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# Adaptive Fault Estimation and Fault-Tolerant Control for Nonlinear System With Unknown Nonlinear Dynamic

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**ABSTRACT** This paper investigates the problems of fault estimation and fault-tolerant control for nonlinear system. The nonlinear part of the system is assumed to be unknown. Based on the adaptive approximation technique, which can be performed by fuzzy logic systems or neural networks, an adaptive fault estimation observer is designed, and the fault and the system state can be estimated simultaneously. Based on the estimation information, the observer-based fault-tolerant controller is designed. In this paper, the parameter matrices of the adaptive law and the observer and the controller can be calculated by slowing LMIs. To verify the proposed scheme, a simulation example is provided at the end of this paper.

**INDEX TERMS** Adaptive approximation, adaptive fault estimation observer, observer-based fault-tolerant control, regional pole placement.

## I. INTRODUCTION

In modern industrial systems, the safety of system operation is becoming more and more important. However, various types of faults are inevitable, which can affect the safe operation of the system and may cause significant losses. Fault diagnosis and fault-tolerant control are common techniques for handling faults, and have become hot topics in recent decades [1]–[8].

Generally speaking, fault diagnosis can be divided into three methods: model-based method, signal-based method and date-based method. Compared with the signal-based method and the date-based method, model-based fault diagnosis method can make full use of the dynamic information of the system model, and the system model can be constructed by considering the system operation mechanism. Recently, many researchers have considered the model based fault diagnosis and fault-tolerant control, and many results have been reported, such as [9]–[15]. As it pointed in [16], fault diagnosis is composed of three parts: fault detection, fault isolation and fault estimation. In [17], a fault detection filter has been designed for a class of nonlinear network systems. A robust filter-based fault detection method has been proposed in [18], where both the $H_\infty$ performance and $H_-$ performance have been considered to increase the robustness and sensibility of the proposed detection technique. In [19], an observer based fault detection method has been reported for nonlinear system with fault and limited communication capacity. For switched nonlinear system, the adjustable dimension observer based fault detection method has been reported [20], [21]. The problem of fault detection for closed-loop control system has been considered in [22]. For multi-agent systems, fault detection methods have been reported by [23], [24], and these methods can be applied for fault isolation. A new hierarchical fault detection and fault isolation method has been reported by [25] for complex industrial processes. In [26], the problems of fault detection and it based fault-tolerant control have been considered. The integrated design of fault detection, isolation, and control has been reported by [27] for Markovian jump systems.

It should be noted that fault estimation can obtain more information of fault, such as the fault size and shape, and the information is useful in considering the fault-tolerant control. Compared with fault detection and fault isolation,

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fault estimation is more challenging, and has become a hot topic in recent years [28]–[34]. In [35], a fault estimation observer has been designed to estimate the actuator fault, and corresponding fault-tolerant control method has been proposed. A dynamic unknown input observer-based sensor fault estimation method has been reported by [36]. In [37], [38], fault estimation methods have been reported to reconstruct the actuator and sensor faults, and observer-based fault-tolerant controllers have been designed. Note that above results were based on the full order observer. Based on the reduced order observer, fault estimation problem has been considered in [39]. The problem of dissipativity-based fault estimation has been addressed by [40].

It should be pointed out that the system model is very important in above model-based fault estimation and fault-tolerant control methods, since the imprecise model may cause the methods to fail. On the other hand, factors such as disturbance and uncertainty lead that we cannot know all system dynamic information accurately [41], [42]. Under such a background, this paper considers the problems of fault estimation and fault-tolerant control for a class of system with unknown nonlinear dynamic. The main contributions of this paper are as follows:

1) For the nonlinear system with unknown nonlinear dynamic and unmeasurable system state, an adaptive approximation-based fault estimation observer is designed to reconstruct the fault and the system state, simultaneously. The approximation method can be performed by fuzzy logic systems or neural networks. LMI regions are introduced to improve the estimation performance. 2) Utilizing the estimation information, an observer-based fault-tolerant control method is proposed. The parameter matrices of the observer and the controller can be obtained by solving LMIs. The fault-tolerant controller and the observer are designed separately, which can reduce the computation complexity.

In this paper, Section 2 is problem description. Observer and controller design method are provided in Section 3 and Section 4. Simulation study is listed in Section 5. Section 6 is the Conclusions.

## II. PROBLEM DESCRIPTION

In this paper, the following nonlinear system with unmodeled nonlinear dynamic is considered:

$$\dot{x}(t)=Ax(t)+Bu(t)+Hh(x(t))+Ed(t) \tag{1}$$
$$y(t)=Cx(t) \tag{2}$$
$$y_c(t)=C_cx(t) \tag{3}$$

in which $x(t)\in R^{n}$, $u(t)\in R^{n_u}$, $d(t)\in R^{n_d}$, $y(t)\in R^{n_y}$, $y_c(t)\in R^{n_c}$ are the state, input, disturbance, measurement output and controlled output, respectively. $h(x(t))\in R^{n_h}$ represents the nonlinear part of the system, which is assumed to be unknown. And the parameter matrices in above system are assumed to be constant real matrices. Without loss of generality, it is assumed that above system is controllable and observable. If there is additive fault in the actuator, the system can be described as

$$\dot{x}(t)=Ax(t)+B(u(t)+f(t))+Hh(x(t))+Ed(t) \tag{4}$$
$$y(t)=Cx(t) \tag{5}$$

where $f(t)\in R^{n_f}$ represents the actuator fault.

In this paper, the main objectives are: 1) Design an observer to estimate the system state and actuator fault. 2) Based on the estimation information, design an observer-based fault-tolerant controller.

The following lemmas are useful.

*Lemma 1 [6]:* For the matrix $U$, assume that $\lambda_i$ represents the eigenvalue of the $U$. If there is a symmetric positive definite matrix $V=V^T>0$, such that the following LMIs hold,

$$\begin{bmatrix} -\alpha V & UV \\ * & -\alpha V \end{bmatrix}<0$$
$$\begin{bmatrix} sin(\beta)(UV+VU^T) & cos(\beta)(UV-VU^T) \\ * & sin(\beta)(UV+VU^T) \end{bmatrix}<0$$

then $\lambda_i\in D(\alpha,\beta)$, where $D(\alpha,\beta)$ represents the conic sector region, whose center is $(0,\ 0)$, radius is $\alpha$ and the angle of sector of the disc region is $2\beta$.

*Lemma 2 [35]:* Assume that $X$ and $Y$ are two matrices with appropriate dimensions, then

$$X^TY+Y^TX\le \tau X^TX+\frac{1}{\tau}Y^TY$$

where $\tau>0$ is a constant.

Note that the nonlinear dynamic in (4) is unknown, then the methods proposed in [35]–[39] cannot be used directly. In fact, it can be found that the nonlinear function $h(x(t))$ can be described as

$$h(x(t))=\theta^T\varphi(x(t))+w(t) \tag{6}$$

where $\varphi(x(t))$ is the basis function, $\theta$ is the weight coefficient,

$$w(t)=h(x(t))-\theta^T\varphi(x(t))$$

is the approximation error.

*Remark 1:* It should be noted that above approximation method can be performed by fuzzy logic systems or neural networks, and both of them are with the same form of (6). As it pointed in [43], $\varphi(x(t))$ in (6) can be selected as sigmoids, Gaussians or fuzzy knowledge basis, $\theta$ represents the ideal weight vector of fuzzy logic systems or neural networks with the approximation error $w(t)$. Similar approximations can be found in [43], [44]. 9 According to (6), the system dynamic (4) can be described as

$$\dot{x}(t)=Ax(t)+B(u(t)+f(t))+H\theta^T\varphi(x(t))+Hw(t)+Ed(t) \tag{7}$$

## III. FAULT ESTIMATION OBSERVER DESIGN

To estimate the system state and actuator fault, the following observer will be considered:

$$\begin{aligned}\dot{\hat{x}}(t)=&A\hat{x}(t)+B(u(t)+\hat{f}(t))\\ &+H\hat{\theta}^T\varphi(\hat{x}(t))+K_1(y(t)-\hat{y}(t))\end{aligned} \tag{8}$$

$$\dot{\hat{f}}(t)=K_2(y(t)-\hat{y}(t)) \tag{9}$$
$$\hat{y}(t)=C\hat{x}(t) \tag{10}$$

where $\hat{x}(t)$, $\hat{f}(t)$, $\hat{\theta}$, $\hat{y}(t)$ are the estimations of $x(t)$, $f(t)$, $\theta$ and $y(t)$. $K_1$ and $K_2$ are observer gain matrices to be designed. In addition, $\hat{\theta}$ can be updated online, and the undated law will be provided later.

Let $e_x(t) = x(t) - \hat{x}(t)$, $e_f(t) = f(t) - \hat{f}(t)$. According to the designed observer and the system dynamic, we can obtain the following error dynamic:

$$\begin{aligned}\dot{e}_x(t)&=\dot{x}(t)-\dot{\hat{x}}(t)\\&=Ae_x(t)+Be_f(t)+H(\theta^T\varphi-\hat{\theta}^T\hat{\varphi})+Ed(t)\\&\quad+Hw(t)-K_1(y(t)-\hat{y}(t))\\&=(A-K_1C)e_x(t)+Be_f(t)+H\tilde{\theta}^T\hat{\varphi}\\&\quad+H\theta^T\tilde{\varphi}+Ed(t)+Hw(t)\\&=(A-K_1C)e_x(t)+Be_f(t)+H\tilde{\theta}^T\hat{\varphi}\\&\quad+Ed(t)+H\bar{w}(t)\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{e}_f(t)&=\dot{f}(t)-\dot{\hat{f}}(t)\\&=\dot{f}(t)-K_2(y(t)-\hat{y}(t))\\&=\dot{f}(t)-K_2Ce_x(t)\end{aligned} \tag{12}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\varphi} = \varphi - \hat{\varphi}$, $\hat{\varphi}$ and $\varphi$ represent $\varphi(\hat{x}(t))$ and $\varphi(x(t))$, $\bar{w}(t) = \theta^T\tilde{\varphi} + w(t)$.

Let $e(t) = \begin{bmatrix} e_x(t) \\ e_f(t) \end{bmatrix}$, then we have

$$\dot{e}(t) = (\bar{A} - \bar{K}\bar{C})e(t) + \bar{H}\tilde{\theta}^T\hat{\varphi} + \bar{E}\bar{d}(t) \tag{13}$$

where

$$\begin{aligned}\bar{A}&=\begin{bmatrix} A & B \\ O_{n_f\times n} & O_{n_f\times n_f}\end{bmatrix}, \quad \bar{K}=\begin{bmatrix} K_1 \\ K_2\end{bmatrix},\\ \bar{C}&=\begin{bmatrix} C & O_{n_y\times n_f}\end{bmatrix}, \quad \bar{H}=\begin{bmatrix} H \\ O_{n_f\times n_h}\end{bmatrix},\\ \bar{E}&=\begin{bmatrix} E & H & O_{n\times n_f} \\ O_{n_f\times n_d} & O_{n_f\times n_h} & I_{n_f}\end{bmatrix},\\ \bar{d}(t)&=[d^T(t),\ \bar{w}(t),\ \dot{f}^T(t)]^T\end{aligned}$$

Based on above analysis, it can be found that $\dot{f}$ is treated as a disturbance, and it is a common handling method, which can be found in [12], [29], [32], [35], [39]. Thus, to obtain a satisfied estimation performance, it is assumed that the fault $f$ is with a slow changing property. In fact, this assumption existed definitely or potentially in many reported results, such as [12], [29], [32], [35], [39].

*Theorem 1:* For given constant $\gamma > 0$ and matrix $M > 0$, the error dynamic (13) is asymptotically stable with the $H_\infty$ performance $\gamma$, that is,

$$\lim_{t\to\infty} e(t) = 0, \quad if\ \bar{d}(t) = 0 \tag{14}$$

$$\int_0^t e^T(s)e(s)ds < \gamma^2 \int_0^t \bar{d}^T(s)\bar{d}(s)ds, \quad if\ \bar{d}(t) \neq 0 \tag{15}$$

if the adaptive law satisfies

$$\dot{\hat{\theta}} = \hat{\varphi}(y(t) - \hat{y}(t))^T N^T M \tag{16}$$

and there is symmetric positive definite matrix $P = P^T > 0$ and matrices $Q$ and $N$, such that

$$\begin{bmatrix} \Phi + \Phi^T + I & P\bar{E} \\ * & \gamma^2 I\end{bmatrix} < 0 \tag{17}$$

$$P\bar{H} = \bar{C}^T N^T \tag{18}$$

where $\Phi = P\bar{A} - Q\bar{C}$. The observer gain matrix $\bar{K} = \begin{bmatrix} K_1 \\ K2 \end{bmatrix} = P^{-1}Q$.

*Proof:* Let

$$V(t) = e^T(t)Pe(t) + tr(\tilde{\theta}M^{-1}\tilde{\theta}^T) \tag{19}$$

Note that $\dot{\theta} = 0$. Thus,

$$\begin{aligned}\dot{V}(t)&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{H}\tilde{\theta}^T\hat{\varphi}+2e^T(t)P\bar{E}\bar{d}(t)\\&\quad-2tr(\tilde{\theta}M^{-1}\dot{\hat{\theta}}^T)\end{aligned} \tag{20}$$

According to adaptive law (16), we have

$$\begin{aligned}\dot{V}(t)&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{H}\tilde{\theta}^T\hat{\varphi}+2e^T(t)P\bar{E}\bar{d}(t)\\&\quad-2tr(\tilde{\theta}M^{-1}(\hat{\varphi}(y(t)-\hat{y}(t))^TN^TM)^T)\\&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{H}\tilde{\theta}^T\hat{\varphi}+2e^T(t)P\bar{E}\bar{d}(t)\\&\quad-2tr(\tilde{\theta}M^{-1}M^TNe_y(t)\hat{\varphi}^T)\end{aligned} \tag{21}$$

where $e_y(t) = y(t) - \hat{y}(t)$.

Note that $e_y(t) = Ce_x(t) = \bar{C}e(t)$. Thus,

$$\begin{aligned}\dot{V}(t)&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{H}\tilde{\theta}^T\hat{\varphi}+2e^T(t)P\bar{E}\bar{d}(t)\\&\quad-2tr(\tilde{\theta}N\bar{C}e(t)\hat{\varphi}^T)\end{aligned} \tag{22}$$

Note that

$$tr(\tilde{\theta}N\bar{C}e(t)\hat{\varphi}^T) = e(t)^T\bar{C}^TN^T\tilde{\theta}^T\hat{\varphi}$$

According to (18), we have

$$\begin{aligned}\dot{V}(t)&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{H}\tilde{\theta}^T\hat{\varphi}+2e^T(t)P\bar{E}\bar{d}(t)\\&\quad-2e(t)^T\bar{C}^TN^T\tilde{\theta}^T\hat{\varphi}\\&=e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t)\\&\quad+2e^T(t)P\bar{E}\bar{d}(t)\end{aligned} \tag{23}$$

For the case that $\bar{d}(t) = 0$, it can be found that

$$\dot{V}(t) = e^T(t)(P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T)e(t) \tag{24}$$

Obviously, if (17) hold,

$$P(\bar{A}-\bar{K}\bar{C})+(P(\bar{A}-\bar{K}\bar{C}))^T < 0$$

that is, $\dot{V}(t) < 0$, which implies that

$$\lim_{t\to\infty} e(t) = 0$$

For the case that $\bar{d}(t) \neq 0$, let

$$J(t) = \dot{V}(t) + e^T(t)e(t) - \gamma^2 \bar{d}^T(t)\bar{d}(t)$$

Based on (23), we have

$$\begin{aligned} J(t) &= e^T(t)(P(\bar{A} - \bar{K}\bar{C}) + (P(\bar{A} - \bar{K}\bar{C}))^T)e(t) \\ &\quad + 2e^T(t)P\bar{E}\bar{d}(t) + e^T(t)e(t) - \gamma^2 \bar{d}^T(t)\bar{d}(t) \\ &= e^T(t)(P(\bar{A} - \bar{K}\bar{C}) + (P(\bar{A} - \bar{K}\bar{C}))^T + I)e(t) \\ &\quad + 2e^T(t)P\bar{E}\bar{d}(t) - \gamma^2 \bar{d}^T(t)\bar{d}(t) \end{aligned} \tag{25}$$

Thus, if (17) holds, we have

$$J(t) = \dot{V}(t) + e^T(t)e(t) - \gamma^2 \bar{d}^T(t)\bar{d}(t) < 0$$

That is,

$$\int_0^t [\dot{V}(s) + e^T(s)e(s) - \gamma^2 \bar{d}^T(s)\bar{d}(s)]ds < 0 \tag{26}$$

Under the zero initial condition, above inequality means that

$$V(t) + \int_0^t e^T(s)e(s)ds - \gamma^2 \int_0^t \bar{d}^T(s)\bar{d}(s)ds < 0 \tag{27}$$

Since $V(t) \geq 0$, (27) implies that

$$\int_0^t e^T(s)e(s)ds < \gamma^2 \int_0^t \bar{d}^T(s)\bar{d}(s)ds$$

That is, the error system is asymptotically stale with $H_\infty$ performance $\gamma$. ■

*Remark 2:* It should be noted that there is a equation constraint (18) in Theorem 1. Thus, it is not easy to calculate the observer gain matrices based on the LMI tool in Matlab. To overcome this problem, (18) can be rewritten approximatively as

$$\begin{bmatrix} -\varepsilon I & N\bar{C} - \bar{H}^T P \\ * & -\varepsilon I \end{bmatrix} < 0 \tag{28}$$

where $\varepsilon$ is a small constant. Thus, the observer gain matrices can be obtained by solving the LMIs (17) and (28). More details about this method can be found in [45], where similar handling method has been used.

In this paper, fault estimation observer is designed for nonlinear system with unknown nonlinear dynamic. Based on theorem 1, it can be found that the fault and the unknown nonlinear dynamic can be estimated simultaneously. Once the actuator fault occurs, the approximations of the unknown nonlinear dynamic will be affected, which may lead to the output estimation error increase. However, based on the designed observer and the adaptive law, it can be found that the output estimation error is existed in the designed observer and the adaptive law, and it can adjust the estimation (approximation) results. In fact, similar process have been reported in [44], [45].

In the error dynamic (13), the eigenvalues of $\bar{A} - \bar{K}\bar{C}$ can affect the estimation performance. Thus, in order to obtain better estimation results, the regional pole placement conditions are introduced.

*Theorem 2:* For given constant $\gamma > 0$ and matrix $M > 0$, if the adaptive law is selected as (16), and there is symmetric positive definite matrix $P = P^T > 0$ and matrices $Q$ and $N$, such that (17)-(18) and the following LMIs hold,

$$\begin{bmatrix} -\alpha P & \Phi \\ * & -\alpha P \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} sin(\beta)(\Phi + \Phi^T) & cos(\beta)(\Phi - \Phi^T) \\ * & sin(\beta)(\Phi + \Phi^T) \end{bmatrix} < 0 \tag{30}$$

then the error dynamic (13) is asymptotically stable with the $H_\infty$ performance $\gamma$, and $\lambda_i \in D(\alpha, \beta)$, where $\lambda_i$ represent the eigenvalues of $\bar{A} - \bar{K}\bar{C}$, $\Phi = P\bar{A} - Q\bar{C}$, $D(\alpha, \beta)$ is defined as it in Lemma 1. The observer gain matrix $\bar{K} = \begin{bmatrix} K_1 \\ K2 \end{bmatrix} = P^{-1}Q$.

*Proof:* Based on Theorem 1, we only need to prove that (29)-(30) mean that $\lambda_i \in D(\alpha, \beta)$.

According to Lemma 1, if there is matrix $V^T = V > 0$, such that

$$\begin{bmatrix} -\alpha V & \breve{\Phi} \\ * & -\alpha V \end{bmatrix} < 0$$

$$\begin{bmatrix} sin(\beta)(\breve{\Phi} + \breve{\Phi}^T) & cos(\beta)(\breve{\Phi} - \breve{\Phi}^T) \\ * & sin(\beta)(\breve{\Phi} + \breve{\Phi}^T) \end{bmatrix} < 0$$

then $\lambda_i \in D(\alpha, \beta)$, where $\breve{\Phi} = (\bar{A} - \bar{K}\bar{C})V$, $\lambda_i$ represent the eigenvalues of $\bar{A} - \bar{K}\bar{C}$.

Let $V = P^{-1}$. Pre- and post-multiplying by $diag\{P, \ P\}$ and its transpose in above inequalities, it can be found that above inequalities are equivalent to (29)-(30).

That is, if (29)-(30) hold, $\lambda_i \in D(\alpha, \beta)$. ■

## IV. FAULT-TOLERANT CONTROLLER DESIGN

In this section, the observer based fault-tolerant controller will be designed. It is assumed that the nonlinear dynamic $h(x(t))$ is Lipschitz with respect to $x(t)$, and satisfies the zero initial condition, that is $h(0) = 0$. In addition, supposed that the Lipschitz constant $c$ is known. In fact, there are more than one Lipschitz constant for a nonlinear function, and we can select a big $c$ to satisfy this assumption.

The following observer-based controller is considered in this paper:

$$u(t) = -K_c\hat{x}(t) - \hat{f}(t) \tag{31}$$

where $K_c$ is the controller gain to be designed.

Substituting (31) into (4), we have

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(-K_c\hat{x}(t) - \hat{f}(t) + f(t)) \\ &\quad + Hh(x(t)) + Ed(t) \\ &= (A - BK_c)x(t) + Hh(x(t)) + Be_f(t) \\ &\quad + BK_c e_x(t) + Ed(t) \end{aligned} \tag{32}$$

Let $v(t) = [e_x^T(t) \ e_f^T(t) \ d^T(t)]^T$, and

$$G = [BK_c \ \ B \ \ E]$$

then we have

$$\dot{x}(t) = (A - BK_c)x(t) + Hh(x(t)) + Gv(t) \tag{33}$$

*Theorem 3:* For given constant $\gamma$, the closed-loop system (33) is asymptotically stable with the $H_\infty$ performance $\gamma$, that is,

$$\lim_{t\to\infty} x(t) = 0, \quad if\ v(t) = 0 \tag{34}$$

$$\int_0^t y_c^T(s)y_c(s)ds < \gamma^2 \int_0^t v^T(s)v(s)ds, \quad if\ v(t) \neq 0 \tag{35}$$

if there is symmetric positive definite matrix $\check{P} = \check{P}^T > 0$ and matric $Q$, constant $\tau > 0$, such that

$$\begin{bmatrix} \Upsilon & BQ & B & E & c\check{P} & \check{P}C_c^T \\ * & \gamma_2^2(I-2\check{P}) & O & O & O & O \\ * & * & -\gamma_2^2 I_{nf} & O & O & O \\ * & * & * & -\gamma_2^2 I_{nd} & O & O \\ * & * & * & * & -\tau I & O \\ * & * & * & * & * & -I_{n_c} \end{bmatrix} < 0 \tag{36}$$

where $\Upsilon = A\check{P} - BQ + (A\check{P} - BQ)^T + \tau HH^T$. The controller gain matrix $K_c = Q\check{P}^{-1}$.

*Proof:* Let

$$V(t) = x^T(t)Px(t)$$

where $P = P^T > 0$. Then we have

$$\begin{aligned} \dot{V}(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &= x^T(t)(P(A-BK_c) + (A-BK_c)^T P)x(t) \\ &\quad + 2x^T(t)PHh(x(t)) + 2x^T(t)PGv(t) \end{aligned} \tag{37}$$

Note that

$$2x^T(t)PHh(x(t)) \leq \tau x^T(t)PHH^T Px(t) + \frac{1}{\tau}h^T(x(t))h(x(t)) \tag{38}$$

Since $h(x(t))$ is Lipschitz with respect to $x(t)$, then we have

$$||h(x(t)) - h(0)|| \leq c||x(t) - 0|| = c||x(t)||$$

Note that $h(x(t))$ satisfies the zero initial condition, i.e., $h(0) = 0$, then,

$$||h(x(t))|| \leq c||x(t)||$$

which implies that

$$h^T(x(t))h(x(t)) \leq c^2 x^T(t)x(t)$$

Thus, (38) means that

$$\begin{aligned} 2x^T(t)PHh(x(t)) &\leq \tau x^T(t)PHH^T Px(t) + \frac{1}{\tau}c^2 x^T(t)x(t) \\ &= x^T(t)(\tau PHH^T P + \frac{c^2}{\tau}I)x(t) \end{aligned}$$

Thus,

$$\begin{aligned} \dot{V}(t) &= x^T(t)(P(A-BK_c) + (A-BK_c)^T P)x(t) \\ &\quad + 2x^T(t)PHh(x(t)) + 2x^T(t)PGv(t) \\ &\leq x^T(t)(P(A-BK_c) + (A-BK_c)^T P \\ &\quad + \tau PHH^T P + \frac{c^2}{\tau}I)x(t) + 2x^T(t)PGv(t) \end{aligned} \tag{39}$$

For the case that $v(t) = 0$, we have

$$\begin{aligned} \dot{V}(t) &= x^T(t)(P(A-BK_c) + (A-BK_c)^T P)x(t) \\ &\quad + 2x^T(t)PHh(x(t)) \\ &\leq x^T(t)(P(A-BK_c) + (A-BK_c)^T P \\ &\quad + \tau PHH^T P + \frac{c^2}{\tau}I)x(t) \end{aligned} \tag{40}$$

It is not difficult to find that (36) means that

$$P(A-BK_c) + (A-BK_c)^T P + \tau PHH^T P + \frac{c^2}{\tau}I < 0$$

where $\check{P} = P^{-1}$. That is, $\dot{V}(t) < 0$, which implies that

$$\lim_{t\to\infty} x(t) = 0$$

For the case that $v(t) \neq 0$, let

$$J(t) = \dot{V}(t) + y_c^T(t)y_c(t) - \gamma^2 v^T(t)v(t)$$

According to (39), we have

$$\begin{aligned} J(t) &\leq x^T(t)(P(A-BK_c) + (A-BK_c)^T P \\ &\quad + \tau PHH^T P + \frac{c^2}{\tau}I)x(t) + 2x^T(t)PGv(t) \\ &\quad + y_c^T(t)y_c(t) - \gamma^2 v^T(t)v(t) \\ &= x^T(t)(P(A-BK_c) + (A-BK_c)^T P \\ &\quad + \tau PHH^T P + \frac{c^2}{\tau}I + C_c^T C_c)x(t) \\ &\quad + 2x^T(t)PGv(t) - \gamma^2 v^T(t)v(t) \\ &= \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \Psi_{11} & PG \\ * & -\gamma^2 I_{n_v} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \end{aligned} \tag{41}$$

where $\Psi_{11} = P(A-BK_c) + (A-BK_c)^T P + \tau PHH^T P + \frac{c^2}{\tau}I + C_c^T C_c$, and $n_v$ represents the dimension of $v(t)$, and it can be found that $n_v = n + n_f + n_d$.

Let $\Psi = \begin{bmatrix} \Psi_{11} & PG \\ * & -\gamma^2 I_{n_v} \end{bmatrix}$. Note that $G = [BK_c \ \ B \ \ E]$, then

$$PG = [PBK_c \ \ PB \ \ PE]$$

Thus,

$$\Psi < 0$$

can be rewritten as

$$\begin{bmatrix} \Psi_{11} & PBK_c & PB & PE \\ * & -\gamma_2^2 I_n & O & O \\ * & * & -\gamma_2^2 I_{nf} & O \\ * & * & * & -\gamma_2^2 I_{nd} \end{bmatrix} < 0 \tag{42}$$

Note that $\check{P} = P^{-1}$. Pre- and post-multiplying by $diag\{\check{P},\ \check{P},\ I_{n_f},\ I_{n_d}\}$ and its transpose in above inequality, we have

$$\begin{bmatrix} \check{P}\Psi_{11}\check{P} & BK_c\check{P} & B & E \\ * & -\gamma_2^2(\check{P}\check{P}) & O & O \\ * & * & -\gamma_2^2 I_{nf} & O \\ * & * & * & -\gamma_2^2 I_{nd} \end{bmatrix} < 0 \tag{43}$$

Based on Lemma 2,

$$I + \breve{P}\breve{P} \geq \breve{P} + \breve{P}$$

That is

$$\breve{P}\breve{P} \geq 2\breve{P} - I$$

Thus,

$$-\breve{P}\breve{P} \leq I - 2\breve{P}$$

Hence, if the following inequality holds, (43) holds.

$$\begin{bmatrix} \breve{P}\Psi_{11}\breve{P} & BK_c\breve{P} & B & E \\ * & \gamma_2^2(I-2\breve{P}) & O & O \\ * & * & -\gamma_2^2 I_{n_f} & O \\ * & * & * & -\gamma_2^2 I_{n_d} \end{bmatrix} < 0 \quad (44)$$

Note that $\breve{P}\Psi_{11}\breve{P} = (A - BK_c)\breve{P} + \breve{P}(A - BK_c)^T + \tau HH^T + \frac{c^2}{\tau}\breve{P}\breve{P} + \breve{P}C_c^T C_c\breve{P}$. Based on Schur complement, above inequality is equivalent to:

$$\begin{bmatrix} \bar{\Psi} & BK_c\breve{P} & B & E & c\breve{P} & \breve{P}C_c^T \\ * & \gamma_2^2(I-2\breve{P}) & O & O & O & O \\ * & * & -\gamma_2^2 I_{n_f} & O & O & O \\ * & * & * & -\gamma_2^2 I_{n_d} & O & O \\ * & * & * & * & -\tau I & O \\ * & * & * & * & * & -I_{n_c} \end{bmatrix} < 0 \quad (45)$$

where $\bar{\Psi} = (A - BK_c)\breve{P} + \breve{P}(A - BK_c)^T + \tau HH^T$.

It can be found that (36) is equivalent to (45). Thus, if (36) holds, we have $\Psi < 0$, which implies that

$$J(t) = \dot{V}(t) + y_c^T(t)y_c(t) - \gamma^2 v^T(t)v(t) < 0$$

Similar to the proof of Theorem 1, (36) means that the closed-loop system (33) is asymptotically stable with the $H_\infty$ performance $\gamma$. ■

*Remark 3* In this paper, it can be found that the fault estimation observer and the fault-tolerant controller are designed separately. As it pointed in [32], compared with the integrated design method reported in [37], the separate design method can reduce the computation complexity.

## V. SIMULATION STUDY

Here, the simulation example is provided to show the effectiveness of the proposed method.

*Example:* Consider the system with the same form as (4)-(5), and the parameter matrices are selected as

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & -1 & 0 & 0 \\ 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 & 0.2 \\ 1 & -1.2 \\ -1 & 1 \\ 0 & 1 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0.2 & 1 \\ 1.2 & 1 \\ 0.5 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

The parameter matrix in the controlled output equation (3) is assumed to be $C_c = [1\ 2\ 1\ 1]$. The nonlinear dynamic $h(x(t))$ is assumed to be $h(x(t)) = sin(x_4(t))$. In this paper, $h(x(t))$ is assumed to be unknown, that is, it does not appear in the designed observer and the controller. In addition, it is supposed that the fault is $f(t) = [f_1^T(t)\ \ f_2^T(t)]^T$, and $f_1(t) = f_2(t) = 0$ for $t < 5$, $f_1(t) = 1 - e^{-5(t-5)}$ and $f_2(t) = -3$ for $t \geq 5$. Note that the fault considered in this example is with a slow changing property, since $\dot{f}$ may affect the estimation results. In fact, similar faults have been considered in [12], [32], [39]. Assume that the disturbance $d(t) = [d_1^T(t)\ \ d_2^T(t)]^T$, where $d_1(t) = cos(2t+1)e^{-t}$ and $d_2(t) = sin(2t-3)e^{-t}$. The system initial value is assumed as $x(0) = [0.9,\ 0.8,\ -0.2,\ -1.3]^T$.

In this example, similar to [45], the basis function $\varphi$ is selected as the hyperbolic tangent function. Assume that the Lipschitz constant $c = 2$. The observer initial value $\hat{x}(0)$ and $\hat{\theta}(0)$ are selected as zero. The $H_\infty$ performance of the observer and the controller are selected as 1.732 and 2.236, $M$ in (16) is selected as $M = 2$, small constant $\varepsilon$ in (28) is selected as $\varepsilon = 0.00001$, and the LMI region $D(\alpha, \beta)$ is chosen as $D(10, \pi/12)$. Based on Theorem 2 and Theorem 3, we can obtain the observer gain matrices and the controller gain matrix as follows:

$$K_1 = \begin{bmatrix} 12.4554 & -1.3062 & -0.3075 \\ 4.0371 & 7.6169 & -1.6552 \\ -3.9995 & -2.0033 & 6.4107 \\ 6.0147 & 0.9770 & 4.4923 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 18.8136 & 2.5995 & -5.8239 \\ 11.9503 & 2.2214 & 12.0052 \end{bmatrix},$$

$$K_c = \begin{bmatrix} 7.2874 & 6.0767 & 2.6017 & 2.7318 \\ 6.2565 & 1.3345 & 2.5316 & 1.8272 \end{bmatrix}.$$

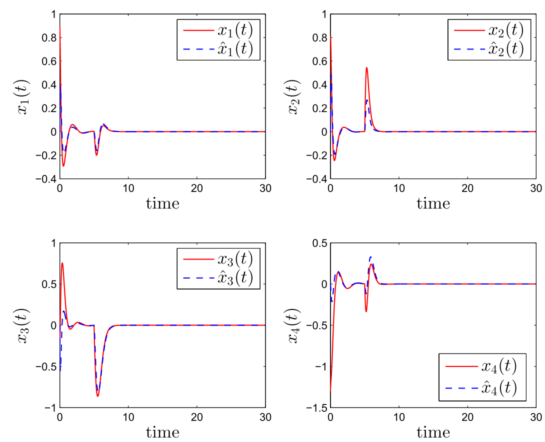


**FIGURE 1. The system states and their estimations.**

The simulation results are provided in Fig.1-Fig.3. In Fig.1, it can be found that the designed observer can reconstruct the system states, and the designed controller can stabilize the system states, where solid lines are the system states and dashed lines are the estimations of them. Fig.2 represents

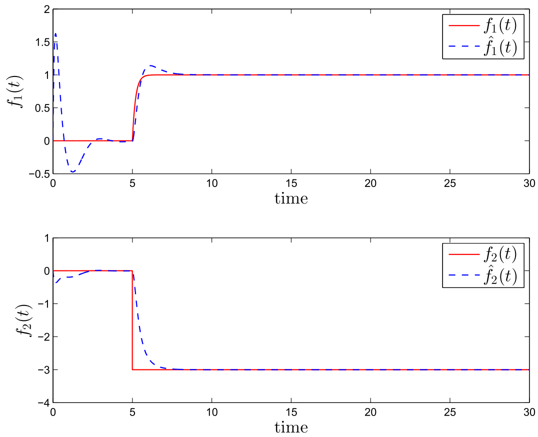


**FIGURE 2.** **The faults and their estimations.**

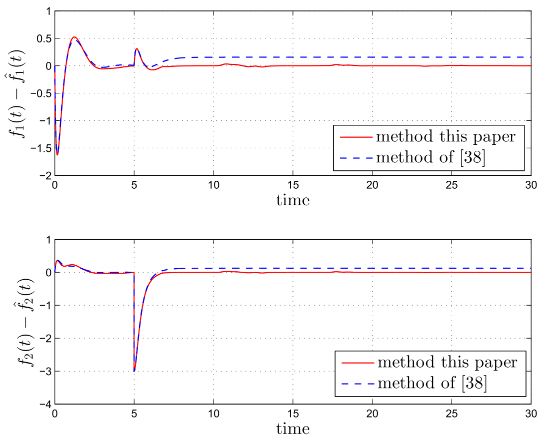


**FIGURE 3.** **The fault estimation errors.**

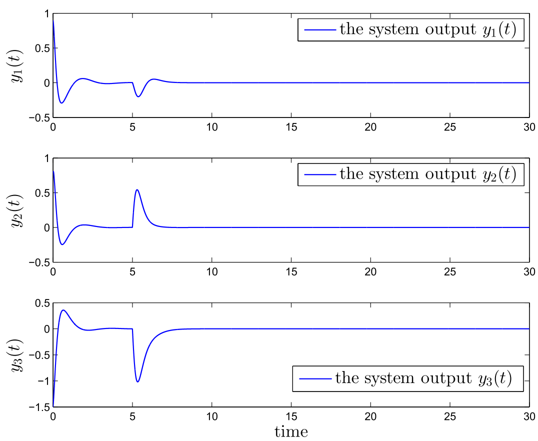


**FIGURE 4.** **The system output $y(t)$.**

the fault estimation results, where the solid lines represent the faults and dashed lines are fault estimations. According to Fig.2, it can be found that the faults can be estimated by the proposed observer. To illustrate the effectiveness of the proposed method, the fault estimation comparison results are provided in Fig.3. To verify the observer performance, in this figure, the control input is assumed to be zero. In Fig.3, the solid lines represent the fault estimation errors obtained by our observer, and the dashed lines are the fault estimation results obtained by the method proposed in [38], where the adaptive approximation is not considered. In Fig.4-5, the system output $y(t)$ and the controlled output $y_c(t)$ are provided. It can be found that the designed controller can ensure that the controlled output approaches to zero, even if there is fault occurs in the system.

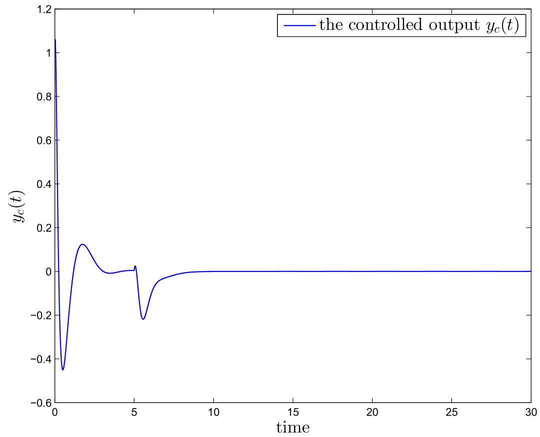


**FIGURE 5.** **The controlled output $y_c(t)$.**

## VI. CONCLUSION

In this paper, the fault estimation observer and fault-tolerant controller design methods have been proposed for a class of system with unknown dynamic. Adaptive approximation method, which can be performed by fuzzy logic systems or neural networks, has been introduced to approximate the unknown nonlinear dynamic in the system. The designed adaptive fault estimation observer can estimate the fault and the system state simultaneously. LMI region has been introduced to adjust the estimation performance. The observer-based fault-tolerant controller has been proposed to stabilize the system. The observer and the controller are designed separately, which can reduce the computation complexity. The parameter matrices of the observer and the controller can be obtained by solving LMIs, this implies that we can calculate the parameter matrices by MATLAB easily. At last, a simulation example has been added to verify the proposed method. It should be noted that this paper assume that the system model is partially known, and the problem of fault-tolerant control for the system with completely unknown dynamic will been considered in our further work.

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