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Performance Analysis and Power Allocation for NOMA-Based Hybrid Satellite-Terrestrial Relay Networks With Imperfect Channel State Information

SILIN XIE, BANGNING ZHANG, DAOXING GUO, AND BING ZHAO
College of Communications Engineering, Army Engineering University of PLA, Nanjing 210007, China

Corresponding author: Bangning Zhang (bangning_zhang@sina.com)

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ABSTRACT In this paper, we study the multiple-user non-orthogonal multiple access (NOMA) scheme in the hybrid satellite-terrestrial relay networks (HSTRN). The proposed system model takes into account both the decode-and-forward (DF) and amplify-and-forward (AF) protocols at the relay and the imperfection of channel state information (CSI) at all nodes. We analyze the outage performance and investigate the power allocation problem to ensure fairness among users. Specially, we derive the closed-form expressions and the asymptotic expressions at high signal-to-noise-ratios (SNR) region for the outage probability of each user. Based on the asymptotic expressions, the considered non-convex power allocation problem is approximated to a generalized linear fractional programming problem. A low-complexity algorithm is developed to yield an optimal solution. Simulation results demonstrate the validity of theoretical results. The impacts of the channel estimation error and channel fading parameters on the outage performance are analyzed. Comparisons between NOMA and orthogonal multiple access (OMA), as well as between DF and AF protocols are also shown.

INDEX TERMS Hybrid satellite-terrestrial relay networks (HSTRN), non-orthogonal multiple access (NOMA), outage probability, fairness, power allocation.

I. INTRODUCTION

Satellite communication system has been considered as an important element of the future radio access network owing to its comprehensive coverage. The combination of terrestrial and satellite networks is an economical and practical solution to provide global service to users with higher reliability. In this regard, hybrid satellite-terrestrial relay networks (HSTRN) have been proposed as an integration of satellite communication and terrestrial relay transmission to improve reliability [1], [2].

There are many works focusing on the performance of HSTRN. In [3], the authors analyzed the performance of the hybrid satellite-terrestrial system with an amplify-and-forward (AF) relay. An extension of this work was studied in [4], in which the authors considered a more general system model with co-channel interference and multiple relays. The decode-and-forward (DF) relay in the hybrid satellite-terrestrial system was also considered in [5] and the average symbol error rate (SER) was derived. In [6], the authors investigated the beamforming in the AF-based hybrid satellite-terrestrial network, where the relay and destination are equipped with multiple antennas. The authors in [7] also considered multi-antenna techniques with multi-antenna satellite and multi-antenna users and analyzed performance in terms of the ergodic capacity, outage probability, and SER. In [8], the authors investigated the multiuser HSTRN with opportunistic scheduling and derived the analytical expressions for the ergodic capacity and outage probability. In the above references, the channel and hardware are considered to be ideal. A joint channel estimation and detection theme in HSTRN was proposed in [9] and SER and ergodic capacity

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were derived with imperfect channel state information (CSI). The effect of hardware impairments on the performance of two-way satellite multi-terrestrial relay networks was considered in [10].

Due to the limited resource, e.g. power and spectrum, the existing communication system will face great challenges when the data traffic grows more and more rapidly. Non-orthogonal multiple access (NOMA) has been proposed and has become an attractive topic recently, which has notable superiority compared with conventional orthogonal multiple access (OMA). Unlike OMA, a NOMA scheme permits multiple users to be multiplexed in the power domain. Superposition coding (SC) utilized at the transmitter and successive interference cancellation (SIC) utilized at the receiver make users share the spectrum simultaneously [11]. Generally, the user with a poorer channel is allocated more power, so that it can decode its signal directly regarding other signals as noise. The user that experiences better channel decodes and cancels stronger signal of the other user before detecting its own signal.

There have been numerous studies on NOMA because of the advantages of NOMA in terms of massive connectivity, low latency and high spectral efficiency [12]. In [13], the performance of the NOMA system was investigated in a cellular downlink scenario with randomly deployed users, showing that NOMA can achieve superior performance in terms of ergodic sum rate. The authors in [14] investigated the outage balancing taking into account power allocation, decoding order selection, and user grouping. In [15], the authors presented that the additional power should be allocated to the user with best channel condition to maximize the sum rate of users, while other users allocated with minimum power to maintain their minimum requirements of service. The authors in [16] and [17] investigated the fair-NOMA approach, in which each user's capacity is always greater than or equal to the capacity that can be achieved using OMA. In [18], a joint power and rate allocation was proposed to minimize the total transmission power with network throughput constraint. The application of NOMA in device-to-device (D2D) communication was developed in [19]. The cooperative NOMA scheme was studied in [20] and [21] in which the user with better channel is regarded as a relay to forward the messages of other users' with poorer channels. In [22], the authors investigated the performance of NOMA-based AF relay networks. Outage probability and ergodic sum capacity were derived in a NOMA system in [23], where only one of the users is assisted by a DF relay. In [24], unmanned aerial vehicles (UAVs) were used as a DF relay to improve the quality of service (QoS) of the ground user with NOMA scheme.

For relaying communication systems, some recent works have considered imperfect conditions for practical applications [25], [26]. These works were expanded to NOMA scheme in [27]–[29]. The authors in [30] studied the outage performance in a underlay cognitive radio NOMA system with a DF relay, considering imperfect CSI at receiver. In [31], the authors considered the impact of imperfect SIC on the ergodic sum capacity.

Several works have focused on the application of NOMA in HSTRN. Integrated terrestrial-satellite networks were researched in [32]–[34], where NOMA is only employed in terrestrial networks. The performance of the hybrid terrestrial-satellite networks was analyzed where NOMA is also utilized in satellite-terrestrial link applying DF [35], [36] and AF relays [37]. However, only [36] has considered the impact of imperfect CSI. And all the aforementioned researches have not mentioned the fairness among users in NOMA-based HSRTN.

Motivated by the above discussions, we consider a NOMA-based HSTRN, in which the satellite provides service for multiple users through a terrestrial relay. We assume that CSI is imperfect at all nodes. Additionally, both DF and AF protocols are considered. The main contributions of this paper are summarized as follows.

- We propose a practical system model for the dual-hop downlink HSTRN with a multiple-user NOMA theme. Perfect CSI is considered to be unavailable at each node. At the transmitter, only statical CSI is available. A pilot-based channel estimation is applied for receivers, based on which the exact expressions for received instantaneous signal to interference and noise ratios (SINR) at the receivers are given.
- The closed-form expression for the outage probability of each user is derived with both DF and AF protocols. To obtain more insights into the system performance, the asymptotical expressions are also provided. The difference between the DF and AF protocols and the impairment of the estimation on the performance are evaluated.
- Considering the fairness among users, the power allocation is studied to minimize the maximum outage probability of users. Based on the asymptotical expressions, a low-complexity iterative algorithm is proposed.

The rest of this paper is organized as follows. In Section II, the system and channel models are described. Moreover, applying a pilot-based channel estimation, the analytic expressions for received SINR at users are derived. The closed-form expressions of analytic and asymptotic expressions for the outage probability are derived in Section III. In Section IV, we study the power allocation and propose an iterative algorithm. Simulation results are presented in Section V. Finally, Section VI provides a brief conclusion of the paper.

II. SYSTEM AND CHANNEL MODELS

We consider the dual-hop downlink HSTRN shown in Fig. 1, where a satellite (S) serves M users with the assistance of a relay (R) in one spot beam implementing an M-user NOMA scheme. The direct links are assumed to be unavailable between S and users due to heavy shadowing, e.g., mobile terminals locate in buildings or the lines of sight (LoS) are

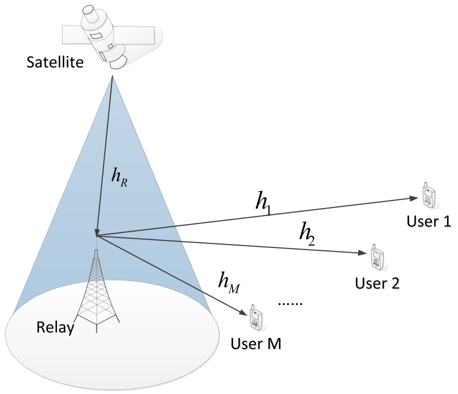

FIGURE 1. The system model of the hybrid satellite-terrestrial relay networks.

shadowed by trees, mountains and other obstacles. The relay receives and forwards the signals to guarantee the reliability of communications. The channel fading coefficient h_R of S-R link is modeled as shadowed-Rician fading distribution. The fading coefficient h_k of the channel between R and user k follows independent distributed Nakagami-m fading. All the nodes are equipped with a single antenna. We assume that perfect CSI is unavailable at all nodes.

The communication occurs in two time slots. During the first time phase, S transmits a superposed signal x to R, which can be expressed as

$$x = \sum_{i=1}^{M} \sqrt{\alpha_i} x_i, \tag{1}$$

where α_i and x_i are the power allocation coefficient and signal transmitted to user i. The signals are normalized as $E\left\{|x_i|^2\right\} = 1, i = 1, 2, \ldots, M$. Noting that instantaneous CSI is unknown to S, S needs to allocate the power based on the statistical CSI. Without loss of generality, we assume that the users are ordered by their distances between R and themselves as Fig. 1 and the channel fading of user i is more serious than that of user j while $i < j$, i.e., $E\left\{|h_1|^2\right\} \leq E\left\{|h_2|^2\right\} \leq \cdots \leq E\left\{|h_M|^2\right\}$. Hence the power is allocated to users in the opposite order with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_M$.

The received signal at R can be represented as

$$y_R = \sqrt{P_S} h_R x + w_R, \tag{2}$$

where P_S devotes the total transmit power at S and w_R devotes the additive white Gaussian noise (AWGN) with the power density σ_R^2. The probability density function (PDF) of $|h_R|^2$ is expressed as [2]

$$f_{|h_R|^2}(x) = \alpha_R e^{-\beta_R x} {}_1F_1(m_R; 1; -\delta_R x), \tag{3}$$

where $\alpha_R = (2b_R m_R)^{m_R}/(2b_R m_R + \Omega_R)^{m_R}/(2b_R)$, $\beta_R = 1/(2b_R)$, $\delta_R = \Omega_R/(2b_R)/(2b_R m_R + \Omega_R)$, $2b_R$ and Ω_R are the average power of the multipath component and line-of-sight (LoS) component, respectively, and m_R ($m_R > 0$) denotes the Nakagamim parameter of LoS. The function ${}_1F_1(\cdot)$ is the confluent hypergeometric function [38, Eq.(9.14.1)].

L training symbols $s_i, i = 1, 2, \cdots, L$ are transmitted to R with the same power P_S for CSI estimation. Received training symbols at R can also be expressed as

$$r_i = \sqrt{P_S} h_R s_i + n_{r,i}, \tag{4}$$

where $n_{r,i} \sim \mathcal{CN}(0, \sigma_R^2)$. During the second time slot, R forwards the received signal to all users. In this paper, we consider both AF and DF protocols.

A. DF RELAY

Assume that DF protocol is adopted at R. R needs to estimate the channel fading coefficient before decoding. The estimation of h_R is expressed as

$$\hat{h}_R = h_R + e_R = \frac{1}{P_S L} \sum_{i=1}^{L} r_i s_i^* = h_R + \frac{1}{P_S L} s_i^* n_i, \tag{5}$$

where e_R is the estimation error. From (5), we have $e_R = \frac{1}{P_S L} s_i^* n_i \sim \mathcal{CN}(0, \sigma_{eR}^2)$, where $\sigma_{eR}^2 = \sigma_R^2/(P_S L)$. R performs SIC with estimated channel coefficient $\hat{h}_R$. The decision variable for symbol x_1 at R can be expressed as [39]

$$\mathbf{\Lambda}_{R,1} = \arg\min_{\tilde{x}_1} \left\{ \| y_R - \sqrt{P_S} \alpha_1 \hat{h}_R \tilde{x}_1 \|^2 \right\}, \tag{6}$$

where $\tilde{x}_1$ is the detected symbol. Substituting (1) and (5) into (6), we have

$$\mathbf{\Lambda}_{R,1} = \arg\min_{\tilde{x}_1} \left\{ \left\| \sqrt{P_S \alpha_1} h_R (x_1 - \tilde{x}_1) + w_{R,1} \right\|^2 \right\}, \tag{7}$$

where

$$w_{R,1} = \sum_{i=2}^{M} \sqrt{P_S \alpha_i} h_R x_i - \sqrt{P_S \alpha_1} e_R \tilde{x}_1 + w_R. \tag{8}$$

Instantaneous SINR of symbol x_1 at R can be written from (7) and (8) as

$$\gamma_{R,1} = \frac{\rho_s \alpha_1 |h_R|^2}{\rho_s \sum_{i=n+1}^{M} \alpha_2 |h_R|^2 + \frac{\alpha_1}{L} + 1}, \tag{9}$$

where $\rho_s = P_S/\sigma_R^2$ devotes the transmitting SNR. Suppose that $x_i, i = 1, 2, \cdots, n-1$ have been decoded correctly and been canceled with estimated $\hat{h}_R$. The decision variable for symbol x_n can be expressed as

$$\begin{aligned} \mathbf{\Lambda}_{R,n} &= \arg\min_{\tilde{x}_n} \left\{ \left\| y_R - \sum_{i=1}^{n-1} \sqrt{P_s \alpha_i} \hat{h}_R x_i - \sqrt{P_S \alpha_n} \hat{h}_R \tilde{x}_n \right\|^2 \right\} \\ &= \arg\min_{\tilde{x}_n} \left\{ \left\| \sqrt{P_S \alpha_n} h_R (x_n - \tilde{x}_n) + w_{R,n} \right\|^2 \right\}, \end{aligned} \tag{10}$$

where

$$w_{R,n} = \sum_{i=n+1}^{M} \sqrt{P_S \alpha_i} h_R x_i - \sum_{i=1}^{n-1} \sqrt{P_S \alpha_i} e_R x_i - \sqrt{P_S \alpha_n} e_R \tilde{x}_n + w_R. \tag{11}$$

Then the SINR of x_n is

$$\gamma_{R,n} = \frac{\rho_s \alpha_n |h_R|^2}{\sum_{i=n+1}^{M} \rho_s \alpha_i |h_R|^2 + \sum_{i=1}^{n} \frac{\alpha_i}{L} + 1}. \quad (12)$$

Specially, when $n = M$, we have

$$\gamma_{R,M} = \frac{\rho_R \alpha_M |h_R|^2}{\frac{1}{L} + 1}. \quad (13)$$

After decoding all the symbols, R broadcasts the superimposed signal of detected symbols, as well as L training symbols to M users with power P_R and power allocation β_i, where $\beta_1 > \beta_2 > \cdots > \beta_M$ and $\sum_{i=1}^{M} \beta_i = 1$. If all the messages are decoded correctly at R, the received signal at user k is written as

$$y_k = \sum_{i=1}^{M} \sqrt{P_R \beta_i} h_k x_i + w_k, \quad (14)$$

where w_k is AWGN with variance σ_k^2. Here we set $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_M^2 = \sigma_R^2$. The PDF of h_k is written as

$$f_{|h_k|^2}(x) = \frac{m_k^{m_k} x^{m_k - 1}}{\Gamma(m_k) \Omega_k^{m_k}} e^{-m_k x / \Omega_k}, \quad (15)$$

where (m_k, Ω_k) is the Nakagami parameter. Only user 1 can decode its message directly. SIC is performed for user k, $k > 1$ to decode $x_1, x_2, \cdots, x_{k-1}$ before decoding its own message. Following the same channel estimation and detector manner as described above, the received SINR of x_n at user k is expressed as

$$\gamma_{k,n} = \frac{\rho_r \beta_n |h_k|^2}{\sum_{i=n+1}^{M} \rho_r \beta_i |h_k|^2 + \sum_{i=1}^{n} \frac{\beta_i}{L'} + 1}, \quad (16)$$

where $1 \le n \le k \le M$ and $\rho_r = P_R / \sigma_k^2$.

B. AF RELAY

When an AF protocol is used at R, R amplifies and broadcasts the received signals to all users with constant power P_R. Channel estimation and decoding are unnecessary at R. The received signal of user k is written as

$$\begin{aligned} y_k^A &= \sqrt{P_R} G h_k (\sqrt{P_S} h_R x + w_R) + w_k \\ &= \sqrt{P_S} \sqrt{P_R} g_k x + \sqrt{P_R} h_k G w_R + w_k. \end{aligned} \quad (17)$$

where G is the amplifying factor defined as $G = 1/\sqrt{P_s |h_R|^2 + \sigma_{w_R}^2}$ and $g_k = G h_k h_R$. The pilots are also amplified and sent to users, written as

$$r_{k,i} = \sqrt{P_S} \sqrt{P_R} g_k s_i + \sqrt{P_R} h_k G n_{R,i} + n_{k,i}. \quad (18)$$

From (18), we can get the estimation of g_k as

$$\begin{aligned} \hat{g}_k &= \frac{1}{\sqrt{P_S P_R} L} \sum_{i=1}^{L} r_{k,i} s_i^* \\ &= g_k + \frac{G h_k}{\sqrt{P_S}} \sum_{i=1}^{L} s_i^* n_{R,i} + \frac{1}{\sqrt{P_S P_R}} \sum_{i=1}^{L} s_i^* n_{k,i} \\ &= g_k + e_k. \end{aligned} \quad (19)$$

SIC is performed with estimated $\hat{g}_k$ for receivers except for user 1. Be similar to (10), the decision variable for symbol x_n at user k can be expressed as

$$\begin{aligned} \mathbf{\Lambda}_{k,n} &= \arg\min_{\tilde{x}_n} \left\{ \left\| y_k - \sum_{i=1}^{n-1} \sqrt{P_s P_R \alpha_i} \hat{g}_k x_i - \sqrt{P_s P_R \alpha_i} \hat{g}_k \tilde{x}_n \right\|^2 \right\} \\ &= \arg\min_{\tilde{x}_n} \left\{ \left\| \sqrt{P_S P_R \alpha_n} g_k (x_n - \tilde{x}_n) + w_{k,n} \right\|^2 \right\}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} w_{k,n} = &\sqrt{P_S P_R} g_k \sum_{i=n+1}^{M} \sqrt{\alpha_i} x_i + \sqrt{P_R} h_k G w_R + w_k \\ &- \sum_{i=1}^{n-1} \sqrt{P_S P_R \alpha_i} e_k x_i - \sqrt{P_S P_R \alpha_n} e_k \tilde{x}_n. \end{aligned} \quad (21)$$

Substituting (19) and (21) into (20), the received SINR at user k can be derived as (22) shown at the bottom of this page

III. OUTAGE ANALYSIS

The outage probability of user k is defined as the probability that the instantaneous achievable rate falls below the data rate requirement, which is an important metric of the fading channel networks. In this paper, we normalize the bandwidth to 1. Therefore, the requirement of data rate is relative to an SINR threshold as $R_{th,i} = \frac{1}{2} \log_2 (1 + \phi_i)$ in dual-hop relay networks. It's worth noting that, the outage of user k also occurs if any x_n, $n < k$ is not detected correctly due to the application of SIC.

A. DF PROTOCOL

When a DF protocol is applied, the outage probability of user k is written as

$$P_{out,k}^D = 1 - P\left[E_{R,k} \cap E_m\right] = 1 - P\left[E_{R,k}\right] P\left[E_k\right] \quad (23)$$

where $E_{R,k} = E_{R,1}^c \cap E_{R,2}^c \cap \cdots \cap E_{R,k}^c$, $E_n = E_{k,1}^c \cap E_{k,2}^c \cap \cdots \cap E_{k,k}^c$, $E_{R,n}^c$ represents the event $\gamma_{R,n} \ge \phi_n$ and $E_{k,n}^c$ represents

$$\gamma_{k,n}^A = \frac{\rho_s \rho_r |h_r|^2 |h_k|^2 \alpha_n}{\sum_{i=n+1}^{M} \rho_s \rho_r |h_R|^2 |h_k|^2 \alpha_i + \left(1 + \frac{1}{L} \sum_{i=1}^{n} \alpha_1\right) (\rho_r |h_k|^2 + \rho_s |h_R|^2 + 1)} \quad (22)$$

the event $\gamma_{k,n} \geq \phi_n$. The probability of $E_{R,n}^c$ is derived as

$$P\left[\gamma_{R,n} \geq \phi_n\right] = P\left[\left(\rho_s\alpha_n - \phi_n\rho_s\sum_{i=n+1}^{M}\alpha_i\right)|h_R|^2 \geq \phi_n\left(\sum_{i=1}^{n}\frac{\alpha_i}{L_1}+1\right)\right] \underset{(a)}{=} P\left[|h_R|^2 \geq \frac{\phi_n\left(\sum_{i=1}^{n}\frac{\alpha_i}{L_1}+1\right)}{\rho_s\alpha_n - \phi_n\rho_s\sum_{i=n+1}^{M}\alpha_i}\right], \quad n<M, \tag{24}$$

and

$$P\left[\gamma_{R,M} \geq \phi_M\right] = P\left[|h_R|^2 \geq \frac{\phi_M\left(\frac{1}{L_1}+1\right)}{\rho_s\alpha_M}\right]. \tag{25}$$

Step (a) is under the condition with $\alpha_n > \phi_n\sum_{i=n+1}^{M}\alpha_i$. In other cases, x_n can not be decoded at any node. Let $\theta_n = \frac{\phi_n\left(\sum_{i=1}^{n}\frac{\alpha_i}{L_1}+1\right)}{\alpha_n-\phi_n\sum_{i=n+1}^{M}\alpha_i}$ $(n<M)$, $\theta_M = \frac{\phi_M\left(\frac{1}{L_1}+1\right)}{\alpha_M}$ and $\theta_k^* = \max\{\theta_1, \theta_2, \cdots, \theta_k\}$. Then we have

$$P\left[E_{R,k}\right] = P\left[|h_R|^2 \geq \frac{\theta_k^*}{\rho_s}\right] = 1 - F_{|h_R|^2}\left(\frac{\theta_k^*}{\rho_s}\right), \tag{26}$$

where $F_{|h_R|^2}(\cdot)$ is the cumulative distribution function (CDF) of $|h_R|^2$ and is given by [7] as

$$F_{|h_R|^2}(x) = \alpha_R\sum_{k=0}^{\infty}\frac{(m_R)_k\,\delta_R^k}{(k!)^2\,\beta_R^{k+1}}\gamma\left(k+1, \beta_R x\right), \tag{27}$$

where $\gamma\left(a, x\right) = \int_0^x e^{-t}t^{a-1}dt$ [38, Eq.(8.350.1)]. Similar to (26), we have

$$P\left[E_{k,k}\right] = P\left[|h_k|^2 \geq \frac{\eta_k^*}{\rho_r}\right] = 1 - F_{|h_k|^2}\left(\frac{\eta_k^*}{\rho_r}\right), \tag{28}$$

where $\eta_n = \frac{\phi_n\left(\sum_{i=1}^{n}\frac{\beta_i}{L_2}+1\right)}{\beta_n-\phi_n\sum_{i=n+1}^{M}\beta_i}$ $(j<M)$, $\eta_M = \frac{\phi_M\left(\frac{1}{L_2}+1\right)}{\beta_M}$, $\eta_k^* = \max\left\{\eta_{k,1}, \eta_{k,2}, \cdots, \eta_{k,k}\right\}$ and $\beta_j > \sum_{i=j+1}^{M}\phi_i\beta_i$. $F_{|h_k|^2}(\cdot)$ devotes the CDF of $|h_k|^2$ and is presented as

$$F_{|h_k|^2}(x) = \frac{\gamma\left(m_k, \frac{m_k}{\Omega_k}x\right)}{\Gamma\left(m_k\right)}. \tag{29}$$

With the help of (26) and (28), (23) can be expressed as

$$P_{out,k}^D = \begin{cases} 1-\left(1-F_{|h_R|^2}\left(\frac{\theta_k^*}{\rho_s}\right)\right)\left(1-F_{|h_k|^2}\left(\frac{\eta_k^*}{\rho_r}\right)\right), \\ \qquad \alpha_n > \phi_n\sum_{i=n+1}^{M}\alpha_i, \beta_n > \phi_n\sum_{i=n+1}^{M}\beta_i, \quad \forall n \leq k \\ 1, \qquad otherwise \end{cases} \tag{30}$$

Substituting (27) and (29) into (30), we can get the exact expression for the outage probability with DF protocol. From (30), we can see that $P_{out,k}$ will increase if θ_k^* or η_k^* increases. To achieve better performance, θ_k^* and η_k^* should be as small as possible. Obviously, the minimum values of θ_k^* and η_k^* will be the same. To facilitate the analysis, we set $\alpha_k = \beta_k$ in the remainder of this paper.

B. AF PROTOCOL

Considering the AF strategy, the outage probability is written as

$$P_{out,k}^A = 1 - P\left[\gamma_{k,n}^A \geq \gamma_{th,n}, \forall n \leq k\right]. \tag{31}$$

The probability of the event $\gamma_{k,n}^A \geq \gamma_{th,n}$ is derived as

$$P\left[\gamma_{k,n}^A \geq \gamma_{th,n}\right] = P\left[|h_k|^2 \geq \frac{\theta_n}{\rho_r}, |h_R|^2 \geq \frac{\theta_n(\rho_R|h_k|^2+1)}{\rho_s(\rho_r|h_k|^2-\theta_n)}\right]. \tag{32}$$

The condition $\alpha_n > \phi_n\sum_{i=n+1}^{M}\alpha_i$ is also necessary here. When $|h_k|^2 \geq \theta_n/\rho_r$, $\theta_n(\rho_R|h_k|^2+1)/\rho_s(\rho_r|h_k|^2-\theta_n)$ is a monotonic increasing function of θ_n. As a result, (31) can be derived as

$$P_{out,k}^A = 1 - P\left[|h_k|^2 \geq \frac{\theta_k^*}{\rho_r}, |h_R|^2 \geq \frac{\theta_k^*(\rho_R|h_k|^2+1)}{\rho_s(\rho_r|h_k|^2-\theta_k^*)}\right], \tag{33}$$

where θ_k and θ_k^* have been defined as (26). Rewrite (33) as

$$P_{out,k}^A = P\left[|h_k|^2 < \frac{\theta_k^*}{\rho_r}\right] + P\left[|h_k|^2 \geq \frac{\theta_k^*}{\rho_r}, |h_R|^2 < \frac{\theta_k^*(\rho_R|h_k|^2+1)}{\rho_s(\rho_r|h_k|^2-\theta_k^*)}\right] = F_{|h_k|^2}\left(\frac{\theta_k^*}{\rho_r}\right) + \int_{\frac{\theta_k^*}{\rho_r}}^{\infty} f_{|h_k|^2}(x)F_{|h_R|^2}\left(\frac{\theta_k^*(\rho_r x+1)}{\rho_s(\rho_r x-\theta_k^*)}\right)dx. \tag{34}$$

The former term of (34) is defined in (29). The latter one can be derived with the help of [40] and [37] by utilizing the series representation of $\gamma\left(n, x\right) = (n-1)!\left(1-e^{-x}\sum_{m=0}^{n-1}\frac{x^m}{m!}\right)$ [38, Eq.(8.352.6)]. The exact expression for the outage probability of user k is shown as (35) at the top of the next page, where $\Gamma\left(a, x\right) = \int_x^{\infty} e^{-t}t^{a-1}dt$ [38, Eq.(8.350.2)] and $K_v(\cdot)$ is the modified Bessel function [38, Eq.(3.471.9)(8.432.6)]. In the derivation of (35), the fading parameter m_k is constrained to take integer values.

C. ASYMPTOTIC OUTAGE PROBABILITY

At high SNR region, we have $\rho_s, \rho_r \to \infty$ and $\theta_k^*/\rho_s, \eta_k^*/\rho_r \to 0$. We can obtain that

$$F_{|h_R|^2}\left(\frac{\theta_n^*(\rho_R x+1)}{\rho_s(\rho_r x-\theta_n^*)}\right) \simeq F_{|h_R|^2}\left(\frac{\theta_n^*}{\rho_s}\right). \tag{36}$$

Substituting (36) into (34), the outage probability in AF-relay networks is written as

$$P_{out,k}^A \simeq F_{|h_k|^2}\left(\frac{\theta_k^*}{\rho_r}\right) + F_{|h_R|^2}\left(\frac{\theta_k^*}{\rho_s}\right)\left[1-F_{|h_k|^2}\left(\frac{\theta_k^*}{\rho_r}\right)\right] = P_{out,k}^D. \tag{37}$$

(37) shows that the asymptotic outage probability with AF protocol in the considered networks will have the same expression as that in DF relay networks when $\theta_k = \eta_k$. Rewriting $\gamma\left(a, x\right)$ as its series representation

$$P_{out,k}^{A} = \frac{\gamma\left(m_k, \frac{m_k\theta_k^*}{\Omega_k\rho_r}\right)}{\Gamma(m_k)} + \frac{\alpha_R}{\Gamma(m_k)}\sum_{l=0}^{\infty}\frac{(m_R)_l\,\delta_R^l}{l!\beta_R^{l+1}}\Gamma\left(m_k, \frac{m_k\theta_k^*}{\Omega_k\rho_r}\right) - \frac{2\alpha_R m_k^{m_k}}{\Omega_k^{m_k}\Gamma(m_k)}e^{-\frac{m_k\theta_k^*}{\Omega_k\rho_r}-\frac{\rho_r^2\beta_R}{\theta_k^*\rho_s}}\sum_{l=0}^{\infty}\frac{(m_R)_l\,\delta_R^l}{l!\beta_R^{l+1}}\sum_{n=0}^{l}\frac{1}{n!}$$
$$\times\left(\frac{\beta_R\rho_r}{\theta_n^*\rho_S}\right)^n\sum_{i=0}^{n}\binom{n}{i}\rho_r^i\sum_{j=0}^{m_k+i-1}\binom{m_k+i-1}{j}\left(\frac{\theta_n^*}{\rho_r}\right)^{m_k+i-1-j}\left(\frac{\Omega_n\theta_n^*(\theta_n^*+1)}{\beta_R^{-1}\rho_r\rho_s m_n}\right)^{\frac{j-n+1}{2}}K_{j-n+1}\left(2\sqrt{\frac{m_n\theta_n^*(\theta_n^*+1)}{\beta_R^{-1}\rho_r\rho_S\Omega_n}}\right) \quad (35)$$

[38, Eq.(8.354.1)] and ignoring high order terms, we can obtain

$$\gamma(a, x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{a+n}}{n!(a+n)} \simeq \left.\frac{x^a}{a}\right|_{x\to 0}. \quad (38)$$

Then the approximate expressions of the CDFs of $|h_R|^2$ and $|h_k|^2$ are given by

$$F_{|h_R|^2}(x) \simeq \alpha_R x, \quad (39)$$

and

$$F_{|h_k|^2}(x) \simeq \frac{m_k^{m_k-1}}{\Omega_k^{m_k}\Gamma(m_k)}\left(\frac{\theta_k^*}{\rho_r}\right)^{m_k}. \quad (40)$$

Substituting (39) and (40) into (30), the asymptotic outage probability with DF or AF relay is derived as

$$P_{out,k} \simeq \frac{\alpha_R\theta_k^*}{\rho_s} + \frac{m_k^{m_k-1}}{\Omega_k^{m_k}\Gamma(m_k)}\left(\frac{\theta_k^*}{\rho_r}\right)^{m_k} - \frac{\alpha_R m_k^{m_k-1}\theta_k^{*m_k+1}}{\Omega_k^{m_k}\Gamma(m_k)\rho_s\rho_r^{m_k}}. \quad (41)$$

Furthermore, if we set $\rho_s = \rho_r = \rho$ and ignore higher order items of $1/\rho$, the asymptotic expression for the outage probability of user k with DF or AF relay can be approached as

$$P_{out,k} \simeq \kappa_k\theta_k^* = \begin{cases}\frac{\alpha_R}{\rho}\theta_k^*, & m_k > 1\\ \left(\frac{\alpha_R}{\rho} + \frac{1}{\rho\Omega_k}\right)\theta_k^*, & m_k = 1,\end{cases} \quad (42)$$

showing that the the diversity order is 1 for all the users.

IV. POWER OPTIMIZATION

Fairness is an important and challenging problem with practical interest in a multi-user system. For this purpose, we study the power allocation problem to minimize the maximum outage probability of all users. Obviously, the problem can not be solved directly since the exact expression of $P_{out,k}^{D}$ or $P_{out,k}^{A}$ is complicated and not convex. For the sake of simplicity, we use its asymptotic representation at high SNR region instead of the analytical expression, which is suitable for both DF and AF relay networks.

The min-max problem is formulated as follows:

$$\begin{aligned}&\min_{\boldsymbol{\alpha}}\max_{k}\; P_{out,k}\\ &s.t.\; \sum_{k=1}^{M}\alpha_k = 1\\ &\qquad \alpha_k > 0, \quad k = 1, 2, \cdots, M.\end{aligned} \quad (43)$$

where $P_{out,k}$ is given as (42). As discussed above, users are ordered by their statistical CSI between R and themselves. We can assume that $\kappa_m \geq \kappa_n$ when $m < n$. Setting $\mu_n(\boldsymbol{\alpha}) = \kappa_n\phi_n\left(\sum_{i=1}^{n}\frac{\alpha_i}{L}+1\right)$ and $\nu_n(\boldsymbol{\alpha}) = \alpha_n - \phi_n\sum_{i=n+1}^{M}\alpha_i$, the problem in (43) can be expressed as

$$\begin{aligned}&\min_{\boldsymbol{\alpha}}\max_{k}\; \frac{\mu_k(\boldsymbol{\alpha})}{\nu_k(\boldsymbol{\alpha})}\\ &s.t.\; \sum_{k=1}^{M}\alpha_k = 1\\ &\qquad \alpha_k > 0, \quad k = 1, 2, \cdots, M.\end{aligned} \quad (44)$$

Note that $\mu_k(\boldsymbol{\alpha})$ and $\nu_k(\boldsymbol{\alpha})$ are both linear functions of $\boldsymbol{\alpha}$. Therefore, the optimization problem is transformed into a generalized linear fractional programming problem, which can be solved by introducing an additional variable λ as

$$G(\lambda) = \min_{\boldsymbol{\alpha}}\max_{k}\; \mu_k(\boldsymbol{\alpha}) - \lambda\nu_k(\boldsymbol{\alpha}). \quad (45)$$

The optimal solution λ^* of Problem (44) is also the solution of $G(\lambda) = 0$, which can be obtained by iterative methods such as bisection and Dinkelbach [41], [42] algorithms.

Considering the specific characteristics of the functions, we propose a new algorithm based on bisection procedure with low complexity. Firstly, we express the problem equivalently as

$$\begin{aligned}&\min_{\boldsymbol{\alpha},\lambda}\; \lambda\\ &s.t.\; \sum_{k=1}^{M}\alpha_k = 1\\ &\qquad \frac{\mu_k(\boldsymbol{\alpha})}{\nu_k(\boldsymbol{\alpha})} \leq \lambda\\ &\qquad \alpha_k > 0, \quad k = 1, 2, \cdots, M.\end{aligned} \quad (46)$$

Generally, we assume the SINR threshold satisfies $\phi_k > 1/L$. Then the constraints in (48) can be rewritten as

$$\alpha_k \geq \frac{\kappa_k\phi_k}{\lambda}\left(\frac{1}{L}+1\right) + \left(1-\frac{\kappa_k}{\lambda L}\right)\phi_k\sum_{i=k+1}^{M}\alpha_i, \quad 1 \leq k < M \quad (47)$$

and

$$\alpha_M \geq \frac{\kappa_M\phi_M}{\lambda}\left(\frac{1}{L}+1\right) \quad (48)$$

in which

$$\frac{\kappa_k}{\lambda L} \leq \frac{\alpha_k - \phi_k \sum_{i=k+1}^{M} \alpha_i}{\phi_k \left(\sum_{i=1}^{k} \alpha_i + L\right)} < \frac{1}{\phi_k L} < 1. \quad (49)$$

We could draw a conclusion from (47) and (48) that the lower bound of α_k will increase if $\sum_{i=k+1}^{M} \alpha_i$ becomes larger. Particularly, α_M has a constant lower bound as $\alpha_M \geq \frac{\kappa_M \phi_M}{\lambda}\left(\frac{1}{L}+1\right)$. Towards this direction, we state Theorem 1.

Theorem 1: For any $\lambda \in (0, 1)$, devote $\tilde{\boldsymbol{\alpha}} = \{\tilde{\alpha}_1, \cdots, \tilde{\alpha}_M\}$ defined as (50). *If $\mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}) < \lambda$, we have $\lambda^* \in [\mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}), \lambda]$, otherwise $\lambda^* \in [\lambda, \mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}),]$, where λ^* is the optimal solution of* (46) *and*

$$\tilde{\alpha}_k = \begin{cases} \frac{\kappa_M \phi_M}{\lambda}\left(\frac{1}{L}+1\right), & k = M \\ \frac{\kappa_k \phi_k}{\lambda}\left(\frac{1}{L}+1\right) + \left(1 - \frac{\kappa_k}{\lambda L}\right)\phi_k \sum_{i=k+1}^{M} \tilde{\alpha}_i, & 1 < k < M \\ 1 - \sum_{i=2}^{M} \tilde{\alpha}_i, & k = 1. \end{cases} \quad (50)$$

Proof: Obviously, $\tilde{\alpha}_k$ $(1 < k \leq M)$ will decrease and $\tilde{\alpha}_1$ will increase with the increase of λ. From the derivation of (50), we can obtain that $\mu_k(\tilde{\boldsymbol{\alpha}})/\nu_k(\tilde{\boldsymbol{\alpha}}) = \lambda$ when $1 < k \leq M$ and $\tilde{\alpha}_k$ $(1 < k \leq M)$ is the lower bound of the power allocation coefficient for user k, while $\tilde{\alpha}_1$ is the upper bound of the coefficient for user 1 under the condition $\mu_k(\tilde{\boldsymbol{\alpha}})/\nu_k(\tilde{\boldsymbol{\alpha}}) \leq \lambda$. If $\mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}) > \lambda$, there will be no feasible solution with given λ. Thus, λ is large than the optimal solution λ^*. In addition, $\mu_k(\tilde{\boldsymbol{\alpha}})/\nu_k(\tilde{\boldsymbol{\alpha}}) \leq \mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}})$ is satisfied for any k, implying that $\lambda^* \leq \mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}})$. When $\mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}) \leq \lambda$, $\tilde{\boldsymbol{\alpha}}$ is a feasible solution. Furthermore, if reset $\lambda = \mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}})$, $\tilde{\alpha}_k$ $(1 < k \leq M)$ needs to be larger and $\tilde{\alpha}_1$ will be too small to meet the constraint. Therefore, we have $\mu_1(\tilde{\boldsymbol{\alpha}})/\nu_1(\tilde{\boldsymbol{\alpha}}) \leq \lambda^* \leq \lambda$. This completes the proof.

With Theorem 1, an improved bisection-based algorithm is proposed as follows. The proposed algorithm provides a way to obtain the optimal solution of Problem (46). Since that it is an equivalent transformation from (43) to (46), the proposed algorithm could guarantees the fairness among different users at high SNR region for both AF-relay or DF-relay.

It can be seen that $\mu_k(\boldsymbol{\alpha})$ is inversely proportional to ρ in Problem (46). Thus the optimal solution is suitable for any transmit power, only depending on the SINR threshold and channel conditions. The algorithm provides a practical scheme to ensure fairness among users with different channel conditions. Compared with existing algorithms, the proposed algorithm also has an advantage in complexity. It has a faster convergence rate than the bisection algorithm in the outer loop iterations. And in each iteration, the complexity is $O(M)$. The conventional bisection method converges with a complexity of $O(\log_2(1/\epsilon))$. In each iteration, the same approach as in Theorem 1 can be used with $O(M)$. The Dinkelbach algorithm can guarantee to converge faster than the bisection

Algorithm 1 Optimal Algorithm of Problem (46)

1: Given a feasible initial solution $\boldsymbol{\alpha}$, initialize the lower bound $\lambda_{low} = 0$ and the upper bound $\lambda_{up} = \max_{1 \leq k \leq M}\left\{\frac{\mu_k(\boldsymbol{\alpha})}{\nu_k(\boldsymbol{\alpha})}\right\}$;
2: **repeat**
3: Set $\lambda = \frac{1}{2}(\lambda_{low} + \lambda_{up})$;
4: Obtain $\boldsymbol{\alpha}$ as (50);
5: **if** α_1 satisfies then
6: set $\lambda_{up} = \lambda$, $\lambda_{low} = \max\left\{\frac{\mu_1(\boldsymbol{\alpha})}{\nu_1(\boldsymbol{\alpha})}, \lambda_{low}\right\}$
7: **else**
8: set $\lambda_{low} = \lambda$, $\lambda_{up} = \min\left\{\lambda_{up}, \frac{\mu_1(\boldsymbol{\alpha})}{\nu_1(\boldsymbol{\alpha})}\right\}$;
9: **until** $|\lambda_{up} - \lambda_{low}| < \epsilon$
10: Return the optimal power allocation $\boldsymbol{\alpha}^* = \boldsymbol{\alpha}$

method. Whereas, in each iteration of the Dinkelbach algorithm, a linear programming problem needs to be solved by standard convex optimization techniques with a complexity of $O(M^3)$. In addition, the methodology proposed above can also be applied for accurate optimization using the exact expressions. We can rewrite the constraint $P_{out,k}^{D} \leq \lambda$ or $P_{out,k}^{A} \leq \lambda$ as $\theta_k^* \leq \zeta_k$, where ζ_k is the solution of $P_{out,k}^{D} = \lambda$ or $P_{out,k}^{A} = \lambda$. Afterward, a similar feasible solution can be obtained. However, ζ_k can not be solved directly. Using Newton's or bisection method will bring an additional complexity of $O(M \log_2(1/\epsilon))$.

V. NUMERICAL RESULTS

Some numerical results are presented in this section. The satellite link is subject to different shadowed-Rician fading, including infrequent light shadowing (ILS) with $(b_R, m_R, \Omega_R) = (0.158, 19.4, 1.29)$, average shadowing (AS) with $(b_R, m_R, \Omega_R) = (0.126, 10.1, 0.835)$, and frequent heavy shadowing (FHS) with $(b_R, m_R, \Omega_R) = (0.063, 0.739, 8.97 \times 10^{-4})$ [43]. The terrestrial links follow independent Nakagami-m distribution with $m_1 \leq m_2 \leq \cdots \leq m_M$ and $\Omega_1 \leq \Omega_2 \leq \cdots \leq \Omega_M$. Ω_k is modeled as $\Omega_k = d_k^{-\delta}$ where d_k is the distance between R and user k and δ is the pathloss exponent with $\delta = 2$. Without loss of generality, we assume that Ω_M is normalized to unity, i.e., $\Omega_M = 1$ and $\Omega_k = (d_k/d_M)^{-\delta}$, and all the users have the same SINR threshold requirements with $\phi_1 = \cdots = \phi_M = \phi = 3dB$.

Fig. 2 and Fig. 3 show the outage performance of a two-user NOMA scheme. The satellite link undergoes average shadowing. The distance between R and users is set as $d_1 = 2d_2$. The Nakagami parameters are given by $m_1 = 1$ and $m_2 = 2$. The power allocation is set as $(\alpha_1, \alpha_2) = (0.8, 0.2)$.

Fig. 2 demonstrate the exact and asymptotic outage probability versus transmitting SNR ρ, in which the length of pilots is set as $L = 10$. The two asymptotic curves are obtained based on (41) and (42), respectively. From Fig. 2, we can see that the analytical results agree well with the Monte Carlo simulations, which shows the correctness of our analytical analysis. The figure also shows that the outage performance

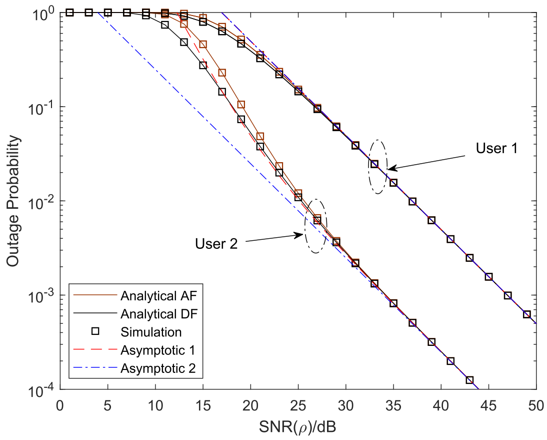

FIGURE 2. Outage probability of of each user versus SNR ρ with $M = 2$.

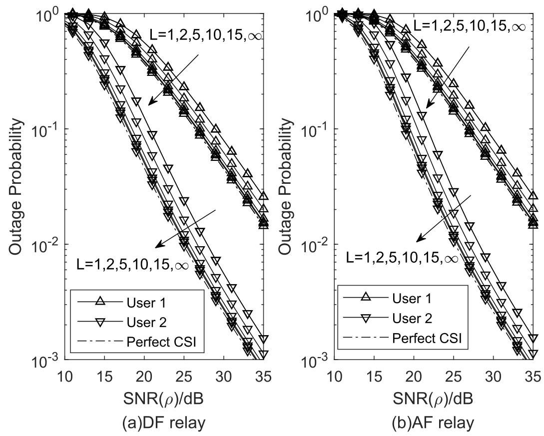

FIGURE 3. Outage probability versus SNR ρ with various L.

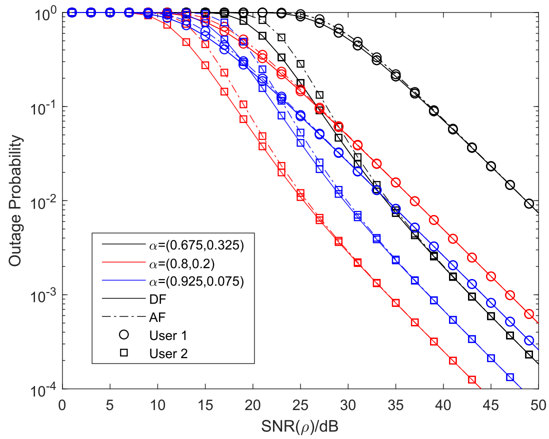

FIGURE 4. Outage probability versus SNR ρ with different power allocation.

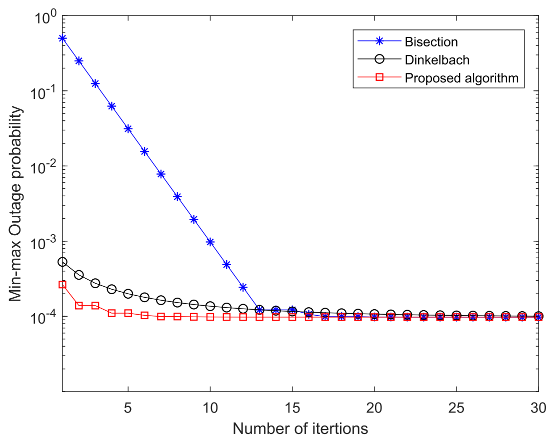

FIGURE 5. The min-max outage probability versus the number of iterations.

for both users with DF protocol outperforms that with AF protocol. However, the gaps are little at high SNR regime, owing to their same asymptotic expressions. What's more, the asymptotic curves calculated by (41) are closer to exact curves than those calculated by (42), because that (42) has ignored the m_k-order term of (41) when $m_k > 1$.

Fig. 3 illustrates the impact of estimation error term of CSI at receiver on outage probability. We consider different numbers of pilot symbols with $L = 1, 2, 5, 10, 15, \infty$, where the case with perfect CSI is demonstrated for reference as $L = \infty$. Other illustrations are the same as Fig. 2. When the length of the pilots increases, the outage performance becomes better and tends to perfect conditions, e.g., a 3dB gain can be obtained for user 2 with DF relay if the number of the pilot symbols increases from 1 to 10 when the outage probability is 10^{-2}. However, the rate of the performance benefit will reduce when L is getting larger. In fact, it will be almost meaningless when L is larger than 10 owing to the extra redundancy. As a result, we assume $L = 10$ in the following simulations.

In Fig. 4, we present the outage performance with different power allocation. It can be observed that the outage probability of user 1 decreases with larger α_1, while the outage performance with $\alpha = (0.8, 0.2)$ is better than those in other cases for user 2. This can be explained by the decoding principle of NOMA. For user 1, larger α_1 means more power for the message and less inference, i.e., smaller θ_1^*, since it decodes its symbols directly and regards x_2 as inference. For user 2, the performance depends on the larger one between θ_1 and θ_2 due to the application of SIC, which is the reason why too larger or too smaller α_2 will decrease the performance. It is clear that the power allocation is an important factor that affects the outage performance and the fairness among users greatly. The optimal power allocation is necessary.

Fig. 5 compares the convergence properties of the proposed algorithm with those of conventional algorithms. In a 6-user system with AS, the value of the objective function versus the number of iterations is plotted. As the number of iterations increases, the results of the three algorithms tend to converge. Obviously, the bisection algorithm has the slowest convergence speed. The Dinkelbach algorithm converges faster than bisection algorithm. However, its accuracy increases slowly when the number of iterations is large. The proposed

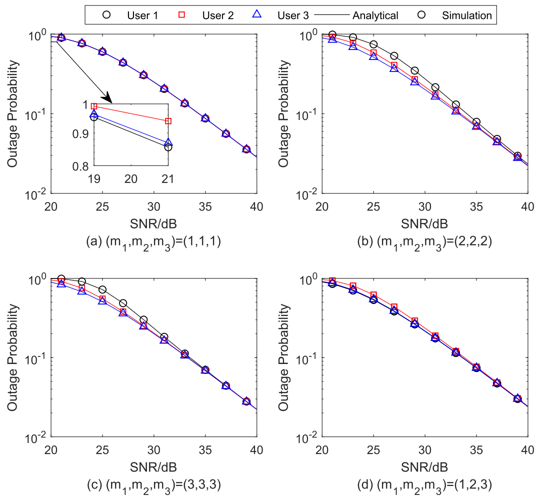

FIGURE 6. The min-max outage probability versus SNR ρ with various m_k and DF relay.

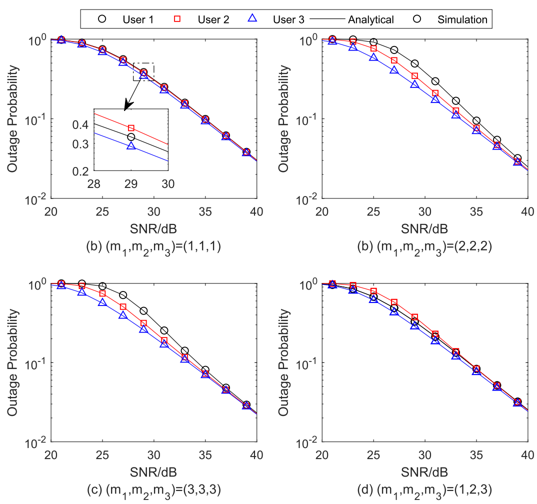

FIGURE 7. The min-max outage probability versus SNR ρ with various m_k and AF relay.

algorithm converges with the least times of iterations. Furthermore, the complexity in each iteration is also less than or equal to the other two algorithms.

Fig. 6 and Fig. 7 show the performance of each user with proposed power allocation in a 3-user system with AS in S-R link and $(d_1, d_2, d_3) = (3, 2, 1)$. Different fading parameters m_k of terrestrial links are considered. Clearly, the proposed algorithm guarantees fairness among all users very well with both DF and AF protocol at high SNR regime. With small ρ, there are still some differences. Greater fairness is achieved with $(m_1, m_2, m_3) = (1, 1, 1)$ than those in other cases. Moreover, the proposed power allocation in DF-relay networks offers better fairness than that in AF-relay networks. It is because that the optimal solution is obtained based on the asymptotic expressions for the outage expression at high SNR region. With DF protocol or $(m_1, m_2, m_3) = (1, 1, 1)$, the asymptotic expressions are closer to the exact ones. From the comparison between Fig. 6 and Fig. 7, it is easy to see that

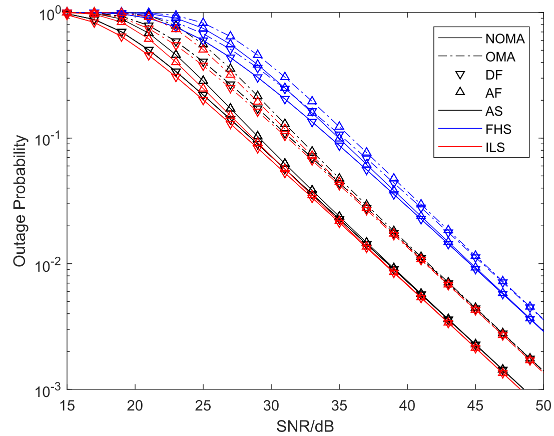

FIGURE 8. The min-max outage probability versus ρ with different S-R scenarios.

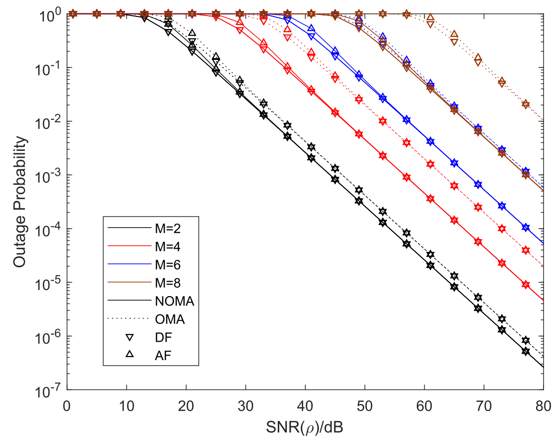

FIGURE 9. The min-max outage probability versus ρ with various M.

a DF relay will bring more benefits than the AF protocol with larger m_k.

Fig. 8 compares the min-max outage probability between NOMA and OMA with different fading scenarios between S and R, where TDMA is used as an example of OMA and is also optimized to guarantee fairness power among users with similar approaches. We assume $M = 3$, $m_1 = m_2 = m_3 = 1$ and $(d_1, d_2, d_3) = (3, 2, 1)$. It can be seen that better performance will be achieved if S-R link has a better quality. NOMA performs OMA in all considered cases. The superiority of NOMA is stronger with IFS/AS than that with FHS. The outage curves with IFS and AS are quite close due to the fact that κ_k changes little in the two scenarios, which impacts the outage performance in terms of coding gain, although it is irrelevant to the diversity gain. Moreover, the curves with DF protocols outperform those with AF protocol in the considered conditions in line with previous conclusions.

In Fig. 9, we investigate the variation of the system performance with different user numbers. In order to facilitate the analysis, we assume that the S-R link follows AS. The fading in R-user satisfies $m_1 = m_2 = \cdots = m_M = 1$ and d_k $(k < M)$ is assumed randomly between 1 and

20 and sorted in decrease turn. The advantage of NOMA over OMA increases as the number of users increases. For example, when outage probability is 1×10^{-4}, there is an about 2dB gain for NOMA compared with OMA. While the performance gap will increase to be about 10dB for $M = 10$. However, the increasing number of users will bring higher computational complexity. Therefore, a trade-off between performance and complexity is needed in the practical application. Furthermore, the performance gaps between DF and AF are almost constant for different user numbers.

VI. CONCLUSION

This paper has studied a multiple-user NOMA theme in the two-hop HSRTN with imperfect CSI. Training symbols are implanted for channel estimation. Both DF and AF protocols are considered in this paper. Specially, we have derived the closed-form expressions for the outage probability, as well as the asymptotic results at high SNR region. The power allocation problem has been investigated to ensure fairness among users. Since that the outage probability is not a concave function of the power factor, we have approximated the problem to a generalized linear fractional programming problem based on the asymptotic expressions for the outage probability. A low-complexity iterative algorithm has been developed to solve the problem. Simulation results have demonstrated the effectiveness of the theoretical results and the proposed algorithm, and made a comparison between DF and AF protocols. We also have confirmed that NOMA can achieve better fairness performance than OMA in terms of the outage probability with our proposed power allocation scheme in the considered networks. In this work, we have mainly investigated one NOMA pair of HSRTN in a spot beam. For future works, the multi-beam satellite communication and the combination of NOMA and conventional OMA will be attractive. The application of multi-antenna in NOMA-based HSRTN is also deserved to be concerned.

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SILIN XIE received the B.S. degree from Tsinghua University, Beijing, China, in 2013, and the M.S. degree from the PLA University of Science and Technology, Nanjing, China, in 2016. He is currently pursuing the Ph.D. degree with the Army Engineering University of PLA, Nanjing. His research interests include satellite communication, nonorthogonal multiple access cooperative relay networks, and multiuser communication systems.

BANGNING ZHANG received the B.S. and M.S. degrees from the Institute of Communications Engineering (ICE), Nanjing, China, in 1984 and 1987, respectively. He is currently a Full Professor and the Head of the College of Communications Engineering. He has authored or coauthored more than 80 conference and journal articles and granted more than 20 patents in his research areas. His current research interests include communication antijamming technologies, microwave technologies, satellite communications systems, cooperative communications, and physical layer security. He has served as a Reviewer for several journals in communication field.

DAOXING GUO received the B.S., M.S., and Ph.D. degrees from the Institute of Communications Engineering (ICE), Nanjing, China, in 1995, 1999, and 2002, respectively. He is currently a Full Professor and a Ph.D. Supervisor with the PLA University of Science and Technology. He has authored or coauthored more than 40 conference and journal articles and granted more than 20 patents in his research areas. His current research interests include satellite communications systems and transmission technologies, communication antijamming technologies, and communication antiinterception technologies, including physical layer security and so on. He has served as a Reviewer for several journals in communication field.

BING ZHAO received the B.S. and M.S. degrees from the Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2007 and 2009, respectively. She is currently a Full Lecturer with the Army Engineering University of PLA. She has been granted more than ten patents in her research areas. Her current research interests include satellite communication systems and transmission technologies.