Received July 25, 2019, accepted August 29, 2019, date of publication September 16, 2019, date of current version October 4, 2019.
Digital Object Identifier 10.1109/ACCESS.2019.2941513

Optimization of (*R*, *Q*) Policies for Assembly Inventory Systems With Operating Flexibility

PENG LI AND DI WU
Faculty of Economics and Management, Xi'an University of Technology, Xi'an 710048, China
Corresponding author: Peng Li (lipenggn@xaut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 71401133, and in part by the Shaanxi Province Social Science Foundation under Grant 2018S44.

ABSTRACT Guaranteed-Service Approach (GSA) was used to set safety stock for multi-echelon inventory systems. This approach assumes that each stock can use operating flexibility measures such as expediting and overtime to fulfill excessive customer demand superior to a bound as a supplement to its safety stock. In this paper, we consider a continuous review assembly inventory system with Poisson final demand and fixed order costs at each stock controlled by a (R, Q) policy. We use the GSA to optimize the policy with the consideration of operating flexibility costs and fixed order costs. A deterministic mathematical programming model is established for the problem. And the model is solved by a line search for finding the optimal target cycle service level (CSL) to customer and an iterative procedure for solving the model when the target CSL is given. Moreover, we analyze the optimality conditions for the extended GSA model and obtain some important properties in given conditions. Numerical experiments on randomly generated instances demonstrate the efficiency of the procedure and confirm the solution presented in this paper.

INDEX TERMS Dynamic programming, guaranteed-service approach, inventory management, multi-echelon inventory system.

I. INTRODUCTION

Effective management of inventories in a supply chain is critical for the firms in the chain to assure a high service level to their customers at the minimal costs. As such supply chain can be modeled as a multi-echelon inventory system, one important issue of its management is to find an optimal inventory policy of the system.

Over last two decades, two competing approaches have emerged in multi-echelon inventory theory: stochastic-service approach (SSA) and guaranteed-service approach (GSA), which were introduced by [1] and [2], respectively. These two approaches differ in demand treatment and service time characteristics. In the SSA, it is assumed that any demand of a stock is immediately satisfied if its on-hand inventory is sufficient to fulfill the demand. Otherwise, the unsatisfied demand will be backlogged and satisfied later after the replenishment of the stock. In this case, a stochastic delay to fulfill the unsatisfied demand will occur. The service time of the stock, which is defined as the lead time for fulfilling its demand, is thus stochastic. In contrast, the GSA assumes that each stock can quote a deterministic service time to fulfill each customer demand. That is, the stock can always fulfill a customer demand in a given lead time. This is achieved by using some sort of emergency measure (referred to as operating flexibility hereafter) such as expediting and overtime to ensure the excessive customer demand superior to a pre-specified bound is also satisfied within the lead time. Since for each stock, the amount of safety stock to hold depends on the timespan for which the safety stock is used to protect against demand variability, the service time of the stock is thus a deterministic decision variable although its demand is stochastic. The authors of [3] compared the two approaches for a two-level distribution system, the results show that the difference between the two approaches is quite small in terms of costs and the GSA outperforms the SSA for the systems with moderate costs of operating flexibility, long processing time at the warehouse and high service level at retailers.

Since the GSA formulates the safety stock optimization problem of a multi-echelon inventory system as a deterministic mathematical programming problem rather than a

The associate editor coordinating the review of this manuscript and approving it for publication was Nikhil Padhi.

stochastic problem in the SSA, it greatly simplifies the problem and makes the latter much easier to be solved. However, the original GSA does not explicitly model the costs of using operating flexibility measures to fulfill excessive demand superior to a pre-specified demand bound [4]. The ignorance of the operating flexibility costs may make the GSA model unable to reflect the reality, since expediting and overtime usually lead to additional costs in practice. In addition, the original GSA does not consider fixed order costs for placing orders at each stock. In reality, such order costs often exist, which may include the costs for placing and delivering orders. For these two reasons, in this paper we try to extend the original GSA so that it can consider both fixed order costs and operating flexibility costs.

In this paper, we consider a continuous review assembly system with Poisson final demand and fixed order costs at each stock, where each stock of the system is controlled by an echelon (R, Q) policy. This means that an order of Q units is placed every time when the inventory position (=on hand inventory +outstanding order -backorders) of a stock reaches a reorder point R. We extend the GSA by considering the effects of operating flexibility on the material flow and the total cost of the system. Firstly, we derive a deterministic mathematical programming model for the optimization of an echelon (R, Q) policy for the system under the GSA. Secondly, we propose a method for solving the model based on a line search for finding the optimal target cycle service level (CSL) to customer. Moreover, we analyze the optimality conditions for the extended GSA model and obtain some important properties. Numerical experiments on randomly generated instances show the efficiency of the iterative procedure and confirm the solution presented in this paper.

The rest of this paper is organized as follows: Section 2 contains a literature review. Section 3 describes the assembly inventory system considered and the original GSA assumptions, and provides a mathematical model for the optimization of an echelon (R, Q) policy of the system with considering the operating flexibility costs. Section 4 presents an iterative procedure for solving the model and analyzes the optimality conditions for the extended GSA model. Computational results are presented and analyzed in Section 5. Section 6 concludes the paper with some remarks on future research.

II. RELATED LITERATURE

The GSA was originated by a fundamental work of [2]. In that work, the authors studied a single stock with random but bounded demand, controlled by a base-stock policy. It is proved that the bound of the demand during the lead time of the stock can be used to set its base-stock level. The author of [5] extended the model to serial inventory systems and proved that the optimal inventory policy of the systems is an ''all or nothing'' policy. The authors of [4] extended the previous work to more general multi-echelon inventory systems. More works on GSA can be found in [6]–[13]. Recently, the authors of [14] proposed solution methods to solve the GSA model under arbitrary cost functions. The authors of [15] considered the stochastic lead times into the GSA model, and presented efficient algorithms to solve. The authors of [16] extended the formulation presented in [17] to general acyclic systems and showed that the computational complexity increases significantly with differentiated service times. More comprehensive survey of GSA can be found in [18] and [19]. Note that these works did not explicitly consider the effects and the costs of using operating flexibility measures in their GSA models.

Only few studies have been conducted regarding the impact of using operating flexibility measures in GSA model. The authors of [3] considered a two-level distribution system with a particular type of operating flexibility measure, i.e., express delivery, which can speed up the process of delivery from the warehouse to the retailers and make use of inflow materials, they assumed unit cost associated with this operating flexibility measure and provided an extension of GSA model to minimize inventory costs of the whole distribution system. Their simulation results demonstrated the relevance of the operating flexibility cost assumption. The authors of [20] proposed a stochastic GSA model with recourse for a supply chain that uses another type of operating flexibility measure, i.e., outsourcing. The authors of [18] also considered outsourcing and assumed such measure is only applied in the demand level of a multi-echelon inventory system. In that paper, they evaluated the service level that results from carrying safety stocks and showed that if demand is truncated at the demand stage, there exists a gap between the effectively observed service level and the target service level. The authors of [21] extended their previous work and presented a GSA model which consider the capacity constraints of outsourcing, they compared their model with the original GSA model and the SSA model proposed in [22]. The experimental results demonstrated that their model is more cost-effective.

From the above literature review, we can see two types of operating flexibility measures have been studied in the GSA framework, one includes express delivery, expediting and overtime, which speeds up the production and distribution process ([3]), another turns to external sources ([18], [20] and [21]). The first type of operating flexibility measures makes the original unbounded demand of the final level propagated towards the upstream level of a supply chain, whereas the second type uses outsourcing to handle the excessive final demand superior to a specified demand bound, and only part of the demand within the bound propagated in the system.

Although the GSA was primarily applied to safety stock placement, it can also be used to optimize the (R, Q) policy for a multi-echelon inventory system, because for each stock controlled by an echelon (R, Q) policy in the system, its reorder point R is strongly related to its safety stock. The authors of [23] is the first paper to use the GSA to optimize an echelon (R, Q) policy for assembly inventory systems with fixed order costs but does not explicitly model the effects of using operating flexibility measures when external demand exceeds the specified demand bounds. This study shows that the consideration of operating flexibility effects in assembly

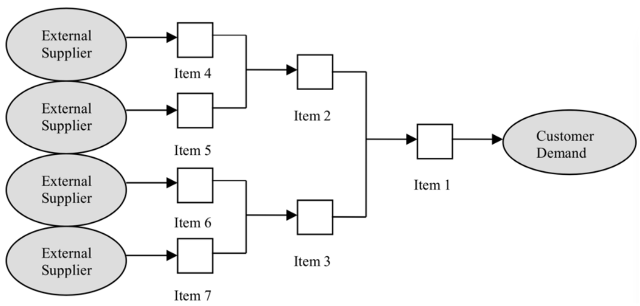

FIGURE 1. An assembly inventory system with 7 items (stocks).

systems makes the GSA model more realistic. Compared with other previous works on considering operating flexibility measures in the GSA framework, this paper deals with a more complicated system with fixed order costs controlled by an echelon (R, Q) policy. Moreover, most relevant studies only consider the effects of combining operating flexibility measures with safety stocks to address demand variations, but ignore the cost impact of such measures, this paper provides a model that considers both the effects and the costs of operating flexibility measures for assembly systems and a deeper analysis of the model.

III. GUARANTEED-SERVICE APPROACH

This section first presents the assembly inventory system considered and the original GSA assumptions to provide the reader with a foundation regarding the GSA, and then formulates a new mathematical programming model for the optimization of an echelon (R, Q) policy of the system with the consideration of the effects of operating flexibility measures on its material flow and its total cost.

A. COMMON ASSUMPTIONS AND CHARACTERISTICS

This paper considers a continuous review assembly inventory system with multiple intermediate items (components and sub-assemblies) and a single end item. The network structure of the system is defined by its bill-of-materials (BOM) which is a tree whose root node corresponds to the end item, as illustrated in Figure 1. All components at the highest level of the BOM are purchased from outside suppliers, these components are assembled into a finished product (end item) at the lowest level of the BOM. Hereafter, the stock of item i in the system is also called stock i, $i = 1,2,\ldots,N$. It is assumed that the outside suppliers never run of stock. Let N denote the number of items (stocks) in the system, $N > 3$, and A be the set of all components at the highest level of the BOM, where a component is called at the highest level if it has no predecessor. These items (stocks) are numbered from 1 to N, where item (stock) 1 represents the end item (end stock). Moreover, it is assumed that customer demand occurs only at the end item (stock) and follows a Poisson process with the average demand rate λ.

One major assumption of the GSA is that if customer demand during a lead time exceeds a pre-specified upper bound, excessive part of the demand superior to the bound will be fulfilled by using operating flexibility measures such as expediting and overtime rather than fulfilled normally from the stocks of the considered system. With this assumption, the system is regarded as one facing a bounded demand although the real customer demand is not bounded. Note that the bound is not defined directly on the demand of each time unit but on the lead time demand, i.e., the total demand occurred during the lead time. Since the lead time is a decision variable in the GSA, the bound is defined as a function of the lead time.

Let d_t and $d[t_1, t_2)$ denote the customer demand at time t and the total customer demand from time t_1 to time t_2 (not including time t_2), with $t_2 \geq t_1$, respectively. Since the customer demand of the assembly system is stationary, the lead time demand $d[t\text{-}\tau, t)$ with $\tau \geq 0$ can also be briefly denoted by $d(\tau)$. For this lead time demand, its upper bound to be specified can be denoted by $D(\tau)$. We assume that the excessive part of the lead time demand superior to $D(\tau)$ will be fulfilled by using operating flexibility measures in the system.

As in the original GSA, the lead time demand bound $D(\tau)$ is determined by the system's target CSL α to final customer, that is, $D(\tau)$ is the minimum number satisfying the following condition:

$$p\{d[t-\tau, t) \leq D(\tau)\} \geq \alpha \tag{1}$$

where $p\{.\}$ denotes the probability.

In the GSA literature, most studies use a normal distribution to describe the external demand process. Since the considered system assumes that customer demand only occurs at the end item and the demand follows a Poisson process with average demand rate λ, then, $D(\tau)$ can be calculated by

$$\sum_{k=0}^{D(\tau)} \frac{[\lambda\tau]^k e^{-\lambda\tau}}{k!} \geq \alpha \tag{2}$$

The GSA assumes that each stock i quotes and guarantees an outbound service time S_i to its immediate downstream stock, and an inbound service time SI_i to its immediate upstream stocks. That is, demand that arrives at time t and that is smaller than the demand bound must be filled at $t + S_i$ with 100% service level. The inbound service time SI_i is the time required by stock i to receive its ordered products from its immediate upstream stocks after the placement of the corresponding order. In addition, a given production time T_i is also defined at each stock i, which represents the time from the arrival of all materials required for the production of a product to the completion of the production and ready to serve a demand. In the GSA, the parameters of the inventory policy for a system are determined by the outbound service time, inbound service time and production time of each stock.

Under the setting presented above, at time t, stock i observes its demand and places an order to its upstream stocks. If stock i does not hold inventory, the earliest time that it can satisfy the demand is $t+SI_i + T_i$. The GSA guarantees that stock i satisfy the demand at time $t + S_i$. This implies that

TABLE 1. Notations used for the problem formulation.

Assembly inventory system	
N	the number of stocks in the assembly inventory system
i	stock index, i=1,2,…,N
t	time index
$s(i)$	the set of immediate successor of stock i
$SUC(i)$	the set consisting of stock i and all its successors
C_i	the cardinality of $SUC(i)$
$P(i)$	the set of the immediate predecessors of stock i
$PRE(i)$	the set consisting of stock i and all its predecessors
A	the set of all components at the highest level of the system
Demand	
λ	average demand rate of the customer demand
$d_i(t)$	demand realization of stock i at time t
$d[t\text{-}L_i, t)$	the lead time demand over L_i units of time of stock i, i=1,2,…,N
$D_i(\tau)$	upper bound imposed on the lead time demand over τ units of time
Time parameters and variables	
T_i	production time of stock i
S_i	outbound service time of stock i
SI_i	inbound service time of stock i
L_i	net lead time of stock i
M_i	maximum replenishment time of stock i
s_1	upper bound imposed on the outbound service time of the end stock
Performance measures	
h_i^e	unit echelon on-hand inventory holding cost at stock i
h_i	unit on-hand inventory holding cost at stock i
c_i	fixed order cost for placing each order by stock i to its customer
p	cost for using operating flexibility measures to fulfill each unit of excessive customer demand
α	cycle service level (CSL) of the system
β	fill rate of the system, which is the percentage of customer demand (in quantity) fulfilled normally by using the on-hand inventory of the system without resorting to any operating flexibility measure
Inventory policy parameters	
r_i	installation reorder point of stock i
R_i	echelon reorder point of stock i
Q_i	order size of stock i
Inventory state variables (evaluated before demand occurs)	
$I_i(t)$	on-hand inventory of stock i at time t
$I_i^e(t)$	echelon on-hand inventory of stock i at time t
$IL_i(t)$	echelon inventory level of stock i at time t
$IP_i(t)$	echelon inventory position of stock i at time t

if $t + S_i \geq t{+}SI_i + T_i$, stock i can always satisfy the demand. Otherwise if $t + S_i < t{+}SI_i + T_i$, stock$i$ has to hold a certain amount of inventory to satisfy the demand occurred between $t+S_i$ and $t{+}SI_i+T_i$, the length $SI_i+T_i\text{-}S_i$ of the time interval $[t + S_i, t{+}SI_i + T_i]$ is thus called the net lead time of stock i.

B. MODEL FORMULATION

For the assembly inventory system considered, we establish a new mathematical programming model for the optimization of an echelon (R, Q) policy under the GSA by extending the original GSA proposed in [23] to take account of the cost of using operating flexibility measures and their effects on the material flows of the system. The notations given in Table 1 will be used in the formulation of the model.

The objective of the problem is to minimize the average total cost of the system per time unit in the long run, i.e., the sum of the inventory holding costs, fixed order costs and the operating flexibility costs at the end stock (stock 1). Specifically, the three types of costs can be formulated as follows:

Fixed order costs Since β is assumed to be the percentage of customer demand (in quantity) fulfilled normally by the on-hand inventory of the end stock, then, for each time unit the average customer demand fulfilled normally is $\lambda\beta$. Therefore, the average fixed order cost per unit of time for stock i can be formulated as $\frac{c_i\lambda\beta}{Q_i}$.

Operating flexibility costs As we know, 1-β can be regarded as the percentage of customer demand fulfilled by resorting to operating flexibility measures. Therefore, the average operating flexibility cost per time unit can be formulated as $p\lambda(1\text{-}\beta)$.

Inventory holding costs The inventory holding costs of stock i are considered at all stocks, and for each stock i, its average holding cost can be formulated as $h_i^e \times E[I_i^e]$ for i =1,2,..., N.

Summarizing the above three costs, the average total cost of the system per unit of time can be formulated as

$$\sum_{i=1}^{N}(\frac{c_i\lambda\beta}{Q_i}+h_i^e\times E[I_i^e])+p\lambda(1-\beta) \tag{3}$$

To obtain a mathematical expression for $E[I_i^e]$, some analysis is needed. Under the GSA, each stock i has no backorder because of using operating flexibility, then, the following equation can be derived

$$I_i^e(t)=IP_i^e(t-L_i)-d[t-L_i,t) \tag{4}$$

Define $\hat{d}[t-L_i,t)$ as the lead time demand fulfilled normally by the on-hand inventory of the stock. Since $100\beta\%$ of the total demand is fulfilled normally, then

$$E\left[\hat{d}[t-L_i,t)\right]=\beta\lambda L_i \tag{5}$$

Furthermore, it is assumed that all excessive demands are satisfied without incurring inventory holding costs. This assumption is reasonable since the occurrence of excessive demand implies zero on-hand level in the stock considered. With this assumption, we can ignore excessive demand in the calculation of expected holding cost $E[I_i^e]$. That is, $d[t-L_i,t)$ can be replaced by $\hat{d}[t-L_i,t)$ when calculate $E[I_i^e]$ according to (5). Since$IP_i^e(t)$ is uniformly distributed over the interval $[R_i+1, R_i+Q_i]$ in steady state, then:

$$E[IP_i^e]=\frac{1}{Q_i}\sum_{j=1}^{Q_i}(R_i+j)=R_i+\frac{1+Q_i}{2} \tag{6}$$

According to [23], we can prove that there exists an optimal solution with R_i given by

$$R_i=\sum\nolimits_{j\in SUC(i)}D(SI_j+T_j-S_j)+\sum\nolimits_{j\in SUC(i)}Q_j-Q_i-C_i \tag{7}$$

Then, we can derive $E[I_i^e]$ as follows:

$$\begin{aligned}E[I_i^e]&=E[IP_i^e(t-L_i)-\hat{d}[t-L_i,t)]\\&=R_i+\frac{1+Q_i}{2}-\lambda\beta L_i\\&=\sum\nolimits_{j\in SUC(i)}D(SI_j+T_j-S_j)+\sum\nolimits_{j\in SUC(i)}Q_j\\&\quad+\frac{1-Q_i}{2}-C_i-\lambda\beta(SI_i+T_i-S_i)\end{aligned} \tag{8}$$

With (3) and (8) and referring to the original GSA proposed in [4], the problem P of finding the optimal S_i, SI_i and Q_i to minimize the total cost of safety stock in the assembly systems can be formulated as follows:

$$\begin{aligned}&P:\text{Minimize}\\&\sum_{i=1}^{N}\{\frac{c_i\lambda\beta}{Q_i}+h_i^e*[\sum\nolimits_{j\in SUC(i)}D(SI_j+T_j-S_j)-\lambda\beta\\&\quad\times(SI_i+T_i-S_i)\\&+\frac{1+Q_i}{2}-C_i]+\sum\nolimits_{j\in PRE(i)}h_j^eQ_{s(i)}\}+p\lambda(1-\beta)\end{aligned}$$

$$s.t.$$

$$Q_i=m_{s(i)i}Q_{s(i)}\ for\ i=1,2,...,N \tag{9}$$

$$SI_i+T_i-S_i\geq 0\ for\ i=1,2,...,N \tag{10}$$

$$SI_i\geq\max\{S_j, j\in P(i)\}for\ i=1,2,\cdots,N \tag{11}$$

$$0\leq S_1\leq s_1 \tag{12}$$

$$Q_i,\ m_{s(i)}\geq 0\ and\ integer\ for\ i=1,2,\ldots,N \tag{13}$$

$$SI_i,S_i\geq 0\ and\ integer\ for\ i=1,2,\ldots,N \tag{14}$$

Constraint (9) is the integer-ratio constraint between the order sizes of any two successive stocks. Constraint (10) assures that the net lead time of each stock is nonnegative. Constraint (11) implies that the inbound service time of each stock must equal to or greater than the outbound service time of any of its immediate upstream stocks. Constraint (12) imposes an upper bound s_1 on the outbound service time of the end stock (stock 1), where s_1 may be given by final customers. Constraint (13) and (14) imply that all the decision variables must be integer.

Note that the target CSL α is also a decision variable of model P although it does not explicitly appear in the model, because its objective function depends on $D(SI_j+T_j\text{-}S_j)$ which in turn depends on α. The fill rate β depends on the inventory policy, the net lead time, and the Poisson demand rate of the end item, it also depends on α. By observing the objective function of model P, if optimal α and β are known, $p\lambda(1\text{-}\beta)$ becomes constant and the model P can be decomposed into two independent sub-models, one with decision variables Q_i and the other with decision variables SI_i and S_i, and the two sub-models are called the order size decision sub-problem and the reorder point decision sub-problem or the Q-problem and R-problem for short, respectively. The Q-problem has an objective function composed of all Q-dependent cost terms and constraints (9) and (13), whereas the R-problem has an objective function composed of all R-dependent cost terms and linear constraints (10), (11), (12) and (14).

IV. SOLUTION METHODOLOGY

In this section, we will present a procedure to solve model P for the optimization of (R, Q) policy. The procedure is based on a line search of the optimal target CSL α and the calculation of the corresponding fill rate β. Moreover, after analyzing the model, we get some important properties about the structure of an optimal solution of the model.

A. LINE SEARCH

To solve model P, the remaining tasks are to find optimal target CSL α and the corresponding fill rate β. The optimal α can be found by a line search over its domain, i.e., over the interval [0, 1], since $0\leq\alpha\leq 1$. We implement the line search by solving model P for each possible value of α (referred as model $P(\alpha)$), and if β is known, model $P(\alpha)$ can be efficiently solved by decomposition, i.e., by solving two sub-problems, Q-problem and R-problem. However, the fill rate β always depends on the (R, Q) policy, the (net) lead time L, and the Poisson demand rate λ of the end stock (stock 1), and the first

TABLE 2. Parameters setting for the instances.

Parameter	Description	Values
h_i^e	echelon inventory holding cost of stock $i\in\{1,2,3,4,5,6,7\}$	$U[1,5]$
c_i	fixed order cost of stock $i\in\{1,2,3,4,5,6,7\}$	$h_i^e\times U[10,20]$
p	operating flexibility cost of the system	$r\times h_1$, r=10,20,50
T_i	production time of stock $i\in\{1,2,3,4,5,6,7\}$	$U[1,5]$
s_1	upper bound of the outbound service time of the end stock (item 1)	$U[1,3]$
λ	demand rate of the system	$U[1,10]$

three parameters R, Q, and L can only be obtained by solving model $P(\alpha)$, which in turn depends on β. To overcome the difficulty caused by the interdependence of β and the three parameters in solving model $P(\alpha)$, we propose an iterative procedure to solve model $P(\alpha)$ based on guessing the value of β in each iteration. Since β is usually larger than α and close to β when α approaches 1, it is initialized to α in the procedure. As soon as the value of β does not change in two successive iterations, we have got the real β and the optimal (R, Q) policy for the system can be obtained by solving model $P(\alpha)$ at the last iteration of the procedure. The main steps of the procedure are given as follows:

Procedure BETA for solving P
Step 0: Set $\beta := \alpha$;
Step 1: Solve the Q-problem and the R-problem to get the values (R_i, Q_i) for each stock i;
Step 2: Calculate the real fill rate $\beta^*$of the system for the given (R, Q) policy;
Step 3: If $\beta^* = \beta$, stop. Otherwise, set $\beta := \beta^*$ and go to Step 1.

To implement the above procedure, a method for calculating the fill rate β in Step 2 is needed when the (R, Q) policy is given.

Let α^* denote the real CSL of the system, α^* is defined as the percentage of customer orders (in number of orders) fulfilled normally by the on-hand inventory of the end stock of the system without resorting to operating flexibility measures. The real CSL α^* may be larger than the target CSL α because of the nature of the (R, Q) policy used. In the system, after each inventory replenishment of the end stock, its inventory position will be brought to a level in the interval $[D(L_1), D(L_1)\text{-}1+Q_1]$. This level may be larger than $D(L_1)$ if $Q_1 > 1$.

For the considered assembly system with Poisson demand, each customer demand (order) contains only one unit if it occurs, so the number of backorders (orders not fulfilled on-time) equals to the quantity of demand not fulfilled on-time. Therefore, the fill rate of the system is equal to its real CSL α^*, i.e., $\beta = \alpha^*$. Under the conventional GSA framework, it is assumed that all upstream stocks (all stocks other than the end stock) never run out of stock facing a bounded lead time demand at the end stock. With this assumption, α^* can be calculated by only considering the end stock (stock 1).

After each inventory replenishment, the echelon inventory position of stock 1 will be within the interval $[R_1+1, R_1+Q_1]$, where $R_1 = D(L_1)\text{-}1$, i.e.,$L_1 = SI_1+T_1\text{-}S_1$. Since this echelon inventory position is uniformly distributed in this interval, α^* can be calculated as

$$\alpha^* = \frac{1}{Q_1}\sum_{IP=R_1+1}^{R_1+Q_1} p(d(SI_1+T_1-S_1) \le IP) \qquad (15)$$

where $SI_1 + T_1\text{-}S_1$ and $d(SI_1 + T_1\text{-}S_1)$ are the net lead time and the net lead time demand of the end stock, respectively.

Therefore, for the considered system with Poisson demand of rate λ, we can derive

$$\beta = \alpha^* = \frac{1}{Q_1}\sum_{IP=R_1+1}^{R_1+Q_1}\sum_{k=0}^{IP}\frac{[\lambda(SI_1+T_1-S_1)]^k e^{-\lambda(SI_1+T_1-S_1)}}{k!} \qquad (16)$$

B. PROPERTIES OF THE MODEL

This paper presents an extended GSA model for the considered assembly system, in which the fixed order costs and the effects of operating flexibility on the material flows of the system are incorporated. In this section, we analyze the optimal solution of the GSA model in-depth. Firstly, we study the characteristics of inbound and outbound service times in an optimal solution of the model, and the following propositions can be derived.

Proposition 1: For model P, there always exists an optimal solution such that the outbound service time of the end stock (stock 1) equals s_1, and the inbound service times of all components at the highest level are 0, That is

$$S_1 = s_1,$$
$$SI_i = 0, i \in A.$$

Proof: Since the cost terms and the constraints related to outbound and inbound service times in model P are all included in the objective function and constraints of the R-problem, the optimal values of the service times of model P can be derived by solving the sub-problem, thus, to prove this proposition, we only need to consider the R-problem which has the following objective function:

$$\sum_{i=1}^{N}[D(SI_i+T_i-S_i)\times \sum\nolimits_{j\in PRE(i)} h_j^e - h_i^e\lambda\beta(SI_i+T_i-S_i)] \qquad (17)$$

TABLE 3. The optimal solution of solving model P for the 10 instances (s_1 <T_1).

No.	s_1	T_1	β^*	$SI^*_{1\text{-}7}$	$S^*_{1\text{-}7}$	$LT^*_{1\text{-}7}$	$Q^*_{1\text{-}7}$	R_1	*time*
1	2	3	0.9834	{0,0,0,0,0,0,0}	{2,0,0,0,0,0,0}	{1,1,3,2,2,3,2}	{3,3,3,12,9,9,9}	7	0.229
2	3	4	0.9854	{0,0,0,0,0,0,0}	{3,0,0,0,0,0,0}	{1,1,1,4,5,3,3}	{3,3,6,6,6,6}	12	0.186
3	3	4	0.9474	{2,0,0,0,0,0,0}	{3,2,2,0,0,0,0}	{3,0,3,3,3,5,4}	{1,2,2,4,2,2,4}	26	0.18
4	4	5	0.9639	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{1,2,3,1,3,2,5}	{2,4,4,8,8,8,4}	9	0.171
5	4	5	0.9764	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{1,5,2,4,4,5,4}	{2,2,4,12,6,4,4}	17	0.188
6	4	5	0.9701	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{1,4,1,5,3,2,5}	{2,2,4,4,6,4,4}	7	0.192
7	3	5	0.9828	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{2,1,5,5,5,2,5}	{2,4,4,8,12,8,8}	22	0.174
8	3	5	0.9879	{1,0,0,0,0,0,0}	{3,1,1,0,0,0,0}	{3,1,0,3,5,3,2}	{4,4,4,12,12,12,12}	34	0.228
9	4	5	0.9750	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{1,4,3,1,2,5,5}	{2,4,6,8,12,6,12}	18	0.203
10	3	4	0.9855	{0,0,0,0,0,0,0}	{3,0,0,0,0,0,0}	{1,5,5,1,2,3,4}	{4,4,4,4,8,12,12}	14	0.388

According to (8) and the objective function of model P, (17) is equal to the inventory holding costs of the considered system plus (or minus) a term only depending on Q. If all Q are given, the inventory holding costs increase as each $SI_i + T_i\text{-}S_i$ increases. This implies that (17) also increases when $SI_i + T_i\text{-}S_i$ increases. As a result, the value of (17) will decrease with the increase of the outbound service time S_i if SI_i and T_i are given. This implies that the optimal outbound service time of the end stock S_1 must take its maximum possible value. For the R-problem, the constraints involving the outbound service time of the end stock are $SI_1 + T_1\text{-}S_1 \geq 0$ and $S_1 \leq s_1$.Then, in an optimal solution, the optimal outbound service time of the end stock can be determined by

$$S_1 = Min\{SI_1 + T_1, s_1\}$$

(1) if $s_1 < T_1$, it is certain that $s_1 < SI_1 + T_1$, then, the optimal outbound service time $S_1 = Min\{SI_1 + T_1, s_1\} = s_1$;

(2) otherwise, if $s_1 \geq T_1$, the following two cases may happen: If $SI_1 + T_1 \geq s_1$, then, $S_1 = Min\{SI_1 + T_1, s_1\} = s_1$. Otherwise, if $SI_1 + T_1 < s_1$, there is an optimal solution with S_1 and SI_1 so that $S_1 = Min\{SI_1 + T_1, s_1\} = SI_1 + T_1 < s_1$. Suppose that the optimal objective value of the R-problem is x. At this solution, the inventory holding cost of the end stock is zero. We can construct a new solution with $S_1^{'} = s_1$, $SI_1^{'} = s_1 - T_1$, and the inbound and outbound service times for all other stocks being the same as in the optimal solution. Note that the new solution increases SI_1 from $S_1\text{-}T_1$ to $s_1\text{-}T_1$, this only expands the feasible range of inbound and outbound service times at its immediate predecessor stocks, but does not change the optimal inbound and outbound service times of other stocks, this can be proved by considering the constraint of (11). That is, the new solution, with the change of the value of S_1 and SI_1, has no effect on the inbound and outbound service times of the other stocks and is thus feasible. Moreover, this new solution has the same objective value x for the R-problem and zero inventory holding cost of the end stock. Then, the new solution with $S_1 = s_1$ is also an optimal solution. Thus, there always exists an optimal solution such that $S_1 = s_1$.

Next, we consider the optimal solution regarding the inbound service times of all components stocks at the highest level, i.e., all stocks i with $i \in A$. Firstly, there is an optimal solution with S_i and SI_i for each stock i so that the objective value of the R-problem is x.

For this optimal solution, if $SI_i \leq S_i$ for some $i \in A$, we can construct a new solution with $SI_i^{'} = 0$, $S_i^{'} = S_i - SI_i$, and the inbound and outbound service times for all other stocks being the same as in the optimal solution. This new solution is feasible, because for all constraints related to stock i, we have $SI_i^{'} \geq 0$, $SI_i^{'} + T_i - S_i^{'} = SI_i + T_i - S_i \geq 0$ and $0 \leq S_i^{'} \leq S_i$. This new solution has the same objective value x for the R-problem, then, there exists an optimal solution with $SI_i = 0$ for all components at the highest level.

Otherwise, if $SI_i > S_i$ for some $i \in A$, we can also construct another solution, such that $SI_i^{'} = SI_i - S_i$ and $S_i^{'} = 0$. This solution is also feasible, because for all constraints related to stock i, we have $SI_i^{'} + T_i - S_i^{'} \geq 0$ and $S_i^{'} \geq 0$. The solution has the same objective value x for the R-problem, then, this solution is also optimal. Moreover, if we decrease $SI_i^{'}$ to $SI_i^{''} = 0$ and set $S_i^{''}$ to 0, we can get a new solution which satisfies all constraints related to stock i, with the objective value no larger than x for the R-problem, because the objective value of this problem will not increase with the increase of SI_i, so this new solution is an optimal solution with $SI_i^{''} = 0$. Thus, there exists an optimal solution with $SI_i = 0$ for all components at the highest level, and this proposition is thus proved. □

Proposition 2: For model P with $s_1 < T_1$, there always exists an optimal solution such that the end stock has a positive net lead time, a nonnegative reorder point, and the fill rate β less than 1. This implies that if $s_1 < T_1$, the operating flexibility measures are always used to fulfill the excessive customer demand superior to a pre-specified demand bound. That is,

$$SI_1 + T_1 - S_1 > 0,$$
$$R_1 > -1,$$
$$0 < \beta < 1.$$

Proof: Consider an optimal solution and the end stock, we first prove that if $SI_1 + T_1\text{-}S_1 = 0$, $s_1 \geq T_1$ must be satisfied. In this case, we can construct an optimal solution such that $SI_1 + T_1 = S_1$, then we have $S_1 \leq s_1$ and $S_1 = SI_1 + T_1$. This implies that $SI_1 + T_1 \leq s_1$. Since $SI_1 \geq 0$, in order to satisfy $SI_1 + T_1 \leq s_1$, $s_1 \geq T_1$ must hold. Therefore, if $s_1 < T_1$, there always exists an optimal solution such that the end stock has a positive net lead time $SI_1 + T_1\text{-}S_1 > 0$; Next,

TABLE 4. The optimal solution of solving model *P* for the 10 instances ($s_1 \geq T_1$).

No.	s_1	T_1	β^*	$SI^*_{1\text{-}7}$	$S^*_{1\text{-}7}$	$LT^*_{1\text{-}7}$	$Q^*_{1\text{-}7}$	R_1	time
1	2	1	1	{1,0,0,0,0,0,0}	{2,1,1,0,0,0,0}	{0,2,2,2,1,2,2}	{3,3,3,12,6,6,6}	-1	0.165
2	2	2	1	{0,0,0,0,0,0,0}	{2,0,0,0,0,0,0}	{0,3,3,1,1,3,2}	{2,4,2,8,8,8,6}	-1	0.166
3	3	2	1	{1,0,0,0,0,0,0}	{3,1,1,0,0,0,0}	{0,0,3,5,5,2,1}	{3,3,3,6,6,6,9}	-1	0.181
4	3	1	1	{2,0,0,0,0,0,0}	{3,2,2,0,0,0,0}	{0,1,0,5,5,2,4}	{2,4,2,8,12,6,6}	-1	0.187
5	4	4	1	{0,0,0,0,0,0,0}	{4,0,0,0,0,0,0}	{0,3,3,3,5,5,2}	{3,6,6,12,12,6,6}	-1	0.164
6	4	1	1	{3,1,1,0,0,0,0}	{4,3,4,1,1,1,1}	{0,0,0,3,0,0,1}	{2,6,4,12,12,12,12}	-1	0.173
7	4	3	1	{1,0,0,0,0,0,0}	{4,1,1,0,0,0,0}	{0,0,4,5,4,4,2}	{1,2,2,1,5,3,4,6}	-1	0.203
8	3	2	1	{1,0,0,0,0,0,0}	{3,1,1,0,0,0,0}	{0,2,4,3,1,3,3}	{1,2,2,4,4,2,4}	-1	0.185
9	3	1	1	{2,0,0,0,0,0,0}	{3,2,2,0,0,0,0}	{0,0,1,4,2,4,4,}	{2,4,2,4,4,6,6}	-1	0.193
10	3	3	1	{0,0,0,0,0,0,0}	{3,0,0,0,0,0,0}	{0,3,4,2,1,1,3}	{3,6,3,6,6,9,6}	-1	0.194

TABLE 5. Impact of the operating flexibility costs *p*.

No.	*r*	*β**	*Fixed order costs*	*Inventory holding costs*	*operating flexibility costs*	*total costs*	*time*
1	10	0.975	241.477	821.696	9.996	1073.17	0.239
	20	0.975	241.477	821.696	19.994	1083.17	0.201
	50	0.975	241.477	821.696	39.988	1103.16	0.186
2	10	0.982	154.819	513.147	1.276	669.242	0.271
	20	0.982	154.819	513.147	2.552	670.519	0.237
	50	0.982	154.819	513.147	6.381	674.347	0.242
3	10	0.973	178.906	608.764	1.355	789.025	0.198
	20	0.973	178.906	608.764	2.709	790.38	0.239
	50	0.973	178.906	608.764	6.774	794.444	0.232
4	10	0.947	134.055	326.234	7.893	468.181	0.179
	20	0.947	134.055	326.234	15.785	476.074	0.224
	50	0.947	134.055	326.234	39.463	499.752	0.241
5	10	0.988	208.281	1029.49	3.444	1241.22	0.263
	20	0.988	208.281	1029.49	6.889	1244.66	0.24
	50	0.988	208.281	1029.49	17.222	1235	0.267
6	10	0.991	166.806	731.778	4.194	902.778	0.288
	20	0.991	166.806	731.778	8.389	906.973	0.269
	50	0.991	166.806	731.778	20.972	919.556	0.237
7	10	0.954	67.018	234.174	2.301	303.493	0.217
	20	0.954	67.018	234.174	4.601	305.793	0.221
	50	0.954	67.018	234.174	11.502	312.694	0.206
8	10	0.982	142.029	383.232	1.276	526.538	0.25
	20	0.982	142.029	383.232	2.552	527.814	0.257
	50	0.982	142.029	383.232	6.381	531.643	0.239
9	10	0.976	158.451	430.065	1.435	589.951	0.201
	20	0.976	158.451	430.065	2.869	591.386	0.211
	50	0.976	158.451	430.065	7.174	595.691	0.216
10	10	0.971	96.245	289.417	0.59	388.608	0.241
	20	0.971	96.245	289.417	1.178	386.252	0.187
	50	0.971	96.245	289.417	2.945	386.841	0.223

we consider the reorder point R_1 of the end stock at an optimal solution, i.e., $R_1 = D(SI_1 + T_1\text{-}S_1)\text{-}1$. According to (2), $D(\tau)$ always increases when $\lambda\tau$ increases. Since $\tau = SI_1 + T_1\text{-}S_1 > 0$, then, we have $R_1 > D(0)\text{-}1=0\text{-}1=\text{-}1$.

In the above analysis, β can be determined by the following equation:

$$1-\beta = \frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=i+1}^{\infty}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}\cdot\frac{k-i}{k}$$
$$= \underbrace{\frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=i+1}^{\infty}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}}_{\text{term 1}}$$
$$-\underbrace{\frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=i+1}^{\infty}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}\cdot\frac{i}{k}}_{\text{term 2}};$$

Note that 1-β in the equation is the percentage of customer demand (in quantity) fulfilled by using operating flexibility measures.

Moreover, the first term in the equation can be simply rewritten as follows:

$$\frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=i+1}^{\infty}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}=1+e^{-\lambda\tau}-\frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=1}^{i}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!},$$

and the second term can be written as:

$$\frac{1}{Q}\sum_{i=R+1}^{R+Q}\sum_{k=i+1}^{\infty}\frac{(\lambda\tau)^k e^{-\lambda\tau}}{k!}$$
$$\cdot\frac{i}{k} = \frac{e^{-\lambda\tau}}{Q}[\sum_{i=R+1}^{R+Q} i\sum_{k=1}^{\infty}\frac{(\lambda\tau)^k}{k!}\cdot\frac{1}{k} - \sum_{i=R+1}^{R+Q} i\sum_{k=1}^{i}\frac{(\lambda\tau)^k}{k!}\cdot\frac{1}{k}]$$

we can derive that when τequals to 0, the values of the first term and the second term are 2 and 0, respectively. In this

TABLE 6. Impact of the upper bound s_1.

No.	s_1	β*	*Fixed order costs*	*Inventory holding costs*	*operating flexibility costs*	*total costs*	*time*
1	1	0.986	77.571	323.439	5.34	406.556	0.448
	3	0.98	77.125	274.931	7.841	359.896	0.43
	5	0.979	76.996	226.708	8.594	312.198	0.456
2	1	0.984	81.712	512.094	12.417	606.223	0.268
	3	0.98	81.369	418.688	15.722	515.779	0.241
	5	0.975	80.926	325.298	19.994	426.218	0.296
3	1	0.969	64.343	276.154	4.684	345.181	0.286
	3	0.973	64.617	239.574	4.064	308.255	0.305
	5	0.947	62.922	184.683	7.893	255.498	0.291
4	1	0.978	112.01	447.894	10.875	570.778	0.279
	3	0.983	112.59	414.703	8.341	535.635	0.266
	5	0.971	111.081	363.935	14.931	489.946	0.26
5	1	0.987	182.781	879.968	16.845	1079.59	0.268
	3	0.986	182.781	788.564	16.845	988.19	0.275
	5	0.984	182.358	697.276	19.697	879.331	0.263
6	1	0.984	114.172	607.235	4.728	726.134	0.354
	3	0.979	113.592	487.17	6.227	606.989	0.434
	5	0.964	139.645	340.19	10.831	490.665	0.445
7	1	0.983	122.217	676.649	9.826	808.692	0.269
	3	0.98	122.524	599.658	8.592	730.775	0.365
	5	0.975	121.551	501.048	12.962	635.095	0.258
8	1	0.989	209.293	1320.89	15.651	1545.83	0.413
	3	0.988	209.151	1068.73	16.662	1294.54	0.471
	5	0.983	207.99	795.553	24.89	1028.43	0.53
9	1	0.984	81.712	512.094	12.417	606.223	0.287
	3	0.982	81.574	377.761	13.747	473.082	0.238
	5	0.975	80.926	325.298	19.994	426.218	0.23
10	1	0.99	188.852	971.767	17.409	1178.03	0.276
	3	0.989	188.765	771.328	18.212	978.305	0.285
	5	0.985	187.978	554.622	25.434	768.033	0.304

case, β is 1. Moreover, since β decreases when τ increases, then β is less than 1 when τ $=SI_1 + T_1$-S_1 >0, that is, 0< β <1 when the net lead time of the end stock is positive. This proposition is thus proved. □

Proposition 3: For model P with $s_1 \geq T_1$, there always exists an optimal solution such that the end stock (stock 1) has zero-positive net lead time, reorder point R_1 of -1, and fill rate 1. That is,

$$SI_1 + T_1 - S_1 = 0,$$
$$R_1 = -1,$$
$$\beta = 1.$$

Proof: We first consider the optimal value of the net lead time at the end stock, i.e., $SI_1 + T_1$-S_1. According to the previous analysis, we can derive that the optimal value of outbound service time at the end stock is determined by (11). Moreover, two cases, $SI_1 + T_1 < s_1$ and $SI_1 + T_1 \geq s_1$ may happen. Case 1: $SI_1 + T_1 < s_1$. In this case, the optimal S_1 is equal to $SI_1 + T_1$, that is, $SI_1 + T_1$-S_1 = 0; Case 2: $SI_1 + T_1 \geq s_1$. In this case, the optimal S_1 is equal to s_1, and we have proved that there exists an optimal solution such that $S_1 = s_1$ and $SI_1 = s_1$-T_1, then $SI_1 + T_1$-$S_1 = (s_1$-$T_1) + T_1$-s_1 = 0 can be derived. Therefore, we have proved that for model P with $s_1 \geq T_1$, there always exists an optimal solution such that the end stock has zero-positive net lead time, i.e., $SI_1 + T_1$-S_1 =0. Since $SI_1 + T_1$-S_1 =0, we have $D(0)$=0 and $R_1 = D(SI_1 + T_1$-$S_1)$-1=0-1=-1. In addition, the fill rate β is equal to 1 when $SI_1 + T_1$-S_1 =0 has already been proved in the proof of proposition 2. This proposition is thus proved.□

It should be noted that in Proposition 3, the fill rate $\beta = 1$ is obtained under the assumption that all the upstream stock always quotes a service time to its successors that it can always satisfy the customer demand under the GSA framework. However, according to [18], the effectively observed CSL of a supply chain facing an unbounded stochastic external demand, i.e., the CSL of the supply chain obtained when it holds safety stocks defined according to the GSA and face the unbounded demand, is usually less than the target CSL α used for specifying the lead time demand bounds. The difference (gap) between the two CSLs is due to the fact that the CSL observed at a demand stage in a supply chain is affected by the lead time demand bounds applied at its upstream stages. This may happen when the net lead times of upstream-downstream stages are different from the net lead time of the demand stage. Applying to our case, the observation of [13] implies that if all operating flexibility measures are ignored, the effectively observed CSL of the studied assembly system is usually less than the CSL used to define its lead time demand bounds at each stock. Similarly, the effectively observed fill rate (the real fill rate) of the system is usually less than the theoretical fill rate β obtained

TABLE 7. Impact of the CSL *α*.

No.	α	β*	*Fixed order costs*	*Inventory holding costs*	*operating flexibility costs*	*total costs*	*time*
1	0.5	0.938	135.872	508.6	18.643	663.115	0.282
	0.7	0.969	140.452	581.919	9.159	731.53	0.403
	0.8	0.988	114.711	676.199	3.332	794.243	0.323
	0.9	0.996	115.504	764.539	1.283	881.326	0.323
2	0.5	0.938	116.92	569.403	31.0708	717.393	0.332
	0.7	0.969	120.861	663.804	15.2651	799.93	0.307
	0.8	0.987	122.997	737.144	6.698	866.838	0.243
	0.9	0.995	124.013	824.927	2.623	951.563	0.311
3	0.5	0.923	153.764	945.044	38.707	1137.52	0.386
	0.7	0.964	160.642	1056.24	18.075	1234.96	0.329
	0.8	0.976	162.749	1153.18	11.752	1327.69	0.525
	0.9	0.991	165.168	1284.65	4.495	1454.31	0.339
4	0.5	0.959	202.936	1437.26	60.74	1700.93	0.371
	0.7	0.983	202.968	1591.13	25.049	1824.14	0.349
	0.8	0.99	209.387	1687.45	14.986	1911.82	0.327
	0.9	0.995	210.607	1829.57	6.33	2046.51	0.292
5	0.5	0.952	181.558	530.799	84.33	796.686	0.331
	0.7	0.979	186.862	613.202	35.672	835.736	0.344
	0.8	0.987	188.347	663.115	22.05	873.512	0.357
	0.9	0.996	160.415	777.406	6.353	944.175	0.336
6	0.5	0.934	73.462	191.91	26.464	291.836	0.319
	0.7	0.966	76.011	229.416	13.504	318.931	0.438
	0.8	0.983	77.361	256.425	6.637	340.424	0.329
	0.9	0.995	78.294	294.031	1.896	374.22	0.373
7	0.5	0.946	95.401	444.309	43.10	582.81	0.367
	0.7	0.977	81.107	516.617	18.25	615.973	0.286
	0.8	0.984	81.632	552.643	13.188	643.462	0.315
	0.9	0.994	82.541	626.226	4.425	713.192	0.292
8	0.5	0.899	57.723	218.087	15.118	292.928	0.269
	0.7	0.942	62.574	252.365	8.68	323.617	0.288
	0.8	0.967	64.254	294.387	4.885	363.526	0.281
	0.9	0.984	65.363	342.745	2.379	410.488	0.332
9	0.5	0.923	121.825	367.842	38.542	528.209	0.276
	0.7	0.955	113.791	435.063	22.555	571.409	0.406
	0.8	0.975	111.638	480.798	12.496	604.933	0.292
	0.9	0.992	113.541	568.292	4.186	686.019	0.336
10	0.5	0.953	176.601	798.15	58.538	1033.29	0.308
	0.7	0.979	181.557	904.641	25.10	1111.3	0.299
	0.8	0.989	183.171	991.185	14.213	1188.57	0.271
	0.9	0.995	184.344	1086.04	6.298	1276.69	0.29

under the conventional GSA framework, i.e., less than one in the case of Proposition 3.

V. EXPERIMENTAL RESULTS

In order to gain further insights into the performance of the iterative procedure for solving P for a given target CSL a, we conduct comprehensive experiments on randomly generated instances in C++ with Visual Studio 6.0 compile, and all experiments were carried out on a PC with 2.30 GHZ and 8.00 Go RAM.

A. PARAMETER SETTINGS

We consider an assembly system with 7 stocks (see Figure 1). Each stock has only two immediate predecessors and one immediate successor, except for the stocks with no predecessor at the highest level of the BOM and for the end stock with no successor at the lowest level. All the parameters setting is given in Table 2.

B. PROCEDURE FOR SOLVING MODEL P

Since it is uncertain whether the cost function of model P is convex with respect to the target CSL α, in solving model P we vary α from 0.5 to 0.98 and discretize it with an interval of length 0.001 (with precision 0.001) by considering its values $0.5{+}0.001k$, $k = 0, 1, \ldots, 480$. For each possible value of α, we solve model P by using the procedure BETA. The (approximate) optimal solution of model P is obtained by comparing the total costs for all the values of α. In this test, the procedure is evaluated by computational experiments on 10 instances randomly generated as mentioned in subsection A, and for each instance, we calculate six values as the optimal value of β (β^*), and optimal inbound service time (SI^*), optimal outbound service time (S^*), optimal lead time (LT^*), optimal order sizes (Q^*) for each stock i, i =1,2,...,7 and the reorder point at the end stock (R_1). The results are given in Table 3 and Table 4, with the restricted case $s_1 < T_1$ and $s_1 \geq T_1$, respectively.

From the results, we can demonstrate that: (1) the procedure BETA has a good convergence property and is computationally efficient for solving the model P with a given CSL α; (2) for all 10 instances, we can derived that $S_1 = s_1$, $SI_4 = SI_5 = SI_6 = SI_7 = 0$, this solution can verify Proposition 1; (3) for the restricted case: $s_1 < T_1$, from Table 3, the following optimal solutions are also derived: $LT_1 > 0$, $R_1 > 0$ and $\beta > 0$, this solution satisfies Proposition 2; (3) Similarly, Proposition 3 can also be identified by the optimal solution that $LT_1 = 0$, $R_1 = 0$ and $\beta = 1$ in Table 4.

C. SENSITIVITY ANALYSIS

From the experiment results, we identify three important drivers for the optimality of the cost structure: unit operating flexibility cost (p), an upper bound of the outbound service time at the end stock (s_1) and the CSL (α). To assess the effect of the three parameters on the cost structure, three sets of instances are evaluated, and for each instance, except the optimal value of β (β^*), we also compare fixed order costs, inventory holding costs, operating flexibility costs and total costs in each set of instances.

1) UNIT OPERATING FLEXIBILITY COST p

Firstly, we explore the impact of the unit operating flexibility cost on randomly generated instances with $\alpha = 0.8$ and $s_1 = 1$. 10 sets of instances were tested, each set corresponds to a different unit operating flexibility cost p by $r \times h_1$, where $r \in \{10, 20, 50\}$. The results of this test are given in Table 5.

From Table 5, the results indicate that with an increase in the unit operating flexibility cost, operating flexibility costs and total costs increase slightly, other optimal values remain unchanged, since they only depend on the β^*. It is demonstrated that the unit operating flexibility cost (p) has a minor impact on the fill rate $\beta$and the cost structure of the considered system.

2) UPPER BOUND ON OUTBOUND SERVICE TIME AT THE END ITEM s_1

Similarly, we also evaluate 10 sets of randomly generated instances with $\alpha = 0.8$ and $p = 20 \times h_1$, each set corresponds to a different s_1 with 1, 3 and 5, other parameters are given randomly as in Table 2. The results are depicted in Table 6.

From Table 6, we observe that the optimal fill rate β^*, fixed order costs, inventory holding costs always decrease in s_1, whereas operating flexibility costs increases in s_1.

3) CYCLE SERVICE LEVEL (CSL) α

Although the CSL α does not explicitly appear in the GSA presented, it is also a decision variable of model P, this is because its objective function depends on $D(SI+T-S)$ which is in turn depends on α. In addition, another variable, the fill rate β, depends on the inventory policy parameters, the net lead time, and the Poisson demand rate of the end stock. It in turn depends on α. The performance was evaluated by computational experiments on 10 sets of randomly generated instances as presented in Table 2. For each set of instances, four different CSL level varies in 0.5, 0.7, 0.8 and 0.9. The results are given in Table 7.

From Table 7, we can see that the optimal fill rate β^* increases in α, in turn, the fixed order costs, inventory holding costs also increases in α, whereas the operating flexibility costs decreases in α.

VI. CONCLUSION

In this paper, we have studied a continuous review assembly inventory system with Poisson demand, fixed order costs, and controlled by an echelon (R, Q) policy. We used the extended GSA to optimize the parameters of the policy under the assumption that excessive demand beyond a pre-specified bound will be fulfilled by using operating flexibility measures. Different from original GSA proposed by the authors of [4], we also consider fixed order costs for placing orders at each stock and the operating flexibility costs for fulfilling excessive demand. We first formulated a deterministic mathematical model for the inventory policy optimization problem. Then the model is solved by an iterative procedure when the target CSL is given. We also analyze the model and get some important properties about the optimal solution of the model. Experiments results and the sensitivity analysis demonstrate the efficiency of the algorithm and the properties of optimal solutions presented in this paper are also verified to be correct.

This study has demonstrated advantages of the GSA for the optimization of assembly systems with fixed order costs and operating flexibility costs. The results in this paper can be extended to more general multi-echelon inventory systems with other demand process such as normally distributed demand. Moreover, the conclusion in this paper can also be used in the optimization of closed-loop supply chain with the consideration of reverse logistics. These will be our future research topics.

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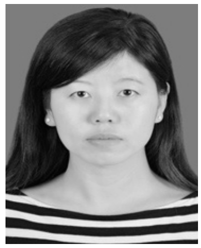

PENG LI received the B.S. degree in management from Zhengzhou University, the M.S. degree in management science and engineering from Northwestern Polytechnical University, China, and the Ph.D. degree in supply chain management from the University of Technology of Troyes, France, in 2013. Since 2014, she has been a Lecturer with the Faculty of Economics and Management, Xi'an University of Technology, China. She has authored one book and more than ten articles. Her research interests include supply chain management, inventory optimization, and logistics management.

DI WU received the B.S. degree in mathematics and applied mathematics from Northwestern Polytechnical University, and the M.S. degree in management science and engineering from the Xi'an University of Technology, in 2016, where he is currently pursuing the Ph.D. degree with the Supply Chain Integration and Service Innovation Institute. He has been engaged in dual-channel reverse supply chain, since 2016, and has published several related papers in the field.