dropouts and signal quantization. The required H_∞ performance and a sufficient condition of stochastic stability for the system are presented by a fuzzy Lyapunov function. In order to obtain the solutions of the filter gains, the application of the CCL procedure is efficient. The effectiveness of the suggested control schemes is illustrated by a simulation example.

DECLARATION

The authors declare that there is no conflict of interests on the research, authorship, and publication of this article.

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