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Distributed Multi-Robot Formation Control Based on Bipartite Consensus With Time-Varying Delays

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ABSTRACT A distributed formation control scheme of multi-robot systems for bipartite consensus under communication delays is presented in this paper. The bipartite consensus protocol under communication delays is first proposed. Under the bipartite consensus protocol, a necessary and sufficient condition is then presented for achieving bipartite consensus of the multi-agent system without time-delays. For the system with time-delays, a sufficient condition to asymptotically achieve the bipartite consensus is proposed. Furthermore, a universal multi-robot formation control protocol with communication delays is put forward according to the given bipartite consensus protocol. As an example, the bipartite consensus protocol under communication delays is applied to multi-robot formation control. Finally, a number of simulations are proposed to demonstrate the theoretical contributions of this work.

INDEX TERMS Formation control, multi-robot systems, bipartite consensus, time-varying delays.

I. INTRODUCTION

Multi-robot systems are more suitable than a single robot for accomplishing complex tasks and varied scenarios due to their cooperative ability. Multi-robot systems have been used in numerous scenarios including searching for survivors in collapsed zones, production in industry, surveillance of suspicious buildings, gas detection in coal mines, disaster rescue, and dangerous object inspection [1]–[3]. Multi-robot systems could also be less expensive than a multi-functional robot. When a number of simpler robots fail, several robots can cooperatively complete the objective by adding redundancy to the proposed system.

Robot's formation control is concerned by numerous scholars recently. Chen proposed a leader-follower formation controller based on RHLF control architecture to discuss multi-robot formation control issues [4]. The leader-follower method of multi-robot formation control was provided based on the internal model in Wang's paper [5], while Ren proposed a distributed coordination scheme via information exchange, achieving multi-robot formation control in the light of the given consensus protocols [6]. Peng presented neural network torque and kinematic controllers to investigate multi-robot formation control issues [7]. Park put forward adaptive control techniques to solve problems of multi-robot formation control [8]. In the light of basis of distributed position estimation, Oh presented a distributed formation control algorithm by using relative position and attitude measurements [9]. Additionally, Hawwary discussed the circular formation control of unicycles by making use of distributed control method on the basis of graph theory [10]. In another study, Alonso presented a distributed formation control method about navigating multi-robot systems in complex environments [11].

In practical applications, consensus-based formation control makes more sense compared with other formation control schemes due to inherent properties and scalability that enables the design formation to achieve objectives, even if some simpler robots fail. Accordingly, the consensus issue is a popular topic in multi-robot formation control and has been

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widely studied [12]–[18]. In the wake of extensively developed consensus theory, many experts found that consensus approaches are more suitable for handling formation control problems. Ren proposed multiple formation control methods on the basis of local neighboring information for linear multi-agent systems [19]. Meanwhile, Hawwary presented circular formation control methods for multi-agent systems [20].

Communication delay is another crucial problem in consensus-based formation control [21]–[24]. Designing an available and reasonable system with time-delay is a critical issue for theoretical research and practical applications. In recent years, the issue is concerned by numerous experts including Saber, who studied the average consensus issue for a dynamic system with communication delay [12]. Ren and Beard expanded the research contents based on the above harvest [25], while research into the consensus problem for 2-order system has been undertaken by some experts including Xie and Wang [26], Sun *et al.* [27], Liu *et al.* [28] and Liu *et al.* [29]. Relative consensus protocol and absolute consensus protocol has also been studied by Sun *et al.* [27] and Liu *et al.* [28], respectively.

In practice, there is both a cooperative and antagonistic relationship between agents. In view of this, the definition of bipartite consensus was put forward by Altanfini [30], in which the condition of achieving the bipartite consensus was also provided. On account of the condition of structural balance, the adjacency matrix was converted by means of a gauge transformation. After that, the bipartite consensus under negative weights was transformed into ordinary consensus problems. Based on Altanfini's research, Zhang analyzed the equivalence relation between ordinary consensus and bipartite consensus, [31]. Hu presented the requirement to guarantee the bipartite consensus [32], while bipartite consensus for the 2-order system was propoded in Li's paper [33], Qin firstly investigated the circular formation control problems using numerical optimization approaches [34]. In addition, other experts improved the corresponding results of bipartite consensus for 2-order system [35]–[37].

In view of previous works, the distributed multi-robot formation control problem is studied based on bipartite consensus under communication delays. The bipartite consensus problem for multi-agent systems is first investigated. The simultaneously cooperative and antagonistic relationship between agents will lead to system instability. Thus, a bipartite consensus protocol under time-varying delays is provided to solve this problem by means of designing the Laplacian matrix. Furthermore, the consensus issue is converted into the stability issue by means of a reduced-order method of the dynamic. Different from [26], bipartite consensus in coopetition networks is also studied in this paper, and a relative damping consensus protocol is adopted. Additionally, different from [33], the bipartite consensus in hypothesis condition under non-uniform time-delays is investigated. Finally, different from [5], [6], [8] and [9], the distributed multi-robot formation control issue is studied based on bipartite consensus under communication delays.

The reminder of this paper is structured as follows. The crucial problem of the paper and the bipartite consensus protocol are proposed in Section II. In Section III, the main results and proofs are provided and discussed. Distributed multi-robot formation control and simulation analysis based on bipartite consensus under communication delays is given in Section IV. Finally, conclusions are provided in Section V.

II. PROBLEM STATEMENT

A. GRAPH THEORY

The weighted graph $G = \{V, E, A\}$ is made up of three parts: a set of nodes $V = \{1, 2, \cdots, n\}$, a set of edges $E = \{(i,j) : i,j \in V, i \neq j\}$ and $A = \left[a_{ij}\right] \in \mathbb{R}^{n\times n}$ which is the weighted adjacency matrix. Denote the cooperative relationship from j to i by $a_{ij} > 0$, and the antagonistic relationship from j to i by $a_{ij} < 0$. Here, we assume that $(i, i) \notin E$ and hence $a_{ii} = 0$. The digraph is called digon sign-symmetric if $a_{ij}a_{ji} \geq 0$, and the digon sign-symmetric of signed digraphs are only considered in this paper. A directed path of the digraph $G(A)$ is denoted by

$$Q = \left\{(i_1,\ i_2),\ (i_2,\ i_3)\cdots,\ \left(i_{q-1},\ i_q\right)\right\} \subset E$$

where for $\forall\, q = 1, 2, \cdots, n$.

B. LINEAR CONSENSUS PROTOCOLS

The dynamics is described by

$$\dot{x}_i = v_i, \quad \dot{v}_i = u_i \tag{1}$$

where $i = 1, 2, \cdots, n$, $x_i \in \mathbb{R}^s$ is the position, $v_i \in \mathbb{R}^s$ is the velocity, $u_i \in \mathbb{R}^s$ represents the control input. Let $s = 1$ to facilitate the research.

The consensus protocol is described by:

$$\begin{aligned} u_i(t) = -\gamma_1 v_i(t) - \sum_{j\in\mathcal{N}_i} \Big\{ & \beta_1 \left|a_{ij}\right| \left[v_i\left(t-\tau_{ij}(t)\right)\right. \\ & \left. - sgn\left(a_{ij}\right) v_j\left(t-\tau_{ij}(t)\right)\right] + \beta_2 \left|a_{ij}\right| \\ & \times \left[x_i\left(t-\tau_{ij}(t)\right) - sgn\left(a_{ij}\right) x_j\left(t-\tau_{ij}(t)\right)\right]\Big\} \end{aligned} \tag{2}$$

where $\gamma_1, \beta_1, \beta_2$ are positive gains, and $0 \leq \tau_{ij}(t) \leq h$, $h > 0$, $\tau_{ij}(t)$ are unknown time-varying delays.

Definition 1: The bipartite consensus of protocol (2) is achieved if $\lim_{t\to\infty}\left[|x_i(t)| - \left|x_j(t)\right|\right] = 0$ and $\lim_{t\to\infty}|v_i(t)| = 0$, $\forall x_i(0), v_i(0)\ (i, j = 1, 2, \cdots, n)$.

When $\tau_{ij}(t) = 0$, the dynamics (1) can be abbreviated as

$$\dot{x} = v, \quad \dot{v} = -\gamma_1 v - \beta_1 L v - \beta_2 L x \tag{3}$$

where $v = [v_1, \cdots, v_n]^T$, $x = [x_1, \cdots, x_n]^T$, Laplacian matrix L is defined by

$$l_{is} = \begin{cases} \sum_{j\in\mathcal{N}_i} |a_{ij}|, & s = i \\ -a_{is}, & s \neq i \end{cases} \tag{4}$$

C. GAUGE TRANSFORMATION AND STRUCTURAL BALANCE

$G(A)$ is structurally balanced, if all nodes can be divided into V_1 and V_2, $V_1 \cap V_2 = \emptyset$, $V_1 \cup V_2 = V$, and $a_{ij} \geq 0$, $\forall i$, $j \in V_f (f \in \{1, 2\})$; $a_{ij} \leq 0$, $\forall i \in V_f$, $j \in V_b, f \neq b (f, b \in \{1, 2\})$.

Denote $\mathcal{P} = \{P | P = diag(\varsigma_1, \cdots, \varsigma_n), \varsigma_i \in \{\pm 1\}, i = 1, \cdots, n\}$. Consider the following transformation with the system (3):

$$\xi = Px, \quad \eta = Pv, \ P \in \mathcal{P} \tag{5}$$

Owing to $P^{-1} = P$, $x = P\xi$, $v = P\eta$, one has

$$\dot{\xi} = P\dot{x} = \eta$$

For system (3), since $\dot{v} = -\gamma_1 v - \beta_1 LP\eta - \beta_2 LP\xi$, it follows that

$$\dot{\eta} = P\dot{v} = -\gamma_1 \eta - \beta_1 L_P \eta - \beta_2 L_P \xi$$

where $L_P = PLP$ is defined as follows

$$l_{P,is} = \begin{cases} \sum_{j \in N_i} |a_{ij}|, & s = i \\ -\sigma_i \sigma_s a_{is}, & s \neq i \end{cases}$$

Lemma 1: [37] If the signed digraph $G(A)$ is structurally balanced and contains a spanning tree, then:

(a) $\exists\ P \in \mathcal{P}$, and all of the elements of PAP are non-negative, here, $\mathcal{P} = \{P | P = diag(\varsigma_1, \cdots, \varsigma_n), \varsigma_i \in \{\pm 1\}, i = 1, \cdots, n\}$;

(b) zero is the single eigenvalue of L.

III. MAIN RESULTS

A. NETWORKS WITHOUT TIME-DELAYS

According to equation (5), system (1) under protocol (2) $(\tau_{i,j}(t) = 0)$ can be described by

$$\dot{\xi} = \eta, \quad \dot{\eta} = -\gamma_1 \eta - \beta_1 L_P \eta - \beta_2 L_P \xi \tag{6}$$

Denote $\eta = [\eta_1, \cdots, \eta_n]^T$, $\xi = [\xi_1, \cdots, \xi_n]^T$, $\psi = [\xi^T, \eta^T]^T$. Equation (6) can be converted into

$$\dot{\psi}(t) = \begin{bmatrix} 0 & I_n \\ -\beta_2 L_P & -\gamma_1 I_n - \beta_1 L_P \end{bmatrix} \psi(t) \tag{7}$$

Remark 1: In view of structural balance for the digraph $G(A)$, $\exists P \in \mathcal{P}$ such that all of the elements of PAP are nonnegative, and zero is the single eigenvalue of $L_P = PLP$. According to equation (5), system (3) can be converted into (7). According to the definition of P and the structural balance for the digraph $G(A)$, $\lim_{t\to\infty} v_i(t) = 0$ iff $\lim_{t\to\infty} \eta_i(t) = 0$. In the circumstances, $\lim_{t\to\infty} [|x_i(t)| - |x_j(t)|] = 0$ iff $\lim_{t\to\infty} \xi_i(t) = \lim_{t\to\infty} \xi_j(t)$, $i, j = 1, \cdots, n$, $\forall x_i(0), v_i(0)$, $i, j \in \mathbb{N}$. Thus, the bipartite consensus of system (1) is asymptotically achieved iff $\lim_{t\to\infty} \eta_i(t) = 0$ and $\lim_{t\to\infty} [\xi_i(t) - \xi_j(t)] = 0$.

Proposition 1: Suppose a digraph $G(A)$ is structurally balanced and contains a spanning tree. System (1) with protocol (2) ($\tau_{ij}(t) = 0$) asymptotically achieves the bipartite consensus iff a positive definite matrix H exists, such that

$$\begin{bmatrix} 0 & I_{n-1} \\ -\beta_2 F L_P E & -\gamma_1 I_n - \beta_1 F L_P E \end{bmatrix} H + H \begin{bmatrix} 0 & I_{n-1} \\ -\beta_2 F L_P E & -\gamma_1 I_n - \beta_1 F L_P E \end{bmatrix} < 0 \tag{8}$$

where $H \in \mathbb{R}^{(2n-1)\times(2n-1)}$, $F = \begin{bmatrix} 1 & -I_{n-1} \end{bmatrix}$, $E = \begin{bmatrix} 0 & -I_{n-1} \end{bmatrix}^T$.

Proof: Suppose

$$\tilde{\xi}_i = \xi_1 - \xi_{i+1}, \tilde{\xi} = [\tilde{\xi}_1, \cdots, \tilde{\xi}_{n-1}]^T \tag{9}$$

$$\tilde{\eta} = [\tilde{\eta}_1, \cdots, \tilde{\eta}_{n-1}]^T \tag{10}$$

One can obtain $\tilde{\xi} = F\xi$, $\tilde{\eta} = F\eta$. Thus

$$\dot{\tilde{\xi}} = \tilde{\eta} \tag{11}$$

$$\dot{\tilde{\eta}} = F\dot{\eta} = -\gamma_1 \tilde{\eta} - \beta_1 F L_P E \tilde{\eta} - \beta_2 F L_P E \tilde{\xi} \tag{12}$$

Therefore

$$\begin{bmatrix} \dot{\tilde{\xi}} \\ \dot{\tilde{\eta}} \end{bmatrix} = \begin{bmatrix} 0 & I_{n-1} \\ -\beta_2 F L_P E & -\gamma_1 I_{n-1} - \beta_1 F L_P E \end{bmatrix} \cdot \begin{bmatrix} \tilde{\xi} \\ \tilde{\eta} \end{bmatrix} \tag{13}$$

It follows that $\lim_{t\to\infty} [\xi_i(t) - \xi_j(t)] = 0$ is that is to $\lim_{t\to\infty} \tilde{\xi}_i(t) = 0$. Thus, the issue of bipartite consensus can be transformed into the stability issue.

Let $\tilde{\psi} = \begin{bmatrix} \tilde{\xi}^T & \tilde{\eta}^T \end{bmatrix}^T$, then

$$\dot{\tilde{\psi}} = \begin{bmatrix} 0 & I_{n-1} \\ -\beta_2 F L_P E & -\gamma_1 I_{n-1} - \beta_1 F L_P E \end{bmatrix} \tilde{\psi} \tag{14}$$

and $\tilde{\psi}(t) = B\psi(t)$, where $B = diag\{F, I_n\}$, and $rank(B) = 2n - 1$. If $\psi_i(t)$ is a linear independent solution of system (7), where $i = 1, 2, \cdots, 2n$, then $B\psi_i$ are the solutions.

Suppose that system (1) with protocol (2) asymptotically achieves the bipartite consensus, i.e., $\lim_{t\to\infty} [\xi_i(t) - \xi_j(t)] = 0$ and $\lim_{t\to\infty} \eta_i(t) = 0$. Then, $\lim_{t\to\infty} B\psi_i(t) = 0$, and accordingly (8) holds.

Moreover, the asymptotic stability of system (14) is achieved if (8) holds, i.e. $\lim_{t\to\infty} \tilde{\psi}_i(t) = \lim_{t\to\infty} B\psi_i(t) = 0$, that is

$$\lim_{t\to\infty} [\xi_1(t) - \xi_i(t)] = 0, \ \lim_{t\to\infty} \eta_i(t) = 0$$

Thus

$$\lim_{t\to\infty} [\xi_i(t) - \xi_j(t)] = \lim_{t\to\infty} [\xi_i(t) - \xi_1(t)] + \lim_{t\to\infty} [\xi_1(t) - \xi_j(t)] = 0$$

i.e. $\lim_{t\to\infty} [|x_i(t)| - |x_j(t)|] = 0$ and $\lim_{t\to\infty} v_i(t) = 0$. Therefore, system (1) with protocol (2) asymptotically achieves the bipartite consensus.

B. NETWORKS WITH TIME-VARYING DELAYS

When $\tau_{ij} \neq 0$, let $\tau_{ij}(t) \in \{\tau_s(t) : s = 1, 2, \cdots, m\}$, where $0 \leq \tau_s(t) \leq k$, $\dot{\tau}_s(t) \leq \rho$, $\tau_s(t)$ are piecewise continuous time-varying delays, then, under protocol (2) and transformation (4), system (1) can be written as

$$\dot{\psi}(t) = \Phi\psi(t) + \sum_{s=1}^{m} L_s\psi(t - \tau_s(t)) \tag{15}$$

where

$$\Phi = \begin{bmatrix} 0 & I_n \\ 0 & -\gamma_1 I_n \end{bmatrix}, \quad \mathcal{L}_s = \begin{bmatrix} 0 & 0 \\ -\beta_2 L_s & -\beta_1 L_s \end{bmatrix}$$

and $L_s = \left[l_{ij}^{(s)}\right]$ are defined as

$$l_{ij}^{(s)} = \begin{cases} \sum\limits_{\kappa=1,\kappa\neq i}^{n} |a_{i\kappa}| I_{\tau_s(\cdot)=\tau_{ij}(\cdot)}, & j = i \\ 0, & j \neq i, \tau_s(\cdot) = \tau_{ij}(\cdot) \\ -\sigma_i\sigma_j a_{ij}, & j \neq i, \tau_s(\cdot) \neq \tau_{ij}(\cdot). \end{cases}$$

$G(A)$ is structurally balanced, Owing to $L_s 1 = 0$, $s = 1, 2, \cdots, m$, and $\sum_{s=1}^{m} L_s = L_P$, it follows from (9) and (10) that, the reduced-order system of (15) can be written as

$$\dot{\tilde{\psi}} = \tilde{\Phi}\tilde{\psi}(t) + \sum_{s=1}^{m} \tilde{L}_s\tilde{\psi}(t - \tau_s(t)) \tag{16}$$

where

$$\tilde{\Phi} = \begin{bmatrix} 0 & E \\ 0 & -\gamma_1 I_n \end{bmatrix}, \quad \tilde{L}_s = \begin{bmatrix} 0 & 0 \\ -\beta_2 F L_s E & -\beta_1 F L_s E \end{bmatrix}$$

Lemma 2: System (2) with protocol (3) asymptotically achieves the bipartite consensus iff every solution of system (16) tends to 0.

Proof: The sufficiency is obvious.

Necessity: Denote $\tilde{\psi}$ is a solution, suppose that $\tilde{\psi} \nrightarrow 0$ when t tends to ∞, and $\tilde{\psi}^T(t) = [\tilde{\xi}_1(t) \cdots \tilde{\xi}_{n-1}(t)\ \tilde{\eta}_1(t) \cdots \tilde{\eta}_n(t)]$. Let $\xi_1(t)$ and $\eta_1(t)$ satisfy $\dot{\xi}_1(t) = \eta_1(t)$. Let $\xi_{i+1}(t) = \xi_1(t) - \tilde{\xi}_i(t)$, $i = 1, 2, \cdots, n-1$. From (16), one has

$$\begin{cases} \dot{\xi}_1(t) = \eta_1(t) \\ \dot{\xi}_{i+1}(t) = \dot{\xi}_1(t) - \dot{\tilde{\xi}}_i(t) = \eta_{i+1}(t) \end{cases} \tag{17}$$

From (16) and $L_s 1 = 0$, one can obtain

$$\begin{aligned} \dot{\tilde{\eta}}(t) &= -\gamma_1\tilde{\eta}(t) - \sum_{s=1}^{m} \beta_2 F L_s E\tilde{\xi}(t - \tau_s(t)) \\ &\quad - \sum_{s=1}^{m} \beta_1 F L_s E\tilde{\eta}(t - \tau_s(t)) \\ &= -\gamma_1 F\eta(t) - \sum_{s=1}^{m} \beta_2 F L_s \left[\xi_1(t - \tau_s(t))\mathbf{1} \right. \\ &\quad \left. + E\tilde{\xi}(t - \tau_s(t))\right] \\ &\quad - \sum_{s=1}^{m} \beta_1 F L_s \left[\eta_1(t - \tau_s(t))|1 \right. \\ &\quad \left. + E\tilde{\eta}(t - \tau_s(t))\right] \\ &= -\gamma_1 F\eta(t) - \sum_{s=1}^{m} \beta_2 F L_s\xi(t - \tau_s(t)) \\ &\quad - \sum_{s=1}^{m} \beta_1 F L_s\eta(t - \tau_s(t)) \end{aligned} \tag{18}$$

From (17) and (18), one can obtain that $\psi(t)$ is a solution of system (15). Owing to $\lim_{t\to\infty} \tilde{\psi}(t) \neq 0$, $\exists\, i, j$, such that $\lim_{t\to\infty} \eta_i \neq 0$ or

$$\begin{aligned} &\lim_{t\to\infty} \left[\xi_i(t) - \xi_j(t)\right] \\ &\quad = \lim_{t\to\infty} [\xi_i(t) - \xi_1(t)] + \lim_{t\to\infty} \left[\xi_1(t) - \xi_j(t)\right] \\ &\quad \neq 0 \\ &\lim_{t\to\infty} \left[\eta_i(t) - \eta_j(t)\right] \\ &\quad = \lim_{t\to\infty} [\eta_i(t) - \eta_1(t)] + \lim_{t\to\infty} \left[\eta_1(t) - \eta_j(t)\right] \\ &\quad \neq 0 \end{aligned}$$

There is a contradiction between the analysis result and the initial assumption. For the solution $\psi(t)$, the bipartite consensus for system (1) cannot be asymptotically achieved.

Lemma 3 [27]: Let $x(t) \in \mathbb{R}^{n\times n}$ be any real differentiable vector function and $B = B^T \in \mathbb{R}^{n\times n}$ be any positive definite constant matrix, one has

$$\begin{aligned} &k\int_{t-\tau_s(t)}^{t} \dot{x}^T(q) B\dot{x}(q)\, dq \\ &\quad \geq [x(t) - x(t - \tau_s(t))]^T B [x(t) - x(t - \tau_s(t))] \end{aligned}$$

where $0 \leq \tau_s(t) \leq k$.

Theorem 1: Let $0 \leq \tau_s(t) \leq k$, $\dot{\tau}_s(t) \leq \rho$, where $k > 0$. Supposing that the signed digraph $G(A_u)$ is structurally balanced and contains a spanning tree. The bipartite consensus of system (1) under protocol (2) is asymptotically achieved if there are positive definite matrices ξ, η_s, $R_s \in \mathbb{R}^{(2n-1)\times(2n-1)}$, such that

$$\begin{bmatrix} \tilde{\Phi}^T\xi + \xi\tilde{\Phi} + \sum\limits_{s=1}^{m}\eta_s - \sum\limits_{s=1}^{m} R_s & \xi\tilde{L} + \tilde{R} \\ \tilde{L}^T\xi + \tilde{R}^T & -(1-\rho)\hat{\eta} - \hat{R} \end{bmatrix} + k^2 \begin{bmatrix} \tilde{\Phi}^T \\ \tilde{L}^T \end{bmatrix} \left(\sum_{s=1}^{m} R_s\right) \begin{bmatrix} \tilde{\Phi} & \tilde{L} \end{bmatrix} < 0 \tag{19}$$

where

$$\hat{\eta} = diag\{\eta_1, \eta_2, \cdots, \eta_s\}, \quad \hat{R} = diag\{R_1, R_2, \cdots, R_s\}$$
$$\tilde{R} = \left[\tilde{R}_1\, \tilde{R}_2 \cdots \tilde{R}_m\right], \quad \tilde{L} = \left[\tilde{L}_1\, \tilde{L}_2 \cdots \tilde{L}_m\right]$$

$\tilde{\Phi}$, $\tilde{L}_s$ are designed as above.

Proof: A Lyapunov-Krasovskii functional is taken by

$$\begin{aligned} V(t) &= \tilde{\psi}^T(t)\xi\tilde{\psi}(t) + \sum_{s=1}^{m}\int_{t-\tau_s(t)}^{t} \tilde{\psi}^T(q)\eta_s\tilde{\psi}(q)\, dq \\ &\quad + k\sum_{s=1}^{m}\int_{-k}^{0}\int_{t+\delta}^{t} \dot{\tilde{\psi}}^T(q) R_s\dot{\tilde{\psi}}(q) dq d\delta \end{aligned} \tag{20}$$

From (16) and $L_s 1 = 0$, one can obtain

$$\begin{aligned}
\dot{V}(t) &= \tilde{\psi}^T(t)\left[\tilde{\Phi}^T\xi + \xi\tilde{\Phi}\right]\tilde{\psi}(t) \\
&\quad + 2\sum_{s=1}^{m}\tilde{\psi}^T(t)\xi\tilde{L}_s\tilde{\psi}(t-\tau_s(t)) \\
&\quad + \sum_{s=1}^{m}\Big[\tilde{\psi}^T(t)\,\eta_s\tilde{\psi}(t) - (1-\dot{\tau}_s(t)) \\
&\quad \times \tilde{\psi}^T(t-\tau_s(t))\,\eta_s\tilde{\psi}(t-\tau_s(t))\Big] \\
&\quad + k\sum_{s=1}^{m}\Big[g\dot{\tilde{\psi}}^T(t)\,R_s\dot{\tilde{\psi}}(t) \\
&\quad - \int_{t-g}^{t}\dot{\tilde{\psi}}^T(q)\,R_s\dot{\tilde{\psi}}(q)\,dq\Big] \\
&\leq \tilde{\psi}^T(t)\left[\tilde{\Phi}^T\xi + \xi\tilde{\Phi}\right]\tilde{\psi}(t) \\
&\quad + 2\sum_{s=1}^{m}\tilde{\psi}^T(t)\xi\tilde{L}_s\tilde{\psi}(t-\tau_s(t)) \\
&\quad + \sum_{s=1}^{m}\tilde{\psi}^T(t)\,\eta_s\tilde{\psi}(t) - \Big[(1-\rho) \\
&\quad \times \sum_{s=1}^{m}\tilde{\psi}^T(t-\tau_s(t))\,\eta_s\tilde{\psi}(t-\tau_s(t))\Big] \\
&\quad + k^2\sum_{s=1}^{m}\dot{\tilde{\psi}}^T(t)\,R_s\dot{\tilde{\psi}}(t) \\
&\quad - \sum_{s=1}^{m}\Big\{\left[\tilde{\psi}(t)-\tilde{\psi}(t-\tau_s(t))\right]^T \\
&\quad R_s\left[\tilde{\psi}(t)-\tilde{\psi}(t-\tau_s(t))\right]\Big\}
\end{aligned} \tag{21}$$

From (21) and (16), one can obtain that $\dot{V}(t) < 0$. Thus, system (16) is asymptotically stable.

From the previous analysis, it can be seen that the bipartite consensus of system (1) with protocol (2) are asymptotically achieved.

IV. FORMATION CONTROL OF MOBILE ROBOT

In this section, the bipartite consensus protocol with time-varying delay is applied for distributed multi-robot formation control.

A. MODEL OF MOBILE ROBOT

The coordinate systems and geometric parameters of mobile robot are provided in Fig. 1, in which R is the radius of driving wheel, and $2l$ denotes the width of the robot platform. The coordinate system XQ_0Y is fixed on robot platform, xOy denotes the world coordinate system, and Q_0 is the centre of robot's two driving wheels. Additionally, Q_C is the barycenter of the robot platform, and b denotes the distance between Q_C and Q_0.

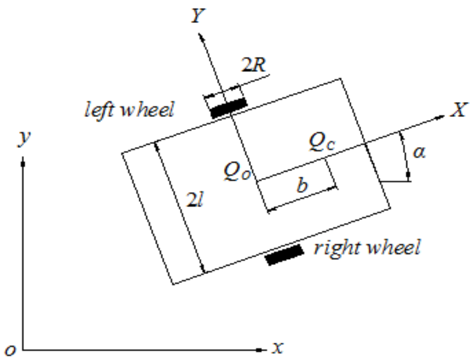

FIGURE 1. Two-wheeled mobile robot.

The configuration of the robot is described by the following generalized coordinates:

$$p = [x, y, \alpha, \phi_r, \phi_l]^T \tag{22}$$

where α denotes the course angle of robot platform, (x, y) mean the coordinates of Q_0, and ϕ_l, ϕ_r denote the corresponding angles of left and right driving wheels.

It is assumed that the moving mode of driving wheels is the pure rolling. Two constraints then occur: the slip phenomenon of driving wheels does not exist, and Q_0 must move along the symmetrical axis.

$$\begin{cases} \dot{y}\cos\alpha - \dot{x}\sin\alpha = 0 \\ \dot{x}\cos\alpha + \dot{y}\sin\alpha + l\dot{\alpha} = R\dot{\phi}_r \\ \dot{x}\cos\alpha + \dot{y}\sin\alpha - l\dot{\alpha} = R\dot{\phi}_l \end{cases} \tag{23}$$

From (23), the kinematic model of the mobile robot can be described as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha} \\ \dot{\phi}_r \\ \dot{\phi}_l \end{bmatrix} = \begin{bmatrix} \frac{R}{2}\cos\alpha & \frac{R}{2}\cos\alpha \\ \frac{R}{2}\sin\alpha & \frac{R}{2}\sin\alpha \\ \frac{R}{2l} & -\frac{R}{2l} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{24}$$

where v_1 and v_2 are angular velocities of two driving wheels.

The three states x,y,α are only considered in this section. The relationship between v_1,v_2 and v, ω is

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & \frac{l}{R} \\ \frac{1}{R} & -\frac{l}{R} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{25}$$

where ω, v denote the angular velocity and the straight line velocity of robot platform at point Q_0, respectively.

Substituting (24) for (25) obtains the ordinary form of a mobile robot

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{26}$$

B. FORMATION CONTROL OF MOBILE ROBOT WITH TWO ACTUATED WHEELS

Based on the above analysis, robot's kinematic equations are described as [31]:

$$\begin{cases} \dot{x} = v\cos\alpha \\ \dot{y} = v\sin\alpha \\ \dot{\alpha} = \omega \\ m\dot{v} = f \\ J\dot{\omega} = \tau \end{cases} \tag{27}$$

where (x, y) denotes robot's Cartesian position, v, ω mean the linear and angular velocity, respectively; α, m are the orientation and the mass of the robot; f, J denote the force and the mass moment of inertia; τ is the torque acted on the robot. Here, friction effects have been neglected.

Another reference is defined here that is not in robot's center of rotation.

$$\begin{cases} x_i = x + b\cos\alpha \\ y_i = y + b\sin\alpha \end{cases} \tag{28}$$

where (x_i, y_i) are the coordinates of Q_c.

When x, y are derived, v and ω can be expressed on the basis of $\dot{x}$ and $\dot{y}$:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\frac{1}{b}\sin\alpha & \frac{1}{b}\cos\alpha \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} \tag{29}$$

We let

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\cos\alpha & \frac{b}{J}\sin\alpha \\ \frac{1}{m}\sin\alpha & \frac{b}{J}\cos\alpha \end{bmatrix}^{-1} \times \begin{bmatrix} v_x + v\omega\sin\alpha + b\omega^2\cos\alpha \\ v_y - v\omega\cos\alpha + b\omega^2\sin\alpha \end{bmatrix} \tag{30}$$

where v_x, v_y denote x and y component of velocity, respectively.

we obtain the following equations of motion:

$$\begin{cases} \dot{x}_i = v_{xi} \\ \dot{v}_{xi} = u_{xi} \\ \dot{y}_i = v_{yi} \\ \dot{v}_{yi} = u_{yi} \end{cases} \tag{31}$$

As follows, the design of u_{xi} and u_{yi} is subsequently discussed on our paper. In addition, (31) means the movable equation about a mobile robot with holonomic constraints.

Seven robots with locomotivity are required to move from initial position to preset destinations in this paper. Meanwhile, Fig. 2 shows the information exchange topology about seven robots, and directed edge ($i \to j$ robot) denotes that $x_i - x_{id}$, $y_i - y_{id}$, $\dot{x}_{id}$ and $\dot{y}_{id}$ can be acquired by the jth robot. For the sake of making (x_{id}, y_{id}) become a ideal destination about the ith robot, a control law between u_{xi} and u_{yi} is put forward as follows:

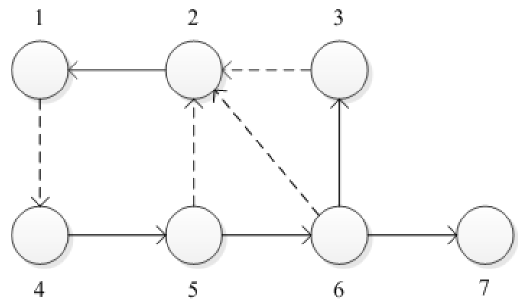

FIGURE 2. Communication topology.

$$\begin{cases} u_{xi}(t) = -\gamma_1 v_{xi} - \sum_{j\in N_i}\{\beta_1 |a_{ij}| [v_{xi}(t-\tau_{ij}(t)) \\ \qquad -sgn(a_{ij}) v_{xj}(t-\tau_{ij}(t))]\} \\ \qquad - \sum_{j\in N_i}\{\beta_2 |a_{ij}| [(x_i - x_{id})(t-\tau_{ij}(t)) \\ \qquad -sgn(a_{ij})(x_j - x_{jd})(t-\tau_{ij}(t))]\} \\ u_{yi}(t) = -\gamma_1 v_{yi} - \sum_{j\in N_i}\{\beta_1 |a_{ij}| [v_{yi}(t-\tau_{ij}(t)) \\ \qquad -sgn(a_{ij}) v_{yj}(t-\tau_{ij}(t))]\} \\ \qquad - \sum_{j\in N_i}\{\beta_2 |a_{ij}| [(y_i - y_{id})(t-\tau_{ij}(t)) \\ \qquad -sgn(a_{ij})(y_j - y_{jd})(t-\tau_{ij}(t))]\} \end{cases} \tag{32}$$

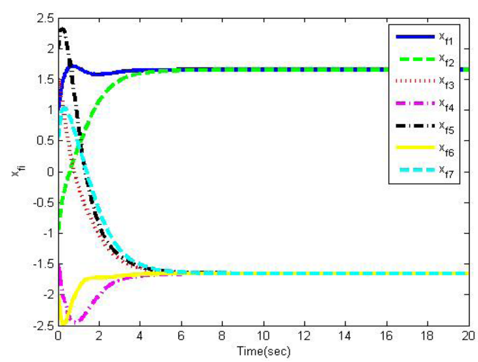

FIGURE 3. x-direction position trajectories of the seven robots with $\tau_{ij} = 0$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

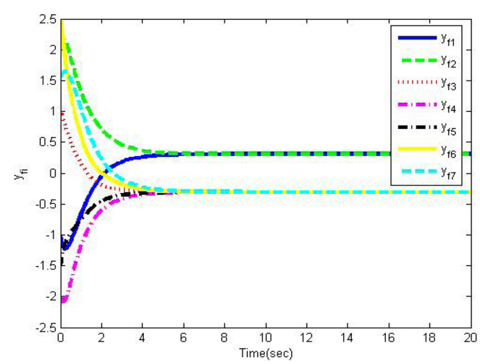

FIGURE 4. y-direction position trajectories of the seven robots with $\tau_{ij} = 0$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

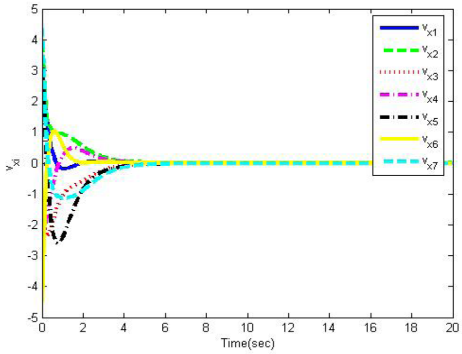

FIGURE 5. x-direction velocity trajectories of the seven robots with $\tau_{ij} = 0$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

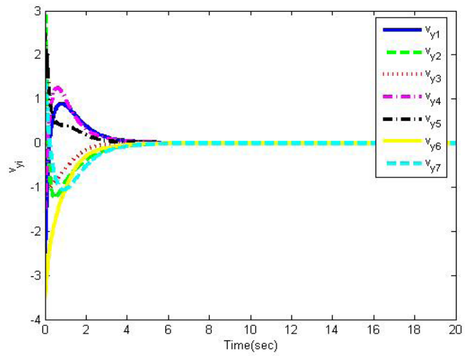

FIGURE 6. y-direction velocity trajectories of the seven robots with $\tau_{ij} = 0$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

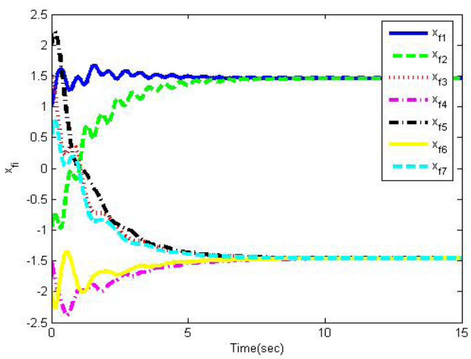

FIGURE 7. x-direction position trajectories of the seven robots with $\tau_{ij} = 0.1474$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

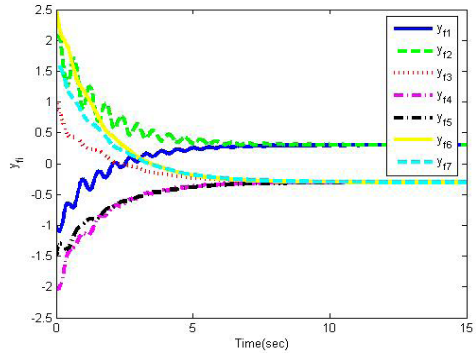

FIGURE 8. y-direction position trajectories of the seven robots with $\tau_{ij} = 0.1474$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

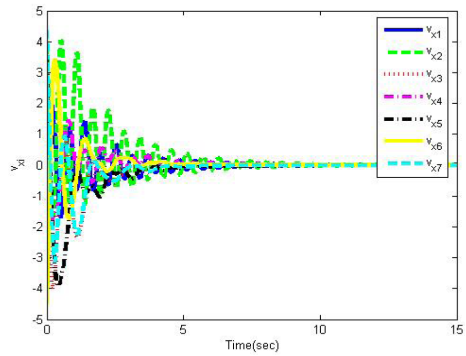

FIGURE 9. x-direction velocity trajectories of the seven robots with $\tau_{ij} = 0.1474$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

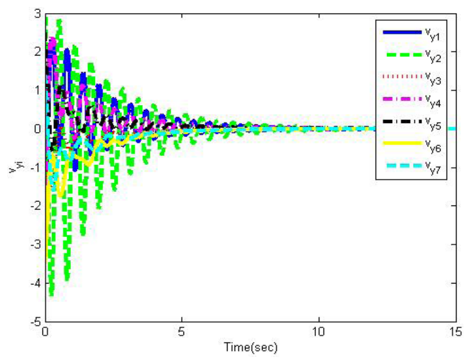

FIGURE 10. Y-direction velocity trajectories of the seven robots with $\tau_{ij} = 0.1474$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

Note that equation (32) is taken advantage of ensuring that an ideal formation among seven robots is maintained in the process of the transition.

C. SIMULATION RESULTS

As shown in Fig. 2, the solid and the dotted line between nodes denote the cooperative and the competitive relationship respectively, and the corresponding weight is taken as 1 and −1. Let $x_f = x_i - x_{id}$, $y_f = y_i - y_{id}$, $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$, $x_f(0)$=[1 −1 1.5 −1.5 2 −2 0.5], $y_f(0)$=[−1 2 1 −2 −1.5 2.5 1.5], $v_x(0)$=[2.5 4 −1 −2.5 3.5 −4.5 4.5], $v_y(0)$=[−2.5 3 −1 −1.5 2.5 −3.5 1.5]. When $\tau_{ij} = 0$, Fig. 3, Fig. 4 and Fig. 5, Fig. 6 respectively illustrate the position and velocity trajectories of each robot under control law (32).

If $\tau_{ij} \neq 0$, let $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$. Solving (19) produces $k = 0.1474$, where k is the upper bound of the time-varying delays. Fig. 7 and Fig. 8 show the motion curves of each robot's position, Fig. 9 and Fig. 10 show the motion curves of each robot's velocity under control law (32). The position trajectories of each robot under control law (32) is illustrated in Fig. 11, in which each robot of the same weight is able to reach their destination.

In the second case, we set $\gamma_1 = 0$, $\beta_1 = 2$, $\beta_2 = 2$. By solving (19), it can be determined that $k = 0.1598$.

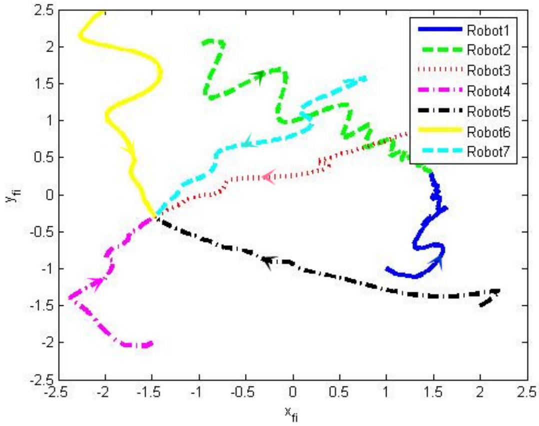

FIGURE 11. Position trajectories of the seven robots with $\tau_{ij} = 0.1474$ and $\gamma_1 = 1$, $\beta_1 = 3$, $\beta_2 = 2$.

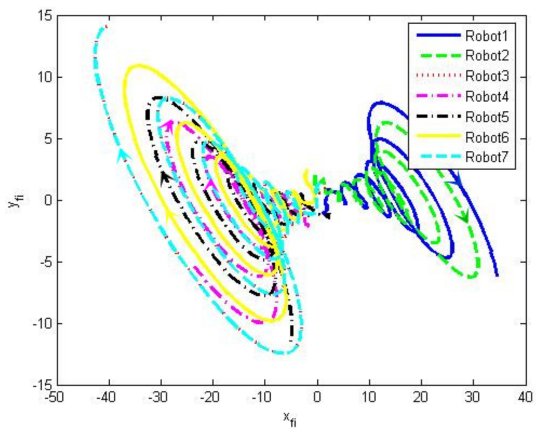

FIGURE 12. Position trajectories of the seven robots with $\tau_{ij} = 0.1598$ and $\gamma_1 = 0$, $\beta_1 = 0$, $\beta_2 = 2$.

The position trajectories of seven robots are provided in Fig. 12. It can be seen that the desired formation among seven robots is maintained even though the information exchange topologies are switching with time [38].

V. CONCLUSION

Distributed multi-robot formation control problems based on bipartite consensus for multi-agent systems with time-varying delays were studied in this paper. Bipartite consensus for multi-agent systems under the second-order dynamics was presented, containing a relationship between agents that is both cooperative and antagonistic. A necessary and sufficient condition was also presented to asymptotically achieve the bipartite consensus for a multi-agent system without time-delays. For the system with time-delays, the bipartite consensus issue was solved by designing Laplacian matrix. Bipartite consensus protocol with time-varying delays was then applied to distributed formation control of multi-robot systems. In future research work, the relationship between the values of the parameters γ_1, β_1, β_2, and the multi-robot formation will be studied.

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