IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received August 24, 2019, accepted September 11, 2019, date of publication September 20, 2019, date of current version October 3, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2942822

A Novel Signal Detector in MIMO Systems Based on Complex Correntropy

PEDRO T. V. DE SOUZA[1], **ALUÍSIO I. R. FONTES**[2], **VINÍCIUS S. V. DE SOUZA**[1], **AND LUIZ F. SILVEIRA**[3]

[1]Engineering and Technology Department, Federal Rural University of Semi-Arid, Pau dos Ferros-RN 59900-000, Brazil
[2]Federal Institute of Education, Science and Technology of Rio Grande do Norte, Pau dos Ferros-RN 59900-000, Brazil
[3]Department of Computer Engineering and Automation, Federal University of Rio Grande do Norte, Natal-RN 59064-741, Brazil

Corresponding author: Pedro T. V. de Souza (pedro.souza@ufersa.edu.br)

This work was supported in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-Brasil (CAPES)-Finance Code 001, and in part by the High-Performance Computing Center at UFRN (NPAD/UFRN).

ABSTRACT In cellular systems, information signals must be transmitted at high rates and with high reliability. One of the possible solutions to meet such criteria is the use of systems with multiple transmitting and/or receiving antenna arranged in the form of a multiple-input, multiple-output (MIMO) system. However, signal processing techniques in MIMO systems are developed under the assumption of transmission on Gaussian channels, which may lead to the decrease of efficiency in non-Gaussian communication scenarios. In this context, the widespread use of MIMO systems in recent years has motivated the development of new processing techniques that can be employed in scenarios that also consider the presence of non-Gaussian noise in communication channels. This work proposes a novel signal detection technique for MIMO systems, which is called maximum correntropy detector (MCD), being adequate to environments characterized by Gaussian and non-Gaussian noise. The introduced approach is based on complex correntropy function and can be seen as a generalization of the maximum likelihood detector (MLD) concept. The MCD is evaluated on Gaussian and non-Gaussian channels, where superior performance is achieved when compared with the classic detectors, without significant increase of the computational complexity.

INDEX TERMS Complex correntropy, maximum correntropy detector, multiple-input multiple-output systems, non-Gaussian noise, signal detectors.

I. INTRODUCTION

In last-generation cellular systems, information signals must be transmitted at high rates with high reliability [1]. However, information signals are affected by various degenerative channel effects, which degrade overall system performance [2]. When a certain interference threshold is reached, any added power to the signal will not result in improvements to the communication system [3]. In this case, minimizing such undesirable effects can be performed using appropriate transmission techniques. Among the existing approaches, systems based on multiple transmitting and/or receiving antennas, named as multiple-input, multiple-output (MIMO) systems, have been consolidating in recent years [4].

Minimizing the degenerative effects of the communication channel in MIMO systems can be accomplished by using a set of transmitting and receiving antennas separated by some wavelengths in a spatial diversity scheme [4]. This arrangement can be particularly useful because it generates information diversity without compromising the transmitted signal bandwidth [5].

In order to improve performance of MIMO systems, several detection and reception techniques of signals have been proposed so far, e.g., maximum likelihood detector (MLD), minimum mean square error (MMSE), zero-forcing (ZF) [6] or robust log-likelihood ratio method [7]. Although these are classical and well-consolidated solutions in the literature, such detectors were proposed for Gaussian communication channels [4], [6]. However, in many practical systems, the noise profile can also be characterized by the presence of outliers, which characterize a non-Gaussian channel [8], [9]. In fact, according to [10]–[12], there is a significant deterioration in system performance due to the influence of non-Gaussian noise, which directly affects the methods based on the second-order statistical moments.

Over time various statistical models have been proposed to describe a non-Gaussian noise. One of the statistical distributions most widely used for this purpose is the stable

The associate editor coordinating the review of this manuscript and approving it for publication was Pietro Savazzi.

distribution [13]. The stable distribution can be understood as a generalization for the Gaussian distribution [14] and by adjusting the free parameters of this distribution, it is possible to define how impulsive is the noise of the modeled channel [15].

Recently, some works have investigated the MIMO transmission technique in non-Gaussian scenarios. The authors in [16] presented an adaptive reception technique based on the use of an impulsive noise level detector. On the other hand, the work developed in [17] introduces an alternative solution to the same problem using an adaptive reception technique through adaptive recursive least mean square (RLS), adaptive normalized least mean square (NLMS), and variable step-size adaptive normalized least mean (VSNLMS) algorithms, which aim to minimize the effect of impulsive noise on the system performance. In [18] the authors used a nonlinear complex Multiple Support Vector Machine Regression (M-SVR) methodology for estimation of fast-fading multipath channel.

In this context, this work introduces a novel detection method for MIMO systems, called Maximum Correntropy Detector (MCD). The proposed detector uses complex correntropy for properly choosing a transmitted symbol on a channel characterized by non-Gaussian noise and time-variant fading. It is also worth mentioning that the correntropy function has been successfully used in various applications involving non-Gaussian signals, e.g., spectral sensing [19], automatic modulation classification [20], pathological voice recognition patterns [21], non-linear system identification [22] and non-linear self-interference cancellation in full-duplex radio systems [23]. The proposed detector employs the maximum complex correntropy criterion for decision making on symbols transmitted on a channel subject to impulsive noise and time-variant fading. The results demonstrate that the proposed reception technique is a generalization of the MLD detector concept, also presenting good performance when compared with the classical methods reported in the literature. Beside, this work investigates the influence of the kernel size, a parameter of the correntropy function, on the performance of the MCD and proposes a method to select this parameter as a function of the measured geometric signal-to-noise ratio.

A. CONTRIBUTIONS

The main contributions of this work include:

1) The analysis of signal detection techniques in MIMO systems considering scenarios characterized by impulsive noise;
2) Introduction of a novel signal detection technique for MIMO systems through the use of correntropy;
3) Analysis of the proposed technique in scenarios characterized by impulsive noise with α-stable distribution and Gaussian channels;
4) Modeling of the adaptive kernel size for the proposed problem;

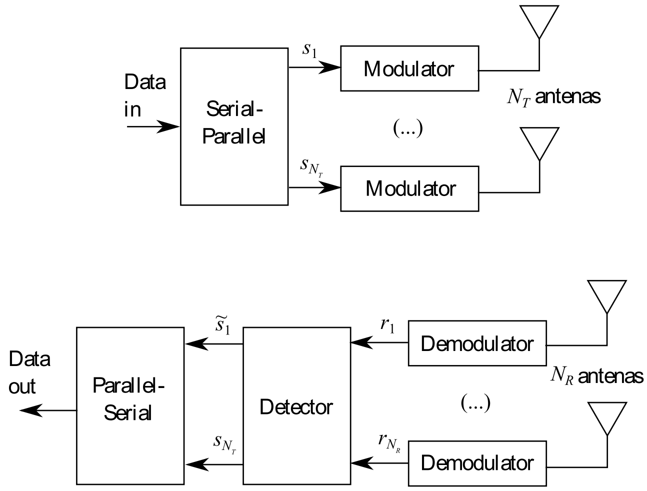

FIGURE 1. Adopted MIMO transmission scheme.

5) Demonstration that MCD is a generalization of the MLD.

B. PAPER ORGANIZATION

The remainder of this paper is organized as follows. Section II presents the system model in terms of the MIMO representation and the non-Gaussian noise model. Section III introduced the mathematical analysis and architecture of the proposed detector. Section IV describes numerical results obtained by simulation. The main conclusions are given in Section V.

II. SYSTEM MODEL

The MIMO digital transmission system is represented in Figure 1 [4]. At the transmitter, the information source is applied to a serial-to-parallel converter, which divides the original sequence into N_T parallel sequences. Each one of the sequences is modulated and transmitted by a different antenna. There are N_R antennas at the receiver, spaced so that the received signals can be considered independent of each other. Each one of the N_R received sequences is demodulated and applied to the MIMO detector. The detector output generates estimates of the transmitted parallel sequences ($\tilde{s}_k$, where $k = 1, \ldots, N_R$), which are applied to a parallel-to-serial converter in order to combine the estimates to obtain the original transmitted signal.

Considering a MIMO system with N_T transmitting antennas and N_R receiving antennas, the k-th symbol received by the m-th antenna is given by [24]:

$$y_m(t) = \sum_{n=1}^{N_T} s_n(t)h_{mn}(t)p(t) + \eta_m(t)$$
$$m = 1, \ldots, N_R, \quad kT_s \leq t < (k+1)T_s, \quad (1)$$

where $s_n(t)$ is the symbol transmitted by the n-th antenna, which is obtained from a phase-shift keying (PSK) or quadrature amplitude modulation (QAM) scheme, $h_{mn}(t)$ is the attenuation due to the channel between the n-th transmitting antenna and the m-th receiving antenna, $\eta_m(t)$ is the channel noise, $p(t)$ is a rectangular pulse, and T_s is the signaling interval.

In this work, it is assumed that the coefficients $h_{mn}(t)$ remain constant during each signaling interval ($kT_s \leq t < (k+1)T_s$). Besides, the transmitter employs a perfect interleaving scheme so that the signaling coefficients $h_{mn}(t)$ are uncorrelated to each other at each interval, being defined by:

$$h_{mn}(t) = h_{mn,r}(t) + jh_{mn,q}(t) \quad kT_s \leq t < (k+1)T_s, \qquad (2)$$

where $h_{mn,r}(t)$ and $h_{mn,q}(t)$ are modeled as Gaussian processes with zero mean and variance 1/2, so that the magnitude of $h_{mn}(t)$ presents a Rayleigh distribution and the phase has uniform distribution over $[0, 2\pi)$ [25].

A. SIGNAL DETECTION

Once received and demodulated, the samples $y_m(t)$ must be applied to a detector in order to estimate the transmitted symbols [26]. Several detectors have been proposed in the literature, while this work adopts the MLD as a reference, which is adequate for minimizing the error probability on channels characterized by additive white Gaussian noise(AWGN) [1]. In this case, symbol $\hat{\mathbf{s}}_{MLD}$ estimated by the detector is given by [24]:

$$\hat{\mathbf{s}}_{MLD} = \underset{\mathbf{s} \in \Omega^{N_T}}{\arg} \min \left| y_m - \sum_{n=1}^{N_T} h_{mn} s_n \right|^2, \qquad (3)$$

where s_n represents one element in the set of possible constellation symbols used in the transmission and Ω is the possible subspace containing all transmitted symbols, i.e., $s_n \in \Omega$.

One possible drawback associated with the MLD lies in the fact that it is not adequate to impulsive noise channels. Besides, the computational complexity increases as the number of receiving antennas also does [24].

B. IMPULSIVE NOISE

Typically, the additive interference model adopted in communication systems is the Gaussian white noise (GWN) [27]. Even though it is a very popular approach, the use of Gaussian distributions to model additive noise cannot be extended to all communication channels [28]–[30]. In fact, for a wide range of communication scenarios, it is more appropriate to take impulsive interference into account as well [31].

Several statistical models have been proposed so far to describe impulsive noise. One of the most popular statistical distributions for this purpose is the α-stable distribution [13]. It can also be understood as the generalized representation of a Gaussian distribution [27]. By adjusting its respective free parameters, it is possible to generate various probability distribution functions, such as Gaussian, Cauchy-Lorentz, or Lèvy.

A random variable with α-stable distribution can be parameterized through a characteristic function defined by [27]:

$$\varphi(t) = \exp\left[j\lambda t - \gamma|t|^{\alpha}(1 + j\beta \operatorname{sign}(t)\omega(t,\alpha))^{\alpha}\right], \qquad (4)$$

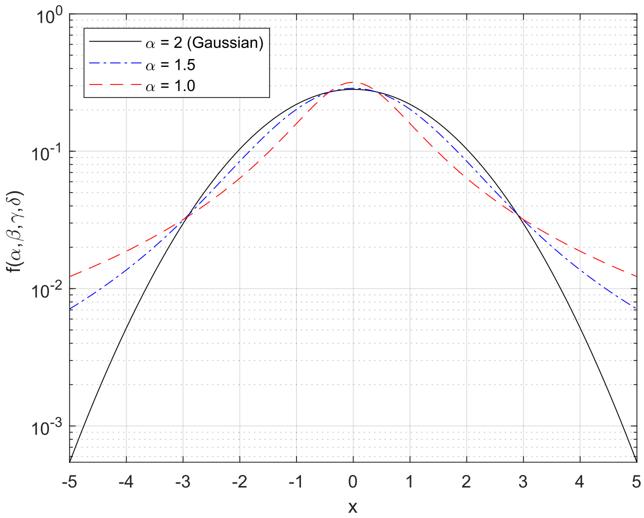

FIGURE 2. Probability distribution function of some α-stable symmetrical distributions as a function of $\beta = \lambda = 0$ and $\gamma = 1$.

where:

$$\omega(t,\alpha) = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right) & \text{when } \alpha \neq 1 \\ \frac{2}{\pi}\log|t| & \text{when } \alpha = 1, \end{cases} \qquad (5)$$

$$\operatorname{sign}(t) = \begin{cases} 1 & \text{when } t > 0 \\ 0 & \text{when } t = 0 \\ -1 & \text{when } t < 0, \end{cases} \qquad (6)$$

being:

$$0 < \alpha \leq 2; -1 < \beta \leq 1; \gamma > 0; -\infty < \lambda < +\infty. \qquad (7)$$

A given variable $X(\alpha, \beta, \gamma, \lambda)$ with α-stable distribution $S_\alpha(\beta, \gamma, \lambda)$ is described by a probability distribution function whose free parameters are $\{\alpha, \beta, \gamma, \lambda\}$ [32]:

1) Parameter α is called characteristic exponent or stability, thus denoting the tail lengthening of the α-stable probability distribution function.
2) Parameter β is the symmetry index or kurtosis, which represents the function symmetry.
3) Parameter γ is the dispersion parameter or scale, corresponding to the statistical dispersion of the function around a central point.
4) Parameter λ is the position of the distribution center.

Figure 2 presents the α-stable probability distribution function for several values of α, also considering parameters $\beta = \lambda = 0$ and $\gamma = 1$.

A particular case that is supposed to be analyzed in this work occurs when $\beta = 0$ and $\lambda = 0$. In this condition, the α-stable distributions are said to be symmetrical [8], while the characteristic function is represented by:

$$\varphi(t) = \exp\left(-\gamma|t|^{\alpha}\right). \qquad (8)$$

Using the α-stable distribution for modeling the impulsive noise is particularly useful due to some properties of this distribution, i.e.:

1) Generalized central limit theorem: the sum of a large number of independent and identically-distributed random $Y_1, Y_2, \ldots, Y_n$ with or without finite variance converges to a random variable X with an α-stable distribution:

$$\frac{Y_1 + Y_2 + \ldots + Y_n}{d_n} + a_n \overset{d}{\Rightarrow} X, \tag{9}$$

where $d_n > 0$, a_n are real constants and $Y \overset{d}{\Rightarrow} X$ denotes the distribution convergence.

2) Stability property: A distribution is said to be α-stable if the independent random variables X, X_1, and X_2 with α-stable distribution meet the stability criterion [13]:

$$v_1 X_1 + v_2 X_2 \overset{d}{=} \mu_1 X + \mu_2, \tag{10}$$

where v_1, v_2, μ_1, e μ_2 are scalars, and $\overset{d}{=}$ represents equal probability distributions.

3) The α-stable distribution is capable of modeling a wide variety of other distributions from its free parameters. Particularly for $\alpha = 2$ and $\beta = 0$, the α-stable distribution becomes a Gaussian distribution, while for $\alpha = 1$ and $\beta = 0$ it corresponds to a Cauchy-Lorentz distribution, i.e.:

$$f_{\alpha=2}(\gamma, \lambda, x) = \frac{1}{\sqrt{4\pi\gamma}} \exp\left[-\frac{(x-\lambda)^2}{4\gamma}\right], \tag{11}$$

$$f_{\alpha=1}(\gamma, \lambda, x) = \frac{1}{\pi[\gamma^2 + (x-\lambda)^2]}. \tag{12}$$

4) The α-stable distribution allows controlling the tail lengthening of the probability density function, while modeling scenarios with higher or lower degree of impulsivity [27], as shown in Figure 2. Besides, the following statement can be easily demonstrated [33]:

$$\lim_{x\to 0} \Pr(X > x) = \gamma^{\alpha} \frac{\Gamma(\alpha)}{\pi} \sin\frac{\pi\alpha}{2}(1+\beta)x^{-\alpha}, \tag{13}$$

where $\Gamma(\cdot)$ is the Gamma function, which is defined by [34]:

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt. \tag{14}$$

In this case, the lower the value of α, the higher the tail weight of the probability distribution function associated with the stable distribution, resulting in scenario with higher impulsivity.

5) Let $X \sim S_\alpha(\beta, \gamma, \delta)$ for $0 < \alpha < 2$, then [13]:

$$\begin{aligned} E[|X|^p] &< \infty \quad \text{when } 0 < p < \alpha \\ E[|X|^p] &\to \infty \quad \text{when } p \geq \alpha, \end{aligned} \tag{15}$$

where $p > 0$, $p \in \Re$. As a consequence of this property, the second-order and higher-order statistical moments do not converge for $\alpha < 2$. Particularly for $\alpha < 1$, the expected value of variable X is not supposed to converge.

Due to the property represented by Equation (15), the α-stable distribution is characterized by an infinite variance if $\alpha < 2$. One of the most effective figures of merit in communication systems is the signal-to-noise ratio (SNR), which relates the signal power to the noise power. Thus, it is essential to replace it for other metrics that are suitable to scenarios characterized by α-stable noise model. A well-established metric for this purpose is the geometric signal-to-noise ratio (GSNR), which is defined by [14]:

$$GSNR = 10\log_{10}\left(\frac{1}{\gamma M}\sum_{k=1}^{M} s_k s_k^*\right), \tag{16}$$

where s_k is the transmitted signal, M is the number of samples of the transmitted signal, and γ is the dispersion parameter.

III. PROPOSED DETECTOR

This work introduces a novel detector for MIMO systems based on complex correntropy, which is called maximum correntropy detector (MCD). The forthcoming subsections are supposed to detail the operation of the proposed detector. Initially, the theory of complex correntropy for random variables is presented, as well as the theoretical background of the MCD. A detection architecture for MIMO systems based on the MCD method is then described in detail.

A. COMPLEX CORRENTROPY FOR RANDOM SIGNALS

Feature extraction is of major importance in random signal processing. In this sense, the use of statistical similarity metrics is a must, e.g., correlation. However, for non-Gaussian random processes or in cases where the analyzed systems are nonlinear, correlation may not be effective [19].

Among the techniques proposed for feature extraction in scenarios where non-Gaussian data or nonlinearity exist, the correntropy function is a prominent choice. It is capable of extracting features from second-order statistical moments by correlation, as well as from higher-order ones, with a computational complexity equivalent to that of correlation [19].

In this work, the complex correntropy function is applied to random variables, which is defined in the form [35]:

$$V_\sigma^c(Q, W) = \mathbb{E}[k_\sigma(Q, W)], \tag{17}$$

where $k_\sigma(\cdot)$ is a positive definite function called kernel, $Q, W \in \mathbb{C}$ are complex random variables such that $Q = X + jZ$ and $W = Y + jS$, with $X, Y, Z, S \in \Re$, and $\mathbb{E}[\cdot]$ is the statistical expectation operator.

This work employs a Gaussian function as kernel due to the inherent symmetry according to [8]:

$$k_\sigma(Q, W) = \frac{1}{2\pi\sigma^2}\exp\left\{-\frac{(Q-W)(Q-W)^*}{2\sigma^2}\right\}, \tag{18}$$

where σ is the kernel size. Using a Gaussian kernel, the complex correntropy defined in Equation (17) can be written as:

$$V_\sigma^c(Q, W) = \frac{1}{2\pi\sigma^2}\mathbb{E}\left[\exp\left(-\frac{(Q-W)(Q-W)^*}{2\sigma^2}\right)\right], \tag{19}$$

where subscript $*$ denotes the complex conjugate.

The calculation of the complex correntropy according to Equation (19) requires the knowledge of the joint probability density function of the random variables Q and W. This problem can be solved by estimating it by means of a Parzen window. Thus, the estimation of the complex correntropy in Equation (19) considering a Gaussian kernel is given by:

$$\hat{V}_\sigma^c(Q, W) = \frac{1}{2\pi\sigma^2}\frac{1}{L}\sum_{m=1}^{L}\exp\left(-\frac{(q_m - w_m)(q_m - w_m)^*}{2\sigma^2}\right), \tag{20}$$

where L is the number of samples of signals q_m and w_m.

The complex correntropy is a similarity metric between two random variables, which assumes a maximum value of $1/(2\pi\sigma^2)$ when $Q = W$. Besides, it can be considered as a generalization of correlation. This is due to the fact that it contains the information regarding infinite even-order moments of random variable $Q - W$. This can be seen by applying the Taylor series expansion to Equation (19). Assuming the complex random variables $Q = X + jZ$ and $W = Y + jS$, it is possible to use the Taylor series expansion to write:

$$\begin{aligned} V_\sigma^C&(Q, W) \\ &= \frac{1}{2\pi\sigma^2}\mathbb{E}\Bigg[1 - \frac{(X-Y)^2}{2\sigma^2} + \frac{(X-Y)^4}{8\sigma^4} \\ &\quad - \frac{(X-Y)^6}{48\sigma^6} + \frac{(X-Y)^8}{384\sigma^8} - \frac{(Z-S)^2}{2\sigma^2} \\ &\quad + \frac{(Z-S)^4}{8\sigma^4} + \frac{(Z-S)^6}{48\sigma^6} + \frac{(Z-S)^8}{384\sigma^8} \\ &\quad + \frac{(X-Y)^2(Z-S)^2}{4\sigma^4} - \frac{(X-Y)^2(Z-S)^4}{16\sigma^6} \\ &\quad - \frac{(X-Y)^4(Z-S)^2}{16\sigma^6} + \frac{(X-Y)^2(Z-S)^6}{96\sigma^8} \\ &\quad + \frac{(X-Y)^4(Z-S)^4}{64\sigma^8} + \frac{(X-Y)^6(Z-S)^2}{96\sigma^8} + \ldots\Bigg] \end{aligned} \tag{21}$$

By grouping the terms containing σ^2 in the denominator and besides, defining h_{σ^4} as a variable containing the high-order terms of the summation, it is possible to write:

$$V_\sigma^C(Q, W) = \frac{1}{2\pi\sigma^2} - \frac{1}{2\pi\sigma^2}\mathbb{E}\left[\frac{(X-Y)^2 + (Z-S)^2}{2\sigma^2}\right] + h_{\sigma^4}, \tag{22}$$

thus,

$$V_\sigma^C(Q, W) = \frac{1}{2\pi\sigma^2} - \frac{1}{4\pi\sigma^4}\mathbb{E}[(Q - W)(Q - W)^*] + h_{\sigma^4}, \tag{23}$$

finally,

$$V_\sigma^C(Q, W) = \frac{1}{2\pi\sigma^2} - \frac{1}{4\pi\sigma^4}R[Q, W] + h_{\sigma^4}, \tag{24}$$

where $R[Q, W] = \mathbb{E}[(Q - W)(Q - W)^*]$ autocorrelation of $(Q - W)$.

One can notice in equation (22) that the higher-order terms represented by h_{σ^4} tend to zero faster than the second-order one as σ increases, what corresponds exactly to the autocorrelation of $(Q - W)$.

B. MATHEMATICAL FOUNDATION OF MCD

The MCD consists in the generalization of the MLD. It can be accomplished through the use of complex correntropy as a similarity measure instead of the mean square error criterion. Thus, the MCD contains information on both second-order statistical moments, notably indicated by the minimized Euclidean distance in the MLD, as well as information on higher-order statistical moments. In addition, the use of the Gaussian kernel reduces the effect of outliers due to the negative exponential argument, which results in the robustness of this technique on channels characterized by impulsive noise.

The MCD lies in selecting a given symbol s_n of a constellation used in the transmission to maximize the maximum complex correntropy criterion defined in Equation (20). This method can be derived adopting q_m and w_m as random variables assuming the following values:

$$q_m = y_m \tag{25}$$

$$w_m = \sum_{n=1}^{N_T} h_{mn}s_n, \tag{26}$$

Thus, the complex correntropy presented in Equation (20) can be written in the form:

$$\hat{V}_\sigma^c(Q, W) = \vartheta\sum_{m=1}^{N_R}\exp\left(-\frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn}s_n\right|^2}{2\sigma^2}\right), \tag{27}$$

where:

$$\vartheta = \frac{1}{2\pi\sigma^2}\frac{1}{N_R}, \tag{28}$$

being σ the kernel size.

The MCD method is based on the fact that correntropy is similarity measure between two random variables, which assumes the maximum value when both of them are identical. Thus, the selected symbol s_n is the one most likely to be transmitted.

Assuming the presence of noise in the communication channel, the MCD will generate an estimate of the transmitted symbol $\hat{\mathbf{s}}_{MCD}$ from the following criterion:

$$\hat{\mathbf{s}}_{MCD} = \underset{\mathbf{s}\in\Omega^{N_T}}{\arg}\ \max\xi(\mathbf{s}, \sigma), \tag{29}$$

where Ω is the possible subspace containing all transmitted symbols and $\xi(\mathbf{s}, \sigma)$ is a function derived from the complex correntropy represented by Equation (19):

$$\xi(\mathbf{s}, \sigma) = \sum_{m=1}^{N_R}\exp\left(-\frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn}s_n\right|^2}{2\sigma^2}\right), \tag{30}$$

where $|\cdot|$ represents the modulus of a complex number.

C. MCD PROPERTIES

This subsection is dedicated to the analysis of some useful properties associated with the approach introduced in this work.

Property 1: - For some value of σ, the MCD is an optimum search process in the order of $|\Omega|^{N_T}$, just like MLD [24]. Therefore, the computational burden of MCD and MLD is nearly the same.

Property 2: - The MCD contains information on second-order moments as associated with MLD, and also information on higher-order statistical moments.

In order to demonstrate Property 2, it is only necessary to expand function $\xi(\mathbf{s}, \sigma)$ as represented by Equation (30) using the Taylor series:

$$\xi(\mathbf{s}, \sigma) = \sum_{m=1}^{N_R} \sum_{k=0}^{\infty} (-1)^k \frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^{2k}}{2\sigma^{2k} k!}, \quad (31)$$

Expanding Equation (31) until term $k = 2$ gives:

$$\xi(\mathbf{s}, \sigma) = \sum_{m=1}^{N_R} \left[\frac{1}{2} - \frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^2}{2\sigma^2} + \sum_{k=2}^{\infty} (-1)^k \frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^{2k}}{2\sigma^{2k} k!} \right]. \quad (32)$$

Analogously, Equation (32) can be rewritten in the form:

$$\xi(\mathbf{s}, \sigma) = \sum_{m=1}^{N_R} \left[\nu(\sigma) + \varphi(\sigma)\right], \quad (33)$$

where:

$$\nu(\sigma) = \frac{1}{2} - \frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^2}{2\sigma^2}, \quad (34)$$

represents the series expansion terms that aggregate first and second-order statistics, while:

$$\varphi(\sigma) = \sum_{k=2}^{\infty} (-1)^k \frac{\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^{2k}}{2\sigma^{2k} k!}, \quad (35)$$

corresponds to the series expansion terms that aggregate high-order statistics.

From Equation (33), it can be stated that $\xi(\mathbf{s}, \sigma)$ aggregates both second-order moments represented by term $\nu(\sigma)$, and high-order moments represented by term $\varphi(\sigma)$.

Property 3: - The MCD contains information on all infinite even-order statistical moments.

This property can be easily observed in Equation (35), as well as that term $\varphi(\sigma)$ contains infinite even-order statistical moments of the random variable $\left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|$.

Property 4: - For high values of the kernel size (σ), the higher-order terms are minimized in relation to the second-order terms.

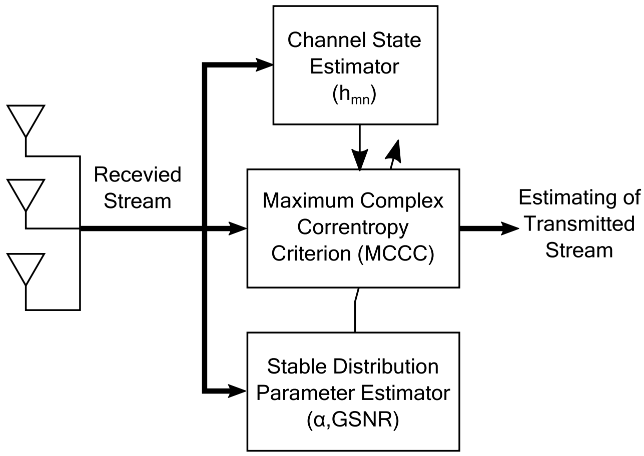

FIGURE 3. Maximum correntropy detector architecture.

This property can be easily observed from the fact that the $(2k)$-th statistical term é weighted by $1/\sigma^{2k}$. Thus, the larger the kernel size σ, the higher the contribution of the second-order term in relation to the higher-order ones, given that high order terms are weighted by factors smaller than the second-order term.

In particular, for a sufficiently large kernel value, the importance of higher-order terms is minimized, resulting in:

$$\xi(\mathbf{s}, \sigma) \approx \frac{1}{2\sigma^2} \sum_{m=1}^{N_R} \sigma^2 - \left|y_m - \sum_{n=1}^{N_T} h_{mn} s_n\right|^2, \quad (36)$$

The analysis of Equation (36) shows that the MCD converges to the MLD for large kernel sizes. In fact, the function maximization process described by Equation (36), corresponds to the same minimization processed expressed by Equation (3) as associated with the MLD.

As will it be further demonstrated by the simulation results, the MCD performance is superior to that of MLD on channels characterized by the presence of impulsive noise. Besides, the same performance is achieved by both methods on channels characterized by AWGN. Since both approaches have similar complexity, in practical systems the MCD can be applied even to non-impulsive channels, since this aspect does not affect the overall system performance in terms of both symbol error rate and computational complexity.

D. SIGNAL DETECTION ARCHITECTURE

Figure 3 presents the signal detection architecture proposed in this work. The "channel array estimation" block obtains estimates for the attenuation coefficients between the transmitter and receiver antennas $\{\tilde{h}_{mn}\}$. On the other hand, the "stable distribution parameter estimator" block estimates the parameters of the noise probability density function and, from a mathematical expression, provides the proposed detector represented by Equation (29) with the appropriate kernel size.

Estimating the kernel size is of major importance when using the MCD receiver. From the proper adjustment of this parameter, it is possible to increase or decrease the importance of higher-order statistical moments in the method performance. In this work, the kernel size is dynamically adjusted, being calculated according to the GSNR and the impulsivity parameter α of the communication channel:

$$\sigma = f(GNSR, \alpha). \quad (37)$$

The kernel size as a function of the aforementioned metrics can be represented by adjusting a polynomial function, i.e.:

$$\sigma(GSNR, \alpha) = p_3(\alpha)(GSNR)^3 + p_2(\alpha)(GSNR)^2 + p_1(\alpha)(GSNR) + p_0(\alpha), \quad (38)$$

where $p_3(\alpha), p_2(\alpha), p_1(\alpha)$, and $p_0(\alpha)$ are quantities related to parameter α of the communication channel. The adjustment of parameters $p_3(\alpha), p_2(\alpha), p_1(\alpha)$,and $p_0(\alpha)$ can be performed from points obtained in computer simulations as it will be further explained in Section 4.1.

The parameters of the stable distribution can be estimated from the method presented in [36], [37]. In this case, the characteristic exponent α and the dispersion parameter γ can be estimated by:

$$\hat{\alpha} = \phi_1(\hat{\nu}_\alpha) \quad (39)$$

$$\hat{\gamma} = \frac{\hat{\eta}_{0.75} + \hat{\eta}_{0.25}}{\phi_3(\hat{\alpha})}, \quad (40)$$

where:

$$\hat{\nu}_\alpha = \frac{\hat{\eta}_{0.95} - \hat{\eta}_{0.05}}{\hat{\eta}_{0.75} - \hat{\eta}_{0.25}}, \quad (41)$$

and $\hat{\eta}_f$ represents f-th quantile of the sample set for the noise η, $\phi_1(\hat{\nu}_\alpha)$, and $\phi_3(\hat{\alpha})$ are tabulated functions as presented in [37].

IV. SIMULATION RESULTS

This section presents the simulation results for the proposed MCD. Symbol error rates (SER) were obtained for different values of channel impulsivity considering two MIMO system configurations.

The evaluated MIMO arrangements employ spatial multiplexing (SM) in the following settings: (i) two transmitting antennas and two receiving antennas; and (ii) four transmitting antennas and four receiving antennas. In each scenario, the performances of the MLD and MCD detectors were analyzed.

The signals transmitted by each antenna are quadrature phase-shift keying (QPSK) modulated with unit energy, and are statistically independent of one each other.

Rayleigh flat fading due to multipath propagation in wireless channels is assumed, as well as a slow Doppler so that the path gains between the transmitting and receiving antennas can be considered constant at each signaling. Such path gains are further considered to be perfectly known at the receiver. In addition, the additive noise model assumed on each receiving antenna is impulsive, following a symmetrical α-stable distribution with $\beta = \delta = 0$ and stability parameter α in the following ranges: $\alpha = \{1.3, 1.5, 1.7, 2.0\}$. For $\alpha = 2$, the modeled channel is Gaussian as presented in Section 2.1.

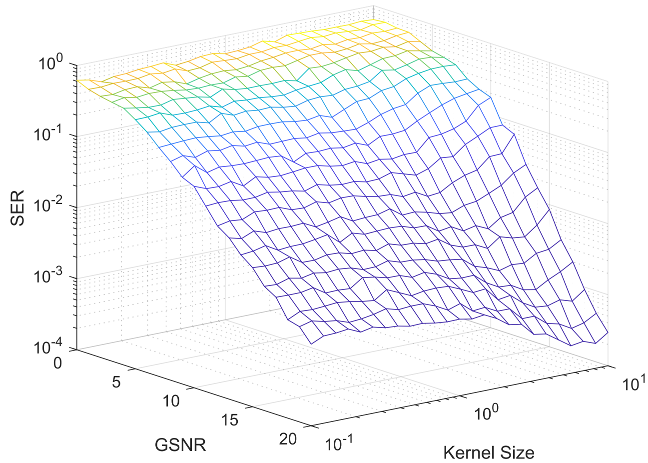

FIGURE 4. Relationship among kernel size, SER, and GSNR for an α-stable channel with impulsive noise.

Impulsive noise parameters were estimated considering 10^3 samples. The samples were generated according to the method proposed by Weron & Weron [8]. The GSNR was varied from 0 dB to 20 dB with steps of 5 dB.

Simulation tests were carried out using the Monte-Carlo method. At each point of the SER curves, at least 100 symbol errors were employed in the estimation. All simulations were performed considering the baseband, while other effects such as the synchronization error were not evaluated.

A. KERNEL SIZE ADJUSTMENT

According to Section 3.4, one of the parameters that must be defined in the proposed detector is the polynomial approximation coefficients for the kernel size determination as presented in Equation (33).

Since a mathematical relationship cannot be established in order to determine the approximation coefficients, such parameters were obtained from a set of surfaces generated from computer simulations, which relate: (i) the SER; (ii) the kernel size used by the proposed architecture (σ); and (iii) the GSNR for a channel with α-stable additive noise with a specific and fixed factor α.

Figure 4 presents one of the surfaces for $\alpha = 1.5$, where each point was obtained for a minimum of 100 symbol errors.

From Figure 4, kernel size were chosen to minimize the SER for a given GSNR. Such values are used to determine the polynomial fit coefficients presented in Equation (33). It is worth mentioning that the coefficients are determined for a specific value of α and depend on this parameter as a consequence. Table 1 presents the coefficients obtained for the values of α investigated in this work.

Figure 5 presents the kernel sizes as a function of the GSNR measured in the channel for $\alpha = \{1.3, 1.5, 1.7, 2.0\}$. Analyzing 5, it is reasonable to state that the kernel size tends to increase as the GSNR also does. This relationship can be

TABLE 1. Relationship among the adjustment coefficients p_3, p_2, p_1, and p_0 and α.

$\alpha \rightarrow$	1.3	1.5	1.7
p_3	7.851×10^{-4}	1.062×10^{-3}	3.366×10^{-4}
p_2	-6.104×10^{-3}	-8.477×10^{-3}	2.159×10^{-2}
p_1	0.1619	0.157	-0.1315
p_0	0.1749	0.2357	0.3495

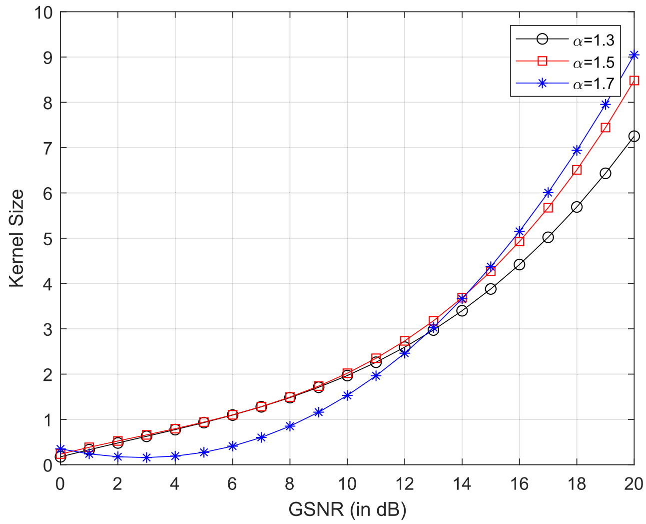

FIGURE 5. Relationship between the optimal kernel size and GSNR for distinct value of the channel impulsivity (α).

justified given that by increasing the GSNR of the channel, the effect of impulsive noise is mitigated, and therefore, the metrics for estimation of transmitted symbols must have a greater influence of the second-order moment.

Another important point to note is that, as seen in Figure 5, the influence of the α parameter on the optimal fit of the kernel size is small. Thus, the detection architecture performance is not very sensitive to small estimation errors of the α parameter. However, there is a strong correlation between the GSNR value and the optimal kernel size. Since GSNR is related to the γ parameter of the impulsive noise model, as indicated by Equation (16), this parameter must be estimated as accurately as possible at the receiver.

B. MCD PERFORMANCE

Using the adjustment rule presented in Equation (38) and the values for the approximation coefficients presented in Table 1, this section presents a set of simulations that compare the SER as a function of the GSNR for the proposed detector with that for MLD considering different channel impulsivity values and two MIMO arrangements.

Figures 6-9 present the relationship of the SER as a function of the GSNR for channels with α-stable noise for $\alpha = 1.3$, 1.5, 1.7, and 2.0, respectively. In each curve, the performance of the proposed detector is compared with that of the MLD for the 2×2 and 4×4 MIMO systems.

From Figures 6-8, considering the performance curves obtained on non-Gaussian channels for $\alpha \neq 2$), it can be

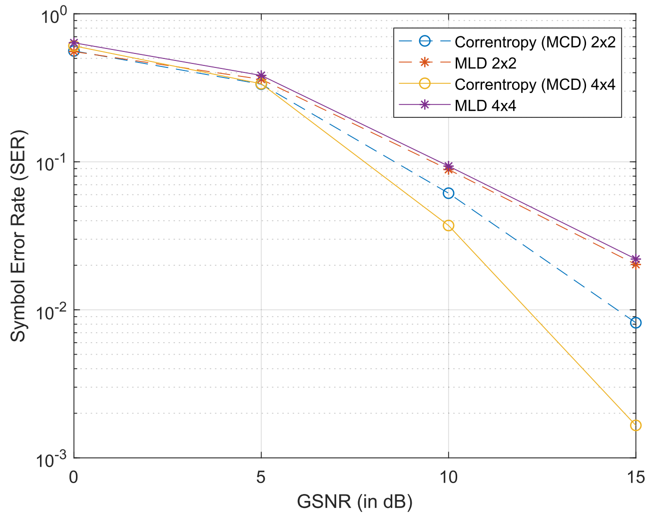

FIGURE 6. SER on channels with α-stable impulsive noise considering $\alpha = 1.3$.

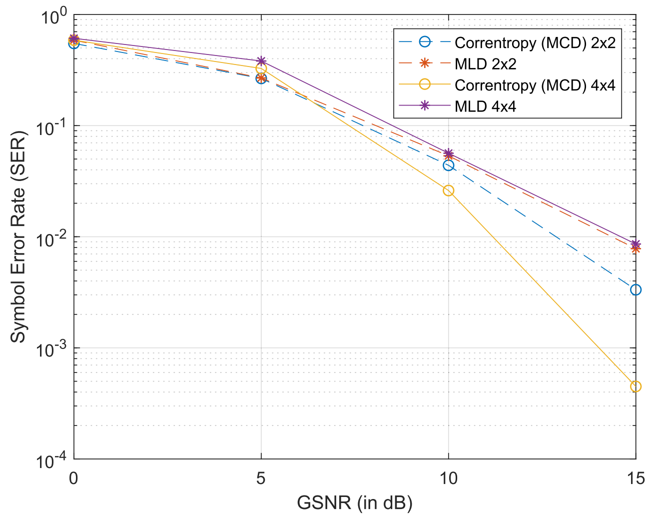

FIGURE 7. SER on channels with α-stable impulsive noise considering $\alpha = 1.5$.

observed that the proposed detector is superior to the MLD for all ranges of the GSNR. At lower GSNRs, the performance of both techniques is similar, but for high GSNRs the gains are equal to about 2 dB and 4 dB for the 2×2 and 4×4 MIMO systems.

Still analyzing Figures 6-8, it is also noted that using more receiving antennas does not lead to the improvement of MLD performance on non-Gaussian channels. On the other hand, the MCD performance is improved as the number of antennas increases. The gain obtained in the 4×4 MIMO system is about 2 dB when compared with the 2×2 MIMO system using the MCD. On the other hand, the gain is nearly null when the MLD is employed instead.

Figure 9 shows the behavior of the MCD and MLD on Gaussian channels for $\alpha = 2$. In this case, the performance of the proposed architecture is equivalent on AWGN channels, to that of the reference system employing the

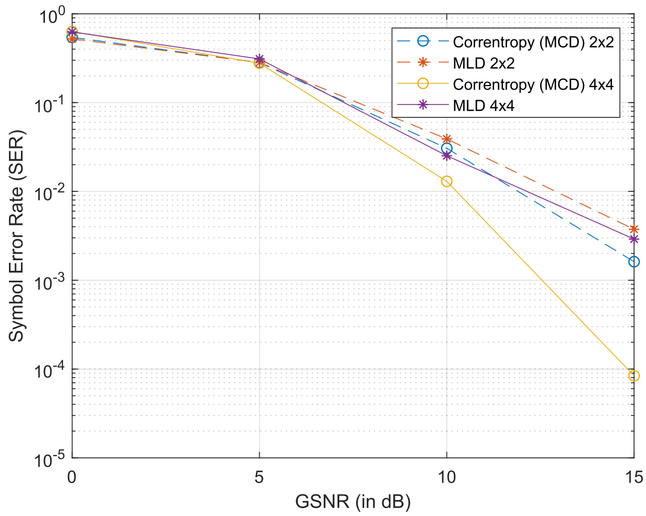

FIGURE 8. SER on channels with α-stable impulsive noise considering $\alpha = 1.7$.

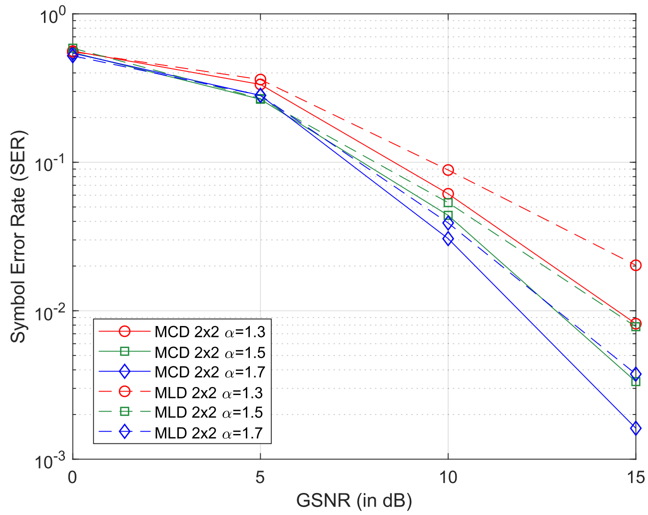

FIGURE 10. SER of the MCD and MLD for distinct channels with α-stable noise in a 2×2 MIMO system.

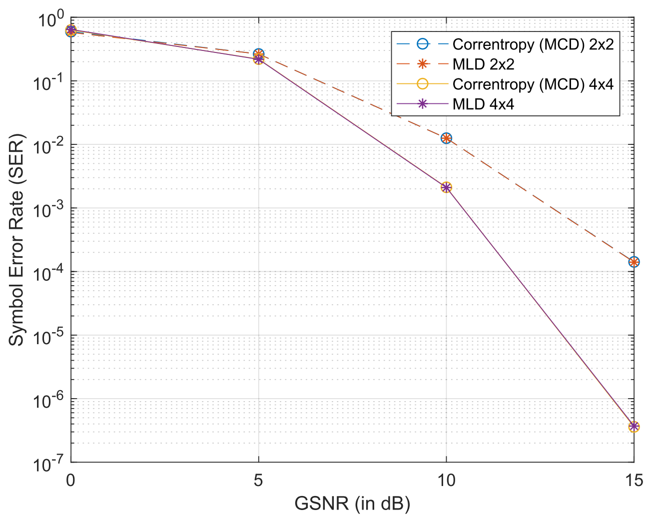

FIGURE 9. SER on channels with α-stable impulsive noise considering $\alpha = 2.0$.

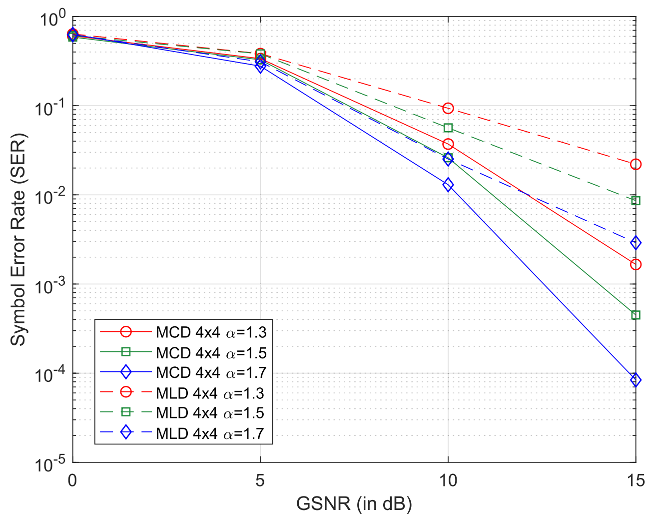

FIGURE 11. SER of the MCD and MLD for distinct channels with α-stable noise in a 4×4 MIMO system.

MLD receiver. This is expected as MLD is an optimal technique for channels with additive Gaussian noise. As a general, it can be stated that the MCD performance will be at most equivalent, but never superior to that of MLD.

Figures 10 and 11 summarize the behavior of the MCD and MLD for distinct degrees of impulsivity of the channel in 2×2 and 4×4 MIMO systems, respectively. It is observed that with the increase of the degree of impulsivity, i.e. when parameter α decreases, the performance of both detectors is impaired. However, the MLD is somewhat more affected than MCD. As an example, from Figure 10, it can be stated that the performance of the MCD with two receiver antennas in a scenario with $\alpha = 1.3$ is equivalent to that of the MLD with two receiver antennas in a scenario with $\alpha = 1.5$, which corresponds to a less impulsive channel. The same behavior occurs in Figure 11, where the performance is further improved since more receiving antennas are employed.

C. DISCUSSION OF RESULTS

In practice, a communication channel has an impulsivity degree that varies over time. Thus, in some less severe situations, parameter α approaches two and the channel is Gaussian. However, in more adverse situations, which may be caused by natural or human effects, the parameter α decreases, thus making the channel non-Gaussian. In such situations, the MLD presents higher error rates leading to burst error. In this same scenario, the MCD would perform better and present reduced burst error.

In this sense, since the MCD presents a computational complexity equivalent to that of MLD, this detection strategy can be used even if the channel is Gaussian, also considering that the MCD would have an equivalent performance to that of the MLD in this scenario.

V. CONCLUSION

In order to increase the robustness of MIMO systems on impulsive noise channels, this work has proposed a novel method for signal detection based on the complex correntropy function, which is so called MCD. The proposed technique can be seen as a generalization of the MLD.

The performance of the introduced approach is evaluated by computer simulation. The results indicate that the MCD achieves superior performance than that of the MLD technique on impulsive noise communication channels, and also equivalent performance on Gaussian channels.

Future work aims to: (i) compare the MCD for MIMO systems with other detection techniques proposed in the literature; (ii) analyze the detector performance in scenarios where the channel is not known by the receiver.

REFERENCES

[1] H. Sung, J. W. Kang, and K. B. Ed Lee, "A simplified maximum likelihood detection scheme for MIMO systems," *IEICE Trans. Commun.*, vol. 89, no. 8, pp. 2241–2244, Aug. 2006.

[2] B. H. Sklar, *Digital Communications: Fundamentals and Applications*. London, U.K.: Pearson Education, 2018.

[3] P. T. V. Souza, "Analysis of wavelet coding in spectrum dynamic access systems," (in Portuguese), M.S. thesis, PPGEEC, Natal, Brazil, 2017.

[4] J. Proakis, J. Saleh, and G. Bauch, *Contemporary Communication Systems Using MATLAB*. Camden, NJ, USA: Nelson Education, 2012.

[5] J. M. L. Duarte, "Algoritmos e arquiteturas VLSI para detectores MIMO com Decisão Suave," M.S. thesis, PPGEEC, Natal, Brazil, 2012.

[6] T. S. Cho, C. G. Kang, and W.-Y. Yang, *MIMO-OFDM Wireless Communications with MATLAB*. Hoboken, NJ, USA: Wiley, 2010.

[7] S. Kalyani and K. Giridhar, "Interference mitigation in turbo-coded OFDM systems using robust LLRs," in *Proc. IEEE Int. Conf. Commun.*, May 2008, pp. 646–651.

[8] H. R. G. Cavalcante, "Frequency acquisition on low-orbit satellite signals using the maximum current criterion," (in Portuguese), M.S. thesis, PPGEEC, Natal, Brazil, 2018.

[9] V. Raj and S. Kalyani, "Blind decoding in α-Stable noise: An online learning approach," Jun. 2019, *arXiv:1906.09811*. [Online]. Available: https://arxiv.org/abs/1906.09811

[10] H. El Ghannudi, L. Clavier, N. Azzaoui, F. Septier, and P.-A. Rolland, "α-stable interference modeling and cauchy receiver for an IR-UWB ad hoc network," *IEEE Trans. Commun.*, vol. 58, no. 6, pp. 1748–1757, Jun. 2010.

[11] J. Li, J. Lu, and J. Zhao, "A robust constant modulus algorithm in alpha-stable noise environments," in *Proc. 10th Int. Conf. Signal Process.*, Oct. 2010, pp. 1589–1592.

[12] S. M. Tahsien, A. Mahmud, N. Jahan, R. Islam, and S. P. Majumder, "Performance limitations of a 2×2 MIMO power line communication system impaired by non-white impulsive and non-Gaussian power line noise," in *Proc. IEEE Int. Conf. Telecommun. Photon.*, Dec. 2015, pp. 1–4.

[13] P. G. Georgiou, P. Tsakalides, and C. Kyriakakis, "Alpha-stable modeling of noise and robust time-delay estimation in the presence of impulsive noise," *IEEE Trans. Multimedia*, vol. 1, no. 3, pp. 291–301, Sep. 1999.

[14] P. Tsakalides and C. L. Nikias, "Maximum likelihood localization of sources in noise modeled as a stable process," *IEEE Trans. Signal Process.*, vol. 43, no. 11, pp. 2700–2713, Nov. 1995.

[15] X. Wang, K. Li, P. Gao, and S. Meng, "Research on parameter estimation methods for alpha stable noise in a laser gyroscope's random error," *Sensors*, vol. 15, no. 8, pp. 18550–18564, 2015.

[16] A. Hakam, M. Khalid, N. A. Aly, S. Jimaa, and S. Al-Araji, "MIMO-OFDM system with impulsive noise reduction technique based on auto level Selection Mechanism," in *Proc. 15th IEEE Int. Conf. Commun. Technol.*, Nov. 2013, pp. 229–233.

[17] A. Hakam, N. A. Aly, M. Khalid, S. Jimaa, and S. Al-Araji, "Impulsive noise reduction in MIMO-OFDM systems using adaptive receiver structures," in *Proc. IEEE 20th Int. Conf. Electron., Circuits, Syst.*, Dec. 2013, pp. 674–677.

[18] A. Charrada, "Nonlinear complex M-SVR for LTE MIMO-OFDM channel with impulsive noise," in *Proc. 7th Int. Conf. Sci. Electron., Technol. Inf. Telecommun.*, Dec. 2016, pp. 10–13.

[19] A. I. R. Fontes, "Correntropy use in the generalization of cyclostationary functions and applications for modulated signal characteristics extraction," (in Portuguese), Ph.D. dissertation, PPGEEC, Natal, Brazil, 2015.

[20] A. I. R. Fontes, A. de M. Martins, L. F. Q. Silveira, and J. C. Principe, "Performance evaluation of the correntropy coefficient in automatic modulation classification," *Expert Syst. Appl.*, vol. 42, pp. 1–8, Jan. 2015.

[21] A. I. R. Fontes *et al.*, "Classification system of pathological voices using correntropy," *Math. Problems Eng.*, vol. 2014, 2014.

[22] C. Zhang, Y. Guo, F. Wang, and B. Chen, "Generalized maximum correntropy-based echo state network for robust nonlinear system identification," in *Proc. Int. Joint Conf. Neural Netw.*, Jul. 2018, pp. 1–6.

[23] Z. Luan, H. Qu, J. Zhao, and B. Chen, "Robust digital non-linear self-interference cancellation in full duplex radios with maximum correntropy criterion," *China Commun.*, vol. 13, no. 9, pp. 53–59, Sep. 2016.

[24] H. Kim *et al.*, "Low-complexity lattice reduction algorithm for MIMO detectors with tree searching," *EURASIP J. Wireless Commun. Netw.*, vol. 2017, no. 1, p. 17, 2017.

[25] A. Papoulis, *Probability, Random Variables, and Stochastic Process*. Tokyo, Japan: McGraw-Hill, 1965.

[26] S. Yang and L. Hanzo, "Fifty years of MIMO detection: The road to large-scale MIMOs," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 4, pp. 1941–1988, 4th Quart., 2015.

[27] M. Shao and C. L. Nikias, "Signal processing with fractional lower order moments: Stable processes and their applications," *Proc. IEEE*, vol. 81, no. 7, pp. 986–1010, Jul. 1993.

[28] S. Kassam, *Signal Detection in Non-Gaussian Noise*. New York, NY, USA: Springer, 2012.

[29] P. L. Brockett, M. Hinich, and G. R. Wilson, "Bispectral characterization of ocean acoustic time series: Nonlinearity and non-Gaussianity," in *Topics in Non-Gaussian Signal Processing*. New York, NY, USA: Springer, 1989, pp. 2–16.

[30] S. C. Schwartz, J. B. Thomas, and E. J. Wegman, *Topics in Non-Gaussian Signal Processing*. New York, NY, USA: Springer, 1989.

[31] D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: New methods an results for class A and class B noise models," *IEEE Trans. Inf. Theory*, vol. 45, no. 4, pp. 1129–1149, May 1999.

[32] R. G. Hernandez, "Stable alpha distribution applied to market risk," (in Portuguese), Ph.D. dissertation, FGV, São Paulo, Brazil, 2016.

[33] M. Lombardi, "Simulation-based estimation methods for α-stable distributions and processes," Ph.D. dissertation, Universita Degli Studi Di Firenze, Dipartimento di Statistica, 2004.

[34] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*, MA, USA: Courier Corporation, 1965.

[35] J. P. F. Guimarães, A. I. R. Fontes, J. B. A. Rego, A. de M. Martins, and J. C. Príncipe, "Complex correntropy: Probabilistic interpretation and application to complex-valued data," *IEEE Signal Process. Lett.*, vol. 24, no. 1, pp. 42–45, Jan. 2017.

[36] E. F. Fama and R. Roll, "Parameter estimates for symmetric stable distributions," (in English), *J. Amer. Stat. Assoc.*, vol. 66, no. 334, pp. 331–338, 1971.

[37] J. H. McCulloch, "Simple consistent estimators of stable distribution parameters," *Commun. Statist. Simul.*, vol. 15, no. 4, pp. 1109–1136, 1986.

PEDRO T. V. DE SOUZA graduated in electrical engineering from the Federal University of Rio Grande do Norte (UFRN). He received the master's degree in computer engineering from the Postgraduate Program in Electrical and Computer Engineering (PPGEEC), Federal University of Rio Grande do Norte (UFRN), where he is currently pursuing the Ph.D. degree in computer engineering with the Graduate Program in Electrical and Computer Engineering (PPGEEC). He is an Assistant Professor I with the Rural Federal University of Semi-Arid—Campus Pau dos Ferros, acting in the areas of digital signal processing, digital communication systems, digital systems, and evolutionary computing.

ALUÍSIO I. R. FONTES graduated in computer engineering from the Federal University of Rio Grande do Norte, in 2010. He received the master's degree and the Ph.D. degree in computer engineering from the Federal University of Rio Grande do Norte, in 2012 and 2015, respectively. He has experience in computer engineering, focusing on computer systems and telecommunications systems, acting on the following subjects: parallel programming, high performance computing, embedded architectures, information theory, correntropy, and cognitive radio. He is currently a Professor with the Federal Institute of Rio Grande do Norte.

VINÍCIUS S. V. DE SOUZA received the degree in computer engineering from the Federal University of Rio Grande do Norte (UFRN) and the master's and Ph.D. degrees in electrical and computer engineering from the Postgraduate Program in Electrical and Computer Engineering (PPGEEC), UFRN. He is currently an Associate Professor III with the Federal Rural University of Semi-Arid (UFERSA), Pau dos Ferros Campus.

LUIZ F. SILVEIRA graduated in electrical engineering from the Federal University of Paraíba (C. Grande), in 2000. He received the master's and Ph.D. degrees in electrical engineering from the Federal University of Campina Grande, in 2002 and 2006, respectively. He has experience in electrical engineering, focusing on telecommunications systems, acting on the following subjects: wavelets, cognitive radio, and energy efficient signal processing. He is currently an Associate Professor with the Department of Computer and Automation Engineering, Federal University of Rio Grande do Norte, and a permanent Faculty Member of the Graduate Program in Electrical and Computer Engineering.