

# Reduced Complexity Detection Schemes for Golden Code Systems

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**ABSTRACT** The Golden code has full-rate and full-diversity. However, its applications are limited because of the very high detection complexity. The complexity of sphere decoding depends on the size of signal set,  $M^2$ , and the depth of search. Meanwhile, the complexity of fast essentially maximum likelihood (ML) detection is still  $\mathcal{O}(M^2)$  for  $M$ -ary quadrature amplitude modulation (MQAM). In this paper, we propose two reduced complexity detection schemes, fast essentially ML with detection subset and sphere decoding with detection subset. Two theoretical bounds on the average bit error probability for the Golden code with MQAM are also formulated in this paper. Simulation results demonstrate that both the fast essentially ML with detection subset and sphere decoding with detection subset agree well with the formulated theoretical bounds and can achieve the error performance of the conventional fast essentially ML detector and sphere decoding.

**INDEX TERMS** Fast essentially ML detection, golden code, QR decomposition based detection, reduced complexity detection, sphere decoding.

## I. INTRODUCTION

Nowadays there is an ever-growing demand to increase data transmission rate and improve communication reliability in wireless communications. Multiple-input multiple-output (MIMO) techniques can be used to increase data transmission rate and/or improve communication reliability. In MIMO systems, there is a trade-off between rate and reliability in terms of the diversity-multiplexing gain [1]. The Golden code is a linear dispersion space-time block code (LD-STBC) with two transmit antennas [2]–[4] and achieves full-rate and full-diversity. It has also been shown to exhibit attractive properties in terms of cubic shaping and non-vanishing minimum determinant due to its algebraic structure [3]. The optimality of the Golden code was studied in [4]. The Golden code has also been considered for index [5], [6] and media-based modulations [7]. An important application of the Golden code is the IEEE 802.16e WiMAX standard [8]. Despite its well-known advantages, extensive application of the Golden code is limited due to the extremely high complexity imposed by maximum likelihood (ML) detection. Specifically, this is because the Golden code transmission matrix contains four complex-valued symbols, resulting in an ML detection

complexity which is proportional to  $\mathcal{O}(M^4)$ , where  $M$  is the modulation order.

In order to reduce the ML detection complexity, a fast ML decoding algorithm has been proposed in [8]. [8] proved that the Golden code is fast-decodable. Furthermore, [8] presented an efficient implementation of the ML detector with a worst-case complexity proportional to  $\mathcal{O}(M^{2.5})$ . Based on [8], an efficient decoding technique based on the dimensionality reduction of the search tree in sphere decoding was proposed in [9]. The worst-case complexity of the proposed scheme in [9] is  $\mathcal{O}(M^{1.5})$ . However, the error performance suffers a 1 dB signal-to-noise ratio (SNR) loss compared to optimal decoding. In [10], a fastest-known near-ML decoding scheme was proposed. In this decoding scheme, the output of the zero-forcing filter is passed through a likelihood-based reliability metric calculator. Symbols deemed reliable are directly decoded from the received signal. The symbols deemed unreliable are decoded via a reduced dimensional ML decoder or near-ML decoder. The near-ML decoder greatly reduces computational complexity; however, there is a possibility that more than two symbols are unreliable. Hence, the computational complexity for more than two unreliable symbols remains very high.

Based on the structure of the Golden code, a fast essentially ML detection scheme was proposed in [11].

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The fast essentially ML detection scheme partitions four complex-valued symbols into two pairs of symbols. Given one pair of symbols, the likelihood maximization function can be easily solved. The detection complexity of the fast essentially ML algorithm is  $\mathcal{O}(M^2)$ . However, the fast essentially ML detection algorithm [11] is only applicable for low-order modulation. For high-order modulation,  $M \geq 16$ , the detection complexity remains impractically high.

Sphere decoding is a detection algorithm that achieves near-ML error performance. The detection complexity of sphere decoding depends on the cardinality of the signal set and the depth of search [12]. In [13], reduced complexity sphere decoding has been proposed to decode the Golden code. The proposed reduced complexity sphere decoding in [13] focused on reducing the search depth. However, detection complexity remains relatively high.

Based on the above, it is evident that if the size of the set of the given pair of symbols is reduced, then the detection complexity of the fast essentially ML detection algorithm in [11] and sphere decoding in [12], [13] may be further reduced. This motivates us to propose two reduced complexity detection schemes for Golden code systems, fast essentially ML with detection subset, which is based on the fast essentially ML detection of [11] and QR decomposition, and sphere decoding with detection subset. The two proposed reduced complexity detection schemes greatly reduce the detection complexity. For example, with a four receive antenna Golden code system, the complexity of the proposed fast essentially ML with detection subset is only  $\mathcal{O}(2 \times 4^2)$  and  $\mathcal{O}(2 \times 4.5^2)$  for  $M$ -ary quadrature amplitude modulation (MQAM) with  $M = 16$  and  $M = 64$ , respectively. Furthermore, the cardinality of signal set in the conventional sphere decoding is reduced from  $M^2$  to 39.1 and 86.2 for 16QAM and 64QAM, respectively. To the best of the authors' knowledge, no other reduced complexity detection algorithms have been investigated in the literature for Golden code systems. Furthermore, two theoretical bounds on the average bit error probability (ABEP) for the Golden code system with MQAM are also presented in this paper.

The remainder of this paper is organized as follows: in Section II, the system model of the Golden code is presented. In Section III, we formulate two theoretical bounds on the ABEP of the Golden code for MQAM. We then present the two reduced complexity detection schemes based on detection subset in Section IV. In Section V, we analyze the computational complexity of the proposed detection schemes and draw comparison with fast essentially ML detection and the conventional sphere decoding. The numerical results are demonstrated in Section VI. Finally, the paper is concluded in Section VII.

*Notation:* Bold lowercase and uppercase letters are used for vectors and matrices, respectively.  $[\cdot]^T$ ,  $(\cdot)^H$ ,  $|\cdot|$  and  $\|\cdot\|_F$  represent the transpose, Hermitian, Euclidean and Frobenius norm operations, respectively.  $\mathcal{D}(\cdot)$  is the constellation demodulator function.  $(\cdot)^{-1}$  is the inverse.  $E\{\cdot\}$  is

the expectation operation.  $\det(\cdot)$  denotes determinant.  $j$  is a complex number.

## II. SYSTEM MODEL

Consider a Golden code system with  $N_T = 2$  transmit antennas and  $N_R$  receive antennas,  $N_R \geq N_T$  [4]. Information bits are grouped into four bit streams,  $\mathbf{b}_i = [b_{i,1} \ b_{i,2} \ \cdots \ b_{i,r}]$ ,  $i \in [1 : 4]$ ,  $r = \log_2 M$ , where  $M$  is the modulation order. Each bit stream  $\mathbf{b}_i$  is then mapped onto a constellation point  $x_i$  of MQAM,  $x_i \in \Omega_M$ , where  $\Omega_M$  is the signal set of MQAM signals. The Golden code transmission matrix is given by [2]:

$$\mathbf{X} = [\mathbf{X}_1 \ \mathbf{X}_2] = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix}, \quad (1)$$

where  $x_{11} = \frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta)$ ,  $x_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})$ ,  $x_{12} = \frac{1}{\sqrt{5}}\alpha(x_3 + x_4\theta)$  and  $x_{21} = \frac{1}{\sqrt{5}}\gamma\bar{\alpha}(x_3 + x_4\bar{\theta})$ , with  $\theta = \frac{1+\sqrt{5}}{2}$ ,  $\bar{\theta} = 1 - \theta$ ,  $\alpha = 1 + j\bar{\theta}$ ,  $\bar{\alpha} = 1 + j(1 - \bar{\theta})$  and  $\gamma = j$ . It is assumed that  $E\{|x_i|^2\} = \varepsilon_\Omega$ ,  $i \in [1 : 4]$ . Let  $x_{11} \in \Omega_G$ , where  $\Omega_G$  is the signal set of  $x_{11}$ . Then Appendix A shows that  $x_{12} \in \Omega_G$ ,  $x_{21} \in \Omega_G$  and  $x_{22} \in \Omega_G$ . For convenience we regard  $x_{ij}$ ,  $i, j \in [1 : 2]$ , as Golden symbols.

The received signal in time slot  $i$ ,  $i \in [1 : 2]$  is given by:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{X}_i + \mathbf{n}_i, \quad (2)$$

where  $\mathbf{y}_i \in \mathbb{C}^{N_R \times 1}$  is the signal vector received in the  $i^{\text{th}}$ ,  $i \in [1 : 2]$  time slot.  $\mathbf{H}_i = [\mathbf{h}_{i,1} \ \mathbf{h}_{i,2}]$  is the channel gain matrix corresponding to the  $i^{\text{th}}$  time slot with  $\mathbb{C}^{N_R \times 1}$  column vectors  $\mathbf{h}_{i,1}$  and  $\mathbf{h}_{i,2}$ .  $\mathbf{n}_i \in \mathbb{C}^{N_R \times 1}$  is the additive white Gaussian noise (AWGN) vector for the  $i^{\text{th}}$  time slot. The entries of  $\mathbf{h}_{i,j}$  and  $\mathbf{n}_i$  are independent and identically distributed (i.i.d.) complex Gaussian random variables (RVs) distributed as  $CN(0,1)$  and  $CN(0, \frac{2\varepsilon_\Omega}{\rho})$ , respectively.  $\frac{\rho}{2\varepsilon_\Omega}$  is the average SNR at each receive antenna.

## III. PROPOSED BOUNDS ON ABEP ANALYSIS OF THE GOLDEN CODE

To the best of the authors' knowledge, the error performance analysis of the Golden code system has not been reported in the open literature. In this section, two bounds on the ABEP are formulated. We refer to these bounds as bound A and bound B. Bound A is based on the transmission of the equivalent single symbol  $x_i$ , where  $x_i \in \Omega_M$ , and bound B is based on the transmission of the equivalent single pair of Golden symbols  $x_{ij}$ , where  $x_{ij} \in \Omega_G$ .

### A. BOUND A

Let  $\hat{\mathbf{X}} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{21} \\ \hat{x}_{12} & \hat{x}_{22} \end{bmatrix}$ , then the conditional pairwise error probability (PEP)  $P(\mathbf{X} \rightarrow \hat{\mathbf{X}} | \mathbf{H}_1, \mathbf{H}_2)$  is defined as the transmitted codeword matrix  $\mathbf{X}$  which is detected as  $\hat{\mathbf{X}}$  at the receiver. Appendix B shows that the bounded PEP corresponds to the assumption that at high SNR only one symbol is detected with errors, while the remaining three symbols are detected correctly. There are two cases for this assumption:

Case 1: Suppose  $x_2$  is detected with errors, while  $x_j$ ,  $j \in [1, 3 : 4]$  are detected correctly. Then (2) may be simplified as:

$$z_i = \mathbf{g}_1^i x_2 + \mathbf{n}_i, \quad i \in [1 : 2], \quad (3)$$

where  $\mathbf{g}_1^1 = \frac{1}{\sqrt{5}}\alpha\theta\mathbf{h}_{1,1}$  and  $\mathbf{g}_1^2 = \frac{1}{\sqrt{5}}\bar{\alpha}\bar{\theta}\mathbf{h}_{2,2}$ . Let  $\sigma_1^2 = \left|\frac{1}{\sqrt{5}}\alpha\theta\right|^2$  and  $\sigma_2^2 = \left|\frac{1}{\sqrt{5}}\bar{\alpha}\bar{\theta}\right|^2$ . The entries of  $\mathbf{g}_1^1$  and  $\mathbf{g}_1^2$  are i.i.d. complex Gaussian RVs distributed as  $CN(0, \sigma_1^2)$  and  $CN(0, \sigma_2^2)$ , respectively.

Case 2: Suppose  $x_1$  is detected with errors, while  $x_j$ ,  $j \in [2 : 4]$  are detected correctly. Then (2) may be simplified as:

$$z_i = \mathbf{g}_2^i x_1 + \mathbf{n}_i, \quad i \in [1 : 2], \quad (4)$$

where  $\mathbf{g}_2^1 = \frac{1}{\sqrt{5}}\alpha\mathbf{h}_{1,1}$  and  $\mathbf{g}_2^2 = \frac{1}{\sqrt{5}}\bar{\alpha}\mathbf{h}_{2,2}$ . Since  $\left|\frac{1}{\sqrt{5}}\alpha\right|^2 = \left|\frac{1}{\sqrt{5}}\bar{\alpha}\right|^2$  and  $\left|\frac{1}{\sqrt{5}}\bar{\alpha}\right|^2 = \left|\frac{1}{\sqrt{5}}\alpha\right|^2$  the entries of  $\mathbf{g}_2^1$  and  $\mathbf{g}_2^2$  are i.i.d. complex Gaussian RVs distributed as  $CN(0, \sigma_2^2)$  and  $CN(0, \sigma_1^2)$ , respectively.

The equivalent models of error performance analysis in either (3) or (4) can be regarded as the transmission of either  $x_1$  or  $x_2$  over non-identical fading channels with fading variances  $\sigma_1^2$  and  $\sigma_2^2$ , respectively. Hence, the maximal ratio combining (MRC) technique with non-identical fading channels [14] can be used to derive the error performance of the above equivalent model.

Based on the exact symbol error probability of MQAM in Equ. (8.10) in [14], and the approximated expression of the Gaussian Q-function using the trapezoidal rule, bound A on the ABEP of MQAM Golden code systems may be derived as:

$$p_e \geq \frac{a}{n \log_2 M} \left[ \frac{1}{2} \prod_{k=1}^2 \left( \frac{2}{2 + \beta_k b \bar{\gamma}} \right)^{N_R} - \left( \frac{a}{2} \right) \times \prod_{k=1}^2 \left( \frac{1}{1 + \beta_k b \bar{\gamma}} \right)^{N_R} + (1 - a) \sum_{i=1}^{n-1} \prod_{k=1}^2 \left( \frac{u_i}{u_i + \beta_k b \bar{\gamma}} \right)^{N_R} + \sum_{i=n}^{2n-1} \prod_{k=1}^2 \left( \frac{u_i}{u_i + \beta_k b \bar{\gamma}} \right)^{N_R} \right], \quad (5)$$

where  $n \geq 6$  is the number of summations for convergence,  $\bar{\gamma} = \frac{\rho}{2\epsilon_\Omega}$ ,  $a = 1 - \frac{1}{\sqrt{M}}$ ,  $b = \frac{3}{M-1}$ ,  $\beta_1 = \sigma_1^2$ ,  $\beta_2 = \sigma_2^2$  and  $u_i = 2 \sin^2\left(\frac{i\pi}{4n}\right)$ .

### B. BOUND B

In subsection A, bound A is derived based on the assumption that at high SNR only one symbol is detected with errors, while the remaining three symbols are detected correctly. In this subsection, we will derive bound B based on the assumption that at high SNR only one pair of Golden symbols

is detected with errors, while the other pair of Golden symbols is detected correctly.

Suppose that the pair of Golden symbols  $(x_{11}, x_{22})$  is detected with errors, while the other pair of Golden symbols  $(x_{12}, x_{21})$  is detected correctly, then (2) may be simplified as:

$$z_i = \mathbf{h}_{i,i} x_{ii} + \mathbf{n}_i, \quad i \in [1 : 2]. \quad (6)$$

Let  $\mathbf{X}_p = (x_{11}, x_{22})$ , where  $p$  is the decimal index of the pair of Golden symbols  $\mathbf{X}_p$ ,  $p \in [1 : M^2]$ . Then bound B on the ABEP for the Golden code system is defined as:

$$p_e \leq \frac{1}{M^{2l}} \sum_{p=1}^{M^2} \sum_{\hat{p} \neq p}^{M^2} e(p, \hat{p}) P(\mathbf{X}_p \rightarrow \mathbf{X}_{\hat{p}}), \quad (7)$$

where  $P(\mathbf{X}_p \rightarrow \mathbf{X}_{\hat{p}})$  is the pairwise error probability (PEP) that the transmitted Golden symbols  $\mathbf{X}_p$  is detected as  $\mathbf{X}_{\hat{p}}$  at the receiver.  $l = \log_2 M^2$  and  $e(p, \hat{p})$  represents the number of bit errors for the associated PEP event.

Since both  $x_{11}$  and  $x_{22}$  convey the same information, we also regard  $x_{11}$  and  $x_{22}$  as a type of space-time labeling diversity (STLD) [15]. The PEP  $P(\mathbf{X}_p \rightarrow \mathbf{X}_{\hat{p}})$  of STLD has been derived in [15], which is given by:

$$P(\mathbf{X}_p \rightarrow \mathbf{X}_{\hat{p}}) = \frac{1}{2n} \left[ \frac{1}{2} \left( 1 + \frac{\bar{\gamma}}{4} d_1 \right)^{-N_R} \left( 1 + \frac{\bar{\gamma}}{4} d_2 \right)^{-N_R} + \sum_{k=1}^{n-1} \left( 1 + \frac{\bar{\gamma}}{4} \frac{d_1}{v_k} \right)^{-N_R} \left( 1 + \frac{\bar{\gamma}}{4} \frac{d_2}{v_k} \right)^{-N_R} \right], \quad (8)$$

where  $d_1 = |x_{11} - \hat{x}_{11}|^2$ ,  $d_2 = |x_{22} - \hat{x}_{22}|^2$ ,  $v_k = \sin^2\left(\frac{k\pi}{2n}\right)$  and as earlier  $n \geq 6$  is the number of summations for convergence.

## IV. REDUCED COMPLEXITY DETECTION SCHEMES FOR THE GOLDEN CODE

The optimal detection for the Golden code is ML detection. However, the complexity of ML detection is proportional to  $\mathcal{O}(M^4)$ . There are two near-ML error performance detection schemes for the Golden code: fast essentially ML detection [11] and sphere decoding [12]. The complexity of the fast essentially ML detection is proportional to  $\mathcal{O}(M^2)$ , while the complexity of sphere decoding also depends on the cardinality of the the Golden symbols,  $M^2$ . As stated earlier, we may reduce the complexity of the detection scheme if we reduce the size of the set of the given pair of symbols. Hence, in this section, we focus on reducing the signal detection size for a given pair of Golden symbols. We firstly present the conventional sphere decoding for the Golden code, then based on the fast essentially ML detector [11] and QR decomposition, we propose a reduced complexity detection scheme for fast essentially ML by employing a detection subset. Finally, we propose a reduced complexity detection with sphere decoding based on detection subset.

### A. SPHERE DECODING OF GOLDEN CODE

Sphere decoding is an algorithm which can achieve the error performance of ML detection. Sphere decoding for general MIMO systems has been documented in detail in [12]. In this paper, we firstly adapt the sphere decoding in [12] to decode the Golden code.

Based on the QR decomposition of  $\mathbf{H}_i$  in (2), we have:

$$\mathbf{H}_i = \mathbf{Q}_i \mathbf{R}_i, \quad i \in [1 : 2], \quad (9)$$

where  $\mathbf{Q}_i$  is a unitary matrix and  $\mathbf{R}_i = [\mathbf{R}_i^1 \ \mathbf{R}_i^{21}]^T$ , where  $\mathbf{R}_i^1$  is an upper-triangular matrix with  $2 \times 2$  nonnegative real diagonal elements and  $\mathbf{R}_i^{21}$  is a zero matrix with  $(N_R - 2) \times 2$  dimension.

Substituting (9) in (2), and multiplying both sides by  $\mathbf{Q}_i^H$ , we have:

$$\mathbf{z}_i = \mathbf{R}_i \mathbf{X}_i + \hat{\mathbf{n}}_i, \quad i \in [1 : 2], \quad (10)$$

where  $\hat{\mathbf{n}}_i = \mathbf{Q}_i^H \mathbf{n}_i$  and  $\mathbf{z}_i = [\mathbf{z}_i^1 \ \mathbf{z}_i^{21}]^T = \mathbf{Q}_i^H \mathbf{y}_i$ ,  $\mathbf{z}_i^1$  is a vector with  $2 \times 1$  dimension and  $\mathbf{z}_i^{21}$  is a vector with  $(N_R - 2) \times 1$  dimension.

Based on (10), the sphere decoding of the Golden code may be formulated as:

$$\left\| \mathbf{z}_i^1 - \mathbf{R}_i^1 \mathbf{X}_i \right\|_F^2 + \left\| \mathbf{z}_i^{21} - \mathbf{R}_i^{21} \mathbf{X}_i \right\|_F^2 \leq r^2, \quad (11)$$

where  $r$  is the radius of sphere decoding.

Let  $\mathbf{p}_i = \mathbf{z}_i^1 - \mathbf{R}_i^1 \mathbf{X}_i$ , where  $\mathbf{p}_i$  actually is a function of  $\mathbf{X}_i$ , then (11) can be written as:

$$\sum_{j=1}^2 |p_{1,j}|^2 + \sum_{j=1}^2 |p_{2,j}|^2 \leq r^2, \quad (12)$$

where  $p_{i,j}$  is the  $j^{\text{th}}$  entry of  $\mathbf{p}_i$ .

The upper-triangular matrix  $\mathbf{R}_i^1$  results in that  $p_{1,1}$  is a function of  $x_{11}$ ,  $p_{1,2}$  is a function of  $x_{12}$ ,  $p_{2,1}$  is a function of  $x_{22}$  and  $p_{2,2}$  is a function of  $x_{21}$ . Typically, when  $j = 2$  we have:

$$|p_{1,2}|^2(x_{11}) + |p_{2,2}|^2(x_{22}) \leq r^2, \quad (13)$$

Note that both  $x_{11}$  and  $x_{22}$  in (13), convey the same information.

Now sphere decoding searches  $x_{11}$  and  $x_{22}$  to meet the constraint in (13), where  $x_{11} \in \Omega_G$  and  $x_{22} \in \Omega_G$ . That is,  $|p_{1,2}|^2(x_{11}) + |p_{2,2}|^2(x_{22})$  lies inside a hyper-sphere of radius  $r$ .

After  $x_{11}$  and  $x_{22}$  are found to meet the constraint in (13), we then have:

$$|p_{1,1}|^2(x_{12}) + |p_{2,1}|^2(x_{21}) \leq r^2 - |p_{2,2}|^2(x_{11}, x_{22}), \quad (14)$$

where  $|p_{2,2}|^2(x_{11}, x_{22}) = |p_{1,2}|^2(x_{11}) + |p_{2,2}|^2(x_{22})$ .

Also note that both  $x_{12}$  and  $x_{21}$  in (14), convey the same information.

Again sphere decoding searches  $x_{12}$  and  $x_{21}$  to meet the constraint in (14), where  $x_{12} \in \Omega_G$  and  $x_{21} \in \Omega_G$ .

### B. PROPOSED FAST ESSENTIALLY ML WITH DETECTION SUBSET

The complexity of detection for the above sphere decoding is proportional to  $L^d$ , where  $L$  is the cardinality of  $\Omega_G$ , and  $d$  is the depth of search. In the MQAM Golden code system  $L$  is large, since  $L = M^2$ . In this subsection, we propose a reduced complexity detection, fast essentially ML with detection subset, which is based on the fast essentially ML detector in [11] and QR decomposition. There is a primary difference between the proposed detector and the fast essentially ML detector in [11]. The fast essentially ML detection scheme exhaustively searches the pair of symbols, while the proposed detector only searches a small subset of the symbol pairs. In the following discussions, we firstly present the fast essentially ML detection scheme of [11] and then present the proposed fast essentially ML with detection subset. The fast essentially ML detector [11] is summarized below.

The received signal given by (2), may be written as:

$$\mathbf{y}_1 = \frac{1}{\sqrt{5}} \alpha \mathbf{h}_{1,1} (x_1 + x_2 \theta) + \frac{1}{\sqrt{5}} \alpha \mathbf{h}_{1,2} (x_3 + x_4 \theta) + \mathbf{n}_1, \quad (15.1)$$

$$\mathbf{y}_2 = \frac{1}{\sqrt{5}} \gamma \bar{\alpha} \mathbf{h}_{2,1} (x_3 + x_4 \bar{\theta}) + \frac{1}{\sqrt{5}} \bar{\alpha} \mathbf{h}_{2,2} (x_1 + x_2 \bar{\theta}) + \mathbf{n}_2. \quad (15.2)$$

Let

$$\mathbf{Y} = [\mathbf{y}_1 \ \mathbf{y}_2]^T, \quad \mathbf{G}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha \mathbf{h}_{1,1} & \alpha \mathbf{h}_{1,2} \\ \bar{\alpha} \mathbf{h}_{2,2} & \gamma \bar{\alpha} \mathbf{h}_{2,1} \end{bmatrix},$$

$$\mathbf{G}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha \theta \mathbf{h}_{1,1} & \alpha \theta \mathbf{h}_{1,2} \\ \bar{\alpha} \bar{\theta} \mathbf{h}_{2,2} & \gamma \bar{\alpha} \bar{\theta} \mathbf{h}_{2,1} \end{bmatrix}, \quad \mathbf{S} = [x_1 \ x_3]^T,$$

$$\mathbf{C} = [x_2 \ x_4]^T \text{ and } \mathbf{N} = [\mathbf{n}_1 \ \mathbf{n}_2]^T.$$

Then (15.1) and (15.2) can be written as:

$$\mathbf{Y} = \mathbf{G}_1 \mathbf{S} + \mathbf{G}_2 \mathbf{C} + \mathbf{N}. \quad (16)$$

Ignoring noise  $\mathbf{N}$  in (16), the pair  $\mathbf{S}$  of symbols can be estimated, given the pair  $\mathbf{C}$  of symbols:

$$\mathbf{S} = (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H (\mathbf{Y} - \mathbf{G}_2 \mathbf{C}). \quad (17)$$

Alternatively, the pair  $\mathbf{C}$  of symbols can be estimated, given the pair  $\mathbf{S}$  of symbols:

$$\mathbf{C} = (\mathbf{G}_2^H \mathbf{G}_2)^{-1} \mathbf{G}_2^H (\mathbf{Y} - \mathbf{G}_1 \mathbf{S}). \quad (18)$$

The condition to choose either (17) or (18) depends on  $\det(\mathbf{G}_1^H \mathbf{G}_1) > \det(\mathbf{G}_2^H \mathbf{G}_2)$  or  $\det(\mathbf{G}_1^H \mathbf{G}_1) < \det(\mathbf{G}_2^H \mathbf{G}_2)$  [11]. If  $\det(\mathbf{G}_1^H \mathbf{G}_1) > \det(\mathbf{G}_2^H \mathbf{G}_2)$ , (17) is used to estimate  $\mathbf{S}$  given the pair  $\mathbf{C}$  of symbols. Otherwise (18) is used to estimate  $\mathbf{C}$  given the pair  $\mathbf{S}$  of symbols.

Note, in [11] either (17) or (18) will be used in the fast essentially ML detection scheme based on  $\det(\mathbf{G}_1^H \mathbf{G}_1) > \det(\mathbf{G}_2^H \mathbf{G}_2)$  or  $\det(\mathbf{G}_1^H \mathbf{G}_1) < \det(\mathbf{G}_2^H \mathbf{G}_2)$ . However, it is recommended that both (17) and (18) are used in the proposed reduced complexity detection scheme.

The complexity of (17) or (18) is  $\mathcal{O}(M^2)$  because ML detection needs to exhaustively search the entire set of Golden



symbols [11]. For high-order modulation,  $M \geq 16$ , the above detection complexity remains very high.

Next, we present the proposed reduced complexity detection scheme.

Based on the Golden code matrix in (1), we have  $x_{11} = \frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta)$ ,  $x_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta})$ ,  $x_{12} = \frac{1}{\sqrt{5}}\alpha(x_3 + x_4\theta)$  and  $x_{21} = \frac{1}{\sqrt{5}}\gamma\bar{\alpha}(x_3 + x_4\bar{\theta})$ . We can easily obtain:

$$x_1 = \frac{\sqrt{5}}{\mu} \left( \frac{\theta x_{22}}{\bar{\alpha}} - \frac{\bar{\theta} x_{11}}{\alpha} \right), \quad (19.1)$$

$$x_2 = \frac{\sqrt{5}}{\mu} \left( \frac{x_{11}}{\alpha} - \frac{x_{22}}{\bar{\alpha}} \right), \quad (19.2)$$

$$x_3 = \frac{\sqrt{5}}{\mu} \left( \frac{\theta x_{21}}{\gamma\bar{\alpha}} - \frac{\bar{\theta} x_{12}}{\alpha} \right), \quad (19.3)$$

$$x_4 = \frac{\sqrt{5}}{\mu} \left( \frac{x_{12}}{\alpha} - \frac{x_{21}}{\gamma\bar{\alpha}} \right), \quad (19.4)$$

where  $\mu = \theta - \bar{\theta}$ .

If  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$  and  $x_{22}$  are known then  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  can be estimated using (19.1) to (19.4).  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$  and  $x_{22}$  can be estimated based on QR decomposition, which will be explained below.

QR decomposition has been discussed in Subsection A.

Let  $\mathbf{z}_i^1 = [z_i^{11} \quad z_i^{12}]^T$ ,  $\mathbf{R}_i^1 = \begin{bmatrix} r_i^{11} & r_i^{12} \\ 0 & r_i^{22} \end{bmatrix}$ . From (10),  $x_{ij}$ ,  $i, j \in [1 : 2]$  can be estimated as:

$$\hat{x}_{12} = z_1^{12}/r_1^{22}, \quad (20.1)$$

$$\hat{x}_{11} = (z_1^{11} - r_1^{12}x_{12})/r_1^{11}, \quad (20.2)$$

$$\hat{x}_{22} = z_2^{12}/r_2^{22}, \quad (20.3)$$

$$\hat{x}_{21} = (z_2^{11} - r_2^{12}x_{22})/r_2^{11}. \quad (20.4)$$

The error performance of the above detection using (19.1) to (19.4) and (20.1) to (20.4) is degraded, therefore the proposed fast essentially ML with detection subset, is based on the above detection together with both (17) and (18). Thus, in the proposed detection scheme, we may form signal subsets based on the estimations from (19.1) to (19.4) and (20.1) to (20.4), then use these signal subsets to replace the whole signal sets in (17) and (18), in order to substantially reduce detection complexity.

**Definition 1:** Given an  $i^{th}$  symbol  $x_i$ , an  $i^{th}$  symbol detection subset<sup>1</sup> (SDS) is defined as  $\Omega(x_i, \delta) = \{x_j, |x_j - x_i|^2 \leq \delta, j \in [1 : M]\}$ .

For example, if the SDS only contains the nearest neighbors, then the average cardinality  $\bar{L}$  is 4 and 4.5 for 16QAM ( $\delta = 4$ ) and 64QAM ( $\delta = 4$ ), respectively. For  $M \geq 16$ , the detection complexity in (17) or (18) can be greatly reduced if the SDS is used to replace the entire signal set.

Hence, the proposed reduced complexity detection scheme is summarized as follows:

<sup>1</sup>Note, based on Definition 1, the worst-case subset cardinality is  $M$  for the fast essentially ML with detection subset.

**Initialization:** Construct an SDS for each symbol  $x_i$ ,  $i \in [1 : M]$ .

**Step 1:** Perform QR decomposition and calculate  $\mathbf{z}_i$  based on (10),  $i \in [1 : 2]$ .

**Step 2:** Estimate  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$  and  $x_{22}$ , using (20.1) to (20.4).

**Step 3:** Estimate  $x_i$ ,  $i \in [1 : 4]$ , using (19.1) to (19.4).

**Step 4:** Determine  $[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]$ , given  $x_2 \in \Omega(x_2, \delta)$  and  $x_4 \in \Omega(x_4, \delta)$  using:

$$\begin{aligned} \tilde{\mathbf{S}}(\mathbf{C}) &= (\mathbf{G}_1^H \mathbf{G}_1)^{-1} \mathbf{G}_1^H (\mathbf{Y} - \mathbf{G}_2 \mathbf{C}), \\ \hat{\mathbf{S}} &= \mathcal{D}(\tilde{\mathbf{S}}(\mathbf{C})), \end{aligned} \quad (21.1)$$

$$[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4] = \min_{\substack{x_2 \in \Omega(x_2, \delta) \\ x_4 \in \Omega(x_4, \delta)}} \{\|\mathbf{Y} - (\mathbf{G}_1 \hat{\mathbf{S}}(\mathbf{C}) + \mathbf{G}_2 \mathbf{C})\|_F^2\}, \quad (21.2)$$

and, determine  $[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]$ , given  $x_1 \in \Omega(x_1, \delta)$  and  $x_3 \in \Omega(x_3, \delta)$  using:

$$\begin{aligned} \tilde{\mathbf{C}}(\mathbf{S}) &= (\mathbf{G}_2^H \mathbf{G}_2)^{-1} \mathbf{G}_2^H (\mathbf{Y} - \mathbf{G}_1 \mathbf{S}), \\ \hat{\mathbf{C}} &= \mathcal{D}(\tilde{\mathbf{C}}(\mathbf{S})), \end{aligned} \quad (22.1)$$

$$[\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4] = \min_{\substack{x_1 \in \Omega(x_1, \delta) \\ x_3 \in \Omega(x_3, \delta)}} \{\|\mathbf{Y} - (\mathbf{G}_1 \mathbf{S} + \mathbf{G}_2 \hat{\mathbf{C}}(\mathbf{S}))\|_F^2\}. \quad (22.2)$$

Then, given  $d_{min}^{24}$ , the minimum distance calculated in (21.2), and  $d_{min}^{13}$ , the minimum distance calculated in (22.2), we choose  $\hat{x}_1$ ,  $\hat{x}_2$ ,  $\hat{x}_3$  and  $\hat{x}_4$  from (21.2) if  $d_{min}^{24} < d_{min}^{13}$ , otherwise we choose  $\hat{x}_1$ ,  $\hat{x}_2$ ,  $\hat{x}_3$  and  $\hat{x}_4$  from (22.2).

### C. PROPOSED SPHERE DECODING WITH DETECTION SUBSET

The detection complexity of the proposed fast essentially ML with detection subset is only  $\mathcal{O}(2 \times \bar{L}^2)$ . Since  $\bar{L} < M$ , a reduction in complexity is evident compared to the fast essentially ML detector of [11]. This motivates us to replace the whole signal set with the subset in (13) and (14) in sphere decoding. In this subsection, we propose a sphere decoding algorithm with detection subset. The detection subset is given in Definition 2.

**Definition 2:** Given a pair of Golden symbols,  $\mathbf{X}_p = (x_{11}, x_{22})$  or  $\mathbf{X}_p = (x_{12}, x_{21})$  in the conventional Golden code. A detection subset<sup>2</sup> of  $\mathbf{X}_p$  is defined as  $\Omega(\mathbf{X}_p, \delta_s) = \{(\hat{x}_{11}, \hat{x}_{22}), |x_{11} - \hat{x}_{11}|^2 |x_{22} - \hat{x}_{22}|^2 \leq \delta_s, \hat{x}_{11}, \hat{x}_{22} \in \Omega_G\}$ .

For example, for  $N_R = 4$ , if we set  $\delta_s = 16$  for 16QAM and  $\delta_s = 28.8$  for 64QAM, then the average cardinality  $\bar{L}_s$  of Golden symbols is reduced from  $16^2$  and  $64^2$  to 39.1 and 86.2 for 16QAM and 64QAM, respectively.

The proposed sphere decoding with detection subset is summarized as follows:

**Initialization:** Construct detection subset for each pair of Golden symbols,  $\mathbf{X}_p = (x_{11}, x_{22})$  and  $\mathbf{X}_p = (x_{12}, x_{21})$ ,  $x_{ij} \in \Omega_G$ ,  $i, j \in [1 : 2]$ .

<sup>2</sup>Note, based on Definition 2, the worst-case subset cardinality is  $M^2$  for the sphere decoding with detection subset.

**Step 1:** Perform QR decomposition and calculate  $z_i$  based on (10),  $i \in [1 : 2]$ .

**Step 2:** Estimate  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$  and  $x_{22}$ , using (20.1) to (20.4).

**Step 3:** Find detection subset for pairs of Golden symbols,  $(x_{11}, x_{22})$  and  $(x_{12}, x_{21})$ .

**Step 4:** Perform sphere decoding with detection subset based on (13) and (14).

### V. COMPLEXITY ANALYSIS

In this section, we discuss the detection complexity for the proposed detection schemes.

For the detection complexity analysis of sphere decoding we focus on the size of signal set to be searched. The conventional sphere decoding exhaustively searches the entire signal set. The size of signal set to be searched for the conventional sphere decoding is  $M^2$ , while the size of signal set to be searched for the proposed sphere decoding with detection subset is  $\bar{L}_s^2$ . For example, with  $N_R = 4$  the size of signal set for the proposed sphere decoding with detection subset is only 39.1 and 86.2 for 16QAM and 64QAM, respectively. It is easily seen that the detection complexity is greatly reduced.

Next, we discuss the detection complexity for proposed fast essentially ML with detection subset. The computational complexity is formulated in terms of floating point operations (flops) [16]–[18], where each addition, subtraction, multiplication, or division counts as a single flop.

#### A. COMPLEX OPERATIONS OF STEP 1

$H_i$  is an  $N_R \times 2$  matrix. Performing a QR decomposition, we require  $8N_R - \frac{16}{3}$  flops for each  $i$  via the Householder algorithm [16]. Computation of  $(Q_i)^H y_i$  in (10), for  $i \in [1 : 2]$ .  $Q_i$  is an  $N_R \times N_R$  matrix, while  $y_i$  is an  $N_R \times 1$  vector. This requires  $N_R^2$  multiplications and  $N_R(N_R - 1)$  additions for each  $i$ . The overall number of flops for Step 1 is:  $\sigma_{Step1} = 4N_R^2 + 14N_R - \frac{32}{3}$ .

#### B. COMPLEX OPERATIONS OF STEP 2

Step 2 estimates  $x_{11}$ ,  $x_{12}$ ,  $x_{21}$  and  $x_{22}$  based on (20.1)–(20.4). The overall number of flops for Step 2 is then:  $\sigma_{Step2} = 8$ .

#### C. COMPLEX OPERATIONS OF STEP 3

Estimating each  $x_i$  needs two multiplications and one addition. The overall flops of Step 3 is:  $\sigma_{Step3} = 12$ .

##### d. Complex operations of Step 4:

In this step, we are required to compute (21.1), (21.2) and (22.1), (22.2). Since the complexity imposed by the computation of (21.1), (21.2) or (22.1), (22.2) is identical, we may analyze (22.1) and (22.2). In (22.1), since the constellation demodulator function represents a one-to-one mapping [16], [17], only the computation of  $(G_2^H G_2)^{-1} G_2^H (Y - G_1 S)$  imposes complexity. The number of flops for  $(G_2^H G_2)^{-1} G_2^H (Y - G_1 S)$  is  $(44N_R + 9)$ . Note, we have employed an LU decomposition to find the inverse of the matrix  $A$  [18]. The detail derivation is in Table 1.

TABLE 1. Complexity analysis of  $(G_2^H G_2)^{-1} G_2^H (Y - G_1 S)$ .

matrix	dimension	operation	No. of flops
$G_2$	$2N_R \times 2$	$G_2^H G_2$	$16N_R - 4$
$G_2^H$	$2 \times (2N_R)$		
$A$	$2 \times 2$	$A^{-1}$	15
$A^{-1}$	$2 \times 2$	$A^{-1} G_2^H$	$12N_R$
$G_2^H$	$2 \times (2N_R)$		
$G_1$	$2N_R \times 2$	$G_1 S$	$6N_R$
$S$	$2 \times 1$		
$Y$	$2N_R \times 1$	$D$	$2N_R$
$B$	$2 \times 2N_R$	$BD$	$8N_R - 2$
$D$	$2N_R \times 1$		

TABLE 2. Complexity analysis of (22.2).

matrix	dimension	operation	No. of flops
$G_1$	$2N_R \times 2$	$G_1 S$	$6N_R$
$S$	$2 \times 1$		
$G_2$	$2N_R \times 2$	$G_2 \hat{C}(S)$	$6N_R$
$\hat{C}(S)$	$2 \times 1$		
$E$	$2N_R \times 1$	$E + F$	$2N_R$
$F$	$2N_R \times 1$		
$Y$	$2N_R \times 1$	$Y - (E + F)$	$2N_R$
$E + F$	$2N_R \times 1$		
$W$	$2N_R \times 1$	$\ W\ _F^2$	$4N_R - 1$

Given  $x_1$  and  $x_3$ , the number of flops for (22.2) is  $(20N_R - 1)$ . The detail derivation is in Table 2.

Since the average cardinality is  $\bar{L}$  the overall number of flops for Step 4 is  $(20N_R - 1)\bar{L} + 44N_R + 9$ .

The overall number of complex operations imposed by the proposed reduced complexity detection is expressed as:

$$\sigma_{proposed} = 4N_R^2 + (20\bar{L} + 58)N_R + 29 - \bar{L} - \frac{32}{3}. \quad (23)$$

Based on the above calculation of flops, the overall number of flops for the fast essentially ML detection algorithm in [11] can be calculated, and is expressed as:

$$\sigma_{fML} = 20M^2 N_R + 44N_R - M^2 + 9. \quad (24)$$

Define the percentage of complexity reduction for the proposed reduced complexity fast essentially ML detection based on detection subset compared to the fast essentially ML detection algorithm [11] as:

$$\beta = \frac{\sigma_{fML} - \sigma_{proposed}}{\sigma_{fML}} \times 100. \quad (25)$$

Suppose the SDS only contains the nearest neighbors, the average cardinality  $\bar{L}$  is 4 and 4.5 for 16QAM and 64QAM, respectively, when  $N_R = 4$ . In comparison to the fast essentially ML detector in [11], for  $N_R = 4$  the proposed fast essentially ML with detection subset results in a 94.5%

and 99.6% complexity reduction for 16QAM and 64QAM, respectively.

$$\text{where } \mathbf{A} = \mathbf{G}_2^H \mathbf{G}_2, \mathbf{B} = (\mathbf{G}_2^H \mathbf{G}_2)^{-1} \mathbf{G}_2^H \text{ and } \mathbf{D} = \mathbf{Y} - \mathbf{G}_1 \mathbf{S}.$$

$$\text{where } \mathbf{E} = \mathbf{G}_1 \mathbf{S}, \mathbf{F} = \mathbf{G}_2 \hat{\mathbf{C}}(\mathbf{S}), \text{ and } \mathbf{W} = \mathbf{Y} - (\mathbf{E} + \mathbf{F}).$$

### VI. NUMERICAL RESULTS

In this section, we firstly present a guideline of using the theoretical bounds for selection of detection subset, and then present the simulation results.

#### A. SELECTION OF SDS

In this subsection, we discuss how to use the theoretical bounds as a guideline to select detection subset for the proposed detection schemes.

The SDS in Definition 1 depends on  $\delta$ . Given an MQAM Golden code the set based on  $\delta$  can be easily generated based on the calculation of Euclidean distance. For example, the set based on  $\delta$  for 16QAM Golden code is  $\{0, 4, 8, 16, 20, 32, 36, 40, 52, 72\}$ . The detection complexity is proportional to the size of  $\delta$ . Hence, a smaller  $\delta$  results in a lower detection complexity. Thus, the objective of selecting the SDS is to choose a small  $\delta$  which can achieve the error performance of the fast essentially ML detection in [11]. Suppose we set  $\delta = 4$ , the SDS only contains the nearest neighbors. Then bound A given by (5) on ABEP of MQAM Golden code system, becomes:

$$p_e \geq \frac{a}{n \log_2 M} \left[ \frac{1}{2} \prod_{k=1}^2 \left( \frac{2}{2 + \beta_k b \bar{\gamma}} \right)^{N_R} + \sum_{i=1}^{n-1} \prod_{k=1}^2 \left( \frac{u_i}{u_i + \beta_k b \bar{\gamma}} \right)^{N_R} + \sum_{i=n}^{2n-1} \prod_{k=1}^2 \left( \frac{u_i}{u_i + \beta_k b \bar{\gamma}} \right)^{N_R} \right]. \quad (26)$$

The SDS in Definition 2 depends on  $\delta_s$ . Similar to the above discussion, bound B given by (7) on ABEP of MQAM Golden code system, becomes:

$$p_e \leq \frac{1}{M^{2l}} \sum_{p=1}^{M^2} \sum_{\mathbf{X}_{\hat{p}} \in \Omega(\mathbf{X}_p, \delta_s)} e(p, \hat{p}) P(\mathbf{X}_p \rightarrow \mathbf{X}_{\hat{p}}). \quad (27)$$

The evaluated bound A based on (5) and bound A based on (26) with  $\delta = 4$  for 16QAM and 64QAM Golden code with  $N_R = 4$  are shown in Fig. 1.

The theoretical results in Fig. 1 show that the error performance of (26) matches the error performance of (5) when SNR is greater than 10 dB and 16 dB for 16QAM and 64QAM Golden codes, respectively. That guides us to set  $\delta = 4$  in the proposed fast essentially ML detection for 16QAM and 64QAM Golden codes with  $N_R = 4$ .

Similarly, the evaluated bound B based on (7) and bound B based on (27) with different  $\delta_s$  for 16QAM and 64QAM Golden code with  $N_R = 4$  are shown in Fig. 2. The theoretical results in Fig. 2 show that the error performance of (27) with

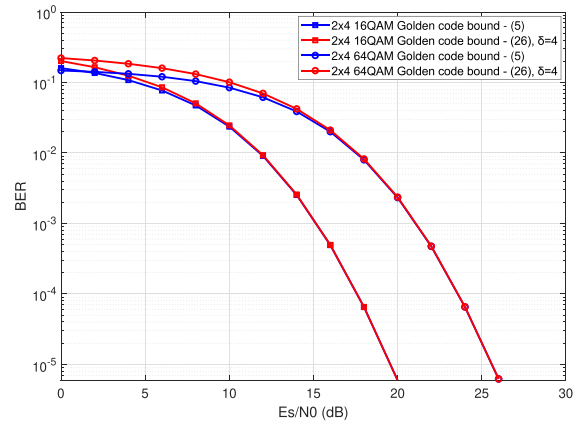


FIGURE 1. Comparison of theoretical bounds (5) and (26) showing choice of  $\delta$ .

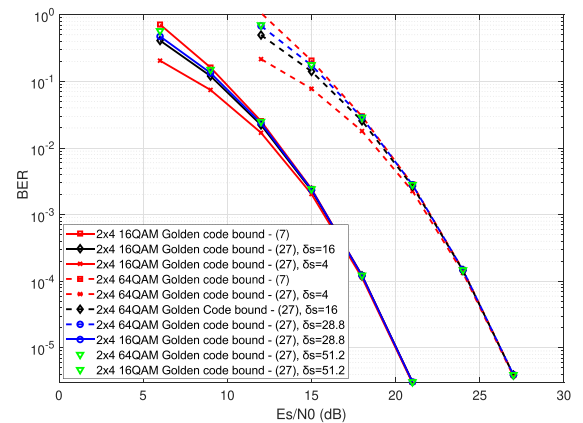


FIGURE 2. Comparison of theoretical bounds (7) and (27) showing choice of  $\delta_s$ .

all  $\delta_s$  almost matches the error performance of (7) when SNR is greater than 15 dB and 21 dB for 16QAM and 64QAM Golden codes. The results in Fig. 2 also show that there is no significant difference of error performance with  $\delta_s = 16$  and  $\delta_s = 28.8$  for 16QAM Golden code and  $\delta_s = 28.8$  and  $\delta_s = 51.2$  for 64QAM Golden code. Taking into account the detection complexity and the error performance of (27) to achieve the error performance of (7), we set  $\delta_s = 16$  and  $\delta_s = 28.8$  for 16QAM and 64QAM Golden code, respectively, in the proposed sphere decoding with detection subset. Note, the above is explained for the case of  $N_R = 4$ . For  $N_R = 3$ , a similar approach may be used.

#### B. SIMULATION RESULTS

In this subsection, we present the simulation results for the proposed sphere decoding with detection subset and the proposed fast essentially ML with detection subset for  $M = 16$  and  $M = 64$  using  $N_R = 3$  and  $N_R = 4$ . For comparison, we also simulate the conventional sphere decoding and the fast essentially ML detection in [11], and plot the formulated theoretical bounds on the ABEP in (5) and (7). The threshold  $\delta$  and  $\delta_s$  for the proposed fast essentially ML with detection

TABLE 3.  $\delta$  and  $\delta_s$  for detection subset.

$M$	$N_R = 3$	$N_R = 4$
16	$\delta = 16, \delta_s = 195.2$ $\bar{L} = 8.25, \bar{L}_s = 135.4$	$\delta = 4, \delta_s = 16$ $\bar{L} = 4, \bar{L}_s = 39.1$
64	$\delta = 20, \delta_s = 195.2$ $\bar{L} = 15.8, \bar{L}_s = 362$	$\delta = 4, \delta_s = 28.8$ $\bar{L} = 4.5, \bar{L}_s = 86.2$

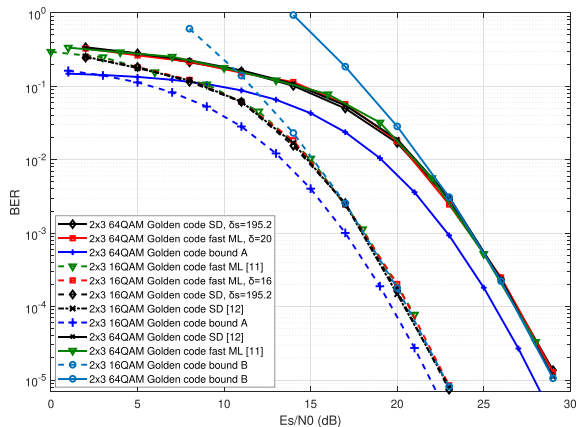


FIGURE 3. Comparison of error performance for 2 x 3, 16QAM and 64QAM Golden code including theoretical bounds.

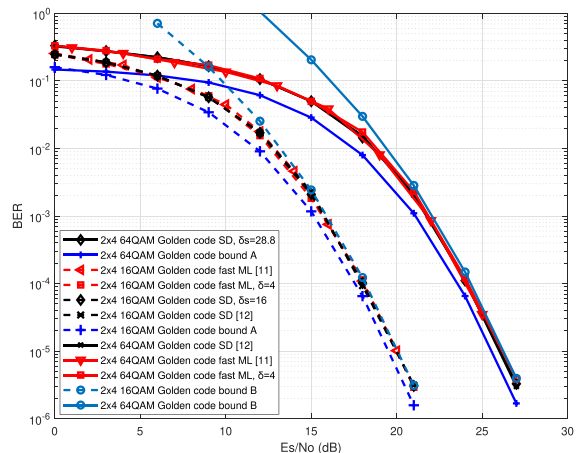


FIGURE 4. Comparison of error performance for 2 x 4, 16QAM and 64QAM Golden code including theoretical bounds.

subset and the proposed sphere decoding with detection subset are shown in Table 3.

The corresponding average subset cardinalities  $\bar{L}$  and  $\bar{L}_s$  of  $M$ QAM symbols and Golden symbols are also shown in Table 3. A Rayleigh frequency-flat fading channel with AWGN, as described in (2) is considered. It is assumed that the channel state information is fully known at the receiver.

Table 3 shows that the average subset cardinalities of  $M$ QAM symbols and Golden symbols decrease as the number of receive antennas increase. Fig. 3 and Fig. 4 show the simulation results for  $2 \times 3$  and  $2 \times 4$  Golden code systems, respectively.

Simulation results presented in Fig. 3 and Fig. 4 show that both fast essentially ML detection and sphere decoding achieve the same error performance. It is further shown that both fast essentially ML with detection subset and sphere decoding with detection subset agree with the formulated theoretical bounds and can achieve the error performance of the conventional fast essentially ML detector and sphere decoding.

### VII. CONCLUSION

Based on the fast essentially ML detection in [11] and QR decomposition, a reduced complexity detection scheme with detection subset for the Golden code with  $M$ QAM was proposed. The proposed scheme matches the error performance of the conventional fast essentially ML detection and only has a complexity  $\mathcal{O}(2 \times 4^2)$  and  $\mathcal{O}(2 \times 4.5^2)$  for 16QAM and 64QAM, respectively. For  $N_R = 4$ , the proposed reduced complexity detection scheme results in a 94.5% and 99.6% complexity reduction for 16QAM and 64QAM, respectively, compared to the fast essentially ML detection in [11]. A reduced complexity detection based on sphere decoding with detection subset was also presented. Two theoretical bounds on the ABEP for  $M$ QAM was further derived and validated by simulation results.

### APPENDIX A

From (1), we have:

$$\frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta) = \beta_1 e^{j\theta_1}x_1 + \beta_2 e^{j\theta_2}x_2, \quad (28)$$

where  $\beta_1 = 0.5257, \beta_2 = 0.8507, \theta_1 = \theta_2 = -31.7175^\circ$ . Again from (1), we also have:

$$\frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta}) = \beta_2 e^{j(\theta_1+90^\circ)}x_1 + \beta_1 e^{j(\theta_1-90^\circ)}x_2. \quad (29)$$

For square  $M$ QAM modulation, if  $x_1 \in \Omega_M$  then  $e^{j90^\circ}x_1 \in \Omega_M$ . Similarly if  $x_2 \in \Omega_M$  then  $e^{-j90^\circ}x_2 \in \Omega_M$ . So if  $x_{11} = \frac{1}{\sqrt{5}}\alpha(x_1 + x_2\theta) \in \Omega_G$ , then  $x_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}(x_1 + x_2\bar{\theta}) \in \Omega_G$ .

Similarly, we can also prove  $x_{12} \in \Omega_G$  and  $x_{21} \in \Omega_G$ .

### APPENDIX B

The signal vector (2) can be rewritten as:

$$y_i = h_{i,1}x_{i1} + h_{i,2}x_{i2} + n_i, \quad i \in [1 : 2]. \quad (30)$$

The conditional PEP  $P(X \rightarrow \hat{X} | H_1, H_2)$  is given by:

$$\begin{aligned} P(X \rightarrow \hat{X} | H_1, H_2) &= P\left(\sum_{i=1}^2 \|y_i - (h_{i,1}x_{i1} + h_{i,2}x_{i2})\|_F^2\right) \\ &\geq \sum_{i=1}^2 \|y_i - (h_{i,1}\hat{x}_{i1} + h_{i,2}\hat{x}_{i2})\|_F^2. \end{aligned} \quad (31)$$



Substituting (30) into (31) based on  $|a + b|^2 \geq |a|^2 - |b|^2$ , (31) is simplified into:

$$\begin{aligned} P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2) &= P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A - B\right) \\ &\geq \sum_{i=1}^2 \|\mathbf{h}_{i,1}(x_{i1} - \hat{x}_{i1}) + \mathbf{h}_{i,2}(x_{i2} - \hat{x}_{i2})\|_F^2. \end{aligned} \quad (32)$$

Let  $A = \|\mathbf{h}_{1,1}(x_{11} - \hat{x}_{11})\|_F^2 + \|\mathbf{h}_{2,2}(x_{22} - \hat{x}_{22})\|_F^2$  and  $B = \|\mathbf{h}_{1,2}(x_{12} - \hat{x}_{12})\|_F^2 + \|\mathbf{h}_{2,1}(x_{21} - \hat{x}_{21})\|_F^2$ .

Again, based on  $|a + b|^2 \geq |a|^2 - |b|^2$ , (32) is further simplified into:

$$P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2) = P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A - B\right) \quad \text{if } A \geq B, \quad (33.1)$$

or

$$P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2) = P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq B - A\right) \quad \text{if } B > A. \quad (33.2)$$

Since  $2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A \geq A - B$ , we have  $P(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A - B) \geq P(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A)$ . Then (33.1) can be bounded as:

$$\begin{aligned} P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2) &= P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A - B\right) \\ &\geq P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq A\right) \quad \text{if } A \geq B. \end{aligned} \quad (34.1)$$

Similarly (33.2) can be bounded as:

$$\begin{aligned} P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2) &= P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq B - A\right) \\ &\geq P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq B\right) \quad \text{if } B > A. \end{aligned} \quad (34.2)$$

The bounded  $P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2)$  in (34.1) is equivalent to assuming that  $x_{12}$  and  $x_{21}$  are detected correctly, while  $x_{11}$  and  $x_{22}$  are detected with errors.

Similarly, the bounded  $P(X \rightarrow \hat{X} | \mathbf{H}_1, \mathbf{H}_2)$  in (34.2) is equivalent to assuming that  $x_{11}$  and  $x_{22}$  are detected correctly, while  $x_{12}$  and  $x_{21}$  are detected with errors.

Suppose that a pair of symbols  $x_{11}$  and  $x_{22}$  are detected with errors, while another pair of symbols  $x_{21}$  and  $x_{12}$  are detected correctly, then (30) can be expressed as:

$$\mathbf{z}_1 = \mathbf{h}_{1,1}x_{11} + \mathbf{n}_1 = \mathbf{g}_{11}x_1 + \mathbf{g}_{12}x_2 + \mathbf{n}_1, \quad (35.1)$$

$$\mathbf{z}_2 = \mathbf{h}_{2,2}x_{22} + \mathbf{n}_2 = \mathbf{g}_{21}x_1 + \mathbf{g}_{22}x_2 + \mathbf{n}_2, \quad (35.2)$$

where  $\mathbf{g}_{11} = \frac{1}{\sqrt{5}}\alpha\mathbf{h}_{1,1}$ ,  $\mathbf{g}_{12} = \frac{1}{\sqrt{5}}\alpha\theta\mathbf{h}_{1,1}$ ,  $\mathbf{g}_{21} = \frac{1}{\sqrt{5}}\bar{\alpha}\mathbf{h}_{2,2}$  and  $\mathbf{g}_{22} = \frac{1}{\sqrt{5}}\bar{\alpha}\theta\mathbf{h}_{2,2}$ .

Let  $\mathbf{X}_{12} = [x_1 \ x_2]$  and  $\hat{\mathbf{X}}_{12} = [\hat{x}_1 \ \hat{x}_2]$ , the conditional PEP  $P(\mathbf{X}_{12} \rightarrow \hat{\mathbf{X}}_{12} | \mathbf{h}_{1,1}, \mathbf{h}_{1,2})$  is defined as the transmitted codeword  $\mathbf{X}_{12}$  which is detected as  $\hat{\mathbf{X}}_{12}$  at the receiver.

Similar to the derivation of (31) to (34) the conditional PEP  $P(\mathbf{X}_{12} \rightarrow \hat{\mathbf{X}}_{12} | \mathbf{h}_{1,1}, \mathbf{h}_{1,2})$  is bounded as:

$$P(\mathbf{X}_{12} \rightarrow \hat{\mathbf{X}}_{12} | \mathbf{h}_{1,1}, \mathbf{h}_{1,2}) = P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq C - D\right) \quad \text{if } C \geq D, \quad (36.1)$$

or

$$P(\mathbf{X}_{12} \rightarrow \hat{\mathbf{X}}_{12} | \mathbf{h}_{1,1}, \mathbf{h}_{1,2}) = P\left(2 \sum_{i=1}^2 \|\tilde{\mathbf{n}}_i\|_F^2 \geq D - C\right) \quad \text{if } D > C, \quad (36.2)$$

where  $C = (\|\mathbf{h}_{1,1}\|_F^2 + \|\mathbf{h}_{2,2}\|_F^2)|x_1 - \hat{x}_1|^2$  and  $D = (\|\mathbf{h}_{1,2}\|_F^2 + \|\mathbf{h}_{2,1}\|_F^2)|x_2 - \hat{x}_2|^2$ .

Again, the bounded  $P(\mathbf{X}_{12} \rightarrow \hat{\mathbf{X}}_{12} | \mathbf{h}_{1,1}, \mathbf{h}_{1,2})$  in (36.1) and (36.2) are equivalent to assuming that either  $x_2$  or  $x_1$  is detected correctly, while  $x_1$  or  $x_2$  is detected with errors, respectively.

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