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Fully Distributed Tracking Control of High-Order Nonlinear Multi-Agent Systems

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ABSTRACT This paper studies the fully distributed tracking problem for high-order nonlinear multi-agent systems (MASs) with directed graph. Unlike global Lipschitz condition, the nonlinear function we considered only needs to be a continuously differential one. A recursive state transformation and adaptive control technique are employed to design the tracking controllers. First, a discontinuous fully distributed tracking controller is developed for MASs. Under this controller, the follower agents track the leader agent asymptotically. Second, a continuous fully distributed tracking controller is purposely presented for MASs to avoid the chattering problem may caused by discontinuous controller. Finally, a numerical example is given to verify the effectiveness of those two fully distributed controllers.

INDEX TERMS Multi-agent systems, fully distributed, tracking control, directed graph, nonlinear system.

I. INTRODUCTION

Past two decades witnessed the rapid development of distributed control of MASs for its practical applications in engineering. Typical designs of distributed controllers for MASs can be found in books [1]–[3] and the references therein. However, those distributed controllers need the global topology information of MASs, such as the spectral set of Laplacian matrix. When a MAS contains too many agents, it is difficult to compute such global topology information. It is a challenge to design distributed controllers for nonlinear MASs without using any global topology information.

For nonlinear MASs, there are three basic assumptions for the nonlinear functions: global Lipschitz assumption, neural network approximation assumption and manifold assumption. The global Lipschitz condition is the most common assumption for nonlinear MASs. Most cooperative control targets of such kind of nonlinear MASs can be achieved by linear state feedback control methods [14]–[17]. A nonlinear function satisfies neural network approximation assumption means that it can be approximated by the product of a known basis function and an unknown neural network weight matrix. The control mechanism is that the unknown neural network weight matrix is cancelled by a neuro-adaptive one [18]–[23]. Manifold assumption is a standard assumption for cooperative output regulation of nonlinear MASs. The manifold assumption is the key to ensure solvability of cooperative output regulation [24]–[27]. Apart from those three assumptions of nonlinear MASs mentioned above, other kinds of assumptions of nonlinear MASs were seldom studied [8], [9], [13].

Fully distributed control, which does not use any global topology information, has attracted many researchers' attention. Yu and his coauthors firstly studied fully distributed cooperative control problems for second-order nonlinear MASs with undirected graph via agent-based adaptive gains [4]. Li *et al.* [5] proposed both edge-based and node-based fully distributed algorithms for general linear MASs. Li also extended edge-based adaptive protocol to MASs with Lipschitz nonlinear dynamics [6]. Based on edge-based adaptive technique, Zhang studied fully distributed robust synchronization of Lur's MASs with incremental nonlinearities in [7]. Papers [4]–[7] only took MASs with undirected graph into account for the reason of symmetrical Laplacian matrix of undirected graph. By using Lyapunov design method, Wang firstly proposed a new kind of adaptive control method for fully distributed tracking problem of first-order nonlinear leader-follower MASs with directed graph in [8], and extended this control method to second-order nonlinear

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MASs [9] and general linear MASs [10]. Based on adaptive control method in [8], Li *et al.* [11] and Wang *et al.* [12] studied fully distributed control for general linear and high-order nonlinear MASs with directed graph. Wang studied the fully distributed tracking problem of nonlinear strict-feedback MASs with directed graph in [13].

Inspired by works mentioned above, this paper investigates the fully distributed tracking problem of high-order nonlinear MASs with directed graph. Unlike global Lipschitz assumption, neural network approximation assumption and manifold assumption, the nonlinear function in this paper only needs to be a continuously differential function. Wang and his coauthors had studied fully distributed tracking problems for first-order and second-order nonlinear MASs with such kind of nonlinearity in [8], [9]. Fully distributed tracking control for high-order nonlinear MASs with continuous differential function is still unsolved. Although fully distributed containment control of high-order nonlinear MASs was studied by Wang in [12], the nonlinear function should satisfy an assumption stricter than global Lipschitz condition. Motivated by backstepping control technique, we firstly give a discontinuous controller. In consideration of chattering problem that may be brought by discontinuous controller, we present a continuous controller to achieve fully distributed tracking task. In the end, simulation examples are employed to verify the proposed fully distributed tracking algorithms.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. PROBLEM STATEMENT

In this paper, we consider fully distributed tracking control problem of high-order nonlinear MASs with directed topology.

Consider a networked system of $N+1$ agents depicted by the following high-order nonlinear dynamic:

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2}, \\ \dot{x}_{i,2} &= x_{i,3}, \\ &\vdots \\ \dot{x}_{i,n} &= f(x_i) + u_i, \quad i = 1, \ldots, N,\ N+1, \end{aligned} \tag{1}$$

where $x_i = [x_{i,1}, \ldots, x_{i,n}]^T \in R^n$, $u_i \in R$ are the state and input of agent i, respectively; $f(\cdot) : R^n \to R$ is a continuously differential function. The agent labelled by $N+1$ is the leader agent, others are the follower agents.

The concerned tracking problem is depicted as follows.

Definition 1: For MASs (1), if there exists a distributed controller $u_i(t)$, such that

$$\lim_{t\to\infty} ||x_i(t) - x_{N+1}(t)|| = 0, \quad \forall i = 1, \ldots, N. \tag{2}$$

then follower agents track leader agent asymptotically.

For leader agent, we have the following Assumptions.

Assumption 1: The state x_{N+1} of leader agent is bounded by an unknown number $b_{N+1} > 0$, i.e., $||x_{N+1}|| \le b_{N+1}$.

Assumption 2: The input u_{N+1} of leader agent is bounded by a known constant $\tau \ge 0$, i.e., $|u_{N+1}| \le \tau$.

B. PRELIMINARIES

In this paper, a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ is used to describe the topology of MASs (1), where $\mathcal{V} = \{v_1, \ldots, v_N, v_{N+1}\}$ denotes the agent set, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ denotes the edge set, and $A = [a_{ij}] \in R^{(N+1)\times(N+1)}$ denotes the adjacency matrix. If agent v_i can get relative state information from agent v_j, then there is a directed edge $(v_i, v_j) \in \mathcal{E}$ and the adjacency weights $a_{ij} > 0$; otherwise $a_{ij} = 0$. In this paper, we only consider directed graph without self-loop, i.e. $a_{ii} = 0$. $d_i = \sum_{j=1}^{N+1} a_{ij}$ denotes the degree of agent v_i and $\mathcal{D} = \text{diag}\{d_1, \ldots, d_{N+1}\}$ denotes the degree matrix of $\mathcal{G}$. Edge sequence $(v_{il}, v_{i(l-1)})$, $l = 1, \ldots, k$, denotes a directed path from agent v_{i0} to agent $v_{i,k}$. Agent v_i is said to be the root of $\mathcal{G}$ if there exists at least one directed path from v_i to any other agent $v_j, j \ne i, j \in \{1, \ldots, N+1\}$ and $d_i = 0$. $L = \mathcal{D} - A$ denotes the Laplacian matrix of graph $\mathcal{G}$.

The following Assumption and lemmas are needed to design fully distributed tracking controllers.

Assumption 3: Leader agent v_{N+1} is the root agent of digraph $\mathcal{G}$.

If Assumption 3 holds, the associated Laplacian matrix L of $\mathcal{G}$ can be expressed into the following form:

$$L = \left[\begin{array}{c|c} L_1 & L_2 \\ \hline 0_{1\times N} & 0 \end{array}\right], \quad L_1 \in R^{N\times N},\ L_2 \in R^{N\times 1}.$$

Lemma 1 [20]: If Assumption 3 holds, then L_1 is nonsingular. Denote $1_N = [1, \ldots, 1]^T$, $q = [q_1, \ldots, q_N]^T = L_1^{-1} 1_N$ and $Q = \text{diag}\{1/q_1, \ldots, 1/q_N\}$. Then, both Q and $QL_1 + L_1^T Q$ are positive definite matrices.

Lemma 2 [28]: If a and b are nonnegative real numbers and p and q are positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$, then $ab \le \frac{a^p}{p} + \frac{b^q}{q}$.

III. DISCONTINUOUS TRACKING CONTROLLER DESIGN

In this sector, we will give a discontinuous controller for MASs (1) to solve fully distributed tracking problem.

Let $\xi_i = [\xi_{i,1}, \ldots, \xi_{i,n}]^T = \sum_{j=1}^{N+1} a_{ij}(x_i - x_j)$ be the local cooperative state of agent i, $i = 1, \ldots, N$. Hence, we get the dynamics of ξ_i as follows:

$$\begin{aligned} \dot{\xi}_{i,1} &= \xi_{i,2}, \\ \dot{\xi}_{i,2} &= \xi_{i,3}, \\ &\vdots \\ \dot{\xi}_{i,n} &= \sum_{j=1}^{N+1} a_{ij}(u_i - u_j) + \sum_{j=1}^{N+1} a_{ij}(f(x_i) - f(x_j)). \end{aligned} \tag{3}$$

For each follower agent, we perform the following state transformation

$$\zeta_i = D\xi_i, \tag{4}$$

where $\zeta_i = [\zeta_{i,1}, \ldots, \zeta_{i,n}]^T$ and satisfies

$$\begin{aligned} \zeta_{i,1} &= \xi_{i,1}, \\ \zeta_{i,2} &= \dot{\zeta}_{i,1} + 2\zeta_{i,1}, \\ &\cdots \\ \zeta_{i,n} &= \dot{\zeta}_{i,n-1} + 2\zeta_{i,n-1}. \end{aligned} \tag{5}$$

From (5), one can easily verify that transformation matrix D is a lower triangular matrix with diagonal elements being 1, i.e.,

$$D = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ d_{2,1} & 1 & 0 & \dots & 0 \\ d_{3,1} & d_{3,2} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_{n,1} & d_{n,2} & d_{n,3} & \dots & 1 \end{bmatrix}, \tag{6}$$

where $d_{i,j}, i = 2, \dots, n, j = 1, \dots, i-1$, are positive constants. Then, we obtain the dynamics of ζ_i as follows:

$$\begin{aligned} \dot{\zeta}_{i,1} &= \zeta_{i,2} - 2\zeta_{i,1}, \\ \dot{\zeta}_{i,2} &= \zeta_{i,3} - 2\zeta_{i,2}, \\ &\vdots \\ \dot{\zeta}_{i,n} &= \sum_{k=1}^{n-1} d_{n,k}\xi_{i,k+1} + \sum_{j=1}^{N+1} a_{ij}(u_i - u_j) \\ &\quad + \sum_{j=1}^{N+1} a_{ij}(f(x_i) - f(x_j)). \end{aligned} \tag{7}$$

Remark 1: For nonlinear MASs, the most typical assumption is global Lipschitz condition (see references [14]–[17]). Since nonlinear function $f(\cdot)$ does not satisfy global Lipschitz condition, the commonly used linear state feedback control method can not complete the tracking control task. Inspired by backstepping control technique of nonlinear systems, we propose recursive state transformation (5). Unlike virtual control law of backstepping control method, transformation (4) is a linear one and transformation matrix D is only determined by dimension n.

Let $e_i = x_i - x_{N+1}$ be the tracking error of agent i. Denote

$$\begin{aligned} e &= [e_1^T, \dots, e_N^T]^T, \\ \xi &= [\xi_1^T, \dots, \xi_N^T]^T, \\ \zeta &= [\zeta_1^T, \dots, \zeta_N^T]^T. \end{aligned}$$

It is easy to obtain the following equation

$$\zeta = (I_N \otimes D)\xi = (L_1 \otimes D)e. \tag{8}$$

If Assumption 3 holds, L_1 is nonsingular. Hence, tracking control problem of MAS (1) is solved if there exists a distributed control law $u_i, i = 1, \dots, N$, such that

$$\lim_{t\to\infty} \zeta(t) = 0. \tag{9}$$

Based on transformation state $\zeta_{i,n}, i = 1, \dots, N$, we propose the following dynamical tracking controller for each follower agent

$$\begin{aligned} \dot{\kappa}_i &= \phi_i(\zeta_{i,n})\zeta_{i,n}^2, \kappa_i(0) = 1, \\ u_i &= -5\kappa_i\psi_i(\zeta_{i,n}^2)\zeta_{i,n} - \tau \operatorname{sgn}(\zeta_{i,n}), \end{aligned} \tag{10}$$

where $\phi_i(\cdot)$ is a nonlinear function determined later, $\psi_i(\cdot) \geq 1$ is a nondecreasing function satisfying $\psi_i^{\frac{2}{5}}(\zeta_{i,n}^2) \geq \max\{\phi_i(\zeta_{i,n}), |\zeta_{i,n}|^{\frac{12}{5}}\}$ and $\operatorname{sgn}(\cdot)$ is the signum function.

Theorem 1: Suppose Assumptions 1, 2 and 3 hold, the fully distributed tracking problem of MASs (1) is solved by discontinuous controller (10).

Proof: Take the Lyapunov function as follows:

$$\begin{aligned} V = \frac{\lambda_0}{2}\sum_{i=1}^N \tilde{\kappa}_i^2 + \frac{1}{2}\sum_{i=1}^N\sum_{k=1}^{n-1}\int_0^{\zeta_{i,k}^2} \beta_i(s)\mathrm{d}s \\ + \sum_{i=1}^N \frac{\kappa_i}{2q_i}\int_0^{\zeta_{i,n}^2} \psi_i(s)\mathrm{d}s, \end{aligned} \tag{11}$$

where λ_0 is the minimum eigenvalue of $QL_1 + L_1^TQ$, $\tilde{\kappa}_i = \kappa_i - \bar{\kappa}_i$ with $\bar{\kappa}_i$ being a constant parameter to be determined, and $\beta_i(\cdot)$ is a continuous nondecreasing positive function to be determined, $i = 1, \dots, N$.

The time derivative of V along with trajectory of (7) and (10) is

$$\begin{aligned} \dot{V} &= \lambda_0\sum_{i=1}^N \tilde{\kappa}_i\dot{\kappa}_i + \sum_{i=1}^N\sum_{k=1}^{n-1}\beta_i(\zeta_{i,k}^2)\zeta_{i,k}(\zeta_{i,k+1} - 2\zeta_{i,k}) \\ &\quad + \sum_{i=1}^N \frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}\dot{\zeta}_{i,n} + \sum_{i=1}^N \frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s. \end{aligned} \tag{12}$$

Since $\beta_i(\cdot)$ is a nondecreasing function, we have

$$\beta_i(\zeta_{i,k}^2)\zeta_{i,k}\zeta_{i,k+1} \leq \beta_i(\zeta_{i,k}^2)\zeta_{i,k}^2 + \beta_i(\zeta_{i,k+1}^2)\zeta_{i,k+1}^2.$$

Hence, for the second term of $\dot{V}$ (12), we obtain

$$\begin{aligned} &\sum_{i=1}^N\sum_{k=1}^{n-1}\beta_i(\zeta_{i,k}^2)\zeta_{i,k}(\zeta_{i,k+1} - 2\zeta_{i,k}) \\ &\qquad \leq -\sum_{i=1}^N\sum_{k=1}^{n-1}\beta_i(\zeta_{i,k}^2)\zeta_{i,k}^2 + \sum_{i=1}^N\beta_i(\zeta_{i,n}^2)\zeta_{i,n}^2. \end{aligned} \tag{13}$$

Denote

$$\begin{aligned} \Xi &= [\kappa_1\psi_1(\zeta_{1,n}^2)\zeta_{1,n}, \dots, \kappa_N\psi_N(\zeta_{N,n}^2)\zeta_{N,n}]^T, \\ \tilde{f}_i &= f(x_i) - f(x_{N+1}) + \sum_{k=1}^{n-1} d_{n,k}(x_{i,k+1} - x_{N+1,k+1}), \end{aligned}$$

and $\tilde{F} = [\tilde{f}_1, \dots, \tilde{f}_N]^T$. For the third term of $\dot{V}$ (12), we get

$$\begin{aligned} &\sum_{i=1}^N \frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}\dot{\zeta}_{i,n} \\ &= \sum_{i=1}^N \frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n} \cdot \Bigg[\sum_{j=1}^{N+1} a_{ij}(u_i - u_j) \\ &\quad + \sum_{j=1}^{N+1} a_{ij}(f(x_i) - f(x_j)) + \sum_{k=1}^{n-1} d_{n,k}\xi_{i,k+1}\Bigg] \\ &= \sum_{i=1}^N\sum_{j=1}^N a_{ij}\tau\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(\operatorname{sgn}(\zeta_{j,n}) - \operatorname{sgn}(\zeta_{i,n})) \\ &\quad + \sum_{i=1}^N a_{i,N+1}\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau\operatorname{sgn}(\zeta_{i,n}) - u_{N+1}) \\ &\quad - 5\Xi^TQL_1\Xi + \Xi^TQL_1\tilde{F}. \end{aligned} \tag{14}$$

Notice that, for any $\zeta_{i,n} \in R$,

$$\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(\mathrm{sgn}(\zeta_{j,n}) - \mathrm{sgn}(\zeta_{i,n})) \le 0,$$
$$\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau\,\mathrm{sgn}(\zeta_{i,n}) - u_{N+1}) \le 0. \tag{15}$$

From Lemma 1, one has

$$-5\Xi^T QL_1\Xi = -\frac{5}{2}\Xi^T(QL_1 + L_1^T Q)\Xi \le -\frac{5}{2}\lambda_0\Xi^T\Xi. \tag{16}$$

By Young's inequality, one gets

$$\Xi^T QL_1\tilde{F} \le \frac{\lambda_0}{2}\Xi^T\Xi + \frac{1}{2\lambda_0}\tilde{F}^T L_1^T QQL_1\tilde{F} \le \frac{\lambda_0}{2}\Xi^T\Xi + \frac{\lambda_1}{2\lambda_0}\tilde{F}^T\tilde{F}, \tag{17}$$

where $\lambda_1 = ||L_1^T QQL_1||$. Since leader state x_{N+1} is bounded and $f(\cdot)$ is continuous differential, there exists a smooth function $\rho(\cdot) > 0$, such that

$$|\tilde{f}_i| \le \rho(e_i, b_{N+1})||e_i||, \quad i = 1, \ldots, N.$$

Notice that $\zeta = (L_1 \otimes D)e$. According to Lemma 7.8 in [30], there exist continuous functions $\vartheta_{i,k}(\cdot) > 0$, such that

$$\tilde{F}^T\tilde{F} = \sum_{i=1}^{N}\tilde{f}_i\tilde{f}_i \le \sum_{i=1}^{N}\sum_{k=1}^{n} b_{i,k}\vartheta_{i,k}(\zeta_{i,k})\zeta_{i,k}^2, \tag{18}$$

with $b_{i,k}$ being the unknown constants related to b_{N+1}, D and L_1.

Since positive function $\psi_i(\cdot)$ is nondecreasing, one can get the following inequality:

$$\frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s \le \frac{1}{2q_i}\phi_i(\zeta_{i,n})\zeta_{i,n}^2\psi_i(\zeta_{i,n}^2)\zeta_{i,n}^2.$$

Letting

$$a = q_i^{-\frac{5}{6}}\lambda_0^{-\frac{5}{6}}\kappa_i^{-\frac{5}{3}}\phi_i^{\frac{1}{6}}\zeta_{i,n}^{\frac{1}{3}}, \quad p = 6,$$
$$b = q_i^{\frac{5}{6}}\lambda_0^{\frac{5}{6}}\kappa_i^{\frac{5}{3}}\phi_i^{\frac{5}{6}}\psi_i\zeta_{i,n}^{\frac{11}{3}}, \quad q = \frac{6}{5}.$$

According to Lemma 2, we obtain

$$\phi_i(\zeta_{i,n})\zeta_{i,n}^2\psi_i(\zeta_{i,n}^2)\zeta_{i,n}^2 \le \frac{1}{6q_i^5\lambda_0^5\kappa_i^{10}}\phi_i(\zeta_{i,n})\zeta_{i,n}^2 + \frac{5q_i\lambda_0\kappa_i^2}{6}\phi_i(\zeta_{i,n})\psi_i^{\frac{6}{5}}(\zeta_{i,n}^2)\zeta_{i,n}^{\frac{22}{5}}.$$

Because $\dot{\kappa}_i \ge 0$, κ_i is nondecreasing and $\frac{1}{\kappa_i} \le 1$. Since $\psi_i(\cdot) \ge 1, i = 1, \ldots, N$, are nondecreasing and satisfy $\psi_i^{\frac{2}{5}}(\zeta_{i,n}^2) \ge \max\{\phi_i(\zeta_{i,n}), |\zeta_{i,n}|^{\frac{12}{5}}\}$, we obtain

$$\frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s \le \frac{1}{12q_i^6\lambda_0^5}\phi_i(\zeta_{i,n})\zeta_{i,n}^2 + \frac{5\lambda_0}{12}\kappa_i^2\psi_i^2(\zeta_{i,n}^2)\zeta_{i,n}^2. \tag{19}$$

Substituting (13) - (19) into (12) gives

$$\begin{aligned}\dot{V} \le& -\sum_{i=1}^{N}\sum_{k=1}^{n-1}\beta_i(\zeta_{i,k}^2)\zeta_{i,k}^2 + \sum_{i=1}^{N}\beta_i(\zeta_{i,n}^2)\zeta_{i,n}^2\\ &+ \frac{\lambda_1}{2\lambda_0}\sum_{i=1}^{N}\sum_{k=1}^{n} b_{i,k}\vartheta_{i,k}(\zeta_{i,k})\zeta_{i,k}^2 - \frac{19\lambda_0}{12}\Xi^T\Xi\\ &+ \lambda_0\sum_{i=1}^{N}\bar{\kappa}_i\phi_i(\zeta_{i,n})\zeta_{i,n}^2 + \frac{1}{12q_i^6\lambda_0^5}\phi_i(\zeta_{i,n})\zeta_{i,n}^2.\end{aligned}$$

Let $b_i = \max_{k=1,\ldots,n} b_{i,k}$ and $\theta_i(s) = \max_{k=1,\ldots,n}\vartheta_{i,k}(s)$, $\forall s \in R$. Choosing

$$\beta_i(\zeta_{i,k}^2) = \frac{\lambda_0}{3} + \frac{\lambda_1}{2\lambda_0}b_i\theta_i(\zeta_{i,k}), \phi_i(\zeta_{i,n}) = 2\theta_i(\zeta_{i,n}).$$

We get

$$\begin{aligned}\dot{V} \le& -\frac{\lambda_0}{3}\sum_{i=1}^{N}\sum_{k=1}^{n-1}\zeta_{i,k}^2 + \frac{\lambda_0}{3}\sum_{i=1}^{N}\zeta_{i,n}^2\\ &+ \frac{\lambda_1}{\lambda_0}\sum_{i=1}^{N} b_i\theta_i(\zeta_{i,n})\zeta_{i,n}^2 - \frac{19\lambda_0}{12}\Xi^T\Xi\\ &+ \lambda_0\sum_{i=1}^{N}\bar{\kappa}_i\phi_i(\zeta_{i,n})\zeta_{i,n}^2 + \frac{1}{12q_i^6\lambda_0^5}\phi_i(\zeta_{i,n})\zeta_{i,n}^2.\end{aligned}$$

Selecting $\bar{\kappa}_i = \frac{\lambda_1}{2\lambda_0^2}b_i + \frac{1}{12q_i^6\lambda_0^6}$. We obtain

$$\begin{aligned}\dot{V} \le& -\frac{\lambda_0}{3}\sum_{i=1}^{N}\sum_{k=1}^{n-1}\zeta_{i,k}^2 + \frac{\lambda_0}{3}\sum_{i=1}^{N}\zeta_{i,n}^2\\ &+ \lambda_0\sum_{i=1}^{N}\kappa_i\phi_i(\zeta_{i,n})\zeta_{i,n}^2 - \frac{19\lambda_0}{12}\Xi^T\Xi.\end{aligned}$$

Note that

$$\Xi^T\Xi = \sum_{i=1}^{N}\kappa_i^2\psi_i^2(\zeta_{i,n}^2)\zeta_{i,n}^2,$$
$$\psi_i(\zeta_{i,n}^2) \ge 1, \quad \kappa_i \ge 1,$$
$$\psi_i^{\frac{2}{5}}(\zeta_{i,n}^2) \ge \phi_i(\zeta_{i,n}).$$

We get

$$\dot{V} \le -\frac{\lambda_0}{3}\sum_{i=1}^{N}\sum_{k=1}^{n-1}\zeta_{i,k}^2 - \frac{\lambda_0}{4}\sum_{i=1}^{N}\zeta_{i,n}^2. \tag{20}$$

According to LaSalle's Theorem (see Theorem 4.4 of [29]), (20) means that ζ will converge to zero asymptotically and adaptive parameters $\kappa_i, i = 1, \ldots, N$ are bounded. Since $\zeta = (L_1 \otimes D)e$, we obtain $\lim_{t\to\infty} e(t) = 0$, i.e., tracking control problem of MASs (1) is solved. Note that $\dot{\kappa}_i \ge 0$. κ_i are not only bounded but also monotonically increasing, i.e., κ_i will converge to some finite values. ■

Remark 2: From (11), one can observe that Lyapunov function V depends on λ_0 and $q_i, i = 1, \ldots, N$, which are global topology information. However, Lyapunov function V are only used to prove that tracking controller (10) can solve fully distributed tracking control problem of MASs (1). Dynamical controller (10) is generated by $\zeta_{i,n}$, which can be got by state transformation (4) of local cooperative state ξ_i. Hence, dynamical controller (10) is a fully distributed controller.

IV. CONTINUOUS TRACKING CONTROLLER DESIGN

Due to the discontinuity of signum function $\mathrm{sgn}(\cdot)$, fully distributed tracking controller (10) is discontinuous as well. In fact, discontinuous control input may yield chattering problem, and even damage the actuator. In this section, we use boundary layer technique to approximate discontinuous function $\mathrm{sgn}(\cdot)$ and σ-modification technique to modify the dynamics of adaptive gains.

We give continuous fully distributed tracking controller as follows

$$\begin{aligned}\dot{\kappa}_i &= \phi_i(\zeta_{i,n})\zeta_{i,n}^2 - \sigma(\kappa_i - 1), \kappa_i(0) = 1,\\ u_i &= -5\kappa_i\psi_i(\zeta_{i,n}^2)\zeta_{i,n} - \tau h(\zeta_{i,n}),\end{aligned} \tag{21}$$

where σ is a small positive number, $h(\cdot)$ is a continuous function defined as:

$$h(s) = \begin{cases}\frac{s}{|s|}, & \text{if } |s| > \delta,\\ \frac{s}{\delta}, & \text{if } |s| \le \delta,\end{cases}$$

with $\delta > 0$ being the width of boundary layer.

Theorem 2: Suppose Assumptions 1, 2 and 3 hold, tracking error e and adaptive gains $\kappa_i, i = 1, \ldots, N$, are uniformly ultimately bounded under continuous tracking controller (21).

Proof: Consider the following Lyapunov function candidate

$$V = \frac{\lambda_0}{2}\sum_{i=1}^{N}\tilde{\kappa}_i^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{n-1}\int_0^{\zeta_{i,k}^2}\beta_i(s)\mathrm{d}s + \sum_{i=1}^{N}\frac{\kappa_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s. \tag{22}$$

Then, we get the time derivative of V as follows

$$\dot{V} = \lambda_0\sum_{i=1}^{N}\tilde{\kappa}_i\dot{\tilde{\kappa}}_i + \sum_{i=1}^{N}\sum_{k=1}^{n-1}\beta_i(\zeta_{i,k}^2)\zeta_{i,k}(\zeta_{i,k+1} - 2\zeta_{i,k}) + \sum_{i=1}^{N}\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}\dot{\zeta}_{i,n} + \sum_{i=1}^{N}\frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s. \tag{23}$$

For the first term of $\dot{V}$ (23), we have

$$\tilde{\kappa}_i\dot{\tilde{\kappa}}_i = \tilde{\kappa}_i\phi_i(\zeta_{i,n})\zeta_{i,n}^2 - \sigma\tilde{\kappa}_i(\kappa_i - 1),$$

and

$$\begin{aligned}-\sigma\tilde{\kappa}_i(\kappa_i - 1) &= -\sigma\tilde{\kappa}_i^2 - \sigma(\bar{\kappa}_i - 1)\tilde{\kappa}_i,\\ &\le -\frac{\sigma}{2}\tilde{\kappa}_i^2 + \frac{\sigma}{2}(\bar{\kappa}_i - 1)^2.\end{aligned} \tag{24}$$

For the third term of $\dot{V}$ (23), we have

$$\begin{aligned}&\sum_{i=1}^{N}\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}\dot{\zeta}_{i,n}\\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\tau\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(h(\zeta_{j,n}) - h(\zeta_{i,n}))\\ &\quad + \sum_{i=1}^{N}a_{i,N+1}\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau h(\zeta_{i,n}) - u_{N+1})\\ &\quad - 5\Xi^T QL_1\Xi + \Xi^T QL_1\tilde{F}.\end{aligned} \tag{25}$$

Notice that, for any $|\zeta_{i,n}| > \delta$,

$$\begin{aligned}&\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(h(\zeta_{j,n}) - h(\zeta_{i,n})) \le 0,\\ &\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau h(\zeta_{i,n}) - u_{N+1}) \le 0.\end{aligned} \tag{26}$$

And, for any $|\zeta_{i,n}| \le \delta$,

$$\begin{aligned}&\tau\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(h(\zeta_{j,n}) - h(\zeta_{i,n})) \le \frac{2}{q_i}\tau\kappa_i\psi_i(\delta^2)\delta,\\ &\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau h(\zeta_{i,n}) - u_{N+1}) \le \frac{2}{q_i}\tau\kappa_i\psi_i(\delta^2)\delta.\end{aligned}$$

Since $\kappa_i = \tilde{\kappa}_i + \bar{\kappa}_i$, one gets

$$\frac{2}{q_i}\tau\kappa_i\psi_i(\delta^2)\delta = \frac{2}{q_i}\tau\tilde{\kappa}_i\psi_i(\delta^2)\delta + \frac{2}{q_i}\tau\bar{\kappa}_i\psi_i(\delta^2)\delta$$

Hence, we get

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}\tau\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(h(\zeta_{j,n}) - h(\zeta_{i,n}))\\ &\quad + \sum_{i=1}^{N}a_{i,N+1}\frac{\kappa_i}{q_i}\psi_i(\zeta_{i,n}^2)\zeta_{i,n}(-\tau h(\zeta_{i,n}) - u_{N+1})\\ &\le \sum_{i=1}^{N}\frac{2}{q_i}d_i\tau\tilde{\kappa}_i\psi_i(\delta^2)\delta + \sum_{i=1}^{N}\frac{2}{q_i}d_i\tau\bar{\kappa}_i\psi_i(\delta^2)\delta\\ &\le \sum_{i=1}^{N}\frac{\sigma}{4}\tilde{\kappa}_i^2 + \varsigma,\end{aligned} \tag{27}$$

where $\varsigma = \sum_{i=1}^{N}\left(\frac{4}{q_i^2\sigma}d_i^2\tau^2\psi_i^2(\delta^2)\delta^2 + \frac{2}{q_i}d_i\tau\bar{\kappa}_i\psi_i(\delta^2)\delta\right)$.

For the fourth term of $\dot{V}$ (23), we have

$$\frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s \le \frac{1}{2q_i}\phi_i(\zeta_{i,n})\zeta_{i,n}^2\psi_i(\zeta_{i,n}^2)\zeta_{i,n}^2 - \frac{\sigma}{2q_i}(\kappa_i - 1)\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s.$$

From the dynamics of κ_i in (21), it is easy to verify that $\kappa_i(t) \ge 1$ for all $t \ge 0$. Both $\kappa_i \ge 1$ and $\psi_i(\cdot) \ge 1$ ensure that $\frac{\sigma}{2q_i}(\kappa_i - 1)\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s \ge 0$. Thus, one gets

$$\frac{\dot{\kappa}_i}{2q_i}\int_0^{\zeta_{i,n}^2}\psi_i(s)\mathrm{d}s \le \frac{1}{2q_i}\phi_i(\zeta_{i,n})\zeta_{i,n}^2\psi_i(\zeta_{i,n}^2)\zeta_{i,n}^2. \tag{28}$$

We only analyse the differences in this proof, the rest analysis is same as that of Theorem 1. Then, we have

$$\dot{V} \leq -\frac{\lambda_0}{3}\sum_{i=1}^{N}\sum_{k=1}^{n-1}\zeta_{i,k}^2 - \frac{\lambda_0}{4}\sum_{i=1}^{N}\zeta_{i,n}^2 - \frac{\sigma}{4}\sum_{i=1}^{N}\tilde{\kappa}_i^2 + \tilde{\varsigma}, \quad (29)$$

with $\tilde{\varsigma} = \varsigma + \sum_{i=1}^{N}\frac{\sigma}{2}(\bar{\kappa}_i - 1)^2$.

According to Ultimate Boundedness Theorem (see Theorem 4.18 of [29]), (29) indicates that ζ will converge to a neighbourhood of origin asymptotically and adaptive parameters κ_i, $i = 1, \ldots, N$ are bounded. ■

Remark 3: In order to develop continuous tracking controller, we introduce continuous function $h(\cdot)$ in fully distributed tracking controller (21). The cost is that tracking error only asymptotically converges into a neighbourhood of origin. According to the definition of $\tilde{\varsigma}$, we can guarantee a satisfactory tracking error e by choosing relatively small δ and σ.

V. SIMULATIONS

We use a numerical example to verify the effectiveness of both discontinuous and continuous tracking controllers (10) and (21) in this section. Consider a nonlinear MAS with six agents described by

$$\begin{aligned}
\dot{x}_{i,1} &= x_{i,2}, \\
\dot{x}_{i,2} &= x_{i,3}, \\
\dot{x}_{i,3} &= -x_{i,2} - 0.5(1 - x_{i,2})x_{i,3} + u_i, \quad i = 1, \ldots, 6.
\end{aligned}$$

It can be easily verified that nonlinear function $(1 - x_{i,2})x_{i,3}$ does not satisfy the common global Lipschitz condition. The topology of MASs is described by following adjacency matrix:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

It can be seen that the leader agent is the root of directed topology.

Since the dimension of agent state is 3, we give the transformation matrix D as follows:

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 4 & 1 \end{bmatrix}.$$

The initial states of leader agent are $[-0.8, 0.5, 0]^T$, and the control input of leader agent is $u_6 = -0.1\cos(t)$. The numerical simulation shows that the leader state is bounded. Hence, the leader agent satisfies both Assumption 1 and Assumption 2. Next, we give simulations for both discontinuous and continuous fully distributed tracking controllers.

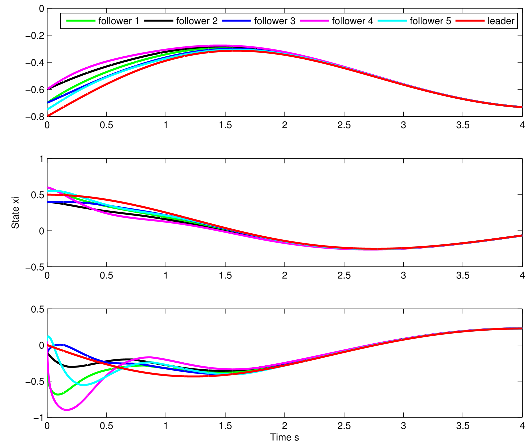

FIGURE 1. **Trajectories of MASs with discontinuous controller** (10).

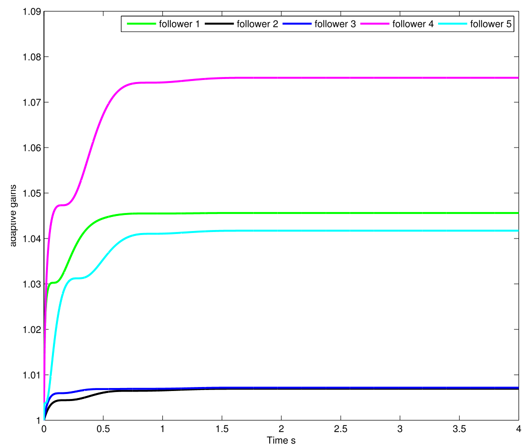

FIGURE 2. **Adaptive parameters κ_i of discontinuous controller** (10).

(i) Discontinuous tracking controller (10).

Since $|u_6| \leq 0.1$, we choose $\tau = 0.1$ and give the following discontinuous tracking controller

$$\begin{aligned}
\dot{\kappa}_i &= (1 + \zeta_{i,3}^2)\zeta_{i,3}^2, \ \kappa_i(0) = 1, \\
u_i &= -5\kappa_i(1 + \zeta_{i,3}^2)^3\zeta_{i,3} - 0.1\mathrm{sgn}(\zeta_{i,3}). \quad (30)
\end{aligned}$$

Under discontinuous fully distributed tracking controller (30), states x_i, adaptive gains κ_i, and inputs u_i of follower agents are shown in Figs. 1 - 3 respectively. One can observe that follower agents track leader agents asymptotically, adaptive parameters converge to some finite values and control input u_i frequently changes its control directions in a short period.

(ii) Continuous tracking controller (21).

We choose modification parameter $\sigma = 0.1$ and width of boundary layer $\delta = 0.1$. The continuous tracking controller is given as follows

$$\begin{aligned}
\dot{\kappa}_i &= (1 + \zeta_{i,3}^2)\zeta_{i,3}^2 - 0.1(\kappa_i - 1), \ \kappa_i(0) = 1, \\
u_i &= -5\kappa_i(1 + \zeta_{i,3}^2)^3\zeta_{i,3} - 0.1h(\zeta_{i,3}). \quad (31)
\end{aligned}$$

Under continuous fully distributed tracking controller (31), the trajectories of MASs, adaptive gains and control inputs

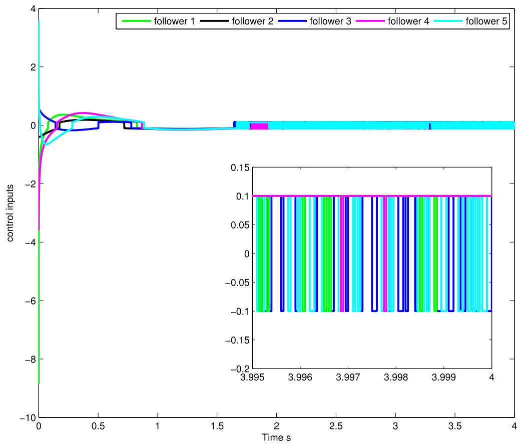

FIGURE 3. **Control inputs u_i of discontinuous controller** (10).

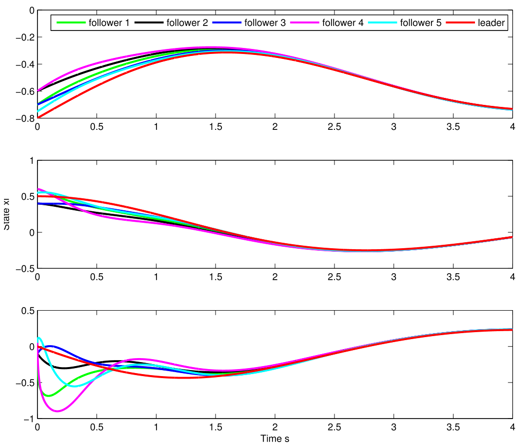

FIGURE 4. **Trajectories of MASs with continuous controller** (21).

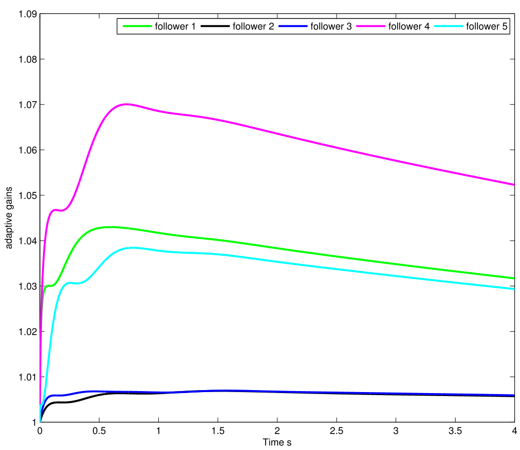

FIGURE 5. **Adaptive parameters κ_i of continuous controller** (21).

are shown in Figs. 4 - 6. We can see that boundary layer and σ-modification techniques guarantee that tracking errors and adaptive parameters κ_i are uniformly ultimately bounded, and control inputs u_i are continuous. From Fig. 5, the σ-modification technique can prevent κ_i from growing infinitely. By comparing Fig. 3 and Fig. 6, we can

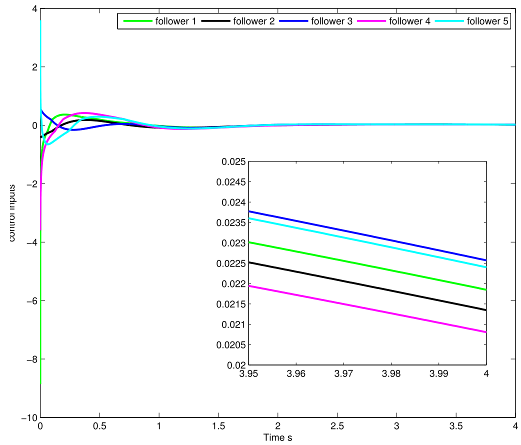

FIGURE 6. **Control inputs u_i of continuous controller** (21).

conclude that continuous tracking controller can avoid frequent changes of control direction effectively.

VI. CONCLUSIONS

Fully distributed tracking problem of high-order nonlinear MASs with directed topology was studied in this paper. We gave two kinds of fully distributed tracking controllers for this problem. The first one is a discontinuous controller which ensures follower agents track leader agent asymptotically. The second one is a continuous controller which only guarantees relatively small tracking error by selecting suitable parameters. At last, a third-order nonlinear MAS is given to verify the proposed tracking controllers.

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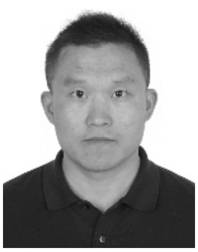

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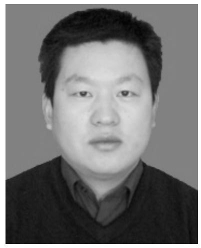

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