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Collaborative Optimization of Dynamic Pricing and Seat Allocation for High-Speed Railways: An Empirical Study From China

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ABSTRACT In order to improve the high-speed trains' service levels and increase their market shares, the Chinese high-speed railway (HSR) enterprise is reforming its ticket pricing strategy. A collaborative model that incorporates seat allocation decision into HSR dynamic pricing problem based on the revenue management theory is proposed, in which the objective is to maximize the total ticket revenue of enterprise under the constrains of price ceilings. A two-stage algorithm is developed to solve practical problems. The first stage solves the optimal price problem, and the second is to obtain the optimal seat allocation decisions. Finally, a case study based on the actual ticket data of Beijing-Shanghai HSR in China is implemented to show the effectiveness of the proposed approach, for which the results show that compared with the fixed price case, the revenue improvement ranges from 4.47% to 4.95% by using dynamic pricing strategy. Also, the case analysis shows that dynamic pricing strategy will lead to an increase in short-haul demands whereas a decrease in long-haul demands.

INDEX TERMS High-speed railway (HSR), dynamic pricing, seat allocation, collaborative optimization, revenue management (RM).

I. INTRODUCTION

With the comprehensive advantages of high speed, safety and comfort, the high-speed railways (HSR) have developed rapidly across the world, especially in China. At present, China has an HSR network over 31,000 kilometers, accounting for two-thirds of total HSR operating mileage in the world, and it is reported that HSR in China has successfully transported more than 10 billion passengers by the end of April 2019.

Taking Beijing-Shanghai HSR (China) as an example, the line totally covers 1,318 kilometers, and connects 24 cities, including Beijing, Tianjin, Jinan, Nanjing, Shanghai and other big cities. It was put into operation in 2011, making the travelling time from Beijing to Shanghai by train reduced from 10 hours to 4.3 hours. Also, compared with air transportation, HSR has the obvious advantage of punctuality, which made the passenger demand for Beijing-Shanghai HSR increase sharply in recent years. The latest statistic shows that by the end of January 2019, the average daily passenger flow of Beijing-Shanghai HSR line has increased from 134,000 to around 500,000 since 2011.

As the railway transportation plays an essential role in China's social and economic development, especially for the long-term over-emphasis on its attributes to basic transportation services, the Chinese railway ticket price used to be controlled at a relatively low price by government, which only depends on the travelling distance and seat class. However, this pricing mechanism makes it impossible for railway enterprises to adjust their product prices according to real-time demand fluctuations; for example, considering the ticket prices of HSR from Beijing to Shanghai, a train departing at 6 am and another departing at 10 am totally share the equal prices although the latter always has a greater passenger demand. Consequently, tickets of some HSR trains are in short supply while other trains are low in passenger

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attendance rate, which obviously leads to an unreasonable use of resources, reduces the service level of HSR and eventually affects the revenue of the HSR enterprise. In order to enhance its market competitiveness and guarantee the sustainable development of the HSR enterprise, Chinese HSR enterprise has been permitted to determine HSR ticket price independently under an upper price constraint since 2016.

Practice on ticket price adjustments in some HSR lines in recent years in China has shown that the flexible pricing strategy can indeed help railway enterprise balance the passenger flows between different trains and increase the revenue level. Revenue management (RM), also known as Yield management or Profit Management, is a management technology for maximizing enterprise's revenue. The core of RM is price discrimination, namely, to determine optimal prices to different customers according to their specific demand characteristics and price elasticities. In other words, RM could help decision-makers sell the right products to the right customers at the right time and at the right price, in which the product prices are always dynamic throughout the whole period.

The first application of RM appeared in American Airlines in 1985, which brought an added profit of more than a billion dollars. After that, the RM strategy also has achieved succeeded in many other fields, such as hotel industry and car rental industry. In view of these successful applications, RM is considered as a practical technique to support modern operating profitably [1].

One of the RM strategy implementations is known as dynamic pricing, which represents the company's ability to sell each unit of a product/service at the maximum price that the potential customer is willing to pay at a specific condition. Although there are some certain similarities in the railway and airline products, the dynamic pricing problem in HSR is more complicated than that in airline industry, due to the computational complexities. Different from a point-to-point flight (always no more than 3 stops in a flight voyage), an HSR train always has a complex stop scheme (always more than 20 stops in the up/down direction). It means that more OD pairs are involved and seat allocation for different OD pairs need to be considered in the railway pricing process, which may not need to be considered in airline cases. Besides, HSR trains always have bigger capacities than airplanes, for example, a CR400 train have 1283 seats while Airbus A380, the biggest passenger plane so far, only has 555 seats, which also makes the HSR dynamic pricing problem more difficult to solve.

The objective of this research is to develop techniques capable of optimizing dynamic pricing and seat allocation problem. The remainder of the paper is organized as follows. In Section II, a literature review related to the dynamic pricing and seat allocation is provided. Section III lists the major assumptions and summarizes the notations, and then describes the price elasticity of HSR passenger demand, based on which a collaborative optimization model of HSR dynamic pricing and seat allocation simultaneously is proposed. Section IV describes a two-stage algorithm for solving the proposed model. A case study based on Beijing-Shanghai HSR line actual ticket data and result discussions is given in Section V. Finally, Section VI concludes the paper.

II. LITERATURE REVIEW

A. DYNAMIC PRICING AND SEAT MANAGEMENT IN AIRLINE INDUSTRY

As mentioned earlier, the research and application of RM first appeared in airline industry and dynamic pricing is the most common implementation of RM strategy. Luo and Peng [2] developed a continuous-time dynamic pricing model for two competitive flights with stochastic control theory and game theory. Zhang and Cooper [3] developed a Markov decision process formulation of a dynamic pricing problem for multiple substitutable flights between the same origin and destination, taking customers' choice behaviors among different flights into account. Otero and Akhavan-Tabatabaei [4] proposed a stochastic dynamic pricing model based on the willingness-to-pay of the customers to fix the price for each type of product in each period, applying phase type distributions and renewal processes to model the inter-arrival time between two customers that book a ticket and the probability that a customer buys a ticket. Santos and Gillis [5] proposed a data-driven modeling framework to estimate the flight pass price, which is a new concept in airlines.

Over the booking horizon, the airline companies also need to decide the number of seats sold at different prices, that is the seat inventory management. You [6] studied a multiple booking class airline-seat inventory control problem and pointed out that the strategy for the ticket booking policy can be reduced to sets of critical decision periods. Zhao and Zheng [7] studied a two-class airline dynamic seat allocation model and showed the relationship between the optimal policy and the static models. Yoon *et al.* [8] considered the joint pricing and seat control problem including a cancellation in airlines under uncertain demands. Kyparisis and Koulamas [9] considered the single-flight leg two-cabin airline RM problem in which there was a flexible partition of the business and economy cabins and determine the optimal cabin partition.

B. DYNAMIC PRICING AND SEAT MANAGEMENT IN RAILWAY TRANSPORTATION SYSTEM

Compared with air transportation, railway transportation has bigger capacities, more intermediate stops and more complicated operational schedules. As a result, the HSR dynamic pricing problem is much more computationally intractable than that in airline industry. Therefore, the method of airline dynamic pricing cannot be applied to the HSR dynamic pricing problem directly.

Some researchers began to summarize the existing pricing strategies in railway industry. Vuuren [10] tried to establish the link between the well-developed economic theory of optimal pricing, and recent empirical results concerning price elasticities of demand and marginal cost estimates for the

TABLE 1. Comparison our research with the literature in HSR.

References	Pricing	Seat allocation	Multiple trains	Multiple ODs	Multiple periods	Solution method
Sato et al. [13]	√	×	√	×	√	Analytical algorithm
Zhang et al. [14]	√	×	√	√	√	Backlog control and pricing optimization algorithm
Qin et al. [15]	√	×	√	√	√	Simulated annealing
Chang et al. [16]	×	√	×	√	×	LINDO software
Luo et al. [17]	×	√	√	√	×	MATLAB optimizer
Yuan et al. [18]	√	√	√	√	×	C# and CPLEX 12.5
Hetrakul et al. [20]	√	√	√	√	√	LINGO 12.0
Yan et al. [21]	√	√	√	√	×	Particle swarm optimization algorithm
Qin et al. [22]	√	√	√	√	√	Heuristic, Artificial bee colony algorithm
This paper	√	√	√	√	√	Analytical, Two-stage algorithm

Netherlands Railways. Armstrong and Meissner [11] provided an overview of the published literature both in passenger and freight rail RM, concluded some available models and gave some possible extensions. Bharill and Rangaraj [12] considered the case of passenger services in the premium segment of Indian Railways and illustrated an application of the principles of RM.

Dynamic pricing strategy for HSR is widely studied under different situations. Wang *et al.* [1] proposed a dynamic optimization model for a single HSR line with multiple prices and time periods. Sato and Sawaki [13] presented a RM model of dynamic pricing for a competitive route, supposing that the passengers were allowed to choose among other transportation modes and that each transportation mode offered the multiple substitutable schedules and the cancellation, no-show and overbooking were incorporated. Zhang *et al.* [14] proposed a revenue-maximization model that integrated both operation planning and pricing dimensions, based on dynamic ticket-pricing, elasticity in passenger demand, and flexible dispatching. Qin *et al.* [15] divided the passenger market according to the different factors affecting passenger choice behaviors, maximized ticketing revenue with expected travel cost as the reference point, and used prospect theory to construct a differentiated pricing model under elastic demand.

Seat allocation is also considered in high-speed railway revenue management (HSRRM). Chang and Yeh [16] presented a multi-objective planning model for generating optimal train seat allocation. Luo *et al.* [17] developed a multi-train seat inventory control model (MSIC) based on the RM theory, introducing OD pair inventory as a constraint by assigning the number of seats to each train and OD pairs to control the seat inventory capacity among different trains. Yuan *et al.* [18] studied a dynamic bid price approach in railway seat inventory control problem, considering multi-dimensional demand.

Based on the existing research results focusing on dynamic pricing and seat management separately in railway transportation, the collaborative optimization was proposed in recent years. Hu *et al.* [19] dealt with the multi-stage determination of price and seat allocation within the booking horizon for the HSRRM problem with multi-train services. Hetrakul and Cirillo [20] jointly considered pricing and seat allocation using multinomial logit and latent class models as discrete choice methods to explain ticket purchase timing of passenger. Yan *et al.* [21] developed a co-optimization model of resource capacity allocation and fare rates of HSR trains in different operation routes. Qin *et al.* [22] proposed an innovative model to optimize the price and seat allocation for HSR simultaneously.

To better highlight the differences between our research and the literature, Table 1 lists the main characteristics in the literature and this paper.

Overall, as for railway RM problem, most models in existing literature are based on a single optimization of either prices or seat allocation for multiple trains between same departure station and destination, e.g. Deng *et al.* [23] considered price optimization only in urban rail transit; Luo *et al.* [17] took seat allocation as the only decision variable. However, RM is a broad term that includes strategies to maximize profit through intelligent control of pricing and capacity [24].

As for HSR, in the process of transportation organization, dynamic pricing and seat allocation are two related problems. The implementation of dynamic pricing strategy will inevitably affect the choice behaviors of passengers for each train, which will influence the seat allocation. Therefore, in order to make full use of the capacity of each train and increase the enterprise's revenue, it is necessary to coordinate and optimize the two decision variables of the HSR ticket price and seat allocation to ensure the rationality of the optimal results.

All in all, taking seat allocations into account in railway RM, and being optimized along with price is a new trend in future studies of the HSRRM problem. Unlike existing studies, we link pricing and seat allocation together for HSRRM problem and propose an analytical algorithm in this paper. Comparing to previous works, our model has the following advantages:

(1) We theoretically propose a collaborative optimization model for HSR dynamic pricing and seat allocation. We apply the idea of maximizing the expected revenue with elastic demand in price optimization process and the principle of maximizing the passenger-kilometers to optimize the seat allocation.

(2) We develop a two-stage algorithm to solve the collaborative optimization problem, in which the two stages solve the optimal price and seat allocation decision problems respectively. It is the first time to introduce transcendental equations to solve the HSR dynamic pricing problem in the first stage, which is always a heuristic algorithm or a software optimizer in existing studies due to the problem's computational complexities.

III. THE COLLABORATIVE OPTIMIZATION MODEL

In this section, we describe the basic setup of our model and lay out our assumptions regarding the model. Then, a collaborative optimization model of dynamic pricing and seat allocation for HSR trains between the same departure station and destination is presented.

A. MODEL ASSUMPTIONS

To simplify the problem to be studied, we now summarize the main assumptions for the joint HSRRM optimization problem of dynamic pricing and seat allocation.

To facilitate the establishment of mathematical models, we need to make some assumptions as follows:

Assumption 1. Train stop schemes and initial passenger demands are given by actual ticket data and the train stop schemes will not be optimized.

Assumption 2. The capacity of each train on the same HSR line are totally equal.

Assumption 3. Different seat classes, overbooking, and cancellations are not considered in this paper.

B. VARIABLE DEFINITIONS

Given an HSR line, the departure station set is R and the arriving station set is S, totally including L tracks, where the passenger departure station $r \in R$, arriving station $s \in S$. For presentation purpose, other symbols are defined as follows.

Parameters and decision variables used throughout this paper are defined in Table 2 and Table 3, respectively.

C. ELASTIC DEMAND FUNCTION

Elastic demand is not only basic for railway passenger transportation organization, but also an important factor of dynamic pricing and seat allocation.

TABLE 2. Parameters.

Symbols	Descriptions
C	Capacity of HSR trains
D	Number of days in ticket pre-sale time
T	Number of periods in ticket pre-sale time
t_{hrs}	Running time for train h from r to s
t	The t^{th} day in pre-sale time
k	The k^{th} time period in pre-sale time
n	Number of operational trains
$\hat{p}_{rs}$	Upper price on segment (r, s)
$\check{p}_{rs}$	Lower price on segment (r, s)
Q_{rs}	Initial passenger demand from r to s
q_{rs}	Elastic demand of passenger choosing HSR trains to travel from r to s
Pr_{hrs}^{k}	Probability for passengers choosing train h to travel from r to s during period k
$q_{hrs}^{k}(p_{hrs}^{k})$	The elastic passenger demand for train h from r to s during period k
$S_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k})$	The expected period tickets amount sold for train h from r to s
$R_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k})$	The expected period revenue for train h from r to s
μ_{hrs}	0-1 judgment variable, when both stations r and s are the stops of train h, then $\mu_{hrs} = 1$, otherwise, $\mu_{hrs} = 0$.

TABLE 3. Decision variables.

Symbols	Descriptions
p_{hrs}^{k}	Price decision variable, the ticket price for train h from r to s during period k
N_{hrs}^{k}	Seat allocation variable, the seat allocation for train h from r to s during period k

Let c_{hrs}^{k} and c_{rs} denote the generalized travel cost for train h and the average generalized travel cost of the line segment (r, s), respectively. Then c_{hrs}^{k} and c_{rs} can be given by

$$c_{hrs}^{k} = p_{hrs}^{k} + \omega t_{hrs}, (r,s) \epsilon RS, k \in [1, T] \tag{1}$$

$$c_{rs} = \frac{1}{n \cdot T} \sum\nolimits_{g=1}^{n} \sum\nolimits_{k=1}^{T} c_{grs}^{k} \tag{2}$$

where k represents the train operating period, and ω is the sensitivity parameter for train running time.

The relationship between actual demand and potential demand meets the typical elastic demand function (Shi et al. [25]):

$$q_{rs} = Q_{rs} \cdot \exp(-\eta c_{rs}), (r, s) \epsilon RS \tag{3}$$

where η is the elastic demand coefficient and Q_{rs} is the original passenger demand between stations r and s.

According to the Logit model, the utility takes the negative of the generalized travel cost. The passengers' choice behavior model between different trains can be established as follows:

$$Pr_{hrs}^{k} = \frac{\exp(-\theta C_{hrs}^{k})}{\sum_{g=1}^{n}\exp(-\theta C_{grs}^{k})}, \forall (r,s), k \in [1,T] \quad (4)$$

where θ is the utility parameter of the Logit model.

Thus, in this paper, the elastic demand function for train h on segment (r, s) during period k is given by

$$q_{hrs}^{k}\left(p_{hrs}^{k}\right) = q_{rs} \cdot [F(k) - F(k-1)] \cdot Pr_{hrs}^{k}, k \in [1,T] \quad (5)$$

where $F(k)$ is the cumulative distribution function of the ticketing, and $F(0) = 0$, that is an integrating the passenger density function with respect to the operating period k.

D. MATHEMATICAL MODEL

This section describes the mathematic model. We first define the expected amount of tickets sold out and the expected operating revenue.

For a single train h, in the period k, let S_{hrs}^{k} be the expected tickets amount and R_{hrs}^{k} be the expected operating revenue. The expected amount of tickets sold out is determined by the minimum of the elastic demand $q_{hrs}^{k}(p_{hrs}^{k})$ and the seat allocation N_{hrs}^{k}; and the expected operating revenue is given by the product of the expected ticket amount $S_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k})$ and the corresponding price p_{hrs}^{k}.

Mathematically, expected ticket amount and expected operating revenue are respectively represented as follows.

$$S_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k}) = \min q_{hrs}^{k}(p_{hrs}^{k}), N_{hrs}^{k} \quad (6)$$

$$R_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k}) = S_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k}) \cdot p_{hrs}^{k} \quad (7)$$

As previously stated, the objective of the dynamic pricing and seat allocation optimization model for HSR is to maximize the transportation enterprise's operating revenue. We now define the total operating revenue R, which is generated from $R_{hrs}^{k}\left(p_{hrs}^{k}, N_{hrs}^{k}\right)$ with all trains in the total operating periods of the HSR system. It is expressed as

$$\max_{p_{hrs}^{k}, N_{hrs}^{k}} R = \sum_{k=1}^{T}\sum_{(r,s)}\sum_{g=1}^{n} \mu_{hrs} \cdot S_{grs}^{k}(p_{grs}^{k}, N_{grs}^{k}) \cdot p_{grs}^{k} \quad (8)$$

s.t.

$$\check{p}_{rs} \le p_{hrs}^{k} \le \hat{p}_{rs}, \forall (r,s), k \in [1,T] \quad (9)$$

$$p_{hrs}^{k-1} \le p_{hrs}^{k}, \forall (r,s), k \in [2,T] \quad (10)$$

$$\sum_{k=1}^{T}\sum_{r=1}^{m}\sum_{s=m+1}^{L+1} N_{hrs}^{k} \le C, m \in [1,L] \quad (11)$$

Objective function (8) is to maximizie the total operating revenue. The set of constraints are given by Eqs. (9)-(11). Constraint (9) guarantees the lower and upper bound of ticket price. Constraint (10) ensures that the ticket price will not be reduced when approaching the train's departure time. Constraint (11) guarantees the passenger flow on the minimum track will not exceed the train capacity.

IV. SOLUTION ALGORITHM

In this section, a two-stage algorithm is developed to solve the proposed objective (8) with bound constraints (9)-(11). The HSR dynamic pricing and seat allocation optimization model is a nonlinear mixed integer programming problem.

To obtain a relatively accurate solution, an analytical algorithm is designed to solve the model, including two stages. The first stage solves the optimal price problem, and then we can get the elastic demand under the optimal price by Eqs. (1)-(5). The second stage is to determine the seat allocation under the principle of maximizing the passenger-kilometers.

As a single train on a single segment (r, s) is the most basic unit in HSR dynamic pricing problem, taking a single train on a single segment (r, s) during period k as an example, the remaining part of this section mainly focuses on the demonstration on the optimization mechanism of the proposed two-stage algorithm. Then the same method could be introduced to solve the extended HSR dynamic pricing problem with multiple trains and multiple time periods as shown in the case study.

A. PRICE DECISION

In the HSR dynamic pricing optimization model, the price decision is made with the goal of maximizing the expected revenue of the train, so the objective function of a single train for a specific OD can be written as in objective (12).

$$\max_{p_{hrs}^{k}, N_{hrs}^{k}} R_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k}) = S_{hrs}^{k}(p_{hrs}^{k}, N_{hrs}^{k}) \cdot p_{hrs}^{k} \quad (12)$$

s.t.

$$\check{p}_{rs} \le p_{hrs}^{k-1} \le p_{hrs}^{k} \le \hat{p}_{rs}, \forall (r,s), \quad k \in [2,T] \quad (13)$$

The maximum revenue can be derived as in (14):

$$R_{hrs}^{k}\left(p_{hrs}^{k}, N_{hrs}^{k}\right) = \min\{q_{hrs}^{k}(p_{hrs}^{k}), N_{hrs}^{k}\} \cdot p_{hrs}^{k} \le q_{hrs}^{k}(p_{hrs}^{k}) \cdot p_{hrs}^{k} \quad (14)$$

So, let

$$\max R_{hrs}^{k}\left(p_{hrs}^{k}, N_{hrs}^{k}\right) = q_{hrs}^{k}(p_{hrs}^{k}) \cdot p_{hrs}^{k} \quad (15)$$

Substituting Eqs. (1)-(5) into (15) yields (16), as shown at the top of the next page.

Let

$$e_0 = \sum_{g=1}^{n-1} \exp\left[\theta\left(-p_{grs}^{k} - \omega t_{grs}\right)\right] \quad (17)$$

where e_0 represents the exponential sum of the other $n-1$ trains, apart from the one to be optimized.

Further, Eq. (16) can be transformed into (18), as shown at the top of the next page.

Then, let

$$\mathrm{A} = \theta + \frac{\eta}{n}, B = \frac{\eta}{n}, C = Q_{rs} \cdot [F(k) - F(k-1)]$$

Thus, Eq. (18) can be transformed into following Equation:

$$R_{hrs}(\cdot) = \frac{\exp\left[\mathrm{A}\left(-p_{hrs}^{k} - \omega t_{hrs}\right) + \mathrm{B}e_0\right] \cdot C \cdot p_{hrs}^{k}}{e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]} \quad (19)$$

$$R_{hrs}^{k}(\bullet) = \frac{\exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right) + \frac{\eta}{n}\sum_{g=1}^{n}\left(-p_{grs}^{k} - \omega t_{grs}\right)\right] \cdot Q_{rs} \cdot \left[F(k) - F(k-1)\right] \cdot p_{hrs}^{k}}{\sum_{g=1}^{n}\exp\left[\theta\left(-p_{grs}^{k} - \omega t_{grs}\right)\right]} \tag{16}$$

$$R_{hrs}(\cdot) = \frac{\exp\left[\left(\theta + \frac{\eta}{n}\right)\left(-p_{hrs}^{k} - \omega t_{hrs}\right) + \frac{\eta}{n}e_0\right] \cdot Q_{rs} \cdot \left[F(k) - F(k-1)\right] \cdot p_{hrs}^{k}}{e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]} \tag{18}$$

$$\frac{\partial R_{hrs}(\cdot)}{\partial p_{hrs}^{k}} = \exp\left[-\mathrm{A}\left(p_{hrs}^{k} + \omega t_{hrs}\right) + \mathrm{B}e_0\right] \cdot C \cdot \frac{\theta \cdot p_{hrs}^{k}\exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right] + \left[e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]\right]\left(1 - \mathrm{A}p_{hrs}^{k}\right)}{\left[e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]\right]^2} \tag{20}$$

The first-order partial derivatives of $R_{hrs}(\cdot)$ with respect to p_{grs}^{k} is given in (20), as shown at the top of this page.

To obtain the optimal price solutions for the total operating revenue, we set the partial derivative of the objective function $R_{hrs}(\cdot)$ with respect to p_{grs}^{k} to zero. Then, we have

$$\theta \cdot p_{hrs}^{k}\exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right] + \left[e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]\right]\left(1 - \mathrm{A}p_{hrs}^{k}\right) = 0 \tag{21}$$

When the number of operating trains n is big enough, then

$$\mathrm{A} = \theta + \frac{\eta}{n} \approx \theta \tag{22}$$

Substituting Eq. (22) into Eq. (21), one obtains

$$\theta \cdot p_{hrs}^{k}\exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right] + \left[e_0 + \exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]\right]\left(1 - \theta p_{hrs}^{k}\right) = 0 \tag{23}$$

The exponential term $\exp\left[\theta\left(-p_{hrs}^{k} - \omega t_{hrs}\right)\right]$ contains the decision variable p_{hrs}^{k} itself in Eq. (23), which is a typical exponential transcendental equation [26]. In view of the fact that the transcendental equation is difficult to be solved by algebraic geometry, we consider transforming the exponential transcendental equation into a Lambert W function [27]–[29], and then using the Lambert W function to solve the original problem.

The Lambert W function as in Eq. (24) can be obtained by the identity transformation from Eq. (23).

$$\exp\left(\theta p_{hrs}^{k} - 1\right) \cdot \left(\theta p_{hrs}^{k} - 1\right) = \frac{\exp\left(-\theta\omega t_{hrs} - 1\right)}{e_0} \tag{24}$$

According to the solution format of Lambert W function in [25], the solution of Eq. (24) can be expressed as in Eq. (25):

$$\theta p_{hrs}^{k} - 1 = W\left(\frac{\exp\left(-\theta\omega t_{hrs} - 1\right)}{e_0}\right) \tag{25}$$

Then, the optimal price can be obtained as in Eq. (26):

$$p^{*} = \frac{W\left(\frac{\exp(-\theta\omega t_{hrs} - 1)}{e_0}\right) + 1}{\theta} \tag{26}$$

Finally, the price decision of train h during the k^{th} period can be determined as in Eq. (27):

$$p_{hrs}^{k} = \begin{cases} \mathrm{p}_{hrs}^{k-1}, & p^{*} < \mathrm{p}_{hrs}^{k-1} \\ p^{*}, & \mathrm{p}_{hrs}^{k-1} \le p^{*} \le \hat{p}_{hrs} \\ \hat{p}_{hrs}, & p^{*} > \hat{p}_{hrs} \end{cases} \tag{27}$$

B. SEAT ALLOCATION

After the price of each train for each time period has been determined, according to Eqs. (1)-(5), the elastic demand of each train can be calculated. Considering the arrival sequential problems of different OD passengers in different time periods, in order to ensure that the train's long-distance capacities will not be prematurely cracked, the maximum passenger-kilometer principle is used to decide the seat allocation for each time period.

According to the differences in the travelling distance of passenger demands, long-haul demands or short-haul demands, the seat allocation can be divided into the full-trip seat allocation and the short-trip seat allocation.

1) THE FIRST T-1 TIME PERIOD

In order to ensure the fairness of ticket purchasing chances for different time periods, the amount of seat allocation for train h during the k^{th} period (N_h^k) is determined by the passenger demand distribution of each time period, as in (28):

$$N_h^k = \frac{\sum_{r,s} q_{hrs}^{k}(p_{hrs}^{k})}{\sum_{\tau=1}^{T}\sum_{r,s} q_{hrs}^{\tau}((p_{hrs}^{\tau}))} \cdot C, k \in [1, T-1] \tag{28}$$

In this paper, the long-haul demands are given priorities to be satisfied, so the seat allocation for full trip of the k^{th} period n_{h-f}^{k} is expressed as:

$$n_{h-f}^{k} = min\left\{N_h^k, q_{h(1,L+1)}^{k}\left(p_{h(1,L+1)}^{k}\right) | k \in [1, T-1]\right\} \tag{29}$$

where $p_{h(1,L+1)}^{k}$ represents the ticket price for train h from the first station to the last station in time period k, and $q^k_{h(1,L+1)}\left(p^k_{h(1,L+1)}\right)$ is the corresponding elastic passenger demand.

In order to avoid the premature cracking of long-distance capacities, the maximum passenger-kilometer principle is used to decide the seat allocation for each time period. More specifically, only when different short-haul passenger demands can completely make up the full trip from the first station to the last station, then the corresponding number of tickets will be sold as short-trip tickets to each OD. Therefore, the short-trip seat allocation in the k^{th} period can be expressed as in Eq. (30):

$$n^k_{h-s} = min\left\{\sum\nolimits_{r=1}^{m}\sum\nolimits_{s=m+1}^{L+1} q^k_{hrs}\left(p^k_{hrs}\right)|\forall m \in [1, L]\,, \right.$$
$$\left. k \in [1, T-1]\,, (r, s) \neq (1, L+1)\right\} \quad (30)$$
$$\text{s.t. } n^k_{h-s} \leq N^k_h - n^k_{h-f} \quad (31)$$

Equation (31) indicates that the short-trip seat allocation is no more than the remaining ticket amount left in the current time period, apart from the full-trip tickets that have been sold preferentially.

2) THE LAST TIME PERIOD

The last period is different from other periods, as there are no following periods any more, which means that there will be no passengers arriving after this period, so there is no need restricting the number of short-trip tickets to be sold in this time period in terms of the goal to achieve the maximum revenue.

The tickets sold during previous periods are all eventually sold in the form of full-trip tickets, sold as full-trip tickets directly (n^k_{h-f}) or sold as short-trip tickets that can make up the full trip (n^k_{h-s}). Therefore, the remaining tickets available for the last period can be obtained as in Eq. (32):

$$N^T_h = C - \sum\nolimits_{k=1}^{T-1}\left(n^k_{h-f} + n^k_{h-s}\right) \quad (32)$$

All tickets in this period can be shared by all stations and be sold on any segments when needed.

V. CASE ANALYSIS

In this section, we perform a case study including several instances based on real ticket data of a major HSR line in China, Beijing-Shanghai HSR, to test the performance of the proposed model and algorithm.

A. DESCRIPTION OF DATA

In this paper, we collected detailed information of trains, prices, running time and average passenger flow from the ticket data of Beijing-Shanghai HSR line in up direction in June, 2017, and then selected 4 typical trains, with different stop schemes, as shown in Figure 1.

All the parameters in the model have been investigated by questionnaires or taken from previous studies, and the values of parameters are given as follows: the train capacity C= 1015, the number of pre-sale days D= 30, the utility parameter θ= 0.012, the coefficient of elasticity η= 0.04

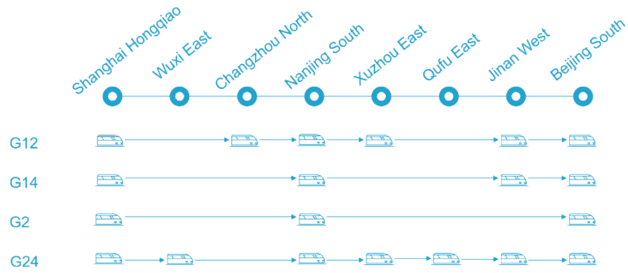

FIGURE 1. The train stop schemes.

referring to [25], the time conversion parameter $\omega = 36$ yuan/hour, the upper and lower price of each OD are 15% higher and 15% lower than the current second-class ticket price respectively.

According to the data statistics, there are more than 3.4 million passengers travelling from Shanghai to Beijing by HSR, which accounts for 21.4% of the total number of tickets for the whole line.

After the statistical analysis and data fitting in Figure 2, we find that the probability distribution function of the amount of tickets sold out in the pre-sale period meets the exponential distribution with the parameter $\lambda = \frac{1}{3}$.

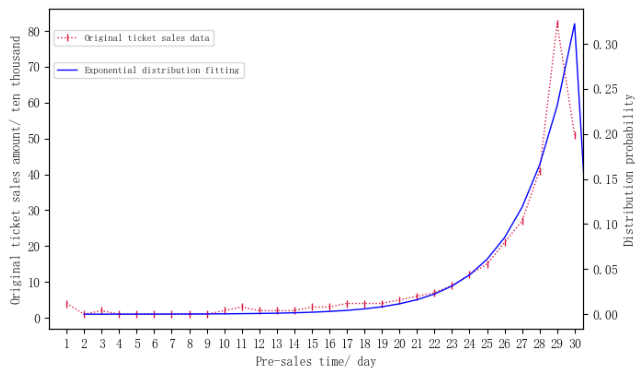

FIGURE 2. Distribution of tickets sold out under a fixed price.

Hence, the probability distribution function and the cumulative distribution function of the amount of tickets sold out under fixed price can be obtained as in Eqs. (33) and (34).

$$f(t) = \frac{1}{3}\cdot\exp\left(\frac{t}{3} - 10\right), \quad 1 \leq t \leq 30 \quad (33)$$
$$F(t) = \int_0^t f(t)\,dt = \frac{\exp\left(\frac{t}{3}\right) - 1}{\exp(10)}, \quad 1 \leq t \leq 30 \quad (34)$$

According to the cumulative distribution function, the accumulated amount of tickets sold out over each day in pre-sale period can be calculated by Eq. (34). Then the pre-sale period is divided into 6 periods, as shown in Figure 3:

The second-class price (used as fixed prices in this paper), running time and initial passenger demand of each train are shown in Table 4-Table 6:

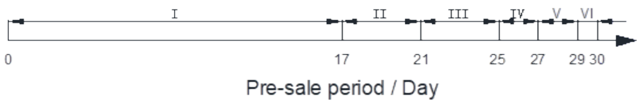

FIGURE 3. The division of the pre-sale period.

TABLE 4. Matrix table of second-class prices (unit: yuan).

	WX	CZ	NJ	XZ	QF	JN	BJ
SH	59.5	74.5	144.5	279	344	398.5	553
WX		19.5	84.5	239	304	354	513.5
CZ			64.5	209	279	334	493.5
NJ				149.5	224	279	443.5
XZ					69.5	134.5	309
QF						59.5	244
JN							184.5

SH=Shanghai, WX=Wuxi, CZ=Changzhou, NJ=Nanjing, XZ=Xuzhou, QF=Qufu, JN=Jinan, BJ=Beijing. And the same as the following tables.

TABLE 5. Matrix table of running time for each train (G12/G14/G2/G24, unit: hour).

	WX	CZ	NJ	XZ	QF	JN	BJ
SH	-/-/-/0.5	0.7/-/-/-	1.2/1.1/1.1/1.2	2.5/-/-/2.5	-/-/-/3.2	3.7/3.4/-/3.8	5.3/5/4.8/5.4
WX		-/-/-/-	-/-/-/0.7	-/-/-/2	-/-/-/2.7	-/-/-/3.3	-/-/-/4.9
CZ			0.5/-/-/-	1.8/-/-/-	-/-/-/-	3/-/-/-	4.6/-/-/-
NJ				1.2/-/-/1.2	-/-/-/1.9	2.4/2.2/-/2.5	4/3.8/3.7/4.1
XZ					-/-/-/0.6	1.2/-/-/1.2	2.7/-/-/2.8
QF						-/-/-/0.5	-/-/-/2.1
JN							1.3/1.6/-/1.5

'-' means the corresponding train does not provide service on that segment, and the same as following tables.

TABLE 6. Matrix table of passenger flow data for each train (G12/G14/G2/G24, unit: passenger).

	WX	CZ	NJ	XZ	QF	JN	BJ
SH	-/-/-/93	142/-/-/-	151/156/203/184	37/-/-/29	-/-/-/33	114/100/-/69	451/747/782/387
WX		-/-/-/-	-/-/-/22	-/-/-/15	-/-/-/12	-/-/-/23	-/-/-/102
CZ			46/-/-/-	20/-/-/-	-/-/-/-	18/-/-/-	145/-/-/-
NJ				43/-/-/46	-/-/-/10	27/15/-/19	146/150/230/159
XZ					-/-/-/5	11/-/-/8	60/-/-/57
QF						-/-/-/11	-/-/-/27
JN							60/100/-/92

B. RESULTS

1) PERFORMANCE OF THE TWO-STAGE ALGORITHM ON REAL DATA INSTANCES

We assume that the fixed price and initial passenger demand are given by the statistics of the ticket data (as shown in Table 4 and Table 6 respectively). Then the total revenue of the case is 2,100,422 yuan.

Considering the influences of factors, such as prices and running time, on passengers' travelling choice behaviors, the collaborative optimization model proposed in this paper is adopted to optimize the pricing and seat allocation for HSR trains.

We evaluate the performance of our approach using real data provided by Table 4- 6 when our two-stage algorithm is used to solve the collaborative optimization problem. Figure 4 gives the characteristics and results of the instances with different initial discounts for trains in the first pre-sale period.

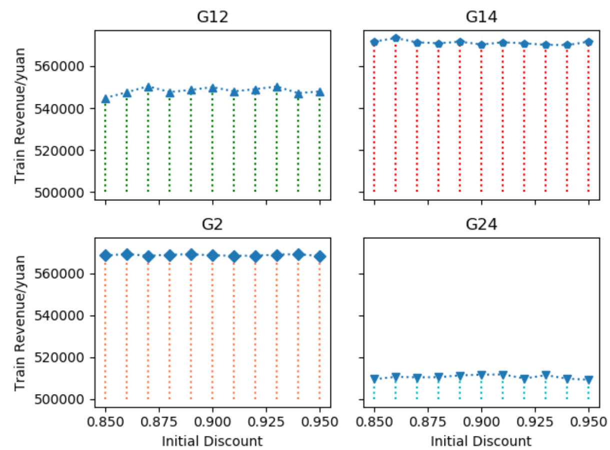

FIGURE 4. Train revenue results under different initial price discounts.

As shown in Figure 4, although the initial price discount varies from 0.85 to 0.95, the final revenue of each train remains stable and it is related to its running time. From Table 5, we can see that in terms of the running time, G2<G14<G12<G24, and correspondingly, the final revenue of each train in Figure 4, G2≈G14>G12>G14.

However, it should be noticed that the passenger demands in different initial price discount cases are significantly unequal, more specifically, lower price with bigger passenger demand and vice versa, because of the price elasticity of passenger demands (shown in Figure 5), which makes it possible for transportation enterprises to adjust its pricing strategies to real passenger demands, high price at peak times, whereas low price at off-peak times, to achieve a better revenue.

2) THE DETAILED RESULTS FOR A SPECIFIC INSTANCE

In this section, we give the detailed dynamic pricing and seat allocation optimization results for the instance with the initial price discounts as shown in Table 7.

Using the running time in Table 5 and the same initial passenger demand in Table 6, we recalculate the elastic passenger demand according to Eqs. (1)-(5) when the ticket price varies in different periods. Finally, the total value of the objective function is 2,201,818 yuan; 4.83% higher than that before optimization.

There are 28 OD pairs involved in the instance, and it is inconvenient to list all them out in this paper.

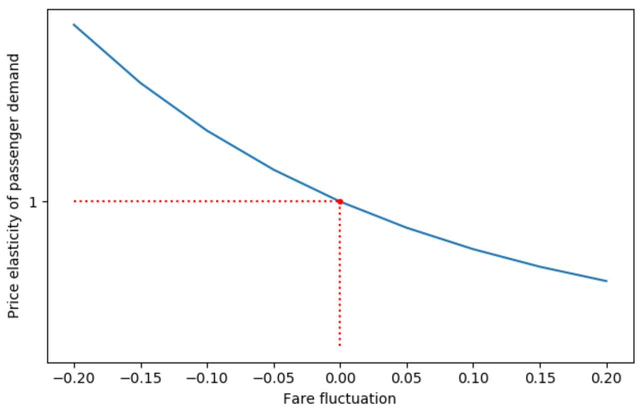

FIGURE 5. Price elasticity of passenger demand.

TABLE 7. Initial discounts for different trains.

Train	Initial discounts
G12	0.90
G14	0.85
G2	0.90
G24	0.90

Here, three special OD pairs, Shanghai-Nanjing, Shanghai-Beijing and Nanjing-Beijing, which are the same segments in all four trains' stop schemes, are listed as examples to show the optimal results of pricing and seat allocation in Table 8.

Table 9 shows the passenger attendance rate of each train on every track of Beijing-Shanghai HSR line.

Table 10 shows the overall demand fluctuations of passenger flow on each segment of Beijing-Shanghai HSR line, compared with the initial demands presented in Table 6.

In the light of above, we can obtain some conclusions as follows:

(1) The passenger demands of G2 and G14 on every track of Beijing-Shanghai HSR line are usually higher than those on G12 and G24.

(2) The number of tickets for each train on the Shanghai-Beijing segment is always equal to the passenger demand, which means that the long-haul travelling demands have been preferentially satisfied in each time period.

(3) Nanjing-Xuzhou is the busiest track with the highest passenger attendance rate amongst all tracks for all trains.

(4) There is an increase in short-haul demands whereas a decrease in long-haul demands, which allows for using the same number of seats to serve more passengers.

Firstly, in Table 5, the running time that G2 and G14 spend on the same OD is less than that of G12 and G24, in other words, the time utilities of G2 and G14 are higher than that of G12 and G24. Therefore, passengers tend to choose G2 and G14 to travel when G2 and G14 can satisfy their travelling demands.

Secondly, from the perspective of the seat allocation, it is always the case that the short-haul travelling demands cannot be immediately satisfied in the first five time periods because

TABLE 8. Optimized OD prices and seat allocation (price unit: yuan, demand unit: passenger, TICKET UNIT: PEICE).

Train	Time period	OD price / demand SH-NJ	SH-BJ	NJ-BJ	Full-trip	Short-trip
G12	I	130/17	497.5/50	399/17	50	45
	II	135/17	519/48	413.5/16	48	42
	III	140/32	541.5/93	428.5/31	93	79
	IV	145/31	565/89	444/30	89	77
	V	150.5/30	589.5/85	460/29	85	75
	VI	156/34	615/106	476.5/33	332	--
G14	I	123/18	470/86	377/18	86	15
	II	127.5/21	504/82	391/20	19	101
	III	132.5/36	541/153	172.5/34	153	50
	IV	137.5/35	580/143	420.5/32	143	60
	V	142.5/35	622/134	436/31	134	69
	VI	148/21	636/130	452/26	204	--
G2	I	130/23	497.5/86	399/26	86	15
	II	136.5/30	535/81	422/36	81	20
	III	143/52	576/151	446/62	151	52
	IV	150/40	619/140	471.5/54	140	40
	V	157.5/38	636/137	498.5/55	137	38
	VI	165.5/28	636/137	510/57	255	--
G24	I	130/21	497.5/43	399/18	43	48
	II	136/20	516/42	414.5/17	42	46
	III	142/38	535/80	431/33	80	82
	IV	148.5/36	555/78	448/32	78	80
	V	155/35	575.5/75	465.5/31	75	77
	VI	162/43	597/93	484/39	364	--

'636 yuan' is the upper price from Shanghai to Beijing.

TABLE 9. Train attendance rate on each track.

	G12	G14	G2	G24
SH-WX	0.869	0.979	0.919	0.786
WX-CZ	0.869	0.979	0.919	0.875
CZ-NJ	0.957	0.979	0.919	0.875
NJ-XZ	0.977	0.994	0.940	0.910
XZ-QF	0.948	0.994	0.940	0.889
QF-JN	0.948	0.994	0.940	0.866
JN-BJ	0.844	0.980	0.940	0.817
Average	0.916	0.986	0.931	0.860

TABLE 10. Matrix table of passenger demand changes on each segment (unit: passenger).

	WX	CZ	NJ	XZ	QF	JN	BJ	Departure
SH	-4	-8	+13	+4	+1	-8	-71	-73
WX		--	+2	+1	+4	+3	-5	+5
CZ			+1	+1	--	+2	-10	-6
NJ				+2	+3	+7	+11	+23
XZ					+1	+4	0	+5
QF						+5	+3	+8
JN							+6	+6
Arrival	-4	-8	+16	+8	+9	+13	-66	-32

of the capacity-cracking defense constraint, so when the constraint is not considered in the last time period, the number of tickets sold to short-trip passengers massively increases whereas the majority of full-trip passengers have booked their tickets in the past five time periods with a lower price.

Thirdly, Shanghai, Nanjing, Jinan and Beijing are the four capital cities along the Beijing-Shanghai HSR line, which means that there will be more passengers travelling between these cities. More specifically, a large quantity of passengers will departure from Shanghai and Nanjing, leaving for Jinan and Beijing in the upward direction. Taking other smaller short-haul travelling demands into account, it eventually turns out that the passenger attendance rate of the Nanjing-Xuzhou track is the highest among all.

And then, in terms of the passenger demand fluctuation, the passenger decline is mainly reflected on the decreasing number of passengers departing from Shanghai, and arriving at Beijing while most of other short-haul demands are increased. However, the final revenue increases, which suggests that accepting short-haul demands provides greater revenue than long-haul demands when using the same capacity.

Finally, more detailed dynamic pricing information can be seen in Figure 6. Figure 6 takes the segment of Shanghai-Beijing as an example, and shows to what degree the ticket price discounts of different trains in different time periods fluctuates based on the fixed ticket price ("553 yuan" in Figure 6).

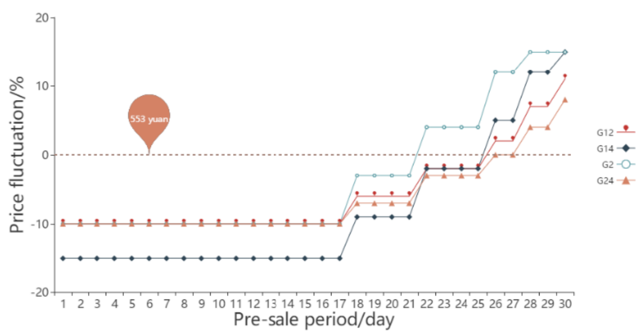

FIGURE 6. **Shanghai-Beijing ticket price fluctuations for different trains in different pre-sale periods.**

At the beginning of the pre-sale period, the price of each train from Shanghai to Beijing is below the fixed ticket price, and there is a continuous increase on price with time going by.

More in detail, G2 first breaks the base price line at the third time period and the price level of G2 is always the highest amongst all trains. This is because comparing the running time of different trains as shown in Table 5, the running time of G2 is the shortest, which means the highest time utility. Also, the initial passenger demand of G2 on the Shanghai-Beijing segment is the biggest in Table 6. With the support of the high time utility and huge passenger demand, the ticket price of G2 is correspondingly the highest.

VI. CONCLUSION

This paper proposes a collaborative optimization model that incorporates the seat allocation decision into HSR dynamic pricing problem, aiming to maximize the revenue of HSR enterprise, and then a two-stage algorithm is given to solve the practical problem. In the first stage, the transcendental equation is introduced to help make the HSR dynamic pricing decision and in the second stage, the optimal seat allocation decision is obtained under the determined prices in the first stage.

In the case study, instances with different initial discounts for trains in the first pre-sale period are designed to testify the proposed methodology. Compared with the fixed price case, the results of the instances show that the final revenue can be improved from 4.47% to 4.95% by using dynamic pricing strategy. As for passenger demand, dynamic pricing strategy will result into an increase in short-haul demands whereas a decrease in long-haul demands in Beijing-Shanghai HSR. In conclusions, this paper has illustrated how a railway enterprise can exploit its existing ticket data to better design its pricing and seat allocation strategies and achieve a higher revenue.

Although the proposed model provides useful insights for HSR dynamic pricing and seat allocation problem, some important features of HSR services and management were still omitted. It is suggested to further study the HSR pricing and seat allocation problem considering different seat classes, different types of passengers, and even the comprehensive optimization of HSR network with multiple lines in the future.

REFERENCES

[1] Y. Wang, B. X. Lan, and L. Zhang, "A revenue management model for high-speed railway," in *Proc. 1st Int. Workshop High-Speed Intercity Railways*, Hong Kong, vol. 147, 2012, pp. 95–103. doi: 10.1007/978-3-642-27960-7_9.

[2] L. Luo and J.-H. Peng, "Dynamic pricing model for airline revenue management under competition," *Syst. Eng. Theory Pract.*, vol. 27, no. 11, pp. 15–25, Nov. 2007. doi: 10.1016/S1874-8651(08)60065-4.

[3] D. Zhang and W. L. Cooper, "Pricing substitutable flights in airline revenue management," *Eur. J. Oper. Res.*, vol. 197, no. 3, pp. 848–861, 2009. doi: 10.1016/j.ejor.2006.10.067.

[4] D. F. Otero and R. Akhavan-Tabatabaei, "A stochastic dynamic pricing model for the multiclass problems in the airline industry," *Eur. J. Oper. Res.*, vol. 242, no. 1, pp. 188–200, 2015. doi: 10.1016/j.ejor.2014.09.038.

[5] B. F. Santos and M. M. D. Gillis, "Optimizing the prices for airline flight passes," *Transp. Res. Procedia*, vol. 37, pp. 266–273, Jan. 2019. doi: 10.1016/j.trpro.2018.12.192.

[6] P.-S. You, "Dynamic pricing in airline seat management for flights with multiple flight legs," *Transp. Sci.*, vol. 33, no. 2, pp. 192–206, 1999. doi: 10.1287/trsc.33.2.192.

[7] W. Zhao and Y.-S. Zheng, "A dynamic model for airline seat allocation with passenger diversion and no-shows," *Transp. Sci.*, vol. 35, no. 1, pp. 80–98, 2001. doi: 10.1287/trsc.35.1.80.10145.

[8] M. G. Yoon, H. Y. Lee, and Y. S. Song, "Dynamic pricing & capacity assignment problem with cancellation and mark-up policies in airlines," *Asia Pacific Manage. Rev.*, vol. 22, no. 2, pp. 97–103, Jun. 2017. doi: 10.1016/j.apmrv.2016.08.004.

[9] G. J. Kyparisis and C. Koulamas, "Optimal pricing and seat allocation for a two-cabin airline revenue management problem," *Int. J. Prod. Econ.*, vol. 201, pp. 18–25, Jun. 2018. doi: 10.1016/j.ijpe.2018.04.021.

[10] D. van Vuuren, "Optimal pricing in railway passenger transport: Theory and practice in The Netherlands," *Transp. Policy*, vol. 9, no. 2, pp. 95–106, 2002. doi: 10.1016/s0967-070x(02)00005-7.
[11] A. Armstrong and J. Meissner, "Railway revenue management: Overview and models (operations research)," Dept. Manage. Sci., Lancaster Univ., Lancaster, U.K., Working Papers MRG/0019, 2010.
[12] R. Bharill and N. Rangaraj, "Revenue management in railway operations: A study of the Rajdhani express, Indian railways," *Transp. Res. A, Policy Pract.*, vol. 42, no. 9, pp. 1195–1207, Nov. 2008. doi: 10.1016/j.tra.2008.03.007.
[13] K. Sato and K. Sawaki, "Dynamic pricing of high-speed rail with transport competition," *J. Revenue Pricing Manage.*, vol. 11, no. 5, pp. 548–559, Sep. 2012. doi: 10.1057/rpm.2011.29.
[14] X. Zhang, L. Li, S. Le Vine, and X. Liu, "An integrated pricing/planning strategy to optimize passenger rail service with uncertain demand," *J. Intell. fuzzy Syst.*, vol. 36, no. 1, pp. 435–448, Feb. 2019. doi: 10.3233/JIFS-181701.
[15] J. Qin, W. Qu, X. Wu, and Y. Zeng, "Differential pricing strategies of high speed railway based on prospect theory: An empirical study from China," *Sustainability*, vol. 11, no. 14, p. 3804, 2019. doi: 10.3390/su11143804.
[16] Y.-H. Chang and C.-H. Yeh, "A multiobjective planning model for intercity train seat allocation," *J. Adv. Transp.*, vol. 38, no. 2, pp. 115–132, 2004. doi: 10.1002/atr.5670380202.
[17] H. Luo , L. Nie , and Z. He , "Modeling of multi-train seat inventory control based on revenue management," in *Proc. Int. Conf. Logistics, Inform. Service Sci. (LISS)*, Jul. 2016, pp. 1–6. doi: 10.1109/LISS.2016.7854371.
[18] W. Yuan, L. Nie, X. Wu, and H. Fu, "A dynamic bid price approach for the seat inventory control problem in railway networks with consideration of passenger transfer," *PLoS ONE*, vol. 13, no. 8, 2018, Art. no. e0201718. doi: 10.1371/journal.pone.0201718.
[19] X. L. Hu, F. Shi, G. M. Xu, R. F. Wu, and T. J. Wei, "Multi-stage pricing and seat allocation model for high-speed rail with multi-train services," in *Proc. Transp. Res. Board 98th Annu. Meeting*, Washington, DC, USA, Jan. 2019, p. 5.
[20] P. Hetrakul and C. Cirillo, "A latent class choice based model system for railway optimal pricing and seat allocation," *Transp. Res. E, Logistics Transp. Rev.*, vol. 61, pp. 68–83, Jan. 2014. doi: 10.1016/j.tre.2013.10.005.
[21] Z.-Y. Yan, X.-J. Li, and B.-M. Han, "Collaborative optimisation of resource capacity allocation and fare rate for high-speed railway passenger transport," *J. Rail Transp. Planning Manage.*, vol. 10, pp. 23–33, Aug. 2019. doi: 10.1016/j.jrtpm.2019.05.001.
[22] J. Qin, Y. Zeng, X. Yang, Y. He, X. Wu, and W. Qu, "Time-dependent pricing for high-speed railway in China based on revenue management," *Sustainability*, vol. 11, no. 16, p. 4272, 2019. doi: 10.3390/su11164272.
[23] L. Deng, Z. Zhang, K. Liu, W. Zhou, and J. Ma, "Fare optimality analysis of urban rail transit under various objective functions," *Discrete Dyn. Nature Soc.*, vol. 2014, Aug. 2014, Art. no. 910736. doi: 10.1155/2014/910736.
[24] W. Lieberman, "The theory and practice of revenue management," *J. Revenue Pricing Manage.*, vol. 3, pp. 384–386, Jan. 2005. doi: 10.1057/palgrave.rpm.5170123.
[25] F. Shi, G.-M. Xu, B. Liu, and H. Huang, "Optimization method of alternate traffic restriction scheme based on elastic demand and mode choice behavior," *Transp. Res. C, Emerg. Technol.*, vol. 39, pp. 36–52, Feb. 2014. doi: 10.1016/j.trc.2013.11.015.
[26] A. Bashirov, "Transcendental functions," in *Mathematical Analysis Fundamentals*, A. Bashirov, Ed. Boston, MA, USA: Elsevier, 2014, pp. 253–305.
[27] D. Veberič, "Lambert W function for applications in physics," *Comput. Phys. Commun.*, vol. 183, no. 12, pp. 2622–2628, Dec. 2012. doi: 10.1016/j.cpc.2012.07.008.
[28] F. Alzahrani and A. Salem, "Sharp bounds for the Lambert W function," *Integral Transf. Special Functions*, vol. 29, no. 12, pp. 971–978, Dec. 2018. doi: 10.1080/10652469.2018.1528247.
[29] A. G. Pakes, "The Lambert *W* function, Nuttall's integral, and the Lambert law," *Statist. Probab. Lett.*, vol. 139, pp. 53–60, Aug. 2018. doi: 10.1016/j.spl.2018.03.015.

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