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Variable Step Size Technique for the Parabolic Equation in Complex Environmental Conditions

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ABSTRACT In this paper, a variable step size method for the PE (Parabolic Equation) is proposed to solve the problem of low computational efficiency in the study of radio wave propagation at a large range of complex environments. Firstly, the relationship between the error and the step size, the frequency and other factors of the SSFT (Split-Step Fourier Transform) solution in the standard atmosphere is deduced, and then the basic selection range of the step size is given for the variable step size method. Secondly, the action mechanism of different environmental factors and the requirement of changing trend for step size are expounded through simulation, and the complex environment of PE application is classified according to the requirement of error. Finally, the electric wave characteristics of typical complex environment are advanced by the method. The simulation results show that compared with the small step method, the variable step size method of the PE can reduce the horizontal sampling point by 55.9% and the calculation time by 69.6% when the accuracy is close to the small step method, and the variable step size method is more accurate than the fixed large size step method. Therefore, the variable step size on the PE can not only ensure the accuracy of calculation, but also reduce the memory and time required, which greatly improves the reliability and efficiency. Moreover, it is suitable for solving the problem of the radio wave propagation in complex environment.

INDEX TERMS Split-step Fourier transform, parabolic equation, radio wave propagation, variable step size method.

I. INTRODUCTION

With the wide applications of electronic information system, such as wireless communication, radar, remote sensing and so on, the electromagnetic environment in space is becoming more and more complex. The problems of radio wave propagation and electromagnetic environment prediction in complex environment have been paid more and more attention [1]–[6]. The PE method can simultaneously deal with the effects of complex atmospheric structure and irregular topography on the propagation of radio waves, and can calculate the attenuation values of different heights at the same distance. Therefore, the PE can not only predict the path propagation loss by point to point, but also predict the coverage of regional electromagnetic characteristics. At the same time, the parabolic equation method is one of the most accurate and effective models for tropospheric wave propagation.

Since Leontovich and Fock deduced the PE from the Helmholtz wave equation and established the theoretical framework for tropospheric wave propagation modeling, the parabolic equation model has been showing its strong adaptability in radio wave propagation [7]. The PE model usually only considers the earth's circular path and the atmospheric structure on the path between transmitting and receiving points because of the vast area in the prediction of radio wave propagation in large-scale complex troposphere environment. At the same time, the 3DPE (three-dimensional PE) model is widely used to calculate RCS (Radar Cross Section) of electrically large targets [3], [4]. The research on tropospheric radio propagation is still in its infancy and exploration stage. It needs to deal with both irregular three-dimensional terrain boundary conditions and corresponding three-dimensional

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atmospheric structure distribution. It is a process of dealing with massive data. It is not only difficult to realize technically, but also needs a high-performance computing platform to support it. In this paper, the 2DPE (two-dimensional PE) method is used to analyze the propagation characteristics of radio waves in a wide range of complex environments.

The solution of PE is mainly numerical solution such as SSFT and FD(finite difference)method [8], [9]. And the adaptive grid is extremely appropriate for the FD due to its flexibility in choosing the size of each step. Therefore, it is suitable to use the FD schemes when dealing with problems of complicated boundaries [10]. But for the PE problem of high frequency and large scale radio wave propagation, the FD algorithm not only solves the PE problem slowly, but also requires high performance of computer. At present, FD algorithm is mainly used to calculate the RCS of targets and predict the propagation characteristics of small-scale radio waves such as urban residential areas. For the PE problem of large-scale radio wave propagation in troposphere, the SSFT algorithm is fast and stable. SSFT is a numerical algorithm for solving PE in frequency domain. Firstly, it was proposed by Hardin and Tappert. The basic idea of SSFT is to separate the pseudo-differential operator from each step in the calculation of PE. Then, combined with the boundary conditions, it carries out Fourier transform operation. Finally, the final solution is obtained by multiplying the refractive index term [8]. Dockery and Kuttler considered the boundary impedance into the SSFT solution, and then the DMFT (Discrete Mixed Fourier Transform) method is proposed later, and then the backward differential discrete Fourier transform method is present to improve the stability of the algorithm [11], [12]. Many scholars are also studying the parabolic equation under irregular terrain in [13], and the bidirectional parabolic equation solution under irregular terrain conditions proposed by Ozgun improves the precision of the parabolic equation in [14]. At present, a lot of researches have been done on parabolic equation in complex environment. He and Chen [15], [16] has analyzed tunnel wave propagation model in complex meteorological environment. Sheng *et al.* [17] has used parabolic equation to analyze millimeter wave characteristics in irregular terrain and rough sea surface complex geographical environment and its application in standard atmosphere, rain and fog complex meteorological environment.

Although SSFT is a fast solution for PE compared to other solutions, it is still inefficient in studying large-scale radio wave propagation problems because it uses fixed-step methods in practical applications. Especially the bidirectional parabolic equations and irregular terrain parabolic equations under complex environmental conditions require a large amount of computation. Many scholars have proposed corresponding algorithms to further improve the computational efficiency of SSFT. Zhang proposed a non-uniform mesh solution for parabolic equations in [18], but did not give the selection range of grids. Chen proposed a variable step size technology for parabolic equations in irregular terrain [19], but did not give the corresponding step size selection range and expound the action mechanism of different environmental factors and the requirement of changing trend for step size.

In view of this, the variable step size solution of the parabolic equation is proposed. The first section introduces the main steps of the PE derivation. In the second part, the PE error is analyzed, and the relationship between the error of the SSFT solution and factors such as the step size and the frequency is derived, which provides theoretical basis for the variable step size. Then, the step selection criteria and the range of step selection in complex environment are given. In the third section, the reliability and efficiency of the variable step size relative and the fixed step size solution are analyzed with simulation results. Variable step size model can further improve the efficiency of SSFT solution while ensuring the accuracy of calculation. It has practical significance in real-time prediction of electromagnetic wave propagation in a wide range of complex environments.

II. OVERVIEW OF FOURIER SPLIT-STEP AND PARABOLIC EQUATION

Assuming that the radio wave propagates in the passive medium, the time harmonic factor of the electromagnetic field is $e^{-i\omega t}$. In the rectangular coordinate system, the space wave satisfies the Helmholtz equation:

$$\frac{\partial^2\psi(x,z)}{\partial x^2}+\frac{\partial^2\psi(x,z)}{\partial z^2}+k^2n^2\psi(x,z)=0 \tag{1}$$

where ψ is scalar electric field (horizontal polarization) or magnetic field (vertical polarization); n is refractive index of medium; k is propagation constant in vacuum.

Assuming that the wave function propagating along the $+x$ axis is substituted for Helmholtz equation and factorized, the forward parabolic equation can be obtained:

$$\frac{\partial u}{\partial x}=-ik(1-Q)u \tag{2}$$

where Q is a pseudo-differential operator, $Q=\sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial z^2}+n^2}$ and $u=e^{-ikx}\psi$.

Different parabolic equations with narrow and wide angles can be obtained by approximating the values Q differently. In this paper, Feit-Fleck approximation is used to obtain the wide angle PE [9]of Feit-Fleck type:

$$\frac{\partial u(x,z)}{\partial x}=ik\left[\sqrt{1+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}}+n(x,z)-2\right]u(x,z) \tag{3}$$

Among them, $u(x,z)$ is the wave function of electric field or magnetic field.

Assume that $\partial n/\partial z=0$, A and B can exchange orders $[AB](u)=[BA](u)$ and the solution of Equation (3) can be expressed as

$$u(x,z)=e^{\Delta x(A+B)}u\left(x_0,z\right) \tag{4}$$

where $A=ik[n(x,z)-2]$ and $B=ik\bullet\sqrt{1+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}}$.

Then, $u(x, z)$ the exponential term can be separated into

$$e^{\Delta x(A+B)} = e^{B\Delta x} e^{A\Delta x} \tag{5}$$

The result of solving the Equation (4) using the power series expansion and the Fourier transform method in [15] is:

$$u(x+\Delta x, z) = e^{ik(n-2)\Delta x}\mathcal{F}^{-1} \times \left\{ e^{i\Delta x\left(\sqrt{k^2-p^2}-k\right)}\mathcal{F}\left[u\left(x_0, z\right)\right]\right\} \tag{6}$$

where Δx is the step size; p is the amount of transform in the frequency domain; $\mathcal{F}(\cdot)$ and $\mathcal{F}^{-1}(\cdot)$ are the Fourier transform and its inverse transform. The $\exp[i\Delta xk(n-2)]$ indicates refractive effect of the medium, which is called the refractive factor; The $\exp\left(i\Delta x\left(\sqrt{k^2-p^2}-k\right)\right)$ indicates the diffraction effect of the obstacle during the radio wave propagation, which is called the diffraction factor; $u(x_0, z)$ indicates the field distribution of the initial field.

It is worth noting, however, that the results obtained using the SSFT method are derived at $\partial n/\partial z = 0$, the refractive index does not vary with the height. However, the atmospheric structure is vertically distributed in a complex tropospheric environment, thus causing $\partial n/\partial z \neq 0$. Therefore, the condition of separation of exponential terms shown in Equation (5) is not satisfied, and the operation of separation of exponential terms will bring errors to the solution.Moreover, it can be seen that the error comes from Δx, A and B.

PF(Propagation factor) and PL(propagation loss) are two commonly used indicators to characterize the effects of real environment on the propagation characteristics of radio waves. The formulas are as follows:

$$PL = -20\log(u(x, z)) + 20\log(4\pi) + 10\log(x) - 30\log(\lambda)$$
$$PF = 20\log(u(x, z)) - 10\log(x) - 10\log(\lambda) \tag{7}$$

where λ is wavelength.

When solving PE with SSFT algorithm, a truncated boundary must be set in a limited height range so that the electromagnetic wave can be fully absorbed when it reaches the upper boundary. In this paper, we add the Cosine-tape (Turkey) window function in z direction [20], [21]:

$$w(z) = \begin{cases} 1, 0 \le z \le \frac{3}{4}z_{\max} \\ \frac{1}{2} + \frac{1}{2}\cos\left[4\pi\left(z - \frac{3}{4}z_{\max}\right)/z_{\max}\right], \\ \frac{3}{4}z_{\max} < z \le z_{\max} \end{cases} \tag{8}$$

where $z_{\max}$ is the maximum calculated height. From the initial distance, the field calculated by the PE at each step is multiplied by $w(z)$.

After calculating the field distribution in the region by SSFT method with variable step size, the data generated are irregular grids, which is inconvenient for subsequent display and utilization. Therefore, in order to process data conveniently, it is necessary to modify the data of large step area to generate regular grid points. The method chosen here is linear interpolation. The interpolation processing of the field is shown in Fig.1.

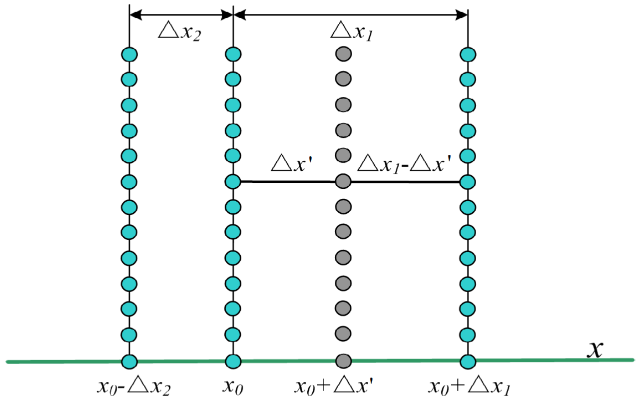

FIGURE 1. The linear interpolation for field distribution.

Linear interpolation is used for field interpolation, which means the field distribution $u\left(x_0 + \Delta x', z\right)$ at $x_0 + \Delta x'$ can be obtained by linear interpolation of field distribution $u(x_0, z)$ and $u(x_0 + \Delta x_1, z)$ at x_0 and $x_0 + \Delta x_1$.

$$u\left(x_0+\Delta x', z\right) = \frac{\Delta x'}{\Delta x_1}u\left(x_0 + \Delta x_1, z\right) + \frac{\Delta x_1 - \Delta x'}{\Delta x_1}u\left(x_0, z\right) \tag{9}$$

Therefore, after SSFT method with variable step size and interpolation processing, the grid points in the field distribution belong to uniform step size, and the corresponding step size is the smallest step in SSFT method with variable step size.

III. ERROR ANALYSIS OF SSFT SOLUTION

Since the SSFT solution is a step-solving process, the step size of each step can be changed, which provides a possibility for variable step size. However, the analysis in the previous section shows that the error of SSFT is related to the choice of step size. If the selected step size is not suitable, it may cause excessive error. Therefore, this section analyzes the error of SSFT, discusses the relationship between error and step size, frequency, etc., and obtains the step size range when the error condition is satisfied.

Assuming that $\partial n/\partial z \neq 0$, so in Equation (4), $[AB](u) \neq [BA](u)$, the error caused by the separation of the exponential terms is denoted by:

$$e_d = e^{i\Delta x(A+B)}u - e^{i\Delta xB}e^{i\Delta xA}u \tag{10}$$

Expanding the Equation (10) by power series and taking the first three items, which can be expressed as:

$$\begin{aligned} e_d &\approx \left[1 + (A+B)\Delta x + \frac{1}{2}(A+B)(A+B)\Delta x^2 \right.\\ &\quad - \left(1 + \Delta xB + \frac{\Delta x^2B^2}{2}\right)\left(1 + \Delta xA + \frac{\Delta x^2A^2}{2}\right)]u \\ &\approx \frac{1}{2}(ABu - BAu)\Delta x^2 \end{aligned} \tag{11}$$

From the equation, we can see that the calculation of $ABu - BAu$ is important for the derivation of the error.

In the process of deriving the SSFT solution of Feit-Fleck-PE,

$$ABu - BAu = \frac{1}{2}\left(\frac{\partial^2 n}{\partial z^2}u + 2\frac{\partial n}{\partial z}\frac{\partial u}{\partial z}\right) \quad (12)$$

Therefore, the error is:

$$|e_d| = \frac{\Delta x^2}{4}\left|\frac{\partial^2 n}{\partial z^2}u + 2\frac{\partial u}{\partial z}\frac{\partial n}{\partial z}\right| \quad (13)$$

In the case of a plane wave, there is $\partial u/\partial z = ik \sin \alpha u$, and taking the Equation (13) gives the following relative error:

$$|e_{dr}| = \frac{\Delta x^2}{4}\left|\frac{\partial^2 n}{\partial z^2} + 2ik \sin \alpha \frac{\partial n}{\partial z}\right| \quad (14)$$

It can be seen from Equation (14) that under the condition that the propagation elevation angle is constant, the error of the separation index term mainly depends on the step size Δx, $\partial^2 n/\partial z^2$, $\partial n/\partial z$ and the wave constant k.

Assuming that the atmospheric environment is a standard atmosphere, therefore dM/dz is a constant, then

$$n = 1 + \left(M_0 + \frac{dM}{dz}z\right) \cdot 10^{-6} \quad (15)$$

where M is the refractive index, $dM/dz \approx -40M/km$, $M_0 \approx 300M$.

Let $dM/dz \cdot 10^{-6} = c$, $M_0 \cdot 10^{-6} = m_0$, then

$$\begin{cases} \frac{\partial n}{\partial z} = c \\ \frac{\partial^2 n}{\partial z^2} = 0 \end{cases} \quad (16)$$

Substituting $\partial^2 n/\partial z^2$, $\partial n/\partial z$ and $k = 2\pi f/v$ into Equation (14). Then, the relationship between the step size Δx, the frequency f and the relative error $|e_{dr}|$ can be expressed as:

$$|e_{dr1}| = \frac{\Delta x^2}{4}\left|i\frac{4\pi f}{v} \sin \alpha c\right| \quad (17)$$

where v is the speed of light.

Therefore, $|e_{dr}|$ is:

$$|e_{dr}| = \frac{4\pi f \Delta x^2 \sin \alpha \cdot 10^{-8}}{v} \quad (18)$$

Assuming the propagation elevation angle $\alpha = 15°$, the relationship between the relative error step size Δx and the frequency f is shown in Fig.2 and Fig.3.

From the above derivation and simulation results, it can be seen that if the relative error is kept constant, the higher the transmission frequency is, the smaller the step size is required; The lower the error at the same frequency, the smaller the step size is. In Fig.3, when the relative error is 1% and the frequency is 4GHz, the required step size is less than 150m. When the frequency is 10GHz, the required step size is less than 100m, which means that the calculation amount is increased by about 30%. So the change of step size has a great influence on the error and the calculation amount. And then the step size under the error condition has a certain selection space, which also defines the basic selection range for the step size.

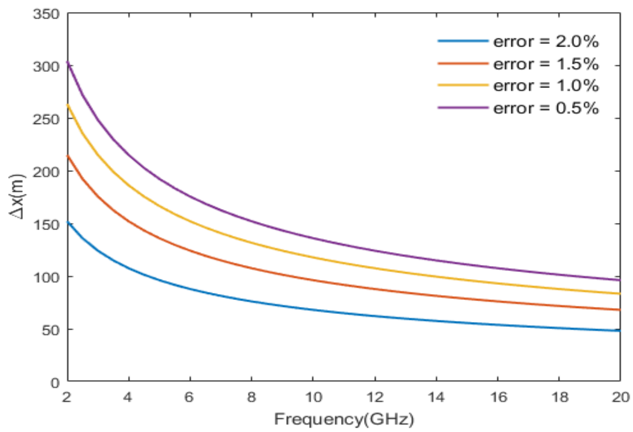

FIGURE 2. Different relative error between the step size and the frequency.

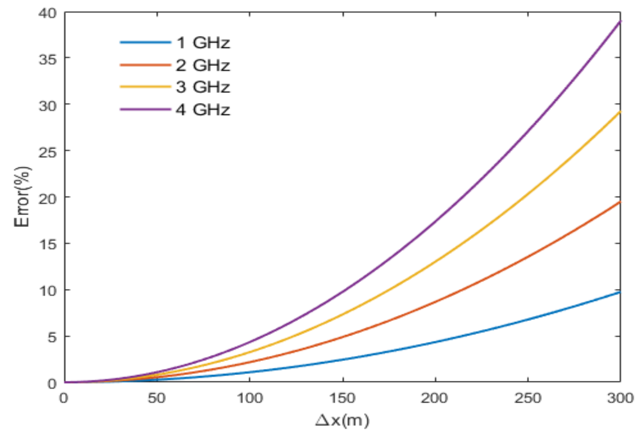

FIGURE 3. Different frequency between the step size and the relative error.

Fig.2 shows that when the frequency is 4GHz and the relative error is 0.5% and 2%, the step size corresponds to 107.38m and 214.75m, respectively. Here the transmitting height of the antenna is 25m and the antenna polarization mode is horizontal polarization. The simulation is designed and established. We can get the propagating factors corresponding to the step size 107.38m and 214.75m at 50km nearly combining linear interpolation in Fig.4. And it is easy to see the basic coincidence. The difference between the two methods accounts for about 1.50% of the corresponding propagation factor, which verifies the accuracy of the method.

When solving the PE based on the SSFT algorithm, the form and the characteristics of SSFT algorithm determine that there are errors in formula solution. They are related to the distribution form of n, the frequency of electromagnetic wave and the step size Δx. In determining the atmospheric refractive index n of the propagation environment, once the

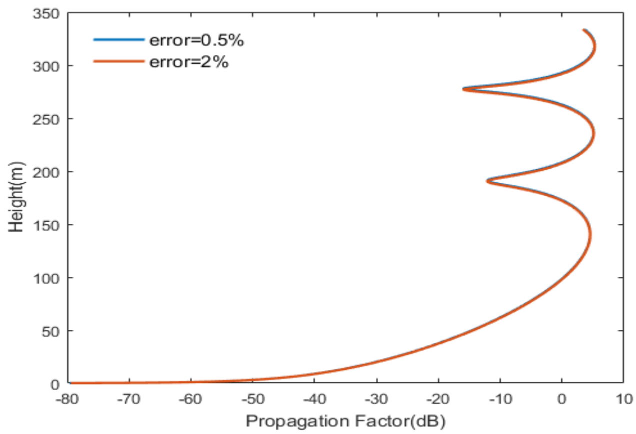

FIGURE 4. **The propagation factors corresponding to the step size 107.38m and 214.75m at 50km.**

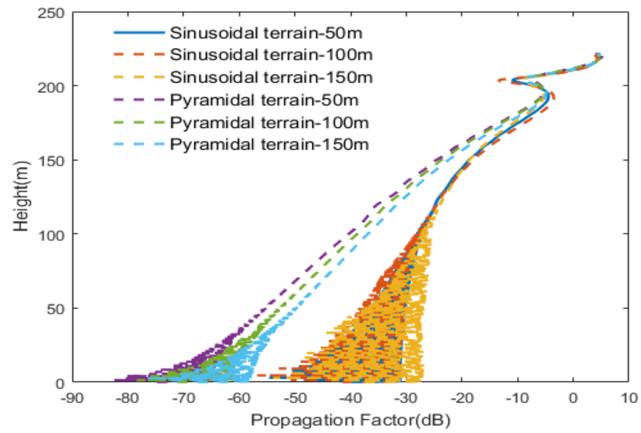

FIGURE 5. **The propagation factors under different mountain shapes (27km).**

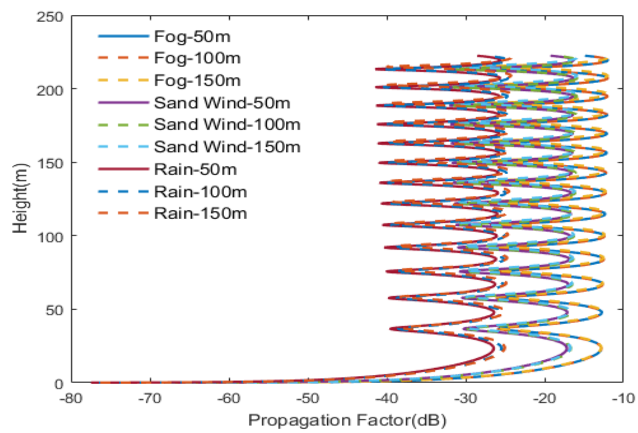

FIGURE 6. **The propagation factors corresponding to different weather conditions (10 km).**

radio frequency is determined, the magnitude of Δx is the main factor affecting the calculation accuracy and the speed.

IV. STEP SELECTION IN COMPLEX ENVIRONMENTS

In the process of tropospheric propagation, the electromagnetic waves are affected by the complex environments such as topography, atmosphere, surface medium, and rough sea surface. Therefore, various modified PE models have been formed to enhance its adaptability. In the process of tropospheric propagation, electromagnetic waves are affected by complex environments. Therefore, various modified PE models have been formed to enhance their adaptability.

A. MOUNTAIN AREA AND STEP SIZE SELECTION

It is of great significance to study the propagation characteristics of the radio waves in the mountainous environments. The refractive index on the mountain area is mainly related to the shape of mountain and the complex meteorological environment through which it propagates. The relationship between refractive index and Δx is analyzed by simulation.

The altitude of the mountain is 150m, and the horizontal distance is 20-25km. The transmitting antenna has a height of 25m and the frequency of 3GHz. The atmospheric environment is the standard atmosphere.

From Fig.5, the irregular terrain in the mountain area is similar to the sinusoidal terrain and the pyramid terrain. Different shapes affect the refractive index of the mountain. Because of the obstruction on the electromagnetic wave forward topography in sinusoidal terrain, the propagation factor in the obstruction area decreases rapidly, forming the shadow area of the electromagnetic wave propagation, and strong interference effect in the lower area. The variation of propagation factors under the pyramid terrain is similar to that under the sinusoidal terrain, but because of the different structure of terrain, there are differences in the electromagnetic wave diffraction. It can be concluded from the analysis that the local slope of the pyramid topography is larger than the sinusoidal topography. Combining the UTD (Uniform Theory of Diffraction) theory, it can be seen that the pyramid topography produces stronger diffraction effect when electromagnetic wave propagates. Therefore, the propagation factor is larger than the sinusoidal terrain. By comparing the propagation factor errors of the two kinds of mountains under different step size Δx, it can be seen that the pyramid terrain of the mountains depends on the step size. So the pyramid terrain need the smaller step size to maintain accuracy.

In this section, the barometric pressure is set to be 1013hPa, and the temperature is 15 °C with the relative humidity of 80%. The visibility for the fog and sand wind mediums is 50m. And Rainfall intensity is 25mm/h. The complex refractive index of fog, sand wind mediums and rain at 30GHz frequency can be calculated from [16], [22]. Fig.6 shows that under different weather conditions, the errors of propagating factors corresponding to different step sizes are different when the distance is 10km. At the same time, when simulating the attenuation of rain (sand wind mediums) in complex environment, it is considered that the intensity of rainfall (sand wind mediums) is uniformly distributed in space. In the actual environment, the intensity of rainfall (sand wind

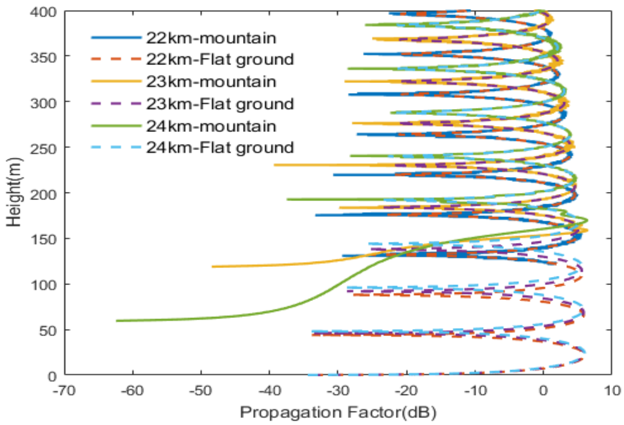

FIGURE 7. The propagation factors corresponding to different weather conditions.

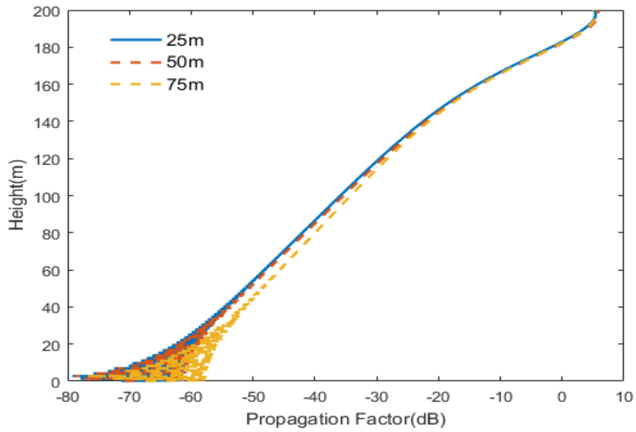

FIGURE 8. The contrast of the different propagating factors in the irregular terrain.

mediums) on the propagation path often changes. Therefore, when using parabolic equation method to calculate rainfall attenuation in real complex environment, it is necessary to take into account the local weather distribution characteristics and seasonal variation. Combined with the step size of the PE method, the weather intensity is calculated at each step, and then the corresponding equivalent dielectric constant is recalculated according to the intensity.

Through simulation analysis, the refractive index (shape) of mountains in mountain environment is related to different atmospheric environments, such as dust storms, rain, fog and so on, all of which are related to Δx. Under the condition of ensuring the accuracy of calculation, the refractive index of mountain (surface undulation) requires a smaller step than that of atmospheric environment. The more intense the topographic fluctuation and the change of atmospheric environment, the smaller the required step size.

B. THE RELATIONSHIP BETWEEN REFRACTIVE INDEX AND HEIGHT IN DIFFERENT ENVIRONMENTS

Here we mainly analyze the relationship between refractive index and height change in common terrain and evaporation duct environment (Except for the above standard atmospheric environment).

As can be seen from Fig.7, the change of refractive index with height is mainly related to the topographic fluctuation in the pyramid topographic environment. The propagation factor above the mountain height at the horizontal distance of 22km is different from the flat terrain. But it coincides basically, and the propagation factor below the terrain height is truncated and disappeared. The propagating factors above 150m (mountain height) at the horizontal distance of 23km and 24km basically coincide with the gentle terrain, and the rapid descent below 150m decreases rapidly at the lowest point with the mountain or the ground. Therefore, in the surface fluctuation environment, the atmospheric refractive index on the unshielded part of the mountain is basically the same as the atmosphere. The refractive index change on the unshielded part of the mountain is corrected by $\sqrt{n^2 - \sin^2 \beta}$ pairs of refractive index n, the β of which is the angle between the terrain and the horizontal plane [23]. The relationship between the change of β with the height of topography and the change of the velocity of β with the height of topography is related to the change on the refractive index of topography with the height of topography.

The evaporation duct is one of the most common atmospheric ducts, which can be expressed as:

$$n_e = 1 + \left(M_0 + \frac{dM}{dz} \left(z - d \ln \frac{z + z_0}{z_0} \right) \right) \cdot 10^{-6} \quad (19)$$

where d is the duct height. When the evaporation waveguide height is $d = 0$, n_e corresponds to the refractive index profile in the standard atmosphere. We can get easily:

$$\begin{cases} \dfrac{\partial n_e}{\partial z} = c - \dfrac{cd}{z} \\ \dfrac{\partial^2 n_e}{\partial z^2} = \dfrac{cd}{z^2} \end{cases} \quad (20)$$

Equation (20) shows that the variation of refractive index with height in the evaporative waveguide environment is related to the waveguide height.

C. ANALYSIS OF THE STEP SIZE SELECTION

In order to describe the relationship between complex environment and step size, the fixed step size of 25m, 50m, 75m, 100m and 150m were selected for comparison under different single environmental conditions. The mechanism of action of different environmental factors and the requirement of changing trend for step size were analyzed.

As shown in Fig.8-11, the propagation factor curves of fixed step size 50m, 75m, 100m and 150m at different horizontal distances is given under different environmental conditions. The fixed step size chosen here is compared with a smaller 50m. The mechanism of action of different environmental factors and the requirement of variation trend on step size are analyzed concretely. In Fig.8, the propagation factor varies with altitude in irregular terrain, the maximum

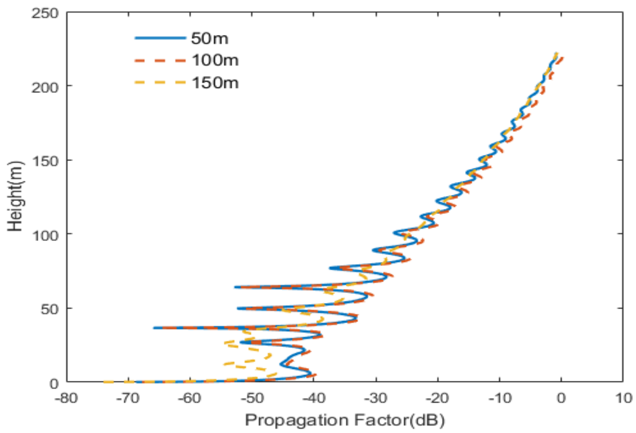

FIGURE 9. The contrast of the atmospheric environment variations with asynchronous propagation factors.

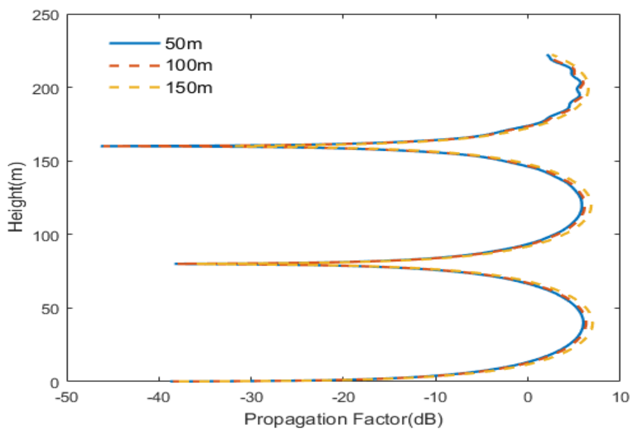

FIGURE 10. The contrast of the atmospheric environment variations with asynchronous propagation factors.

difference of step size of 25m and 75m is about 0.5dB and 5dB, respectively, compared with step length of 50m, and the difference decreases gradually with height rising. The curve basically coincides at altitude of about 200m. Therefore, when the surface changes dramatically, the step length is 50m. In Fig.9, the propagation factor varies with altitude in the environment of severe atmospheric change. At this time, the 150m step method differs about 3dB from the 50m step method, and the 100m step method coincides basically with the step length method. Therefore, the accuracy of selecting 100m step in the environment of severe atmospheric change can be kept close to that of 50m. In Fig.10, it is a height-dependent curve of propagation factor at the interface of two surface media (medium dry land and wetland). However, the maximum difference between 150m step length method and 50m step length method is about 2dB at this time, and the 100m step length coincides with it basically. Therefore, the selection of 100m step length at the interface on two surface media can keep the accuracy close to 50m. In Fig.11, it can be seen that the superimposed area at the junction of atmospheric drastic change and surface medium is about 2dB different

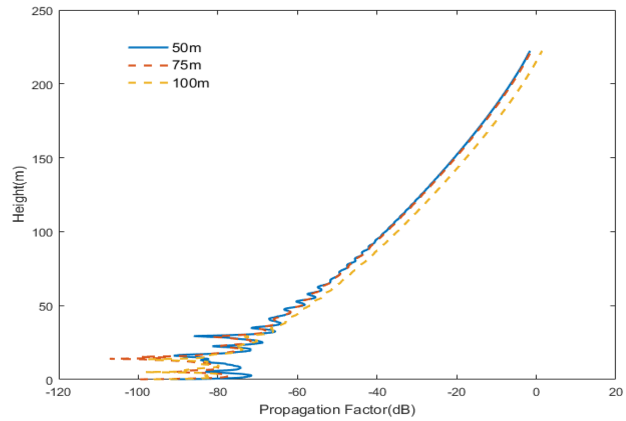

FIGURE 11. The contrast of the asynchronous propagation factors at the junction on two surface media under dramatic changes in the atmospheric environment.

from the step length of 100m and 50m, while the step length of 75m coincides basically with the step length of 50m.

Through the above simulation and analysis, the influence of wave propagation in these different complex environments is not the same, among which surface elevation has the greatest impact, followed by atmosphere, surface media and so on. It also shows that the required step size is not the same in different environments to achieve the same accuracy. That is to say, the smaller step size is required when the topographic elevation changes, while the larger step size is required when the atmospheric and surface medium changes. At present, the most accurate piecewise linear shift transformation method is used to solve the problem of the radio wave propagation in irregular terrain (suitable for general mountainous or hilly areas) while the frequency of electromagnetic wave is unchanged. It mainly modifies the refractive index (corresponding refractive index n), the diffraction factor and the corrected boundary [26]. The atmospheric drastic changes mainly modify the refractive factors of the PE model, and different surface media mainly affect the modified boundary of the PE model. Under the condition of ensuring accuracy, the corresponding step size depend on the more the irregular terrain changes, drastic the atmospheric environment changes and complex the surface medium is, the smaller is.

In these different environments, the influence of radio wave propagation is not the same, which has the greatest impact on the change of surface elevation, followed by the atmosphere, surface medium and rough sea surface.

It means that the required step sizes in different environments are also inconsistent in order to achieve the same accuracy. The surface elevation change requires a small step size, and the atmospheric, surface medium require a larger step size. According to the analysis in the previous section, there is a large choice in the range of error requirements. Combined with the results discussed in the previous section, the range of steps for the variable step solution can be selected.

Here, the complex environment is graded and three basic criteria are proposed for the variable step size on parabolic equation solution:

TABLE 1. Complex environmental classification.

Division level	I	II	III	IV
Complex environment description	Surface elevation changes drastically	Extreme changes in the atmospheric environment	Two kinds of surface medium junction	Simple environment

1) The progress of the step at least meets the requirements for the relative error;

2) The level of complex environment is high, and the choice of smaller steps is longer;

3) The level of complex environment is low, and the choice is larger and the progress is longer.

We can get the classification results of the complex environment in Table1 by the simulations from Fig.7 to Fig.10. Here the typical environment is divided into four levels, corresponding to terrain, atmosphere [24], complex boundary and simple environment, respectively, according to the influence degree of different environment on electromagnetic wave propagation.

Obviously, as the level of the complex environment rises, the changes in the field distribution will also be drastic. To satisfy the precision requirements of radio wave prediction, it will require a smaller step of progress and longer progress. In the region with stable distribution, the requirement for long step progress will be reduced, which is important to the SSFT solution with variable step size.

According to Table 1 and the analysis of the error, it can be concluded that the general requirements for the step-step SSFT solution to improve the step size are as shown as:

$$\Delta x_1 < \Delta x_2 < \Delta x_3 < \Delta x_4 < \Delta x_{max} \tag{21}$$

where Δx_1 is the step that corresponds to a region with a sharp change in surface elevation; Δx_2 is the step size corresponding to the region where the atmospheric environment changes sharply; Δx_3 is the step size corresponding to the boundary region of different surface boundary conditions; Δx_4 is a simple environment corresponding to the step size; Δx_{max} is the maximum step size that can be selected under the error condition in Equation (18).

In practical applications, the step size selection range of the variable step size on SSFT solution is calculated by using Equations (18) and (21). Therefore, it is possible to avoid the precision missing problem on the large step size and the low computational efficiency of the small step size in the fixed environment solution, thereby improving the accuracy and efficiency of the parabolic equation.

Fig.12 and Fig.13 show the variation of propagation factor with height when the wave propagation is at 35km. We can see that the results of variable step size calculation on the parabolic equation are in good agreement with those of two-ray model with comparisons to fixed large step size, which verifies the correctness of the variable step size model. So the variable step size model of the parabolic equation can still achieve high accuracy under complex terrain impedance boundary conditions. In addition, due to the efficient solution of the PE, the prediction of electromagnetic wave propagation over a wide range of different surface boundaries will show its strong advantages.

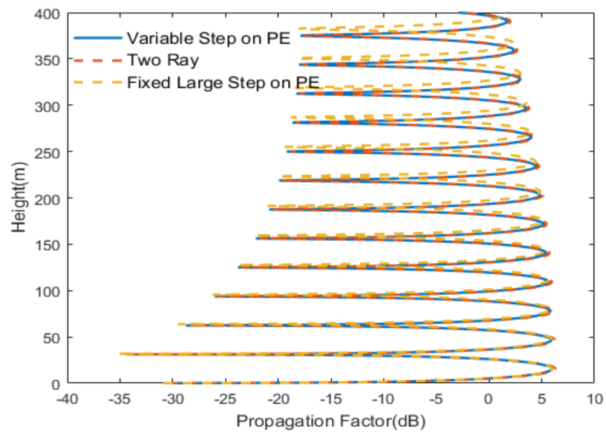

FIGURE 12. The variation of propagation factor with altitude when radio wave propagates is at 35km and the antenna polarization mode is horizontal polarization.

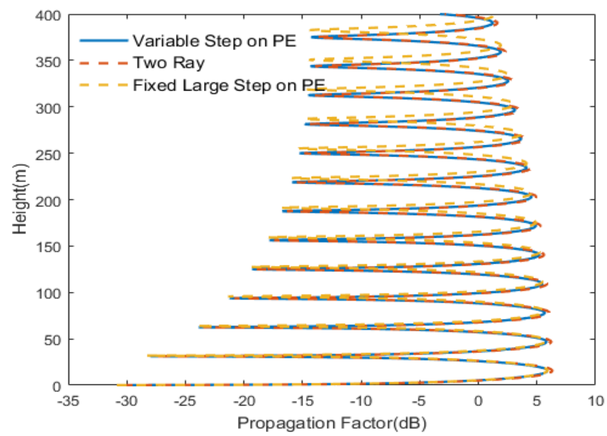

FIGURE 13. The variation of propagation factor with altitude when radio wave propagates is at 35km and the antenna polarization mode is vertical polarization.

V. GUIDELINES FOR GRAPHICS PREPARATION AND SUBMISSION

Through the analysis of the above subsections, in this section uses the fixed step SSFT calculation results are used as a comparison, and the performance of the variable step size solution is simulated and analyzed to illustrate the efficiency and reliability of the variable step size method.

A. SIMULATION CONDITION

1) The change of surface elevation adopts pyramid terrain and sinusoidal terrain as shown in Fig.14, and the variation of terrain is between 20-35km.

2) The variation of surface medium [25], [26]in different distance ranges is shown in Table 2.

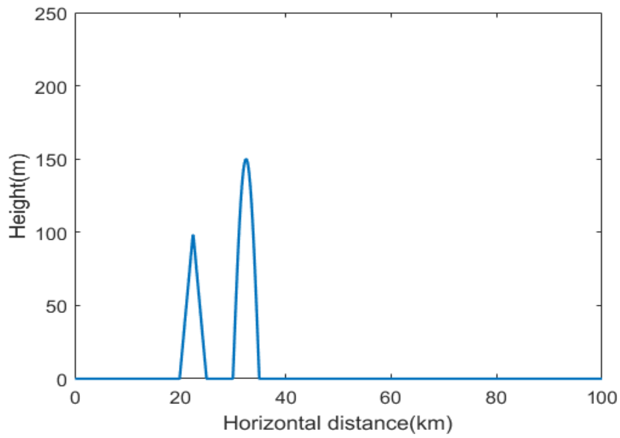

FIGURE 14. Schematic diagram of the irregular terrain.

TABLE 2. Surface medium conditions.

Rangekm	0-40	40-70	70-100
Medium condition	Medium dry ground	Wet ground	Sea surface

TABLE 3. Step size selection in the complex environments.

Range(km)	0-20 25-30 35-40 45-65 75-100	20-25 30-35	40-45	65-75
Complexity level	IV	I	III	II/III
Step size/m	150	50	100	75

3) The atmospheric environment corresponds to the surface medium: 0-70km is the land, the corresponding atmospheric environment is the standard atmosphere. 70-100km is the sea surface environment, and the atmospheric environment is the evaporation duct.

The simulation frequency is 3GHz, and the upper limit of the required error is 1%. The maximum step size can be selected from Equation (18) to be 170m. The variable step size is used to set different steps for the complex environment of the above different regions, and the final step size distribution is shown in Table 3.

According to the complexity level of the environment, different step sizes are selected in different regions, and 20-25km is the region with dramatic change of surface elevation of complexity I, therefore, the minimum step size is selected to be 50m. 40-45km is the surface medium change region with complexity III, therefore, the step size is 100m. The 65-75km is the superimposed area with the complexity II and III, so the smaller step size is 75m. The other area is the simple environment of complexity IV, so the maximum step size is 150m.

B. SIMULATION RESULTS AND ANALYSIS

The frequency of transmitting antenna is 3GHz, the width of 3dB of the pattern of Gauss antenna is 3degrees, the horizontal polarization is 25m, and the maximum propagation distance is 100km.

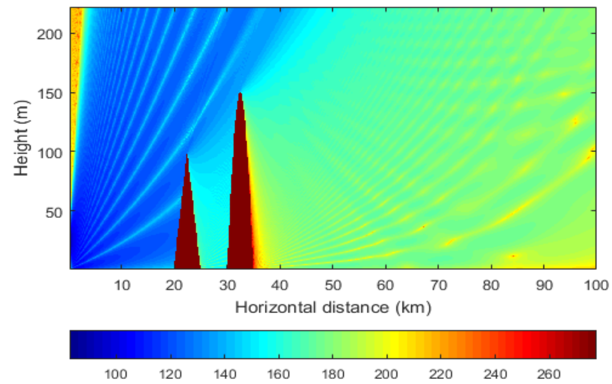

FIGURE 15. The pseudo-color graph of space propagation loss with the variable step size.

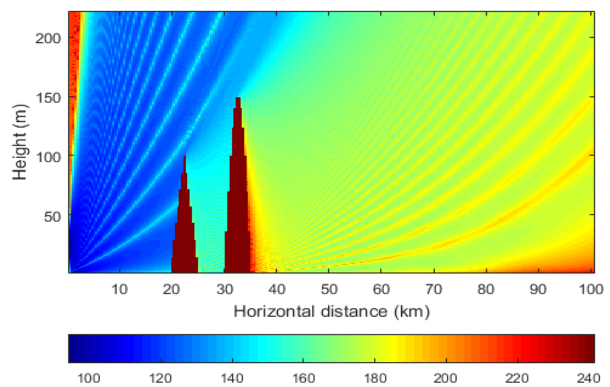

FIGURE 16. The pseudo-color graph of space propagation loss with a step size (150m).

From Fig.15 and Fig.16, it can be seen that the variable step-size model can better describe the terrain boundary than the fixed step-size model. It reflects the diffraction effect of topography on the electromagnetic wave, especially when the terrain is undulating and the distance is longer, the effect is more obvious. In order to verify the reliability of the variable step size SSFT solution, the fixed step sizes of 50m and 100m are selected as the comparison, which is compared with the accuracy of the variable step size on SSFT solution described above.

As shown in Fig.17-20, the propagation factor curve of the variable step size and the fixed step size solution at different horizontal distances is given under different complex environment conditions. As a comparison of the accuracy of SSFT solution with the variable step size. Fig.17 is the curve of horizontal distance 23km propagation factor with height, and the environment complexity is I. However, the variable step size method chooses the minimum step size of 50m, so it is more accurate than the model with a fixed step size of 100m. Fig.18 is a horizontal distance 27km propagation factor with height variation curve. Because it is located on the back of pyramid

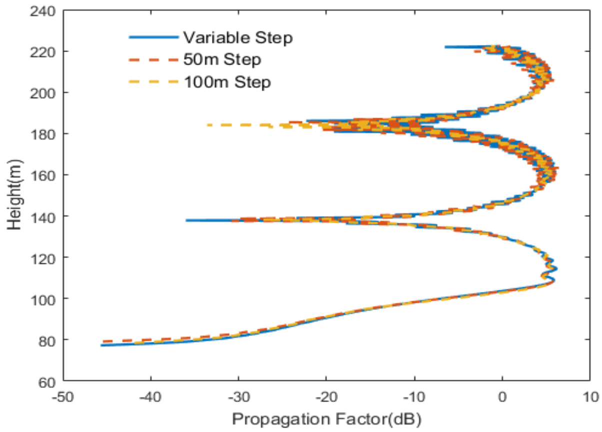

FIGURE 17. The propagation factor of the variable step size and the fixed step size at different horizontal distances (At 23km).

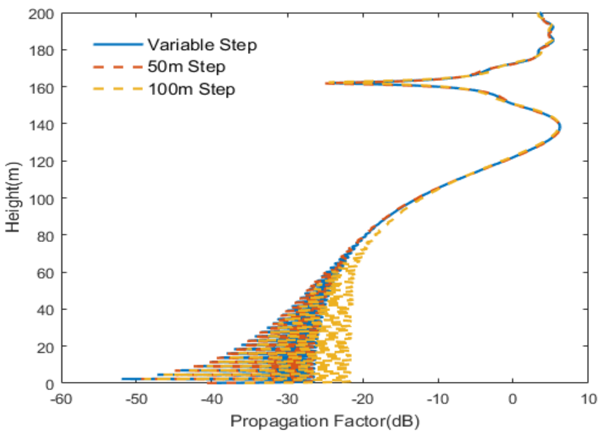

FIGURE 18. The propagation factor of the variable step size and the fixed step size at different horizontal distances (At 27km).

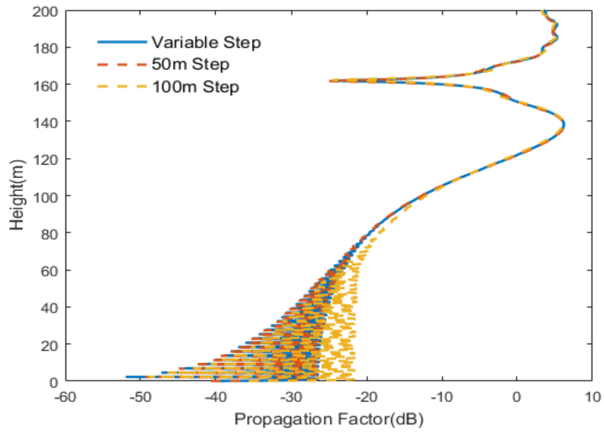

FIGURE 19. The propagation factor of the variable step size and the fixed step size at different horizontal distances (At 41km).

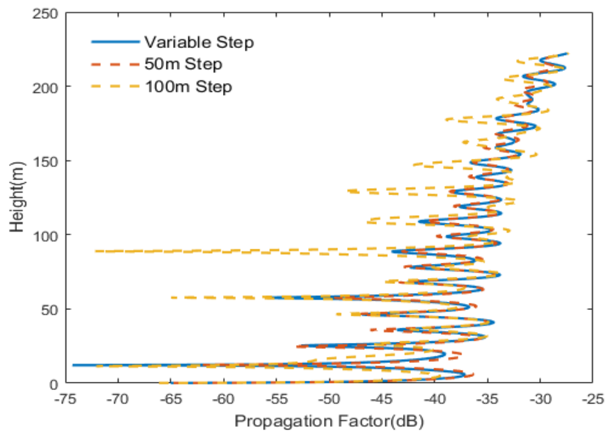

FIGURE 20. The propagation factor of the variable step size and the fixed step size at different horizontal distances (At 60km).

topography, the shadow region of electromagnetic wave propagation will appear. However, because the complexity of the environment is IV, although the variable step size is 150m, it is close to the smaller fixed step size. Compared with the model with larger fixed step size, the smaller error shows the importance of considering the influence of complex terrain. Fig.19 is a horizontal distance 41km propagation factor with height variation curve, where the environment complexity is III, the variable step size selected a larger step (100m), and the precision; Fig.20 is the 60km propagation factor of horizontal distance varies with the height, and the complexity is IV, so the step size is 150m. The errors of the large fixed step size, the smaller step size and the variable step size model indirectly indicate the importance and reliability of PE step size considering different complex environment conditions.

Through the above-mentioned simulation and analysis, it can be seen that the variable step size on SSFT method can achieve the similar results of the smaller fixed step size, thus proving the reliability of the variable step size on SSFT method. At the same time, the variable step-size algorithm can better reflect the reflection, diffraction and refraction of electromagnetic wave after complex environment for the larger fixed step-size algorithm. Secondly, the efficiency of variable step size SSFT method is verified. Fixed step sizes of 50m, 75m and 100m are selected to compare with the efficiency of variable step size SSFT method. Since the maximum calculation range is 100km, the number of horizontal step points can be calculated by SSFT with fixed step size. The calculation time and sampling points in horizontal direction are shown in Fig.21.

In order to verify the efficiency of the variable step size solution, the calculation range is set to 100km. The fixed step sizes of 50m, 75m and 100m are selected as comparisons, and compared with the variable step size efficiency adopted in Table3. The calculation time and efficiency results are shown in Table4.

Table4 is a comparison of the efficiency on the variable step size and the fixed step size. It can be seen that in the range of 100km relative to 50, 75, 100m fixed step solution, the number of horizontal sampling points decreased by 55.9%/33.8%/11.7%, repectively, and the calculation time

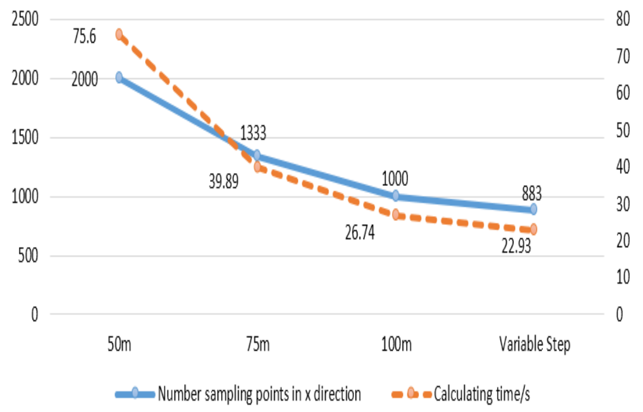

FIGURE 21. **The calculating quantity of parabolic equation with variable step size.**

TABLE 4. **Calculation and efficiency of parabolic equation with variable step size.**

	Step size /m	Number of sampling points in the horizontal direction	Calculating time/s
The fixed step size	50 / 75 / 100	2000 / 1333 / 1000	75.6 / 39.89/ 26.74
The Variable Step Size	Variable	883	22.93
High efficiency	-	55.9%/33.8%/11.7%	69.6%/42.44%/14.1%

decreased by 69.6%/42.44%/14.1%. As shown in Fig.21, the variable step size solution can achieve the same calculation precision as the smaller fixed step size. It shows that the variable step size solution has a large computational efficiency gain without affecting the accuracy. The advantage will be highlighted in the large-scale complex environment radio wave propagation prediction problem.

VI. CONCLUSION

The traditional parabolic equation fixed-step SSFT method is inefficient in long-distance wave prediction and cannot be predicted in real time. Based on the analysis of SSFT solution error, this paper proposes a variable step size solution for SSFT, which can effectively overcome the problem on low computational efficiency of fixed step size. And they improve the adaptability of SSFT solution in various complex environments. By analyzing the propagation factors in different environments and steps, and comparing with the fixed step solution, the reliability and efficiency of the variable step size technique are verified, and the variable step size method can be used without affecting the calculation accuracy. Further improve the efficiency of the parabolic equation, which is of great significance for the study of the propagation characteristics of complex environmental waves.

Moreover, the variable step method is generalized to the three-dimensional case. It is difficult to make sure relationship between Δx and the refractivity. We plan to do further research and other comparisons can be made with real data.

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