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Dynamic Output Feedback Finite-Horizon Control for Markov Jump Systems With Actuator Saturations

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ABSTRACT This paper considers the problem of asynchronous observer-based finite-horizon control of Markov jump systems (MJSs) with actuator saturations. The hidden Markov model is employed to describe asynchronous phenomenon between observer-based controller and the plant, where the observer designed has its own jumping mode that is different from that of the controlled plant. The purpose of this paper is to develop an asynchronous observer-based controller to ensure that an $\mathcal{H}_\infty$ performance index, over a given finite-horizon, can be satisfied for MJSs with actuator saturation. A sufficient condition is derived to guarantee that the $\mathcal{H}_\infty$ performance index can be achieved by using the stochastic Lyapunov function theory and *S-Procedure* lemma. Then, a recursive linear matrix inequality (RLMI) approach is applied to design the gains of the controller and observer. Finally, an example is given to verify the proposed algorithm.

INDEX TERMS Markov jump systems (MJSs), finite-horizon control, actuator saturation, asynchronous control, observer.

I. INTRODUCTION

In the past decades, Markov jump systems (MJSs), as a special type of the hybrid system, can be widely used to describe the dynamics of some practical systems subject to random variations coming from the unpredictable external disturbances, failures or repairs of components. Many results associated with different performance indexes including stabilization problem, passivity, dissipativity, state estimation/filtering and $\mathcal{H}_\infty$ performance, etc., for MJSs have been investigated by employing different approaches. For example, [1] studied the stabilization problem of MJSs subject to time-varying delays and the partially known transition probabilities. The state estimation problem of MJSs has been discussed by employing the sliding-mode control approaches in [2] and the same method has also been developed to consider the dissipativity problem in [3]. Other performances related approaches for MJSs can be found in [4]–[6], just to name a few.

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It is known that in practical control systems, actuators can only provide limited width of transmission signals owing to the physical, safety or technological restrictions, which may lead to amplitude saturations of actuators. Thus, saturation problem for actuators has received increasing attention due to its importance, and some results related to filtering and control have been reported in [7]–[9] and MJSs in [10]–[12]. Unfortunately, in above references, it is implicitly assumed that the states of the controlled system are always available, and the filter and the plant work synchronously. However, the states of the controlled systems are always not available for directed measurement due to external factors. It is often possible to estimate the states by introducing observers that have been widely used when the states of the controlled system are unavailable. For example, [13] employed the dynamic output feedback control to discuss the stabilization problem of singular MJSs. The fault-tolerant sliding-mode-observer problem for MJSs with quantized measurements has been studied in [14] and [15] considered the actuator and sensor faults for MJSs. Robust dynamic output feedback of delayed MJSs has been discussed in [16]. Note that most existing results considered different performances of MJSs over the infinite horizon. Very few results have been presented to consider the observer-based control of MJSs with actuator saturation over a finite horizon, which is one of motivations.

On the other hand, the above mentioned references considered the synchronous phenomenon which means the controller and the plant work synchronously. However, synchronous phenomenon is very difficult and even impossible to hold due to the effects of time delays and packet loss. When the mode information of the controlled system can not be completely accessed to the controller, it leads to the asynchronous phenomenon. Thus, it is worthwhile to pay more attention on the asynchronous control of MJSs. Very recently, the authors of [17]proposed an asynchronous state feedback controller to investigate the passivity problem of MJSs. Then, some associated results for MJSs based on [17] have been published in [18], [19]. However, asynchronous control problem between the controlled plant and the observer has not been adequately investigated, not to mention the case where the actuator saturation and finite horizon are also involved.

Based on above discussions, to the best of the authors' knowledge, very few results have been presented to consider the asynchronous observer-based finite-horizon control of MJSs with actuator saturation. It is, therefore, the purpose of this paper to shorten the gap by using the S-Procedure Lemma and Recursive LMIs(RLMIs) approach. The contributions of this paper can be summarized as follows: (1) The observer is introduced to estimate the states of the systems. An asynchronous control between the controlled system and the observer is considered, where the observer has its own jumping mode different from the jumping mode of the plant. (2) The finite-horizon control is applied to investigate the asynchronous observer of MJSs in the presence of actuator saturation, which has not been well studied in literature. (3) The sufficient condition for MJSs is derived to guarantee that the H_∞ performance can be achieved over a given finite horizon, and an algorithm is given to obtain the gains of the controller and the observer by using RLMIs approach.

II. DEFINITIONS AND PRELIMINARIES

A. SYSTEM DESCRIPTION

Consider the following MJSs with input saturations as

$$\begin{cases} x(k+1) = A_{\theta_k}x(k) + B_{\theta_k}\sigma(u(k)) + D_{\theta_k}w(k), \\ z(k) = E_{\theta_k}x(k) + F_{\theta_k}w(k), \\ y(k) = C_{\theta_k}x(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ denote the state and input of MJSs. $z(k) \in \mathbb{R}^{n_z}$ and $y(k) \in \mathbb{R}^{n_y}$ denote the regulated output and the measurement output. $\sigma(\cdot)$ denotes the saturation function described later. $w(k) \in \mathbb{R}^{n_w}$ is the external disturbance belonging to $l_2\,[0\,N]$. $A_{\theta_k}, B_{\theta_k}, C_{\theta_k}, D_{\theta_k}, E_{\theta_k}, F_{\theta_k}$ are constant matrices with appropriate dimensions.

The stochastic variable θ_k is a Markov chain which are contained in a finite set $\mathcal{S} = \{1, 2, \ldots, s\}$ with the transition probability matrix $\Gamma = [\lambda_{mn}]$ expressed by

$$\Pr\{\theta_{k+1} = n|\theta_k = m\} = \lambda_{mn}, \quad \forall m,\ n \in \mathcal{S}, \tag{2}$$

where $0 < \lambda_{mn} \leq 1,\ \sum_{n=1}^{s}\lambda_{mn} = 1,\ \ \forall n \in \mathcal{S}$.

The saturation function $\sigma(\cdot)$ is defined as

$$\sigma(\varpi) = \begin{pmatrix} \sigma_1^T(\varpi_1) & \sigma_2^T(\varpi_2) & \cdots & \sigma_r^T(\varpi_r) \end{pmatrix}^T, \tag{3}$$

with $\sigma_i^T(\varpi_i) = \mathrm{sign}(\varpi_i)\min\{\varpi_{i,\max}, |\varpi_i|\}$, where $\varpi_{i,\max}$ is the i-th element of the vector $\varpi_{\max}$, the saturation level.

Definition 1 [20]: A nonlinearity $\Omega : \mathbb{R}^n \to \mathbb{R}^n$ is said to satisfy a sector condition if the following equation holds

$$(\Omega(\varpi) - W_1\varpi)^T(\Omega(\varpi) - W_2\varpi) \leq 0, \quad \forall \varpi \in \mathbb{R}^r \tag{4}$$

for some real matrices $W_1, W_2 \in \mathbb{R}^r$, where $W = W_2 - W_1$ is a positive definite symmetric matrix. Then, we can say $\Omega \in [W_1,\ W_2]$.

Similar to [20], [21], if we suppose that there exist two diagonal matrices M_1 and M_2 satisfying $0 \leq M_1 \leq M_2 \leq I$, then the saturation function can be rewritten as the following equation

$$\sigma(u(k)) = M_1u(k) + \Omega_u(u(k)), \tag{5}$$

where $\Omega_u(u(k))$ is a nonlinear vector-valued function satisfying the following sector condition,

$$\Omega_u(u(k))[\Omega_u(u(k)) - Mu(k)] \leq 0, \tag{6}$$

where $M = M_2 - M_1$.

B. ASYNCHRONOUS OBSERVER-BASED CONTROLLER MODEL

In practical systems, the mode information of the system (1) is ususlly unavailable for the controller or observer due to complex environment. Hence, the observer is used to estimate the states of the controlled systems. In this paper, we consider the asynchronous observer-based controller that has its own jumping mode as follows

$$\begin{cases} \bar{x}_o(k+1) = A_{\vartheta_k}\bar{x}_o(k) + B_{\vartheta_k}\sigma(u(k)) \\ \qquad\qquad + L_{\vartheta_k}(y(k) - \bar{y}_o(k)), \\ \bar{y}_o(k) = C_{\vartheta_k}\bar{x}(k), \\ u(k) = K_{\vartheta_k}\bar{x}_o(k), \end{cases} \tag{7}$$

where $\bar{x}(k) \in \mathbb{R}^{n_{\bar{x}_o}}$ denotes the state vector of observer. $\bar{y}_o(k) \in \mathbb{R}^{n_{\bar{y}_o}}$ denote the output. $A_{\vartheta_k}, B_{\vartheta_k}, C_{\vartheta_k}$ are constant matrices with appropriate dimensions. Matrices K_{ϑ_k} and L_{ϑ_k} are the gains of the controller and observer respectively. The stochastic variable ϑ_k, which is similar to (2), is contained in a finite set $\mathcal{G} = \{1, 2, \ldots, G\}$ with transition probability matrix $\Upsilon = [\Xi_{mr}]$ given by

$$\Pr\{\vartheta_k = r|\theta_k = m\} = \Xi_{mr}, \quad \forall m \in \mathcal{S},\ \forall r \in \mathcal{G}, \tag{8}$$

where Ξ_{mr} denotes the probability satisfying $0 < \Xi_{mr} \leq 1$and $\sum_{r=1}^{G}\Xi_{mr} = 1, \forall r \in \mathcal{G}$.

For MJSs (1) and observer (7), define $\delta(k) = x(k) - \bar{x}_o(k)$ as the gap between the states of MJSs (1) and the observer in (7). Then, the dynamic of the resulting system is as follows

$$\begin{cases} \bar{x}_o(k+1) = \mathbb{A}\bar{x}_o(k) + L_r C_m \delta(k) + B_r \Omega_u(u(k)), \\ \delta(k+1) = \mathbb{C}\bar{x}_o(k) + \mathbb{D}\delta(k) + \mathbb{E}\Omega_u(u(k)), \\ z(k) = E_m x(k) + F_m w(k), \end{cases} \tag{9}$$

where

$$\begin{aligned} \mathbb{A} &= A_r + B_r M_1 K_r + L_r C_m - L_r C_r \\ \mathbb{C} &= A_m - A_r + B_m M_1 K_r - B_r M_1 K_r - L_r C_m + L_r C_r \\ \mathbb{D} &= A_m - L_r C_m, \mathbb{E} = B_m - B_r. \end{aligned}$$

The objective of this paper is to design an observer and an controller of the form (7) to guarantee that for a given disturbance attenuation level $\beta > 0$, a positive definite matrix $\Lambda_m, m = 1, \ldots, s$ and an initial state $x(0)$, the $\mathcal{H}_\infty$ performance satisfies the following equation over a given finite-horizon,

$$J = \mathcal{E}\left\{ \|z(k)\|^2_{[0\ N]} - \beta^2 \|w(k)\|^2_{[0\ N]} \beta^2 x^T(0)\Lambda_m x(0) \right\} < 0. \tag{10}$$

Before giving the main result, the following Lemma ($S-$ *Procedure*) is required to be used for proof of achieving $\mathcal{H}_\infty$ performance later.

Lemma 1 [20] ($S -$ Procedure): Let $\mathcal{Y}_0(\rho), \ldots, \mathcal{Y}_p(\rho)$ be quadratic function of $\rho \in \mathbb{R}^n$, $\mathcal{Y}_I(\rho) = \rho^T T_i \rho, i = 0, \ldots, p$ with $T_i^T = T_i$. Then, the implication on $\mathcal{Y}_1(\rho) < 0, \ldots, \mathcal{Y}_p(\rho) \leq 0 \Rightarrow \mathcal{Y}_0(\rho) \leq 0$ holds if there exist $\varsigma_1, \ldots, \varsigma_p > 0$ such that

$$T_0 - \sum_{i=1}^{p} \varsigma_i T_i \leq 0. \tag{11}$$

III. MAIN RESULTS

In this section, we address the asynchronous observer-based $\mathcal{H}_\infty$ control problem of MJSs over a given finite-horizon. The sufficient condition of achieving a prescribed $\mathcal{H}_\infty$ performance is derived by employing Lyapunov based method, *S-Procedure* Lemma and the RLMIs approach.

Theorem 1: Consider the MJSs in (1) and the observer-based controller (7) in the presence of actuator saturations (3). For a given disturbance attenuation level $\beta > 0$, a set of positive scalars $\{\varsigma_1(k)\}_{0\leq k\leq N}$, a positive definite matrix $\Lambda_m, m \in \mathcal{S}$ and gain matrices for the controller and observer $\{A_r(k), r \in \mathcal{G}\}_{0\leq k\leq N}$, $\{B_r(k), r \in \mathcal{G}\}_{0\leq k\leq N}$, $\{C_r(k), r \in \mathcal{G}\}_{0\leq k\leq N}$ and $\{L_r(k), r \in \mathcal{G}\}_{0\leq k\leq N}$, the $\mathcal{H}_\infty$ performance defined in (10) is satisfied for all nonzero $w(k)$ if, for the initial condition satisfies the following equation,

$$\begin{bmatrix} \mathcal{Z}_m(0) & 0 \\ 0 & \mathcal{H}_m(0) \end{bmatrix} < \beta^2 \begin{bmatrix} I \\ I \end{bmatrix} \Lambda_m \begin{bmatrix} I & I \end{bmatrix}, \tag{12}$$

there exist positive definite matrices $\{\mathcal{W}_{mr}(k), m \in \mathcal{S}, r \in \mathcal{G}\}_{0\leq k\leq N}$, $\{\mathcal{F}_{mr}(k), m \in \mathcal{S}, r \in \mathcal{G}\}_{0\leq k\leq N}$, $\{\mathcal{Z}_m(k), m \in \mathcal{S}\}_{0\leq k\leq N+1}$ and $\{\mathcal{H}_m(k), m \in \mathcal{S}\}_{0\leq k\leq N+1}$ such that

$$\sum_{r=1}^{G} \Xi_{mr} \begin{bmatrix} \mathcal{W}_{mr}(k) & 0 \\ 0 & \mathcal{F}_{mr}(k) \end{bmatrix} < \begin{bmatrix} \mathcal{Z}_m(k) & 0 \\ * & \mathcal{H}_m(k) \end{bmatrix}, \tag{13}$$

and

$$\begin{bmatrix} \Pi_{11}^* & \varphi_1^T & \varphi_2^T & \varphi_3^T \\ * & -\hat{\mathcal{Z}}_m^{-1}(k+1) & 0 & 0 \\ * & * & -\hat{\mathcal{H}}_m^{-1}(k+1) & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{14}$$

hold for all $0 \leq k \leq N$, where

$$\begin{aligned} \Pi_{11}^* &= \begin{bmatrix} -\mathcal{W}_{mr}(k) & 0 & \frac{1}{2}\varsigma_1(k) K_r M & 0 \\ * & -\mathcal{F}_{mr}(k) & 0 & 0 \\ * & * & -\varsigma_1(k) I & 0 \\ * & * & * & -\beta^2 I \end{bmatrix}, \\ \varphi_1 &= \begin{bmatrix} \mathbb{A} & L_r C_m & B_r & 0 \end{bmatrix}, \\ \varphi_2 &= \begin{bmatrix} \mathbb{C} & \mathbb{D} & \mathbb{E} & D_m \end{bmatrix}, \\ \varphi_3 &= \begin{bmatrix} E_m & 0 & 0 & F_m \end{bmatrix}, \\ \eta(k) &= \begin{bmatrix} \bar{x}_o^T(k) & \delta^T(k) & \Omega_u^T(u(k)) & w^T(k) \end{bmatrix}^T, \\ \hat{\mathcal{Z}}_m(k+1) &= \sum_{n=1}^{s} \lambda_{mn} \mathcal{Z}_n(k), \\ \hat{\mathcal{H}}_m(k+1) &= \sum_{n=1}^{s} \lambda_{mn} \mathcal{H}_n(k). \end{aligned}$$

Proof: According to (9), one has

$$\begin{aligned} \bar{x}_o(k+1) &= \varphi_1 \eta(k), \\ \delta(k+1) &= \varphi_2 \eta(k), \\ z(k) &= \varphi_3 \eta(k). \end{aligned} \tag{15}$$

The following Lyapunov function candidate is considered

$$V(\bar{x}_o(k), \delta(k), k) = \begin{bmatrix} \bar{x}_o^T(k) \\ \delta^T(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{Z}_m & 0 \\ 0 & \mathcal{H}_m \end{bmatrix} \begin{bmatrix} \bar{x}_o(k) \\ \delta(k) \end{bmatrix}, \tag{16}$$

where $\mathcal{Z}_m = \mathcal{Z}_m(k)$ and $\mathcal{H}_m = \mathcal{H}_m(k)$.

The expectation of there difference equation of (16) is as follows

$$\begin{aligned} \mathcal{E}\{\Delta V(k)\} &= \mathcal{E}\{V(\bar{x}_o(k+1), \delta(k+1), k+1) \\ &\quad - V(\bar{x}_o(k), \delta(k), k)\} \\ &= \mathcal{E}\Big\{ \bar{x}_o^T(k+1)\hat{\mathcal{Z}}_m(k+1)\bar{x}_o(k+1) \\ &\quad - \bar{x}_o^T(k)\mathcal{Z}_m(k)\bar{x}_o(k) - \delta^T(k)\mathcal{H}_m(k)\delta(k) \\ &\quad + \delta^T(k+1)\hat{\mathcal{H}}_m(k+1)\delta(k+1) \Big\} \end{aligned}$$

$$
\begin{aligned}
&= \eta^T(k)\Bigg(\sum_{r=1}^{G}\Xi_{mr}(\varphi_1^T\hat{\mathcal{Z}}_m(k+1)\varphi_1 \\
&\quad +\varphi_2^T\hat{\mathcal{H}}_m(k+1)\varphi_2)\Bigg)\eta(k) \\
&\quad -\bar{x}_o^T(k)\mathcal{Z}_m(k)\bar{x}_o(k)-\delta^T(k)\mathcal{H}_m(k)\delta(k), \quad (17)
\end{aligned}
$$

Then, adding the zero term $z^T(k)z(k) - \beta^2 w^T(k)w(k) - z^T(k)z(k) + \beta^2 w^T(k)w(k)$ to $\mathcal{E}\{\Delta V(k)\}$ yields

$$
\mathcal{E}\{\Delta V(k)\} \le \mathcal{E}\Big\{\eta^T(k)\Pi_k\eta(k) - z^T(k)z(k) + \beta^2 w^T(k)w(k)\Big\}, \tag{18}
$$

where

$$
\begin{aligned}
\Pi_k &= \sum_{r=1}^{G}\Xi_{mr}\Big(\varphi_1^T\hat{\mathcal{Z}}_m(k+1)\varphi_1+\varphi_2^T\hat{\mathcal{H}}_m(k+1)\varphi_2 \\
&\quad +\varphi_3^T I\varphi_3+\Pi_{11}\Big) \\
&= \sum_{r=1}^{G}\vartheta_{mr}\begin{bmatrix}\Pi_{11} & \varphi_1^T & \varphi_2^T & \varphi_3^T \\ * & -\hat{\mathcal{Z}}_m(k+1) & * & * \\ * & * & -\hat{\mathcal{H}}_m(k+1) & * \\ * & * & * & -I\end{bmatrix},
\end{aligned}
$$

$$
\Pi_{11} = \begin{bmatrix} -\mathcal{W}_{mr}(k) & 0 & 0 & 0 \\ * & -\mathcal{F}_{mr}(k) & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -\beta^2 I\end{bmatrix}.
$$

Summing up (18) on both sides from 0 to $N-1$ with respect to k, one has

$$
\begin{aligned}
J &= \mathcal{E}\Big\{\|z(k)\|^2_{[0\ N]} - \beta^2\|w(k)\|^2_{[0\ N]} - \beta^2 x^T(0)\Lambda_m x(0)\Big\} \\
&\le \mathcal{E}\Big\{\sum_{k=0}^{N-1}\eta^T(k)\Pi_k\eta(k) - \mathcal{E}\Big\{\bar{x}_o^T(N)\hat{\mathcal{Z}}_m(N)\bar{x}_o(N) \\
&\quad +\delta^T(N)\hat{\mathcal{H}}_m(N)\delta(N)\Big\} + \mathcal{E}\Big\{\begin{bmatrix}\bar{x}_o^T(0) \\ \delta^T(0)\end{bmatrix}^T \\
&\quad \times\left(\begin{bmatrix}\mathcal{Z}_m(0) & 0 \\ 0 & \mathcal{H}_m(0)\end{bmatrix} - \beta^2\begin{bmatrix} I \\ I\end{bmatrix}\Lambda_m\begin{bmatrix} I & I\end{bmatrix}\right) \\
&\quad \times\begin{bmatrix}\bar{x}_o(0) \\ \delta(0)\end{bmatrix}\Big\}\Big\}. \qquad (19)
\end{aligned}
$$

It is worth noting that $\hat{\mathcal{Z}}_m(N) > 0$, $\hat{\mathcal{H}}_m(N) > 0$ and the with initial condition in (13), if

$$
\eta^T(k)\Pi_k\eta(k) < 0 \tag{20}
$$

holds, then (6) holds. That is to say, the objective of (10) is solved.

From (6),

$$
\Omega_u(u(k))[\Omega_u(u(k)) - Mu(k)] = \eta^T(k)\Theta_k\eta^T(k) \le 0, \tag{21}
$$

where

$$
\Theta_k = \begin{bmatrix} 0 & 0 & -\frac{1}{2}K_r M & 0 \\ * & 0 & 0 & 0 \\ * & * & I & 0 \\ * & * & * & 0\end{bmatrix}.
$$

By using Lemma 1 ($S-Procedure$) in (11), If there exists a positive scalar $\varsigma_1(k)$ ensuring that

$$
\Pi_k - \varsigma_1(k)\Theta_k \le 0 \Rightarrow \Pi_k < 0, \tag{22}
$$

which is equivalent to (14). Thus, the $\mathcal{H}_\infty$ performance index is satisfied.

IV. DESIGN OF OBSERVER-BASED CONTROLLER

In this section, we focus on the gain design of the controller and the observer corresponding to K_r and L_r based on Theorem 1. However, there exist inverse matrices $\hat{\mathcal{Z}}_m(k+1)$ and $\hat{\mathcal{H}}_m(k+1)$, which make the (14) non-convex and not feasible. Thus, we have to convert (14) to a convex linear matrix inequalities by introducing some slack matrices.

Theorem 2: For a given $\beta > 0$, if there is a positive definite matrix $\Lambda_m, m \in \mathcal{S}$, $\{\tilde{\mathcal{W}}_{mr}(k), m \in \mathcal{S}, r \in \mathcal{G}\}_{0\le k\le N} > 0$, $\{\tilde{\mathcal{F}}_{mr}(k), m \in \mathcal{S}, r \in \mathcal{G}\}_{0\le k\le N} > 0$, $\{\tilde{\mathcal{Z}}_m(k), m \in \mathcal{S}\}_{0\le k\le N} > 0$ and $\{\tilde{\mathcal{H}}_m(k), m \in \mathcal{S}\}_{0\le k\le N} > 0$, $\{\vec{K}_r, r \in \mathcal{G}\}_{0\le k\le N}$, $\{\vec{L}_r, r \in \mathcal{G}\}_{0\le k\le N+1}$ and $\Big\{P(k) = \begin{bmatrix} P_1(k) & P_3(k) \\ P_2(k) & P_3(k)\end{bmatrix}\Big\}_{0\le k\le N}$ such that the initial condition

$$
\begin{bmatrix}\mathcal{Z}_m(0) - \beta^2\Lambda_m & -\beta^2 I \\ -\beta^2 I & \mathcal{H}_m(0) - \beta^2\Lambda_m\end{bmatrix} < 0, \tag{23}
$$

and

$$
\sum_{r=1}^{G}\Xi_{mr}\begin{bmatrix}\tilde{\mathcal{W}}_{mr}(k) & 0 \\ 0 & \tilde{\mathcal{F}}_{mr}(k)\end{bmatrix} < \begin{bmatrix}\tilde{\mathcal{Z}}_m & 0 \\ * & \tilde{\mathcal{H}}_m\end{bmatrix}, \tag{24}
$$

$$
\begin{bmatrix}\Omega_{11}^* & \varphi_1^{*T} & \varphi_2^{*T} & \varphi_3^{*T} \\ * & \mathcal{Q} & * & * \\ * & * & \mathcal{R} & * \\ * & * & * & -I\end{bmatrix} < 0 \tag{25}
$$

hold, then the designed controller and observer can guarantee that the $\mathcal{H}_\infty$ performance index defined in (10) holds. Moreover, the gains of the controller and the observer are designed as follows

$$
K_r = \bar{K}_r P^{-1}, \quad L_r = \bar{L}_r P^{-1}, \tag{26}
$$

where

$$
\Omega_{11}^* = \begin{bmatrix} -\tilde{\mathcal{W}}_{mr}(k) & 0 & \frac{1}{2}\varsigma_1(k)\bar{K}_r M & 0 \\ * & -\tilde{\mathcal{F}}_{mr}(k) & 0 & 0 \\ * & * & -\varsigma_1(k)I & 0 \\ * & * & * & -\beta^2 I\end{bmatrix},
$$

$$
\begin{aligned}
\tilde{\mathcal{W}}_{mr}(k) &= P^T\mathcal{W}_{mr}(k)P, \quad \tilde{\mathcal{F}}_{mr}(k) = P^T\mathcal{F}_{mr}(k)P, \\
\tilde{\mathcal{Z}}_m(k) &= P^T\hat{\mathcal{Z}}_m(k)P, \quad \tilde{\mathcal{H}}_m(k) = P^T\hat{\mathcal{H}}_m(k)P, \\
\varphi_1^{*T} &= P\varphi_1^T = \begin{bmatrix} P^T\mathbb{A}^T & \bar{L}_r^T C_m^T & P^T B_r^T & 0\end{bmatrix}^T,
\end{aligned}
$$

$$\begin{aligned}
\varphi_2^{*T} &= P\varphi_2^T = \begin{bmatrix} P^T\mathbb{C}^T & P^T\mathbb{D}^T & P^T\mathbb{E}^T & P^T D_m^T \end{bmatrix}^T, \\
\varphi_3^{*T} &= \varphi_3^T = \begin{bmatrix} P^T E_m^T & 0 & 0 & P^T F_m^T \end{bmatrix}^T, \\
\mathcal{Q} &= \sum_{n=1}^{s} \lambda_{mn} \vec{\mathcal{Z}}_n(k) - P^T - P, \\
\mathcal{R} &= \sum_{n=1}^{s} \lambda_{mn} \vec{\mathcal{H}}_n(k) - P^T - P.
\end{aligned}$$

Proof: Firstly, by premultiplying diag$\{P^{-T}, P^{-T}\}$ and postmultiplying diag$\{P^{-1}, P^{-1}\}$ to both sides of (24), (13) holds.

Then, based on the property $X^T Y^{-1} X - X^T - X \geq -Y$ in [22], one has

$$\begin{aligned}
\mathcal{Q} &= \sum_{n=1}^{s} \lambda_{mn} \vec{\mathcal{Z}}_n(k) - P^T - P \\
&= P^T \hat{\mathcal{Z}}_m(k+1)P - P^T - P \geq -\hat{\mathcal{Z}}_m^{-1}(k+1), \qquad (27)
\end{aligned}$$

where $\vec{\mathcal{Z}}_m(k) = P^T \mathcal{Z}_m(k) P$. Similarly,

$$\begin{aligned}
\mathcal{R} &= \sum_{n=1}^{s} \lambda_{mn} \vec{\mathcal{H}}_n(k) - P^T - P \\
&= P^T \hat{\mathcal{H}}_m(k+1)P - P^T - P \geq -\hat{\mathcal{H}}_m^{-1}(k+1), \qquad (28)
\end{aligned}$$

with $\vec{\mathcal{H}}_m(k) = P^T \mathcal{H}_m(k) P$.

Let

$$\Sigma = \text{diag}\begin{Bmatrix} P^{-1} & P^{-1} & I & I & I & I & I \end{Bmatrix}. \qquad (29)$$

Then, pre- and post-multiplying (25) with Σ^T and Σ, respectively, it is obviously observed from (25) that (14) holds.

The following *Algorithm* shows the steps to design the gain matrices of the controller and observer:

Step 1: Given the $\mathcal{H}_\infty$ performance index β, the positive definite matrices Λ_m and the initial values $x(0)$ and $x_o(0)$. Then, select the initial values for matrices $\mathcal{Z}_m(0)$ and $\mathcal{H}_m(0)$ which satisfies (23) and set $k = 0$.

Step 2: For the sampling instant k, by solving the RLMIs to obtain matrices $\hat{\mathcal{Z}}_m(k+1)$ and $\hat{\mathcal{H}}_m(k+1)$ with known matrices $\mathcal{Z}_m(k)$ and $\mathcal{H}_m(k)$.

Step 3: By solving (25) to compute the gains of the controller and observer in (26) and set $k = k + 1$.

Step 4: If $k < N$, then go to Step 2, otherwise exit.

V. SIMULATION EXAMPLE

Suppose that there exist two jumping modes for MJSs, and the parameter matrices of MJSs (1) is given as follows:

Mode 1:

$$A_1 = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} -0.4 \\ 0.3 \end{bmatrix}, \quad C_1 = 1, \quad F_1 = 0.5.$$

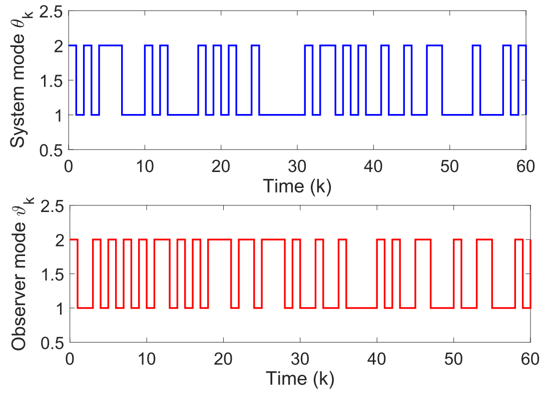

FIGURE 1. Jumping modes for the plant and observer.

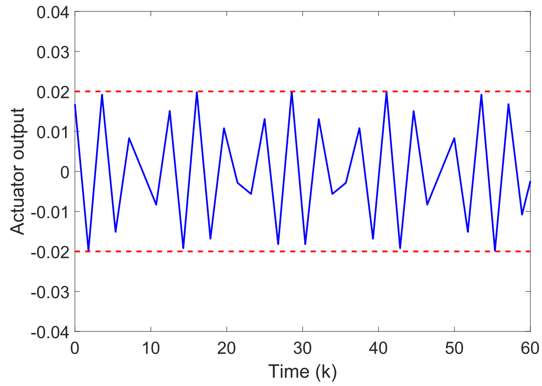

FIGURE 2. Actuator output.

The saturation function is expressed as follows

$$\begin{cases} \sigma(u_1(k)) = u_1(k), & \text{if} -V_{u1n,\max} \leq u_1(k) \leq V_{u1n,\max} \\ \sigma(u_1(k)) = V_{u1n,\max}, & u_1(k) > V_{u1n,\max} \\ \sigma(u_1(k)) = -V_{u1n,\max}, & u_1(k) < -V_{u1n,\max} \end{cases}$$

Mode 2:

$$A_2 = \begin{bmatrix} 0.4 & 0.6 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.3 \\ 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \quad C_2 = 1, \quad F_2 = 0.7,$$

with saturation bounds as

$$\begin{cases} \sigma(u_2(k)) = u_1(k), & \text{if} -V_{u2n,\max} \leq u_2(k) \leq V_{u2n,\max} \\ \sigma(u_2(k)) = V_{u1n,\max}, & u_2(k) > V_{u2n,\max} \\ \sigma(u_2(k)) = -V_{u1n,\max}, & u_2(k) < -V_{u2n,\max} \end{cases}$$

The two jumping modes of the controlled system (1) and the observer (7) are governed by the following transition matrices

$$\Gamma = \begin{bmatrix} 0.4 & 0.6 \\ 0.8 & 0.2 \end{bmatrix}, \quad \Upsilon = \begin{bmatrix} 0.3 & 0.7 \\ 0.65 & 0.35 \end{bmatrix}.$$

In this example, set the $\mathcal{H}_\infty$ performance index $\beta = 1.2$. The saturation values are as $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.02$, and the $M_1 = 0.3$, $M_2 = 0.5$. Initial conditions are set as $x(0) = [0.3 \;\; 0.2]^T$ and $\bar{x}_o(0) = [-0.1 \;\; 0.5]^T$, and let $\Lambda_1 = \Lambda_2 = 1$.

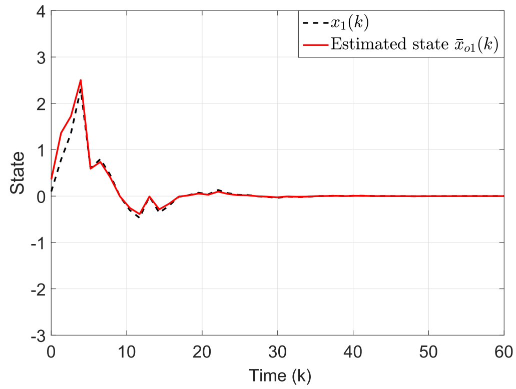

FIGURE 3. The state $x_1(k)$ and the corresponding estimated state $\bar{x}_{o1}(k)$ for Mode 1.

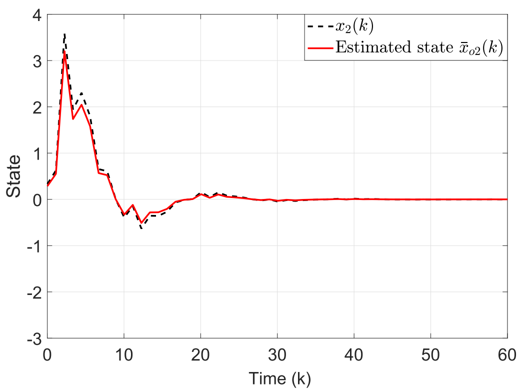

FIGURE 4. The state $x_2(k)$ and the corresponding estimated state $\bar{x}_{o2}(k)$ for Mode 2.

The external disturbance $w(k)$ with the following dynamics

$$w(k) = e^{-0.12k} \sin(2k).$$

Then, based on the above parameters and the proposed algorithms, the gain matrices of the controller and the observer can be calculated as follows:

$$K_1 = \begin{bmatrix} -0.337 & -1.262 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.412 & -0.233 \end{bmatrix},$$

and

$$L_1 = \begin{bmatrix} 0.573 & 0.482 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 1.273 & 0.629 \end{bmatrix}.$$

The possible jumping modes θ_k and ϑ_k occurring asynchronous phenomenon are described in Fig.1, it is clearly observed that the system to be controlled and the observer-based controller have different operations. Based on the above jumping sequence, Fig.2 demonstrates the evolution of the saturated actuator output, and the actuator output does not exceed the lower/upper boundaries. Figs.3-4 depict that the designed observer can effectively estimate the states of the controlled system regardless there are two jumping modes, where Fig.3 depicts the evolution of state $x_1(k)$ and the corresponding estimated state $\bar{x}_{o1}(k)$ for Mode 1, and Fig.4 illustrates the similar evolution for Mode 2. Fig.5 demonstrates the curves of states $x_i(k)$ and the corresponding estimated state $\bar{x}_{oi}(k)$ (i $= 1, 2$) corresponding to saturation values with $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.04$.

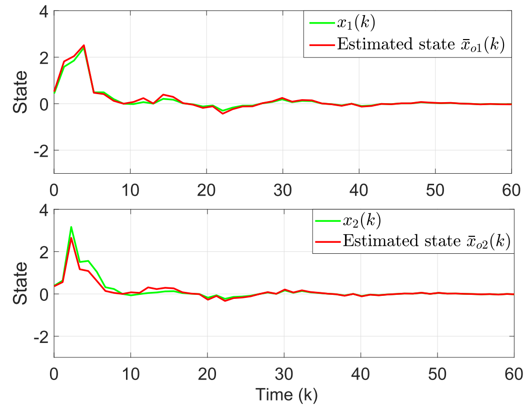

FIGURE 5. States $x_i(k)$ and the corresponding estimated state $\bar{x}_{oi}(k), i = 1, 2$ with saturation value $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.02$.

Note that Figs.3-4 consider the saturation value $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.02$, compared with the $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.02$, Fig.5 describes the evolution of the states $x_i(k)$ and the corresponding estimated state $\bar{x}_{oi}(k), i = 1, 2$ with saturation value $\sigma(u_1(k)) = \sigma(u_2(k)) = 0.04$. It can be seen from Figs.3-4 and Fig.5 that different saturation values have an effect on the evolution of the states $x_i(k)$ and the corresponding estimated state $\bar{x}_{oi}(k), i = 1, 2$. That is to say, the larger the saturation value, the slower the convergence of estimated error.

VI. CONCLUSION

In this paper, the finite-horizon H_∞ problem of Markov jump systems in the presence of actuator saturation is discussed based on the asynchronous phenomenon between the controlled plant and the observer-based controller. The observer-based controller has it own jumping mode, which is different from that of the controlled plant. The sufficient condition in the form of LMIs is derived to ensure that a finite-horizon $\mathcal{H}_\infty$ performance index can be achieved and the gain matrices of the controller and the observer are computed by solving the RLMIs. An example is provided to verify the proposed algorithm.

This paper adopt the asynchronous control approach to discuss the finite-horizon H_∞ problem of Markov jump systems subject to actuator saturations, where the controlled system and the observer run asynchronously. Two different saturation levels are considered to analyze the evolution of the states of the controlled system and the corresponding estimated state. By comparing the different saturation levels, it can be seen that the larger the saturation value, the slower the convergence of estimated error. Then, the sufficient conditions of obtaining the finite-horizon $\mathcal{H}_\infty$ performance index are provided. Finally, we give a simulation example to verify the theoretical analysis.

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