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Design of Adaptive ILC for 2-D FMM Systems With Unknown Control Directions

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ABSTRACT Among the existing adaptive iterative learning control (ILC) work concerning unknown control direction problem, no result is available for the two-dimensional (2-D) dynamical systems. In this paper, an adaptive ILC is developed for a class of 2-D dynamical systems described by the Fornasini-Marchesini model (FMM). Notably, the 2-D FMM system under investigation possesses not only random uncertainties in boundary condition and reference trajectory, but also unknown control direction. Even so, the constructed adaptive ILC combining with a modification mechanism can still guarantee the precise tracking of iteration-variant reference trajectory and the boundedness of all the system signals, as iteration number k tends to infinity. Theoretical analysis and simulation study are given to demonstrate the effectiveness of the developed adaptive ILC.

INDEX TERMS Two-dimensional (2-D) dynamical systems, unknown control direction, adaptive iterative learning control (ILC), the Fornasini-Marchesini model (FMM).

I. INTRODUCTION

As an intelligent and effective control featured with simple structure, iterative learning control (ILC) has received vast application in repeated control process over a finite time interval [1]. Using few system model knowledge, ILC schemes can iteratively learn and compensate the iteration-dependent/independent uncertainties and gain ultimately precise trajectory tracking. Therefore, great interest has been imposed on ILC in the control design of industrial process such as robotic manipulators, chemical plants, lower limb exoskeleton and so on [2], [3], [29]. Then, as we expected, a tremendous amount of ILC algorithms are constantly emerging due to application demand. As pointed out in [3], the limitations of contracting mapping-based ILC, that gradually appeared as the controlled systems encountered non-Lipschitz nonlinearities and random uncertainties in initial value, desired reference and disturbance, give rise to adaptive ILC. Generally, adaptive ILC can well tackle with these uncertainties and non-Lipschitz nonlinearities with the help of the so-called composite energy function to do the

convergence analysis. Recent decades have witnessed much progress in adaptive ILC of nonlinear dynamical systems.

However, most adaptive ILC approaches are developed for one-dimensional (1-D) systems which only rely on one independent variable [2]–[4]. As to the systems that are derived by two or more independent variables, the existing adaptive ILC approaches are becoming ineffective [2]–[4].

Two-dimensional (2-D) dynamical systems, that operate repetitively in finite and given domains, have attracted considerable attention as shown in [5]–[12]. In contrast to 1-D dynamical system, the control objective is no longer tracking curve, but surface in 2-D system. Thus, owing to the more complicated tracking task and no direct analysis tool of 2-D dynamical system, the ILC control design of 2-D dynamical system is much more challenging [10]. To date, some authors have devoted themselves to the study of ILC in 2-D dynamical systems. And they have obtained several ILC works for 2-D dynamical systems [5]–[12]. A fuzzy ILC method was first proposed in [5] and corresponding progresses were considered in [6]–[9]. Cichy *et al.* introduced the ILC method into a kind of 2-D dynamical systems producing by partial differential equations [6]. In [7] and [9], an optimal ILC method was suggested for a 2-D dynamical system described

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by the Fornasini-Marchesini model (FMM) and a P-type ILC method was developed for a 2-D dynamical system represented by Roesser model, respectively. It is worth mentioning that the ILC methods in [5]–[9] are only applicable to the specified systems with identical boundary condition and iteration-invariant reference trajectory. In practice, the random uncertainties in boundary condition and reference trajectory are inevitable due to the complicated environment. Thus, when one of the boundary conditions was set to be iteration-variant, a robust ILC law was introduced in [11]. Furthermore, when random uncertainties in boundary condition and reference trajectory were both considered, authors in [10] proposed an adaptive ILC for 2-D FMM systems. And the objective of perfect tracking was obtained in [10]. With respect to high-order internal model (HOIM) strategy-based iteration-variant reference trajectory, the ILC research in [12] can also achieve perfect tracking when the iterative boundary conditions execute the same HOIM strategy as the reference, while compel the tracking error to a small neighbourhood of reference trajectory under random boundary conditions.

In spite of the promising results in the ILC study of 2-D dynamical systems, it is noticed that all the ILC designs reported in [5]–[12] are basically based on the known control direction. Generally, the applied control direction is assumed to be positive in advance and the convergence property is obtained in view of the fact that the control direction is positive. In fact, since the adaptive technique was introduced to ILC, Xu and Yan have done the pioneering work to tackle with the unknown control direction problem in the field of ILC in 2004 [13]. Then, with the successful application of Nussbaum-type function, several works discussed the adaptive ILC design for uncertain continuous systems were achieved in [14]–[18]. On the other hand, similar discrete adaptive ILC approaches have also been reported in [3], [19], [20]. Nevertheless, it should be noted that the number of tuning parameters and the complexity of controller structure will increase by using Nussbaum-type functions in the adaptive ILC algorithms [3], [13]–[20]. Moreover, one can notice that the transient performance is hard to ensure because of the oscillating gain produced by the Nussbaum-type function based-adaptive ILC control [14]. As a result, there emerged two adaptive ILC methods, which tried to conquer the unknown control direction problems by estimating the control gains directly [21], [22]. By subtly designing two modification mechanisms, an adaptive ILC scheme was suggested to the nonlinear system lacking of control direction information [21]. While in [22], the adaptive ILC method combining with two fuzzy systems was given so that it can successfully solve the unknown control direction problem without resorting to Nussbaum technique. Clearly, without using Nussbaum-type function, the discussion on unknown control direction problem is scarce.

From the above statements, we know that many adaptive ILC works have solved the unknown control direction problems [3], [13]–[22], but none aimed at 2-D dynamical systems. Usually, there are three popular 2-D state space

models which are proposed by Fornasini-Marchesini [23], Roesser [24] and Kurek [25]. Recently, the researches about 2-D FMM systems have been a new focus and some studies in theories and applications of the 2-D FMM systems were recently reported in [10]–[12], [27], [28]. In this paper, for the 2-D FMM system which is subject to unknown control direction, as well as random uncertainties in boundary condition and reference trajectory, an adaptive ILC is appropriately designed. Moreover, we will not resort to the Nussbaum-type function to deal with the unknown control direction, but directly estimate the control gain through the designed adaptive ILC combining with a modification mechanism. The relative contributions of this paper lie in that: (1) Throughout the existing adaptive ILC work regarding the unknown control direction problem [3], [13]–[22], no result was designed for 2-D dynamical systems. Thus, this is the first time to discuss the unknown control direction problem in the ILC design of 2-D dynamical systems; (2) In our design procedure, we estimate the control gain directly, which avoid the need for an application of Nussbaum-type function to accommodate the unknown control direction; (3) To date, few researches but [10] and [12] address the 2-D FMM systems with random uncertainties in boundary condition and reference trajectory. The designed adaptive ILC in this paper can effectively deal with the random uncertainties in boundary condition and reference trajectory, as well as the unknown control direction. As a result, the tracking error is proved to be convergent, while all the system signals maintain bounded in the iteration process

The remainder of this paper is organized as follows. Section II gives the problem formulation. Section III presents the adaptive ILC approach and its convergence analysis is provided in Section IV. Section V is about the simulation study and the conclusion is provided in Section VI.

II. PROBLEM FORMULATION

In this paper, we use the following 2-D linear system represented by Fornasini-Marchesini model (FMM)

$$x_k(i+1, j+1) = A_1 x_k(i+1, j) + A_2 x_k(i, j) + A_3 x_k(i, j+1) + B u_k(i, j) \quad (1)$$

where i and j are non-negative and finite coordinates with $i \in [0, H]$, $j \in [0, V]$ and $k = 0, 1, 2, \dots$ is the k -th iteration of system, $x_k(i, j) \in R$ and $u_k(i, j) \in R$ denote the measurable system state and control input, respectively. A_1, A_2, A_3 , and B are unknown coefficients of the 2-D FMM system.

Assumption 1: The control gain B is nonsingular, i.e., $B \neq 0$, and the sign of B which acts on behave of the control direction is unknown.

Assumption 2: The boundary conditions for (1) are random but bounded, i.e., $x_k(i, 0)$, $i \in [0, H]$, and $x_k(0, j)$, $j \in [1, V]$, are randomly variant with iteration but bounded.

Suppose $x_k^*(i, j)$ is an iteration-variant and realizable reference trajectory of the 2-D FMM system (1). The control objective of this paper is to design an adaptive ILC controller

under Assumptions 1-2 such that the state $x_k(i, j)$ of system (1) can follow the iteration-variant reference trajectories $x_k^*(i, j)$ for $i \in [1, H]$ and $j \in [1, V]$ as k increases and tends to infinity.

Remark 1: It is worth noting that the unknown control direction condition is considered in our paper. Although the unknown control direction problem has generated considerable interest in adaptive ILC literature [3], [13]–[22], they only aimed at 1-D dynamical systems. Thus, no result concerned with unknown control direction is available for 2-D dynamical systems.

Remark 2: In the ILC field of 2-D dynamical systems, by using some prior knowledge about control gain, [10] and [12] considered the random uncertainties in boundary condition and reference trajectory of 2-D FMM systems. In this paper, under the random uncertainties in boundary condition and reference trajectory, an adaptive ILC algorithm is proposed for the 2-D FMM system with unknown control direction. As far as we know, this is the first time to discuss the unknown control direction problem in the ILC design of 2-D FMM system.

III. DESIGN OF AN ADAPTIVE ILC

Rewrite the 2-D FMM system (1) as the following

$$\begin{aligned} x_k(i+1, j+1) &= [A_1 \quad A_2 \quad A_3] \begin{bmatrix} x_k(i+1, j) \\ x_k(i, j) \\ x_k(i, j+1) \end{bmatrix} \\ &\quad + Bu_k(i, j) \\ &= \psi^T \eta_k(i, j) + Bu_k(i, j) \end{aligned} \quad (2)$$

where $\psi = [A_1 \quad A_2 \quad A_3]^T \in R^3$ and $\eta_k(i, j) = [x_k(i+1, j) \quad x_k(i, j) \quad x_k(i, j+1)]^T \in R^3$.

Define $e_k(i, j) = x_k(i, j) - x_k^*(i, j)$ to be the tracking error at k -th iteration. And let $\hat{\psi}_k(i, j)$ and $\hat{B}_k(i, j)$ be the estimated values of ψ and B at the k th iteration, respectively.

Then, design an adaptive ILC algorithm as

$$u_k(i, j) = \frac{-\hat{\psi}_k^T(i, j) \eta_k(i, j) + x_k^*(i+1, j+1)}{\bar{B}_k(i, j)} \quad (3)$$

where $i \in [0, H-1]$ and $j \in [0, V-1]$, and $\bar{B}_k(i, j)$ is designed as following

$$\bar{B}_k(i, j) = \begin{cases} \hat{B}_k(i, j) & \text{if } |\hat{B}_k(i, j)| \geq b_0 \\ \hat{B}_{k-1}(i, j) & \text{otherwise} \end{cases} \quad (4)$$

where b_0 is a positive constant. From the definition of $\bar{B}_k(i, j)$, it is easy to see that $|\bar{B}_k(i, j)| \geq b_0$ if the initial value of $\hat{B}_0(i, j)$ is chosen as $|\hat{B}_0(i, j)| \geq b_0$. While b_0 is only used for analysis, no exact value is needed in the ILC controller.

Remark 3: Different from the adaptive ILC result of 2-D FMM system in [10] that B is assumed to be positive, no prior information about control direction is used in the proposed adaptive ILC algorithm (3) and the modification mechanism about $\bar{B}_k(i, j)$ in (4).

Remark 4: Through the modification mechanism (4), we can guarantee that $\bar{B}_k(i, j)$ is far away from the specified neighborhood of zero. As a result, the singularity problem of the control law is solved by applying the modified $\bar{B}_k(i, j)$ but not the $\hat{B}_k(i, j)$ to the proposed adaptive ILC algorithm (3).

From (3), it can be deduced that

$$x_k^*(i+1, j+1) = \hat{\psi}_k^T(i, j) \eta_k(i, j) + \bar{B}_k(i, j) u_k(i, j) \quad (5)$$

Let $\Psi = [\psi^T \quad B]^T \in R^4$, $\rho_k(i, j) = [\eta_k^T(i, j) \quad u_k(i, j)]^T \in R^4$, and $\hat{\Psi}_k(i, j) = [\hat{\psi}_k^T(i, j) \quad \hat{B}_k(i, j)]^T \in R^4$ being the estimation of Ψ at the k th iteration.

Combining (2) and (5), it yields

$$\begin{aligned} e_k(i+1, j+1) &= x_k(i+1, j+1) - x_k^*(i+1, j+1) \\ &= \psi^T \eta_k(i, j) + Bu_k(i, j) \\ &\quad - [\hat{\psi}_k^T(i, j) \eta_k(i, j) + \bar{B}_k(i, j) u_k(i, j)] \\ &= -[\hat{\psi}_k(i, j) - \psi]^T \eta_k(i, j) \\ &\quad - [\hat{B}_k(i, j) - B] u_k(i, j) \\ &\quad + [\hat{B}_k(i, j) - \bar{B}_k(i, j)] u_k(i, j) \\ &= -\tilde{\psi}_k^T(i, j) \eta_k(i, j) - \tilde{B}_k(i, j) u_k(i, j) \\ &\quad + [\hat{B}_k(i, j) - \bar{B}_k(i, j)] u_k(i, j) \\ &= -\tilde{\Psi}_k^T(i, j) \rho_k(i, j) + \varpi_k(i, j) \end{aligned} \quad (6)$$

where $\tilde{\psi}_k(i, j) = \hat{\psi}_k(i, j) - \psi$, $\tilde{B}_k(i, j) = \hat{B}_k(i, j) - B$, and $\tilde{\Psi}_k(i, j) = [\tilde{\psi}_k^T(i, j) \quad \tilde{B}_k(i, j)]^T$ denote the error of the parameters. And $\varpi_k(i, j) = [\hat{B}_k(i, j) - \bar{B}_k(i, j)] u_k(i, j)$ can be easily obtained. Correspondingly, design the parameter updating law as follows

$$\begin{aligned} \hat{\Psi}_{k+1}(i, j) &= \hat{\Psi}_k(i, j) \\ &\quad + \frac{\rho_k(i, j) [e_k(i+1, j+1) - \varpi_k(i, j)]}{a + \rho_k^T(i, j) \rho_k(i, j)} \end{aligned} \quad (7)$$

where $i \in [0, H-1]$, $j \in [0, V-1]$, and a is a small positive constant for making the denominator $a + \rho_k^T(i, j) \rho_k$ not equal to zero. It is noted that the initial value of $\hat{\Psi}_0(i, j)$ can be arbitrary but meet the condition that $|\hat{B}_0(i, j)| \geq b_0$.

IV. ANALYSIS OF CONVERGENCE

The main result of the proposed adaptive ILC for 2-D FMM system (1) is summarized in the following theorem.

Theorem 1: Considering the 2-D FMM system (1) with unknown control direction and random uncertainties in both boundary condition and reference trajectory, the adaptive ILC algorithm (3), (4), and (7) can guarantee that the tracking errors $e_k(i, j)$, $i \in [1, H]$, $j \in [1, V]$ converge to zero as k tends to infinity and all the system signals maintain bounded during the whole ILC procedure.

Proof: The following proof has three parts. Part 1 is about the boundedness of the involved parameters of (7). And the boundedness of the other system signals is addressed in

Part 2. At last, Part 3 deduces the convergence of the tracking error $e_k(i, j)$ for $i \in [1, H]$ and $j \in [1, V]$.

Part 1. From (6), we get

$$e_k(i + 1, j + 1) - \varpi_k(i, j) = -\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \quad (8)$$

Substituting (8) into (7), it yields

$$\hat{\Psi}_{k+1}(i, j) = \hat{\Psi}_k(i, j) - \frac{\rho_k(i, j)}{a + \rho_k^T(i, j) \rho_k(i, j)} \tilde{\Psi}_k^T(i, j) \rho_k(i, j) \quad (9)$$

Subtracting Ψ from both sides of (9), we have

$$\tilde{\Psi}_{k+1}(i, j) = \tilde{\Psi}_k(i, j) - \frac{\rho_k(i, j)}{a + \rho_k^T(i, j) \rho_k(i, j)} \tilde{\Psi}_k^T(i, j) \rho_k(i, j) \quad (10)$$

Using (10), we have

$$\begin{aligned} & \left\| \tilde{\Psi}_{k+1}(i, j) \right\|^2 - \left\| \tilde{\Psi}_k(i, j) \right\|^2 \\ &= -2 \frac{\left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{a + \rho_k^T(i, j) \rho_k(i, j)} \\ & \quad + \frac{\rho_k^T(i, j) \rho_k(i, j) \left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{\left[a + \rho_k^T(i, j) \rho_k(i, j) \right]^2} \\ & \leq - \frac{\left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{a + \rho_k^T(i, j) \rho_k(i, j)} \leq 0 \end{aligned} \quad (11)$$

Therefore, (11) leads to

$$\left\| \tilde{\Psi}_{k+1}(i, j) \right\|^2 \leq \left\| \tilde{\Psi}_k(i, j) \right\|^2 \leq \dots \leq \left\| \tilde{\Psi}_0(i, j) \right\|^2 \quad (12)$$

Then we can conclude from (12) that $\left\| \tilde{\Psi}_k(i, j) \right\|^2$ is a bounded, non-negative, and non-increasing function with respect to k as $\left\| \tilde{\Psi}_0(i, j) \right\|^2$ is chosen to be bounded. As a result, $\hat{\Psi}_k(i, j)$ is bounded for $i \in [0, H - 1]$, $i \in [0, V - 1]$, and $k = 0, 1, 2, \dots$. That is to say, $\hat{\Psi}_k^T(i, j)$ and $\hat{B}_k(i, j)$ are bounded for $i \in [0, H - 1]$ and $j \in [0, V - 1]$.

Part 2. The following is the boundedness deduction of $u_k(i, j)$ for $i \in [0, H - 1]$, $j \in [0, V - 1]$ and $x_k(i, j)$ for $i \in [0, H]$, $j \in [0, V]$ based on the boundedness of iteration-variant boundary condition $x_k(i, 0)$, $x_k(0, j)$, reference trajectory $x_k^*(i, j)$, $i \in [0, H]$, $j \in [0, V]$, as well as the boundedness of $\hat{\Psi}_k^T(i, j)$ and $\hat{B}_k(i, j)$, $i \in [0, H - 1]$, $j \in [0, V - 1]$.

From the boundedness of the iteration-variant boundary condition and reference trajectory, the boundedness of $\eta_k(0, 0) = [x_k(1, 0) \ x_k(0, 0) \ x_k(0, 1)]^T$ and $x_k^*(1, 1)$ is easily achieved. Thus, from (3), it yields that $u_k(0, 0)$ is bounded. Further, as $u_k(0, 0)$ is bounded, the boundedness of $x_k(1, 1)$ is also obtained from (2). Then, $\eta_k(1, 0) = [x_k(2, 0) \ x_k(1, 0) \ x_k(1, 1)]^T$ is bounded and so is $u_k(1, 0)$ from (3) and $x_k(2, 1)$ from (2). Deducing in the direction of i , we can get the boundedness for all $u_k(i, 0)$, $i \in [0, H - 1]$

and $x_k(i, 1)$, $i \in [0, H]$. Correspondingly, we can also get the boundedness for all $u_k(0, j)$, $j \in [0, V - 1]$ and $x_k(1, j)$, $j \in [0, V]$ in the same way. On the other hand, since $\eta_k(1, 1) = [x_k(2, 1) \ x_k(1, 1) \ x_k(1, 2)]^T$ is bounded, the boundedness of $u_k(1, 1)$ is obtained based on (3). While the boundedness of $x_k(2, 2)$ is also achieved based on (2). Through continuous substitution, we can finally obtain the boundedness of $u_k(i, j)$ for $i \in [0, H - 1]$, $j \in [0, V - 1]$ and $x_k(i, j)$ for $i \in [0, H]$, $j \in [0, V]$.

Part 3. From (11), we have

$$\begin{aligned} \sum_{z=0}^k \frac{\left[\tilde{\Psi}_z^T(i, j) \rho_z(i, j) \right]^2}{a + \rho_z^T(i, j) \rho_z(i, j)} & \leq \left\| \tilde{\Psi}_0(i, j) \right\|^2 - \left\| \tilde{\Psi}_{k+1}(i, j) \right\|^2 \\ & \leq \left\| \tilde{\Psi}_0(i, j) \right\|^2 \end{aligned} \quad (13)$$

Considering the boundedness of Ψ and $\hat{\Psi}_0(i, j)$, it yields

$$\lim_{k \rightarrow \infty} \sum_{z=0}^k \frac{\left[\tilde{\Psi}_z^T(i, j) \rho_z(i, j) \right]^2}{a + \rho_z^T(i, j) \rho_z(i, j)} < \infty \quad (14)$$

That is

$$\lim_{k \rightarrow \infty} \frac{\left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{a + \rho_k^T(i, j) \rho_k(i, j)} = 0 \quad (15)$$

On the other hand, we can get from (9) that

$$\begin{aligned} & \left\| \hat{\Psi}_{k+1}(i, j) - \hat{\Psi}_k(i, j) \right\|^2 \\ &= \frac{\rho_k^T(i, j) \rho_k(i, j) \left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{\left[a + \rho_k^T(i, j) \rho_k(i, j) \right]^2} \end{aligned} \quad (16)$$

Note that

$$\begin{aligned} & \frac{\left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{a + \rho_k^T(i, j) \rho_k(i, j)} \\ &= \frac{\left[a + \rho_k^T(i, j) \rho_k(i, j) \right] \left[\tilde{\Psi}_k^T(i, j) \rho_k(i, j) \right]^2}{\left[a + \rho_k^T(i, j) \rho_k(i, j) \right]^2} \\ & \geq \left\| \hat{\Psi}_{k+1}(i, j) - \hat{\Psi}_k(i, j) \right\|^2 \end{aligned} \quad (17)$$

Combining (15) and (17), the following

$$\lim_{k \rightarrow \infty} \left\| \hat{\Psi}_{k+1}(i, j) - \hat{\Psi}_k(i, j) \right\| = 0 \quad (18)$$

is achieved for $i \in [0, H - 1]$ and $j \in [0, V - 1]$. Notably, we have

$$\lim_{k \rightarrow \infty} \left| \hat{B}_k(i, j) - \hat{B}_{k-1}(i, j) \right| = 0 \quad (19)$$

By using the Schwarz inequality,

$$\begin{aligned} & \left\| \hat{\Psi}_k(i, j) - \hat{\Psi}_{k-n}(i, j) \right\|^2 \\ &= \left\| \left(\hat{\Psi}_k(i, j) - \hat{\Psi}_{k-1}(i, j) \right) \right\|^2 \end{aligned}$$

$$\begin{aligned}
 &+ \left(\hat{\Psi}_{k-1}(i, j) - \hat{\Psi}_{k-2}(i, j) \right) + \dots \\
 &+ \left(\hat{\Psi}_{k-n+1}(i, j) - \hat{\Psi}_{k-n}(i, j) \right) \|^2 \\
 \leq &n \left(\left\| \hat{\Psi}_k(i, j) - \hat{\Psi}_{k-1}(i, j) \right\|^2 \right. \\
 &+ \left\| \hat{\Psi}_{k-1}(i, j) - \hat{\Psi}_{k-2}(i, j) \right\|^2 + \dots \\
 &+ \left. \left\| \hat{\Psi}_{k-n+1}(i, j) - \hat{\Psi}_{k-n}(i, j) \right\|^2 \right) \quad (20)
 \end{aligned}$$

where n is a finite number. Considering (18) and (20), we have

$$\lim_{k \rightarrow \infty} \left\| \hat{\Psi}_k(i, j) - \hat{\Psi}_{k-n}(i, j) \right\| = 0 \quad (21)$$

Using the definition of $\hat{\Psi}_k(i, j)$ and $\bar{B}_k(i, j)$ in (4), we can obtain

$$\lim_{k \rightarrow \infty} \left| \hat{B}_k(i, j) - \bar{B}_k(i, j) \right| = 0 \quad (22)$$

Then, based on (22) and the boundedness of $u_k(i, j)$, it yields

$$\lim_{k \rightarrow \infty} \varpi_k(i, j) = \lim_{k \rightarrow \infty} \left(\hat{B}_k(i, j) - \bar{B}_k(i, j) \right) u_k(i, j) = 0 \quad (23)$$

On the other hand, we have obtained from part 2 that $u_k(i, j)$ and $\eta_k(i, j)$ are bounded for $i \in [0, H - 1]$ and $j \in [0, V - 1]$, thus the boundedness of $\rho_k(i, j) = [\eta_k^T(i, j) u_k(i, j)]^T$ is immediately deduced. From (15), we get

$$\lim_{k \rightarrow \infty} \tilde{\Psi}_k^T(i, j) \rho_k(i, j) = 0 \quad (24)$$

Combining (8) and (24), there is

$$\lim_{k \rightarrow \infty} [e_k(i + 1, j + 1) - \varpi_k(i, j)] = 0 \quad (25)$$

Then, based on (23), we get

$$\lim_{k \rightarrow \infty} e_k(i + 1, j + 1) = 0 \quad (26)$$

That is

$$\lim_{k \rightarrow \infty} e_k(i, j) = 0 \quad (27)$$

where $i \in [1, H]$ and $j \in [1, V]$. It shows that the tracking error will precisely converge to zero as iteration number k goes to infinity. This Theorem is proved.

V. SIMULATION RESULTS

Example: The same example of Darboux equation as in [10], which represents some dynamical plants in gas absorption, water stream heating, and air drying [26], is used for the simulation. However, different to [10], no information including the sign of control gain is known in advance of our simulation. The Darboux equation used in simulation is described as

$$\frac{\partial^2 s(w, t)}{\partial w \partial t} = a_1 \frac{\partial s(w, t)}{\partial t} + a_2 \frac{\partial s(w, t)}{\partial w} + a_0 s(w, t) + b f(w, t) \quad (28)$$

where $w(\text{space}) \in [0, W_f]$ and $t(\text{time}) \in [0, T_f]$ are the independent variables, and $s(w, t)$ is the system output. Letting $x(i, j) = s(i\Delta w, j\Delta t)$ and then doing the discretization of equation (28), we can get the coefficients of the 2-D FMM system (1) as

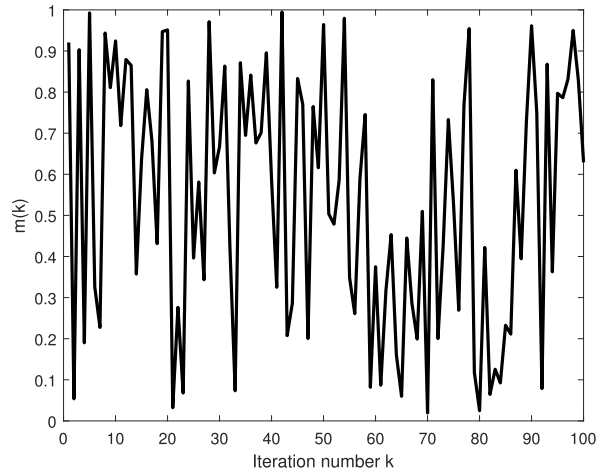


FIGURE 1. Random factor $m(k)$ in the reference trajectory $x_k^*(i, j)$.

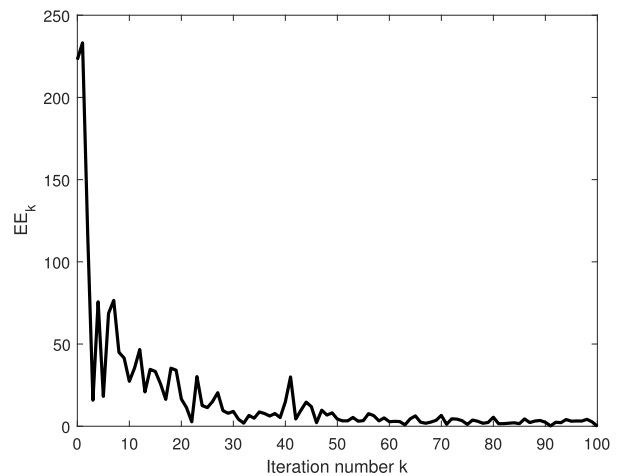


FIGURE 2. The total absolute tracking error EE_k .

$$A_0 = a_0 \Delta w \Delta t - a_1 \Delta w - a_2 \Delta t - 1,$$

$$A_1 = 1 + a_2 \Delta t,$$

$$A_2 = 1 + a_1 \Delta w,$$

$$B = b \Delta w \Delta t.$$

The boundary states of $x_k(i, 0)$ and $x_k(0, j)$ for $i \in [0, 20]$ and $j \in [1, 20]$ will randomly take values at $[0, 1]$. And the reference trajectories are represented by

$$x_k^*(i, j) = m(k) \sin(0.7i + 0.1j) \quad (29)$$

where $m(k)$ denotes the iteration-dependent uncertainties and takes value at $[0, 1]$ randomly.

Choose the same values of parameter $a_0, a_1, a_2, b, \Delta w$, and Δt as to [10]. To illustrate the validity of the proposed adaptive ILC algorithm (3), (4), and (7), no information including the sign of the control gain B is used in the simulation. The constants involved in (4) and (7) are chosen as $b_0 = 0.1$ and $a = 0.01$. Taking the following total absolute tracking error to evaluate the tracking accuracy,

$$EE_k = \sum_{i=1}^{20} \sum_{j=1}^{20} |x_k^*(i, j) - x_k(i, j)| \quad (30)$$

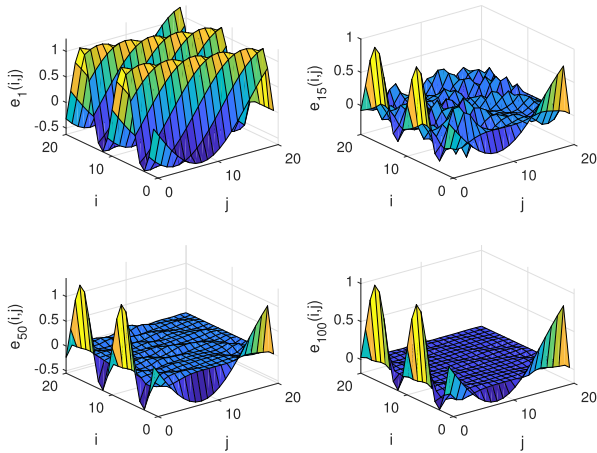


FIGURE 3. The tracking error surfaces at $k = 1$, $k = 15$, $k = 50$, and $k = 100$, respectively.

the corresponding results are given in FIGURE 2 and FIGURE 3.

The random-varying factor $m(k)$ in the reference trajectory $x_k^*(i, j)$ is shown in FIGURE 1. The convergence of tracking error is shown in FIGURE 2. And FIGURE 3 depicts the tracking error surfaces at $k = 1$, $k = 15$, $k = 50$, and $k = 100$, respectively. Clearly, for the Darboux equation system (28) which is subject to unknown control direction as well as random uncertainties in boundary condition and reference trajectory, the precise tracking is achieved.

VI. CONCLUSION

If the prior knowledge of control direction is not available, no result is applicable for the 2-D dynamical systems. In this paper, for the 2-D FMM system possessing unknown control direction as well as random uncertainties in boundary condition and reference trajectory, an adaptive ILC is appropriately designed. Both of the theoretical and simulation analysis have validated the effectiveness of the developed adaptive ILC in this paper. Extensions of the proposed adaptive ILC method to 2-D dynamical systems with unknown control directions, as well as state constraints and nonlinear input characteristics may be considered in the future work.

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