IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received August 17, 2019, accepted September 1, 2019, date of publication September 18, 2019, date of current version October 1, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2942151

Research on a New Singularity-Free Controller for Uncertain Lorenz System

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This work was supported in part by the National Science Foundation of China under Grant 11561029, and in part by the Programs for Foundations of Jiangxi Province of China under Grant GJJ14428, Grant GJJ180749, and Grant 20192BBG70050.

ABSTRACT It is a challenge problem to stably control the well-known Lorenz system with uncertain parameters because of its nonlinearity and singularity. In this paper, by combining Zhang dynamics (ZD) and gradient-algorithm, a novel Zhang-Gradient (ZG) controller is designed and developed for solving the controlling problem of the uncertainty Lorenz system. In order to improve computing efficiency, the stochastic parallel gradient descent algorithm is also introduced to perform an incremental adjustment of the unknown parameters for the uncertain Lorenz system. The presented theoretical analysis in this paper shows that our such presented method could conquer the possible singularity which is a difficult problem in typical backstepping controller design. The computer simulation results exhibit that, the controlled system can be stable globally and the tracking error converges to zero asymptotically, which are further demonstrate the effectiveness and feasibility of our presented ZG controller.

INDEX TERMS Uncertain Lorenz system, Zhang dynamics, backstepping, Lyapunov stability, stochastic parallel gradient descent.

I. INTRODUCTION

The interesting nonlinear phenomenon chaos is widely encountered in practical engineering systems. For example, in chaotic communication [1], by using the extreme sensitivity to initial conditions, the noise-like chaotic signals could be applied to transmit information in a secure and robust way in communication technology research, in which, the useful signal can be hidden into the chaotic signals, and thus protected by this way [2].

However, in some cases, chaotic phenomenon should be avoided and overcome for a stable state in the control fields. Therefore, since the pioneering work by Pecora *et al.* in 1990 [3], [4], many techniques for chaos control have been developed and investigated [5]–[8]. In [9], with the mild assumptions on the partial derivatives of the unknown functions, the developed adaptive neural network control scheme achieves semi-global uniform ultimate boundedness of all the signals in the closed-loop system. In [10], two different methods, feedback control and adaptive control, are used to suppress chaos to an unstable equilibrium or the unstable periodic orbits. The authors in [11] have designed a controller based on predictive principle to stabilize the system trajectories at the unstable equilibrium points successfully.

As a classic and widely used controlling approach, backstepping method was firstly presented by A. Saberi *et al.* in 1990 [12]. It is a kind of systematic and synthetic technique to design a controller for an incertain system by applying a recursive procedure that combines the choice of a Lyapunov function with the design of feedback control [13].

In particular, the control of the well-known Lorenz system has been investigated extensively [14]–[16]. Generally speaking, the problem of controlling Lorenz system is challenged from the following three points.

i) Since the Lorenz system is a nonlinear dynamical system, the methods using linear state feedback cannot guarantee the global stability for the system in some situations [17].
ii) Compared to the active control (i. e., the system parameters are known), it is more difficult to control chaos in the presence of uncertainty in system parameters [18].
iii) Although the Lorenz system can be transformed into a nonlinear system in the "strict-feedback" form, many

The associate editor coordinating the review of this manuscript and approving it for publication was Xi Peng.

control methods (e.g., the typical backstepping indicated in [13]) cannot conquer the singularity problem for a Lorenz system.

This paper intends to control the Lorenz system with consideration of all points mentioned above. Especially, to solving the singularity problem in controlling Lorenz system, Lenz and Obradovic in [19] had presented a global approach with partial linearization. In [20], Zeng and Singh had proposed a well organized adaptive controller based on Lyapunov stability theorem. By exploiting the specific property of the Lorenz system, an adaptive backstepping controller was presented by Wang and Ge [21]. Based on the analysis of the specific structure of the Lorenz system, these methods can solve artfully the singularity problem. Therefore, it may be difficult to directly employ or expand these methods in controlling other similar nonlinear systems.

Recent studies have shown that Zhang dynamics (ZD) is designed and developed for the time-varying problems solving [22]–[24]. By using the combination of ZD and gradient-based dynamics (GD), the authors in [22] have presented a tracking controller in the form of $\dot{u}$ (i. e., the time-derivation of control action u) for a class of nth-order nonlinear systems. Inspired by [22], in this paper, ZD is firstly applied to construct three error functions related to the three Lorenz differential equations by the Lyapunov stability theorem, and then GD is exploited to obtain values of the controller and the system parameters in the form of $\dot{u}$, $\dot{p}_1$, $\dot{p}_2$, and $\dot{p}_3$, respectively. Through the combined method ZG (i. e., the combination ZD and GD), a novel and singularity-free controller (i. e., Zhang-Gradient controller) is designed and developed in this paper for the Lorenz system.

Additionally, in the process of computer simulation, we find that the estimated values of the system parameters are hard to obtain successfully due to the complication of the ultimate deduced-equations (i.e., Eqs. (21)-(23) in Subsection III-B) obtained by Zhang-Gradient controller. Thus, we intend to achieve the approximation of these parameters (i.e., $\dot{p}_1$, $\dot{p}_2$, and $\dot{p}_3$) in use of Stochastic Parallel Gradient Descent (SPGD) method [25], which is based on the gradient descent algorithm and the Simultaneous Perturbation Stochastic Approximation (SPSA) for multivariate stochastic optimization in engineering systems [26]. The convergence and stability of SPGD was deduced preliminarily by Vorontsov and Sivok [25] and Vorontsov and Carhart [27]. As a result, the computation complexity for system parameters is lowered greatly with the update laws defined by SPGD, and simulation results further substantiate this problem can be solved effectively.

The rest of the paper is organized as follows. In Section II, the problem formulation is presented. Section III deals with the details of the ZG principle and exhibits the ZG controller design procedure. Also, in this section the SPGD algorithm is employed to improve the computation efficiency for the estimation of system parameters. In Section IV, one illustrative example is simulated and analyzed to show the exactness and effectiveness of ZG approach. Some final remarks about this paper are given in the last section.

Before ending the introductory section, the main contributions of the paper are listed as follows.

i) Based on the Zhang dynamics and gradient-algorithm, a novel Zhang-Gradient controller is designed and developed for the uncertain well-known Lorenz chaotic system.
ii) To lower the computation complexity, a SPGD design procedure is presented to get the estimated values of the Lorenz system.
iii) Computer simulation results via an illustrative example are presented and analyzed to show the effectiveness of the presented ZG controller with the SPGD computation method.

II. PROBLEM FORMULATION

Lorenz system displays very complex dynamical behaviors and has become a paradigm for chaotic dynamics, together with its mathematical expression as follows.

$$\begin{cases} \dot{x}_1 = p_1 x_2 - p_1 x_1 \\ \dot{x}_2 = -x_1 x_3 + p_2 x_1 - x_2 \\ \dot{x}_3 = x_1 x_2 - p_3 x_3 \end{cases} \tag{1}$$

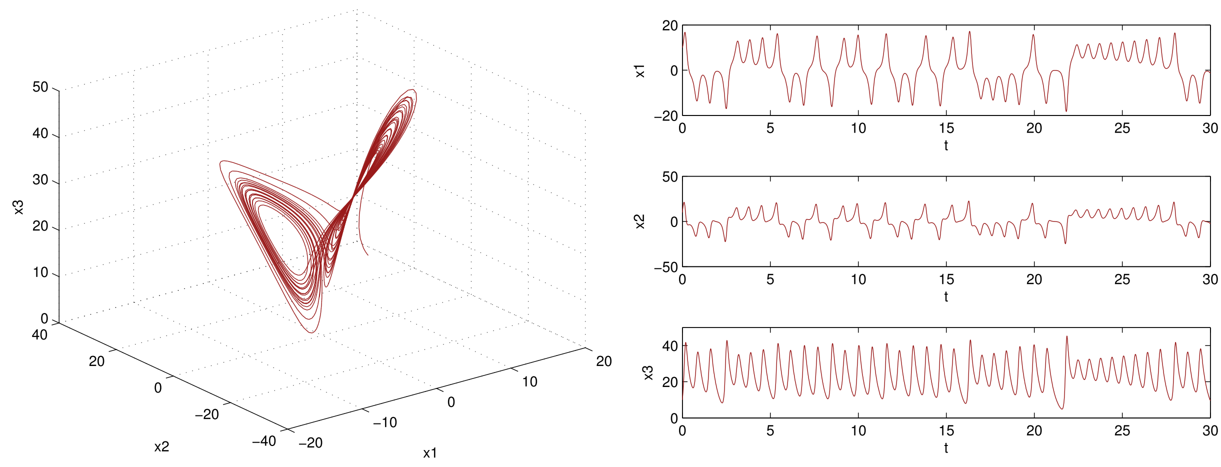
where the states (x_1, x_2, x_3) are needed to be controlled to the stable states, and the system parameters p_1, p_2, p_3 are positive constants and are assumed to be uncertain. The system (1) can generate chaotic behavior in two parameter-pairs, i.e., the first space ($p_1 = 10$, $p_2 = 28$ and $p_3 = 8/3$), and the second space ($p_1 = 16$, $p_2 = 45$ and $p_3 = 4$), which can be shown in Figs. 1 and 2 [28]. The purpose of this work is to design an adaptive Zhang-Gradient controller for system (1) such that all the states (x_1, x_2, x_3) in the controlled system remain ultimately bounded and stabilized, which would be discussed in the simulation part.

Therefore, an external control input $u \in R$ is fed into the third sub-equation in system (1) to form the controlled Lorenz system (2) expressed as follows.

$$\begin{cases} \dot{x}_1 = p_1 x_2 - p_1 x_1 \\ \dot{x}_2 = -x_1 x_3 + p_2 x_1 - x_2 \\ \dot{x}_3 = x_1 x_2 - p_3 x_3 + u \end{cases} \tag{2}$$

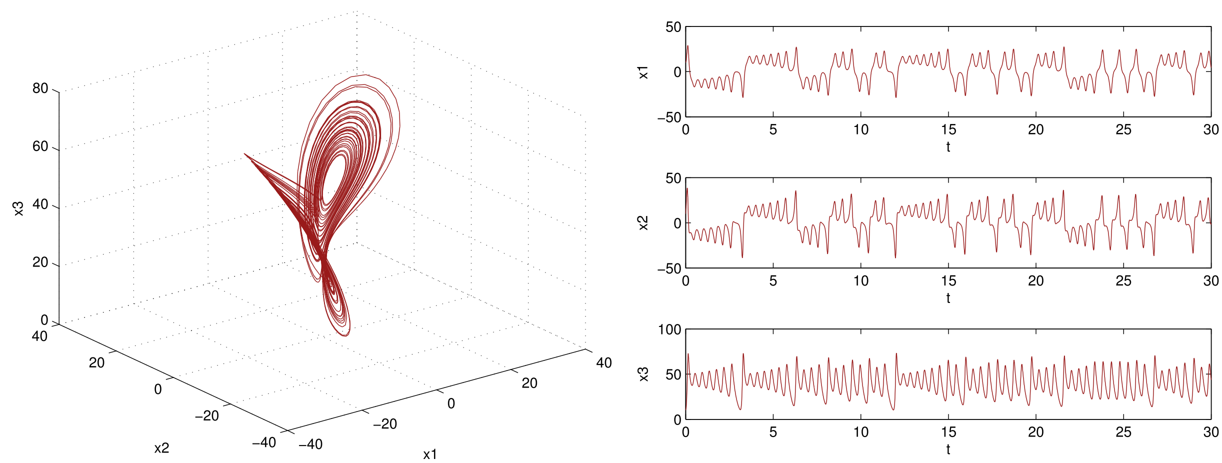
In general, Backstepping is one of the most popular design methods for adaptive nonlinear control because it can guarantee global stability, tracking, and transient performance for a class of strict-feedback system [21]. However, owing to the singularity problem, the Lorenz system could not be controlled directly by using the typical backstepping design procedure. This is because that the controlled Lorenz system (2) can be transformed into the following general strict-feedback form with $n = 3$ (n denotes the number of unknown parameters):

$$\begin{cases} \dot{x}_i = b_i g_i(\bar{x}_i) x_{i+1} + \theta^T F_i(\bar{x}_i) + f_i(\bar{x}_i) \\ \dot{x}_n = b_n g_n(\bar{x}_n) u + \theta^T F_n(\bar{x}_n) + f_n(\bar{x}_n) \\ y = x_1, \end{cases} \tag{3}$$



(a) Phases diagram (b) States diagram

FIGURE 1. **Chaotic behavior of system (1) with the first parameter space ($p_1 = 10, p_2 = 28, p_3 = 8/3$).**



(a) Phases diagram (b) States diagram

FIGURE 2. **Chaotic behavior of system (1) with the second parameter space ($p_1 = 16, p_2 = 45, p_3 = 4$).**

where $i = 1, \cdots, n-1$, the state variables $x = [x_1, x_2, \cdots, x_i]^T \in R^n$, $u \in R$ is the control input, $y \in R$ is the system output, $b \in R_n$ and $\theta \in R_p$ are the vectors of parameters, $g_i(\cdot)$, $F_i(\cdot)$ and $f_i(\cdot)$ are the smooth functions. Thus, by the general strict-feedback form (3), we have the following expressions to match the controlled system (2).

$$\begin{aligned} g_1(x_1) &= 1, \quad g_2(x_1, x_2) = x_1, g_3(x_1, x_2, x_3) = 1 \\ f_1(x_1) &= 0, \quad f_2(x_1, x_2) = -x_2, \\ f_3(x_1, x_2, x_3) &= x_1 x_2, \\ F_1(x_1) &= [x_1, 0, 0]^T, \quad F_2(x_1, x_2) = [0, x_2, 0]^T, \\ F_3(x_1, x_2, x_3) &= [0, 0, x_3]^T, \\ b &= [b_1, b_2, b_3]^T = [p_1, -1, 1]^T, \\ \theta &= [-p_1, p_2, -p_3]^T. \end{aligned}$$

In the procedure of backstepping for (2), in order to achieve the state of x_{i+1}, the following operation (resulting from the first sub-equation of (3)) would be performed,

$$x_{i+1} = \frac{\dot{x}_i - \theta^T F_i(\bar{x}_i) - f_i(\bar{x}_i)}{b_i g_i(\bar{x}_i)}.$$

Evidently, we must make sure that the value of the denominator $b_i g_i(\bar{x}_i)$ is not zero before performing division operation. Consequently, the smooth nonlinear function $g_i(\cdot)$ must be away from zero for avoiding a possible singularity. But for the controlled Lorenz system like the form of (3), $g_2(x_1, x_2) = x_1$. It may take the value of zero. So, the typical backstepping method cannot be directly applied to Lorenz system. In the ensuing sections, a singularity-free Zhang-Gradient controller is presented for the uncertain Lorenz system (2) to guarantee global stability and regulate the state $x_1(t)$ of system (2) to the set-point $x_1^e = 0$ [21].

III. ZHANG-GRADIENT CONTROLLER DESIGN

Before presenting the design procedures of Zhang-gradient controller, in Subsection III-A, we would like to simply introduce the basic idea of ZD method which transforms the stabilization control problem into an error tracking control problem via Lyapunov stability theory. Corresponding to the controlled Lorenz system (2) presented in Subsection III-B, the controller in form of $\dot{u}$ is designed by Zhang-Gradient controller and depicted in detail into an error tracking control problem solving. Subsection III-C presents the update law for $\dot{p_1}$, $\dot{p_2}$ and $\dot{p_3}$ defined by SPGD method by the principle of SPGD method.

A. ZD BASIC DESIGN IDEA

Different from gradient-based dynamics, a special kind of recurrent dynamics has recently been proposed by Zhang *et al.* used for solving online time-varying problems [29]. Such a recurrent dynamics is designed based on an indefinite error-monitoring function instead of a usually norm- or square-based energy function. As for the following nonlinear equation

$$f(x(t), t) = 0. \tag{4}$$

The solution objective is to find $x(t) \in R$ in real time t to make Eq. (4) hold true.

Assume that $x^*(t)$ is denoted by the theoretical solution of (4) at any time instant $t \in [0, -\infty)$. At first, we could construct the following indefinite error function $e(t)$ so as to set up a Zhang dynamics to solve the nonlinear Eq. (4),

$$e(t) := f(x(t), t). \tag{5}$$

Then, the time derivative of $e(t)$, i.e., $\dot{e}(t)$, should be chosen and forced mathematically such that the error function $e(t)$ could converge to zero with the following general form (termed as ZD design formula [29]):

$$\frac{de(t)}{dt} := -\gamma\varphi(e(t)), \tag{6}$$

or equivalently, we have

$$\frac{df}{dt} := -\gamma\varphi(f(x(t), t), \tag{7}$$

where γ is a positive design parameter used to scale the convergence rate (i.e., learning rate) and $\varphi(\cdot)$ is a nonlinear activation function. Generally speaking, any monotonically increasing odd activation function $\varphi(\cdot)$ could be used for the construction of the dynamics model. In this paper, we choose the linear activation function $\varphi(e) = e$.

Remark 1 [29]: If the linear activation function $\varphi(\cdot)$ is used, the exponential convergence with rate γ could be achieved for ZD model (7). In addition, if the power-sigmoid activation function is used, superior convergence can be achieved over the whole error range $(-\infty, +\infty)$, as compared to the linear case.

B. ZHANG-GRADIENT CONTROLLER DESIGN PROCEDURE

In this subsection, the design procedure via ZG method is presented detailedly to avoid the singularity problem caused by $g_2(x_1, x_2) = x_1$ for the uncertain Lorenz system with the following four steps, in which, ZD method is used to get the expression of tracking errors (z_1, z_2, z_3) in the first tree steps, and in the final step, the gradient dynamics (GD) is applied to achieve control input u and the estimated values of the three unknown parameters for the Lorenz system (2).

Step 1: Similar to the backstepping design scheme, we can define $z_1 = x_1$ from the first sub-equation in (2), and then its time-derivative is given by

$$\dot{z}_1 = \dot{x}_1 = p_1x_2 - p_1x_1. \tag{8}$$

According to Zhang *et al*'s design idea, the tracking error z_1 must be converged to zero (i.e., $x_1^e = 0$ presented at the end of Section II), and thus we can construct the update law of the first Zhang error function $z_1(t)$ written as follows.

$$\dot{z}_1 := -\gamma z_1. \tag{9}$$

To stabilize the z_1-subsystem (8), the Lyapunov function candidate can be defined as

$$V_1 = \frac{1}{2}z_1^2,$$

and its derivative is

$$\dot{V}_1 = z_1\dot{z}_1 = x_1(-\gamma z_1) = -\gamma z_1^2. \tag{10}$$

Step 2: Define $z_2 = \dot{z}_1 + \gamma z_1$, which can be rewritten as

$$\begin{aligned} z_2 &= \dot{x}_1 + \gamma x_1 \\ &= p_1x_2 - p_1x_1 + \gamma x_1. \end{aligned} \tag{11}$$

Its derivative is given by

$$\begin{aligned} \dot{z}_2 &= p_1\dot{x}_2 - p_1\dot{x}_1 + \gamma\dot{x}_1 \\ &= p_1p_2x_1 - p_1x_1x_3 - p_1x_2 - p_1^2x_2 \\ &\quad + p_1^2x_1 + \gamma p_1x_2 - \gamma p_1x_1 \end{aligned} \tag{12}$$

In order to make the tracking error z_2 converge to zero, the update law of the second Zhang error function is constructed as $\dot{z}_2 = -\gamma z_2$. Similarly, to stabilize the z_2-subsystem defined by (12), the Lyapunov function candidate can be chosen as

$$V_2 = V_1 + \frac{1}{2}z_2^2,$$

and its derivative is

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 = -\gamma z_1^2 - \gamma z_2^2. \tag{13}$$

Step 3: Define $z_3 = \dot{z}_2 + \gamma z_2$, which can be rewritten as

$$\begin{aligned} z_3 &= \dot{z}_2 + \gamma z_2 \\ &= (p_1p_2 + p_1^2 - 2\gamma p_1 + r^2)x_1 \\ &\quad + (2\gamma p_1 - p_1 - p_1^2)x_2 - p_1x_1x_3. \end{aligned} \tag{14}$$

To simplify the derivation steps, let $k = p_1p_2+p_1^2-2\gamma p_1+r^2$, $q = 2\gamma p_1 - p_1 - p_1^2$. Therefore, Eq. (14) can be rewritten as

$$z_3 = kx_1 + qx_2 - p_1x_1x_3. \tag{15}$$

Its derivative is given by

$$\begin{aligned}\dot{z}_3 &= (k - p_1x_3)\dot{x}_1 + q\dot{x}_2\\ &\quad - p_1x_1(x_1x_2 - p_3x_3 + u)\\ &= -p_1x_1u + p_1p_3x_1x_3 - p_1x_1^2x_2\\ &\quad + (k - p_1x_3)\dot{x}_1 + q\dot{x}_2.\end{aligned} \tag{16}$$

Accordingly, let the update law of the third Zhang error function be $\dot{z}_3 = -\gamma z_3$. Then, the following Lyapunov function candidate is chosen to stabilize the z_3-subsystem (16).

$$V_3 = V_2 + \frac{1}{2}z_3^2 \tag{17}$$

and its derivative is

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 = -\gamma z_1^2 - \gamma z_2^2 - \gamma z_3^2 \tag{18}$$

Till now, we have already completed the deducing steps for (z_1, z_2, z_3) based on Zhang-dynamical (ZD) method and Lyapunov stability theory. Note that, when $z_1 = z_2 = z_3 = 0$, $\dot{V}_3 = 0$. Therefore, $\dot{V}_3$ is negative definite with $\dot{V}_3 \leq 0$. It follows from LaSalle-Yoshizawa Theorem [30], [31] that the equilibrium (0, 0, 0) is global asymptotically stable in the (z_1, z_2, z_3) coordinates. In the next step, the classical gradient-decent algorithm is exploited to obtain the controller $u(t)$ and the estimated parameters $(\hat{p}_1(t), \hat{p}_2(t), \hat{p}_3(t))$.

Step 4: Firstly, a square-based nonnegative energy function is constructed as $\varepsilon := h^2/2$ with h defined as follows.

$$\begin{aligned}h &:= \dot{z}_3 + \gamma z_3\\ &= \dot{z}_3 + \gamma kx_1 + \gamma qx_2 - \gamma p_1x_1x_3\\ &= -p_1x_1u + p_1p_3x_1x_3 - p_1x_1^2x_2\\ &\quad + (k - p_1x_3)\dot{x}_1 + q\dot{x}_2 + \gamma kx_1\\ &\quad + \gamma qx_2 - \gamma p_1x_1x_3.\end{aligned} \tag{19}$$

Then, according to the classical gradient-descent algorithm [32], we have

$$\dot{u} = -\mu\frac{\partial\varepsilon}{\partial u} = -\mu\frac{\partial\varepsilon}{\partial h}\frac{\partial h}{\partial u} = \mu hp_1x_1 \tag{20}$$

$$\begin{aligned}\dot{p}_1 &= -\mu\frac{\partial\varepsilon}{\partial p_1} = -\mu\frac{\partial\varepsilon}{\partial h}\frac{\partial h}{\partial p_1}\\ &= -\mu h(-x_1u + p_3x_1x_3 - x_1^2x_2 - kx_2\\ &\quad + kx_1 + 2p_1x_2x_3 - 2p_1x_1x_3 - \gamma x_1x_3)\end{aligned} \tag{21}$$

$$\dot{p}_2 = -\mu\frac{\partial\varepsilon}{\partial p_2} = -\mu\frac{\partial\varepsilon}{\partial h}\frac{\partial h}{\partial p_2} = -\mu hqx_1 \tag{22}$$

$$\dot{p}_3 = -\mu\frac{\partial\varepsilon}{\partial p_3} = -\mu\frac{\partial\varepsilon}{\partial h}\frac{\partial h}{\partial p_3} = -\mu hp_1x_1x_3. \tag{23}$$

However, it is difficult to obtain the values of p_1, p_2 and p_3 for the complexity of Eqs. (20)-(23). To make the computation tractable, stochastic parallel gradient descent (SPGD) solution is thus used for getting close to the pure gradient solution.

C. SPGD DESIGN PROCEDURE

In [25], [27], M. A. Vorontsov *et al*. preliminarily deduced the convergence and the stability of Adaptive Optics (AO) system based on SPGD (stochastic parallel gradient descent). It is assumed that the system performance metric is $J = J(\gamma)$, the control parameter is $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n\}$, and the change of the system performance is δ_J, where n is the number of control parameters. The SPGD performs an incremental adjustment of control parameter $\{\gamma_j\}$ using a real-time estimation for the gradient $\{\dot{J}_j = \partial J/\partial\gamma_j\}$ with the replacement of $\dot{J} = \delta_J\delta_{\gamma_j}$ in real applications [33], where δ is defined as a perturbation.

In this paper, to compute Eqs. (20)-(23) by using the above-mentioned SPGD method, the energy function ε and the estimated parameters $\{p_j\}$ are corresponding to the minimized performance metric and the increment of control parameter, respectively. Therefore, the metric change δ_ε with $\varepsilon(\cdot)$ is

$$\delta_\varepsilon = \varepsilon(p_1 + \delta_{p_1}, \ldots, p_n + \delta_{p_n}) - \varepsilon(p_1, \ldots, p_n), \tag{24}$$

and

$$p^{k+1} := \{p_j^{k+1}\} = \{p_j^k - \theta_\delta\varepsilon\delta_{p_j}\}, \tag{25}$$

where k is the iteration number and θ is a small and positive constant used to control the convergence speed. Note that, as for the perturbation $\{\delta p_j\}$, we have the following remark.

Remark 2: The perturbation $\{\delta_{p_j}\}$ as random variable is chosen typically as statistically independent variables with zero mean and equal variances: $<\delta_{p_j}\delta_{p_i}>= \sigma^2\delta_{ij}$ and $<\delta_{p_j}>= 0$, where δ_{ij} is the Kronecker symbol.

Furthermore, Eq. (24) can be expanded as a Taylor series form as follows.

$$\delta_\varepsilon = \sum_{j=1}^{n}(\partial\varepsilon/\partial p_j)\delta_{p_j} + 0.5\sum_{j,l}^{n}(\partial^2\varepsilon/\partial p_j\partial p_l)\delta_{p_j}\delta_{p_l} + \ldots,$$

Therefore, we can obtain

$$\delta_\varepsilon\delta_{p_l} = (\partial\varepsilon/\partial p_l)\delta_{p_l}^2 + \psi_l, \tag{26}$$

with the truncation error ψ_l defined as follows.

$$\psi_l = \sum_{j\neq 1}^{n}(\partial\varepsilon/\partial p_j)\delta_{p_j}\delta_{p_l} + 0.5\sum_{j,i}^{n}(\partial^2\varepsilon/\partial p_j\partial p_l)\delta_{p_j}\delta_{p_i}\delta_{p_l} + \ldots.$$

Therefore, Eq. (26) consists of the true gradient $\dot{\varepsilon}_j$ and the noise (i.e., the truncation error) ψ_l.

According to the above-mentioned SPGD method, the gradient $\{\dot{\varepsilon}_j = \partial\varepsilon/\partial p_j\}$ can be replaced by $\{\dot{\varepsilon}_j = \delta_\varepsilon\delta_{p_j}\}$. Then, we could reform the Eqs. (21)-(23) into Eqs. (27)-(29) to achieve the approximate values of the estimated parameters $(\dot{p}_1, \dot{p}_2$, and $\dot{p}_3)$ in form of $\dot{\hat{p}}_1, \dot{\hat{p}}_2$, and $\dot{\hat{p}}_3$.

$$\dot{\hat{p}}_1 = -\mu\frac{\partial\varepsilon}{\partial\hat{p}_1} = -\mu\delta_\varepsilon\delta_{\hat{p}_1} \tag{27}$$

$$\dot{\hat{p}}_2 = -\mu\frac{\partial\varepsilon}{\partial\hat{p}_2} = -\mu\delta_\varepsilon\delta_{\hat{p}_2} \tag{28}$$

$$\dot{\hat{p}}_3 = -\mu\frac{\partial\varepsilon}{\partial\hat{p}_3} = -\mu\delta_\varepsilon\delta_{\hat{p}_3}. \tag{29}$$

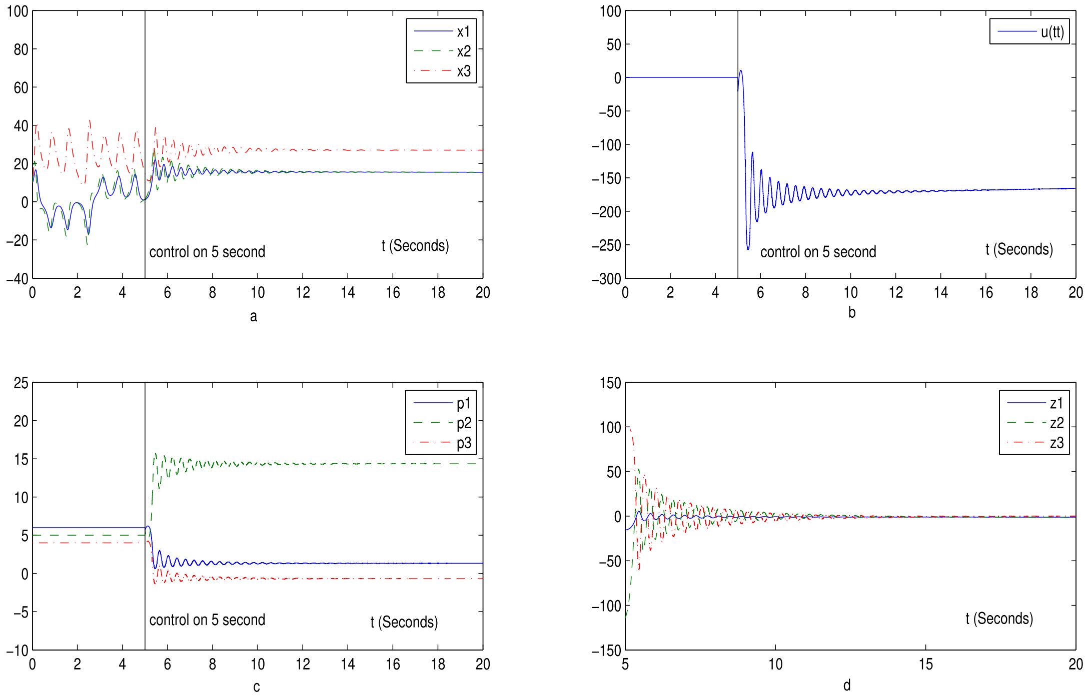


FIGURE 3. Comparison results for the controlled Lorenz system (2) with the initial estimated parameter pair ($\hat{p}_1(t) = 6$, $\hat{p}_2(t) = 5$, $\hat{p}_3(t) = 4$) and the initial perturbations ($\delta_{\hat{p}_1} = 0.25$, $\delta_{\hat{p}_2} = -0.50$, $\delta_{\hat{p}_3} = 0.25$).

IV. SIMULATIVE VERIFICATION

For the purpose of verifying the effectiveness of our presented design procedure, computer simulations have been carried out under the two parameters spaces in this section, i.e., ($p_1 = 10$, $p_2 > 0$, $p_3 = 8/3$) and ($p_1 = 16$, $p_2 > 0$, $p_3 = 4$), in which, the Lorenz system (1) can be at the chaotic state. Our objective is to achieve a correct control variant $u(t)$ to make the tracking errors z_1, z_2 and z_3 converge to zero asymptotically, and make the system states (x_1, x_2, x_3) be able to converge to the stable states.

In the following computer simulation experiments, to obtain and analyze the comparison control results before and after fed into the control variant $u(t)$, in the first 5 minutes, the system (2) is initialized at the chaotic state with $p_1 = 10$, $p_2 = 28$, $p_3 = 8/3$. The initial states are randomly set as $x_1(0) = 10$, $x_2(0) = 10$, $x_3(0) = 10$. At the moment, there is no any control input, i.e., $u(t) = 0$. Evidently, the chaotic states (x_1, x_2, x_3) is not stable, as shown in Figs. 3-5. Then, 5 seconds later, the control input $u(t) \neq 0$ would be fed into the system with different initial estimated parameter pairs $(\hat{p}_1(t), \hat{p}_2(t), \hat{p}_3(t)$) discussed as follows.

Generally speaking, in the practical engineering applications, the parameter pair (p_1, p_2, p_3) is unknown in the Lorenz control system (2). To obtain the control input $u(t)$ and analyze the control results, the parameter pair is estimated and initialized to be ($\hat{p}_1(t) = 6$, $\hat{p}_2(t) = 5$, $\hat{p}_3(t) = 4$), which is assigned randomly as small positive constants, and the initial perturbations $\delta_{\hat{p}_1} = 0.25$, $\delta_{\hat{p}_2} = -0.50$, $\delta_{\hat{p}_3} = 0.25$ are set by Remark 2, which are chosen as the same assignments in the following simulations.

In this case, the control results can be seen in Fig. 3. Evidently, in the first 5 seconds, the Lorenz system (2) is at chaotic state in Fig. 3(a) when the control input $u(t) = 0$ in Fig. 3(b). Naturally, the estimated parameters $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ would not be changed, and it happens that the system (2) would be at a considerable oscillation, as shown in Figs. 3(c) and (d), respectively.

However, after 5 seconds, if the system (2) is controlled by introducing a control input $u(t) \neq 0$, which is decided by Eq. (20) with the initial estimated parameters ($\hat{p}_1(t) = 6$, $\hat{p}_2(t) = 5$, $\hat{p}_3(t) = 4$), the oscillation would vanish when $u(t)$ is stable asymptotically and remains bounded along with the iterations of the estimated parameters $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$, which can be seen in Figs. 3 (b) and (c), respectively. In the meantime, the system (2) arrives at a stable state, and the tracking error is also convergent to zero asymptotically after about 12 seconds, which are corresponding to Figs. 3 (a) and (d), respectively. It shows that the SPGD method is effective to get the values of the estimated parameters $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ by Eqs. (27)-(29). This leads to a fact that the Lorenz system (2) is controlled stably when fed into the control input $u(t)$ decided by $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$.

In addition, studies show that the Lorenz system often generates chaotic signals at certain area of parameter space. Therefore, the system parameters can be initialized near the first chaotic parameter space with the parameters valued usually at ($p_1 = 10$, $p_2 = 28$, $p_3 = 8/3$) and the second parameter space valued at ($p_1 = 16$, $p_2 = 45$, $p_3 = 4$). Assume that the initial estimated parameter pair ($\hat{p}_1 = 8.5$, $\hat{p}_2 = 26$, $\hat{p}_3 = 3.5$) are fetched randomly in the vicinities of $p_1 = 10$, $p_2 = 28$, $p_3 = 8/3$ (i.e., in the first parameters

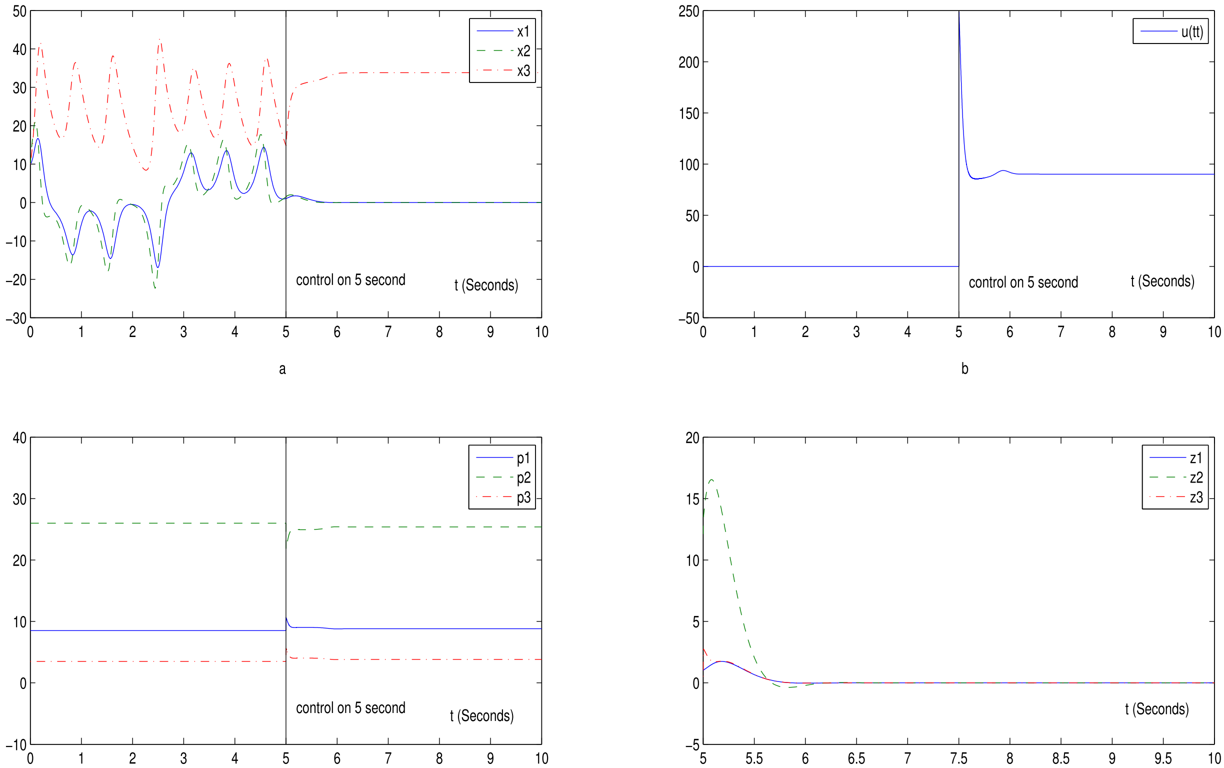


FIGURE 4. Comparison results for the controlled Lorenz system (2) with the initial estimated parameter pair ($\hat{p}_1(t) = 8.5$, $\hat{p}_2(t) = 26$, $\hat{p}_3(t) = 3.5$) and the initial perturbations ($\delta_{\hat{p}_1} = 0.25$, $\delta_{\hat{p}_2} = -0.50$, $\delta_{\hat{p}_3} = 0.25$).

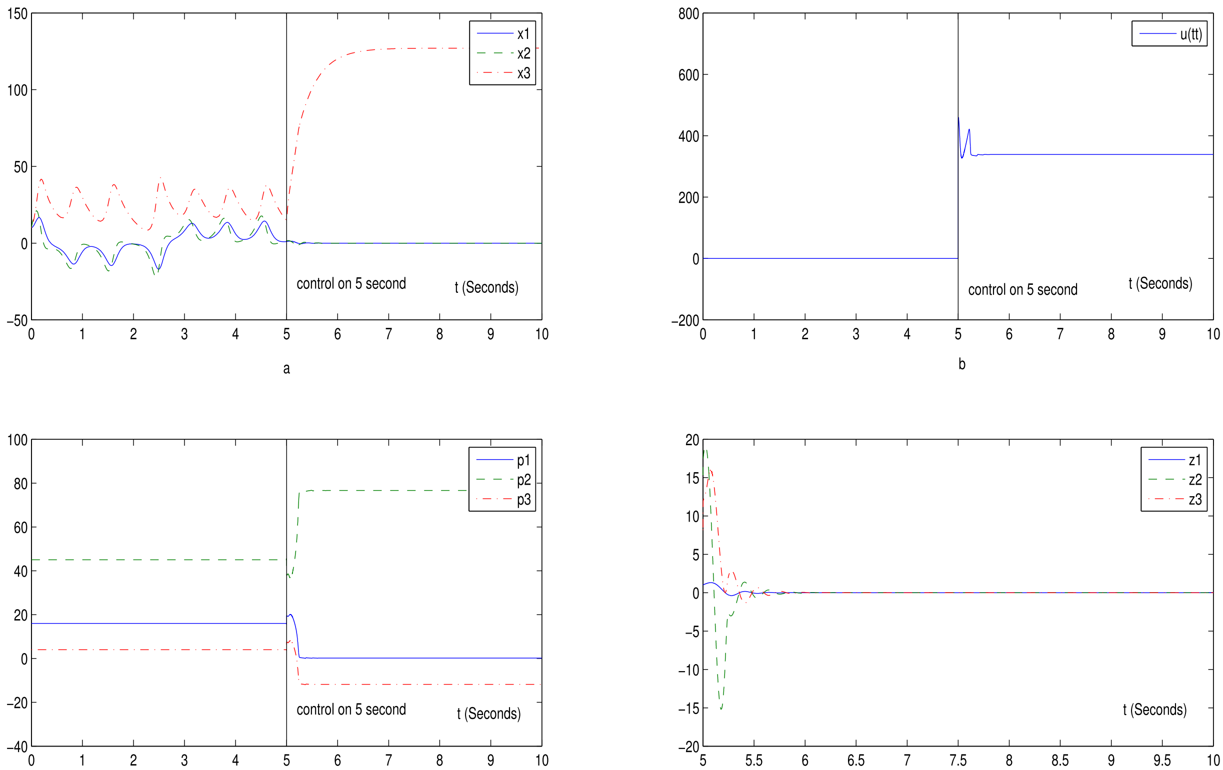


FIGURE 5. Comparison results for the controlled Lorenz system (2) with the initial estimated parameter pair ($\hat{p}_1(t) = 16$, $\hat{p}_2(t) = 45$, $\hat{p}_3(t) = 4$) and the initial perturbations ($\delta_{\hat{p}_1} = 0.25$, $\delta_{\hat{p}_2} = -0.50$, $\delta_{\hat{p}_3} = 0.25$).

space), respectively. The simulation results can be shown in Fig. 4. It is evident that the tracking error quickly converges to zero in less than 1.5 seconds, which is shown in Fig. 4 (d). The system (2) is also controlled stably by the stable control input $u(t)$, and the estimated parameter pair $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ is finally equal to the known chaotic parameter pair (p_1, p_2, p_3). In another case, assume that the initial estimated parameter pair is valued at the second parameters space, i.e., $(\hat{p}_1 = 16, \hat{p}_2 = 45, \hat{p}_3 = 4)$, which is against the fixed system parameters $(p_1 = 10, p_2 = 28, p_3 = 8/3)$. The control results are presented in Figs. 5 (a)-(d). Not surprisingly, the system (2) is also controlled stably. However, the values of $u(t)$ and the estimated parameter pair $(\hat{p}_1 = 16, \hat{p}_2 = 45, \hat{p}_3 = 4)$ are already changed, in contrast with the first parameters space.

In summary, if the estimated parameter pair is valued in the vicinity of the chaotic parameters space, it is equivalent to say that the system parameters are known for (2). The control effect is relatively well. However, in the real applications, the parameters are unknown and must be estimated for the most controlled systems. Therefore, combined with the simulation results, our presented ZG design method can be effectively used to stably control a chaos control system based on the SPGD method.

V. CONCLUSION

Because of the nonlinearity and singularity, many classic controllers cannot be applied directly to the uncertain well-known Lorenz system. In this paper, a novel Zhang-Gradient controller is exploited for the controlling Lorenz system. Firstly, inspired by Zhang dynamics, a tracking error expression is constructed during the recursive process analyssis based on the Lyapunove stability. Then, considering the computation complexity for the system parameters, the SPGD design procedure is presented to obtain the estimated values of system parameters. The numerical results further substantiate that our presented ZG controller can make the controlled Lorenz system stable globally.

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