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Modeling the Impact of Heterogeneous Transmission Powers on the Throughput of IEEE 802.11 Multi-Hop Ad Hoc Networks

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ABSTRACT Transmission power control (TPC) leads to heterogeneous transmission power levels of the nodes in multi-hop wireless ad hoc networks. This eventually results in a quite complicated collision scenario due to hidden and exposed terminal problems, and brings a big challenge for analyzing the saturation throughput of the flows. This paper presents a saturation throughput model for IEEE 802.11 multi-hop ad hoc networks with heterogeneous transmission powers. In the model, one tagged node goes through with three processes, i.e., backoff, freezing and transmission, and each of which is composed of a certain number of continuous fixed length time slots. We put forward a four-dimensional Markov chain model for the behavior of the tagged node in each fixed-length time slot. Under the condition of heterogeneous transmission powers, we consider two types of collisions, i.e., the instantaneous and persistent collisions. Their probabilities are characterized by the one-step transition probability matrix of the Markov chain. The per-flow saturation throughput of the flow is eventually derived through an iterative way. Finally, the accuracy of our model is validated by comparing the analytical values and the simulation results, and the impact of the heterogeneous power levels on the performance of the wireless multi-hop ad hoc networks is effectively analyzed.

INDEX TERMS IEEE 802.11 DCF, Markov chain, heterogeneous transmission powers, multi-hop ad hoc networks, per-flow throughput.

I. INTRODUCTION

In the past decade, wireless ad hoc networks have attracted worldwide attention and popularity due to their desirable features, such as low cost, rapid deployment, self-organization and resistance to destruction etc. In such a network, transmission power control increases the reuse of the spatial channel and improves the overall energy consumption, which consequently prolongs the lifetime of the network. However, transmission power control leads to heterogeneous transmission power levels of the nodes in multi-hop wireless ad hoc networks. This eventually results in a quite complicated collision scenario due to hidden and exposed terminal problems, and brings a big challenge for analyzing the saturation throughput of the flows.

Nowadays, the IEEE 802.11 distributed coordination function (DCF) is the most widely used media access control (MAC) protocol in wireless ad hoc networks. This protocol adopts the carrier sense multiple access with collision avoidance (CSMA/CA) mechanism and the binary exponential backoff (BEB) rules to coordinate the channel access among the competing nodes. For energy saving and throughput improving purposes, many power control protocols using heterogeneous transmission powers are proposed [1]–[3]. Among them, the dynamic distributed power control MAC scheme [1] greatly improves the performance of the network

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by achieving a very high spatial reuse. In this work, we will present a saturation throughput model to analyze the DCF protocol of the multi-hop wireless ad hoc network with heterogeneous transmission powers.

A. MOTIVATIONS

In the past decades, a considerable work is performed to model the DCF protocol mostly for the single-hop scenarios [1]–[4]. These models have a lot in common with regards to the model assumption, i.e., the collisions only occur at the start instant of the frame transmission. This assumption is true in single-hop scenarios. As a result, the saturation throughput of an individual flow in single-hop scenarios can be derived by divide the total network saturation throughput by the number of flows in the network.

However, the situation is quite different in multi-hop scenarios. Collisions induced by the jammers within the physical carrier sense range of the transmitter only occur at the start instant of the packet transmission, whereas collisions induced by the jammers hidden to the transmitter may occur during the whole frame transmission period. Consequently, the saturation throughputs of individual flows in multi-hop scenarios have different values.

In recent research, there are a few works consider the multi-hop scenario. For example, in [11], a three-dimensional Markov chain model is presented to evaluate the multi-hop network. However, they assume all the nodes have the same throughput, which is far from the practical situation. Thus, the accuracy of this work is inaccurate. Literatures [1] and [12] also present two new models to analyze the impact of the heterogeneous transmission powers, but they are not applicable for complex topologies.

In this work, we presented a four-dimensional Markov chain model to analyze the behavior of each node. Thus, the per-flow throughput is more close to the practical situation. Through calculating the collision and transmission probabilities of the proposed model, we derive the per-flow saturation throughput iteratively. By comparing the analytical results and the simulation values, we validated the effectiveness of our proposed model.

B. CONTRIBUTIONS

In this paper, we focus on analyzing the saturation throughput of IEEE 802.11 DCF in the multi-hop networks with heterogeneous transmission powers. The main contributions consist of three aspects as follows.

(1) We present a four-dimensional Markov chain model to analyze the saturation throughput. In the model, one tagged node goes through with three processes, i.e., backoff, freezing and transmission. The behavior of the node in each fixed-length time slot is analyzed.
(2) We derive the closed-form expressions of the collision and transmission probabilities by means of the non-null one-step transition probabilities, and obtain the probability that a tagged node is in the freezing process.
(3) The per-flow saturation throughput is eventually derived through an iterative way. By comparing the analytical results and the simulation values, we first validate the accuracy of our model, then analyze the impact of heterogeneous transmission powers on the performance of the multi-hop ad hoc networks.

C. ORGANIZATION

The rest of this paper is organized as follows. Section II gives the most related works in the literature. In Section III, we present our analytical model followed by the derivation of the expressions of the conditional collision probability, the transmission probability in each fixed-length slot, and the per-flow throughput for a certain probability set of the available transmission power levels. In Section IV, we validate the accuracy of our model and analyze the impact of the transmission power on the per-flow throughput of the multi-hop wireless ad hoc networks numerically. Finally, some conclusions are given in Section V.

II. RELATED WORKS

In recent research, many models are performed to analyze the performance of IEEE 802.11 DCF. One of the most significant models is the one proposed by Bianchi, which uses Markov chain to model the backoff procedure [4]. They assume that collision would occur if two or more nodes transmit packets simultaneously. Through the derivation of the Markov chain model, a closed-form expression of the saturation throughput of a single-hop wireless ad hoc network is obtained. Based on their work, many analytical works have been done to enhance the model by considering the backoff freezing details [6], the capture effects [7] etc.

In [15], Chandra *et al.* presented a thorough IEEE 802.11ad MAC protocol model. They use a three-dimensional Markov chain to analyze the dependencies on the contention period and the number of sectors on the MAC delay and throughput. In [16], the proposed constructive and versatile model framework can handle various types of multi-hop wireless paths, such as the topologies where the nodes are exposed to the well-known hidden terminal problem. The semi-Markov chain-based throughput model given in [17] can work accurately with both single and multi-hop networks with various topologies over a large range of traffic loads.

To solve the hidden terminal problems, Jang *et al.* proposed a new two-dimensional Markov chain model [18]. This model takes into account the interactions between the two stations by jointly modeling the backoff stage of each of the two stations. These models can efficiently analyze the single or multi-hop wireless networks, but they did not consider the impact of heterogeneous transmission powers. Since transmission power control can effectively improve the spatial channel reuse and save energy consumption, it has been widely adopted in IEEE 802.11 DCF. Accordingly, many analytical works are also performed for these networks.

In a single-hop wireless ad hoc network, the impact of the heterogeneous transmission powers on the performance of the network can be modeled by simply considering the capture effect when computing the collision probability [8]. Nyandoro *et al.* analyze the performance of IEEE 802.11 DCF protocol with the assumption that some nodes transmit a higher power than others [5]. They conclude that the differentiation of the quality-of-service (QoS) in IEEE 802.11 WLANs could be achieved by inducing the capture effect. In [9], Patras *et al.* proposed a power-hopping MAC protocol to enhance the throughput. In their work, they also prove that the collision probability is minimized when all power levels are randomly chosen with equal probability.

In a multi-hop wireless ad hoc network, the collision situations under the condition of heterogeneous transmission powers are more complex than those in the single-hop network. A few works have been done to analyze them. Based on the two-dimensional Markov chain model of Bianchi's work [10], a three-dimensional Markov chain model is proposed to calculate the throughput of a single flow in a multi-hop network in [11]. Simply multiplying the single node throughput by the number of nodes, they obtain the overall throughput of the network.

Unlike the single-hop network, each node in the multi-hop network may have various throughputs because of their variable locations. Therefore, the proposed three-dimensional model in [11] cannot effectively model the multi-hop networks. In [14], we present a fixed-length slot based model to analysis the saturation throughput of IEEE 802.11 DCF protocol in multi-hop networks with single transmission power. In this work, we extend the model to analyze the saturation throughput of the network with heterogeneous transmission powers.

In [12], Tsertou and Laurenson pointed the limitation of Bianchi's model and proposed a modeling method by making use of the fixed-length slot to estimate the transmission probability of each slot. However, this work only considers the communication between two transmitting nodes and one receiving node without considering any complex topologies. Based on the notion of fixed-length slot, Garetto et al. proposed a new modeling method by combing Bianchi's model and the continuous time Markov model iteratively [1]. Similarly, this work also failed to be implemented in the complex topologies. In [13], an enhanced TPC algorithm was proposed to maximize the multi-hop end-to-end throughput and reduce the energy consumption of the transmission nodes by adjusting the transmission powers.

In general, the aforementioned works cannot effectively analyze the per-flow throughput under the condition of diverse transmission power levels. Therefore, a more appropriate mode is in need to analyze the saturation throughput of IEEE 802.11 DCF in multi-hop wireless ad hoc networks with heterogeneous transmission powers.

III. PROPOSED FOUR-DIMENSIONAL MARKOV CHAIN MODEL AND PERFORMANCE ANALYSIS

In this section, we describe the proposed four-dimensional Markov chain model for multi-hop networks with heterogeneous transmission powers in detail. In the model, three main processes, i.e., the backoff, transmission and freezing processes, are analyzed. In the analysis, we first demonstrate the reasons which cause the failed transmission situations. Next, the collision and transmission probabilities of one tagged node are derived. Finally, the per-flow throughput is obtained through an iterative method.

A. PROPOSED MARKOV CHAIN MODEL WITH HETEROGENEOUS TRANSMISSION POWER LEVELS

In this paper, we focus on the impact of the heterogeneous transmission powers on the performance of DCF. In the model, all the nodes follow the DCF protocol except that they randomly select a power level x with probability f_x ($x = 1, 2, \ldots, N$) in the transmission process. Since both mechanisms of the IEEE 802.11 DCF protocol consist of the basic handshaking process, we consider the basic access mechanism to facilitate our analysis. This work can be further extended to analyze the RTS/CTS handshaking mechanism.

The proposed four-dimensional Markov chain model is given in Fig. 1. In the model, the state of one node is described as $\{i, j, k, l\}$, and the definitions of these symbols are list in Table 1.

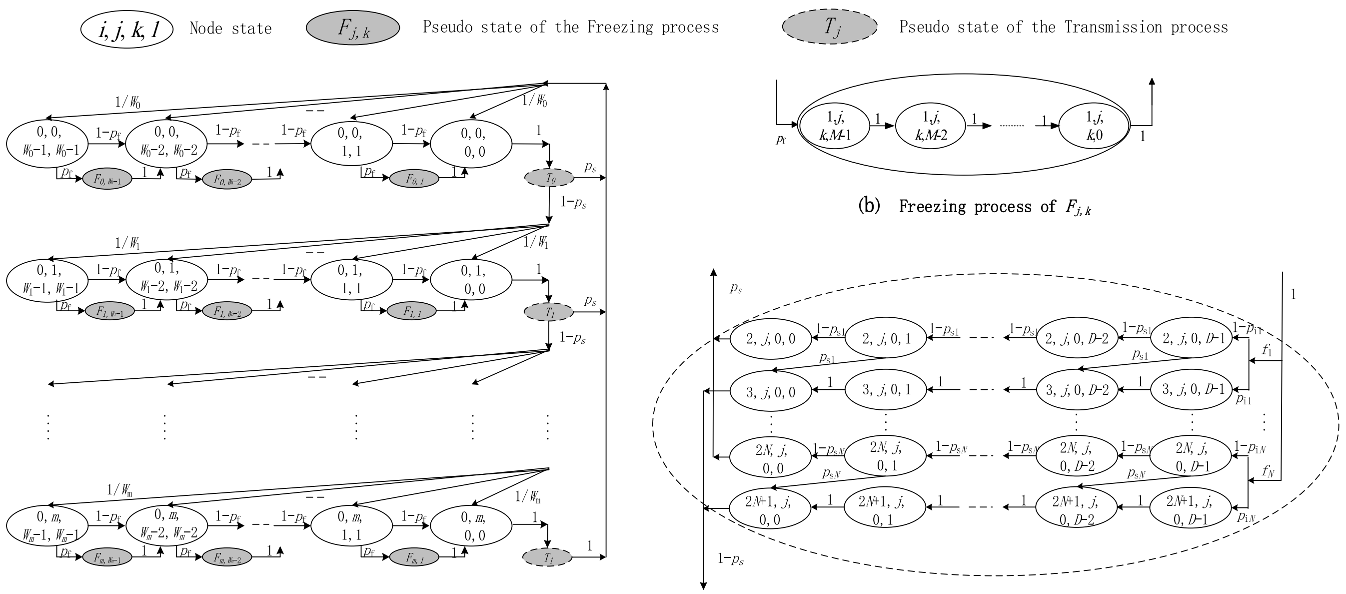
In DCF, a node detects the channel state before its transmission. If the channel keeps idle for a fixed-length time σ, the node starts a backoff slot and keeps sensing the channel. At the end of a backoff slot, the backoff counter k decreases by 1. When $k = 0$, the node enters into the transmission process. During the backoff process, if the channel becomes busy, the backoff counter is freezed, and the node enters the freezing process until the channel is sensed idle again.

Fig. 1(a) gives the overall backoff process, in which one node enters into the freezing process with probability p_f. The freezing and transmission processes are separately described with the pseudo states $F_{j,k}$ and T_j, and their detailed states are given in Figs. 1(b) and 1(c). In Fig. 1(b), the node enters into the next backoff slot with probability 1 in the freezing process. After M slots, the node returns to the backoff process. In Fig. 1(c), the node chooses a random power level x with probability f_x in the transmission process. After D slots, the node goes to the initial backoff process with probability p_s if the packet is successfully transmitted. Otherwise, the node doubles the contention window (CW) and increases j by 1.

According to the BEB mechanism of DCF, if j reaches a specific limitation m', CW is set to be the maximum size. If j exceeds the threshold m, the node drops the packet and goes back to the initial backoff process by resetting $j = 0$. Denote by W_i the value of the contention window size of the i-th backoff stage. Then we have

$$W_i = \begin{cases} 2^i W_0 & 0 \le i \le m' \\ W_{\max} & m' < i \le m, \end{cases} \tag{1}$$

where $m' = \log_2(W_{\max}/W_0)$, $W_{\max}$ is the maximum contention window size. The transmission probabilities of the model in Fig. 1(a) is given in (2). Note that $p(b|a)$ refers to



(a) Four-dimensional Markov chain model

(b) Freezing process of $F_{j,k}$

(c) Transmission process of T_j

FIGURE 1. **Proposed four-dimensional Markov chain model of the multi-hop ad hoc networks with heterogeneous transmission powers.**

the transition probability from state "a" to state "b".

$$\left\{\begin{array}{l}
p(0,j,k,k|0,j,k+1,k+1) = 1 - p_f(n) \\
\qquad\qquad 0 \le j \le m,\ 0 \le k \le W_j - 2 \\
p(1,j,k,M(n)-1|0,j,k,k) = p_f(n) \\
\qquad\qquad 0 \le j \le m,\ 1 \le k \le W_j - 1 \\
p(1,j,k,l|1,j,k,l+1) = 1 \\
\qquad\qquad 0 \le j \le m,\ 1 \le k \le W_j - 1, \\
\qquad\qquad 0 \le l \le M(n) - 2 \\
p(0,i,j,j|1,i,j+1,0) = 1 \quad 0 \le i \le m,\ 0 \le j \le W_i - 2 \\
p(2x,j,0,D-1|0,j,0,0) = 1 - p_{ix}(n) \\
\qquad\qquad 0 \le j \le m \\
p(2x+1,j,0,D-1|0,j,0,0) = p_{ix}(n) \\
\qquad\qquad 0 \le j \le m \\
p(2x,j,0,l-1|2x,j,0,l) = 1 - p_{sx}(n) \\
\qquad\qquad 0 \le j \le m,\ 1 \le l \le D-1 \\
p(2x+1,j,0,l-1|2x,j,0,l) = p_{sx}(n) \\
\qquad\qquad 0 \le j \le m,\ 1 \le l \le D-1 \\
p(0,0,k,k|2x,j,0,0) = 1/W_0 \\
\qquad\qquad 0 \le j \le m,\ 1 \le k \le W_0 - 1 \\
p(0,j+1,k,k|2x+1,j,0,0) = 1/W_{j+1} \\
\qquad\qquad 0 \le j \le m-1, \\
\qquad\qquad 0 \le k \le W_{j+1} - 1 \\
p(0,0,k,k|2x+1,m,0,0) = 1/W_0 \\
\qquad\qquad 0 \le k \le W_0 - 1.
\end{array}\right. \tag{2}$$

TABLE 1. **Symbol definitions.**

Symbol	Definition
i	Process of a tagged node, i.e., i=0 and i=1 represent the backoff and freezing processes, respectively; i=2x and i=2x+1 (i>1, x=1, 2, …, N) separately denote the failed and successful situations of the transmission process when the power level is x.
j	Backoff stage.
k	Backoff time counter.
l	Number of remaining slots in the current process. When the tagged node is in the backoff process, l=k.

The per-flow saturation throughput is defined as the limit reached by the throughput of a flow as the offered load increases. Thus, the per-flow saturation throughput is calculated by

$$S = \frac{\tau_x(n) p_s(n) E[P]}{\sigma}, \tag{3}$$

where $\tau_x(n)$ is the transmission probability of node n at power level x, $p_s(n)$ is the successful transmission probability of a packet, E[P] is the average packet size of the flow, and σ is the size of the fixed-length.

B. BACKOFF PROCESS

Let $p(i,j,k,l)$ be the steady-state probability of a node in state $\{i,j,k,l\}$. Based on Fig. 1(a), we have

$$p(0,j,k,k) = p(0,j,k+1,k+1) + p(0,j-1,0,0)(1-p_s(n))/W_j, \tag{4}$$

where $1 \le j \le m, 0 \le k \le W_j - 2$. Especially, when $k = W_j - 1$, we have

$$p(0,j,W_j-1,W_j-1) = \frac{(1-p_s(n))p(0,j-1,0,0)}{W_j}. \tag{5}$$

Then, the probability that a node is in any state of the backoff process is

$$p(0,j,k,k) = \frac{(W_j - k)(1-p_s(n))p(0,j-1,0,0)}{W_j}, \quad 0 < k \le W_j - 1, \quad 0 < j \le m. \tag{6}$$

If the transmission is failed, the backoff stage increases. We have

$$p(0,j,0,0) = (1-p_s(n))p(0,j-1,0,0), \quad 1 \le j \le m. \tag{7}$$

Then, the probability of any state in the backoff process can be further expressed as a function of $p(0,0,0,0)$, i.e.,

$$p(0,j,k,k) = \frac{(W_j - k)(1-p_s(n))^j p(0,0,0,0)}{W_j}, \quad 0 < k \le W_j - 1, \quad 0 \le j \le m. \tag{8}$$

Hence, we can drive the probability of node n being in the backoff process as follows:

$$\sum_{j=0}^{m}\sum_{k=1}^{W_j-1} p(0,j,k,k) = \frac{p(0,0,0,0)}{2} \times \left[\frac{1-(2(1-p_s(n)))^{m+1}}{1-2(1-p_s(n))}W_0 - \frac{1-(1-p_s(n))^{m+1}}{p_s(n)}\right]. \tag{9}$$

In DCF, the node starts to transmit packet wen $k = 0$. Since the node n chooses a random power level x with probability f_x, the transmission probability of node n at power level x can be derived by

$$\tau_x(n) = f_x \sum_{j=0}^{m} p(0,j,0,0). \tag{10}$$

Note that $\sum f_x = 1$.

C. TRANSMISSION PROCESS

1) SUCCESSFUL TRANSMISSION

As is shown in Fig. 1(b), node n chooses power level x with probability p_x in the transmission process. In the first slot ($l = D-1$), the probability that no collision occurs is $1 - p_{ix}(n)$. In other slots, the probability that no persistent collision slot occurs is $1 - p_{sx}(n)$. We have

$$p(2x,j,0,l) = \begin{cases} p(0,j,0,0)(1-p_{ix}(n)) & l = D-1 \\ p(1,j,0,l+1)(1-p_{sx}(n)) \\ \qquad\qquad\qquad 0 \le l < D-1, \end{cases} \tag{11}$$

where $j \in [0,m]$. With (7) and (11), the probability that the tagged node is in the successful transmission process in a generic time slot at power level x can be obtained.

$$\sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x,j,0,l) = (1-p_{ix}(n))\frac{1-(1-p_{sx}(n))^D}{p_{sx}(n)} \times \frac{1-(1-p_s(n))^{m+1}}{p_s(n)}p(0,0,0,0). \tag{12}$$

2) FAILED TRANSMISSION

At power level x, the failed transmission state of the first slot is transferred from the last state of the backoff process with probability $p_{ix}(n)$. In other slots, the failed transmission state can be transferred either from a successful transmission slot or a backoff slot with probability $p_{sx}(n)$ as is shown in Fig. 1(b). Therefore, we have

$$p(2x+1,j,0,k) = \begin{cases} p(0,j,0,0)p_{ix}(n), & k = D-1 \\ p_{sx}(n)\displaystyle\sum_{l=k+1}^{D-1} p(2x,j,0,l) + p(2x+1,j,0,D-1), \\ \qquad\qquad 0 \le k < D-1 \end{cases} \tag{13}$$

where $j \in [0,m]$. Substituting (7) to (13), we can derive the probability that node n remains in the failed transmission process at power level x in a generic time slot as follows

$$\sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x+1,j,0,l) = (D - (1-p_{c1}(n))\frac{1-(1-p_{c2}(n))^D}{p_{c2}(n)}) \times \frac{1-(1-p_s(n))^{m+1}}{p_s(n)}p(0,0,0,0). \tag{14}$$

The probability that the tagged node is in the transmission process at power level x can be expressed as

$$\tau'_x(n) = \sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x,j,0,l) + \sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x+1,j,0,l). \tag{15}$$

Given the length of the packet, D can be expressed as

$$D = \frac{(t_H + \mathrm{E}[P] + \mathrm{SIFS} + t_{ACK})}{\sigma}, \tag{16}$$

where t_H and t_{ACK} represent the time period for transmitting the head of the packet and the ACK frame, respectively. SIFS is the short interference space.

3) COLLISION PROBABILITY

In this subsection, we calculate the collision probability of the transmission process. Since the collision situation in the first slot differs from that in other slots, p_{ix} and p_{sx} are separately used to denote the collision probabilities of the first and subsequent slots. With heterogeneous transmission powers, the transmission of the tagged node may be collided by the transmissions from the interfering nodes located within a distance of

$$r_{\text{co}} = d\sqrt[4]{\frac{P_{x'}}{P_x}\text{SINR}_{\text{rx}}}, \tag{17}$$

where d is the distance between the transmitter and the receiver, SINR_{rx} is the minimum threshold of the signal-to-interference-plus-noise ratio of a successful transmission at the receiver side, P_x and P_x, are the transmission powers of the tagged node and the interfering node, respectively.

In our model, two types of collisions, i.e., the instantaneous and persistent collisions, are considered as in [10]. The instantaneous collision zone (IZ for short) is defined as the joint area of the physical carrier sense range of the transmitter and the collision range of the receiver, and the persistent collision zone (PZ for short) is the area within the collision range of the receiver and outside of the physical carrier sense range of the transmitter. Besides, we define the acknowledge zone (AZ for short) to be the area that the transmitters are out of PZ and the receivers are within the collision range.

At the first slot of the transmission process, the transmission of a tagged node n is collided in three cases: 1) any other transmitter within IZ transmits a frame at the same time; 2) the transmitters within PZ are in the transmission process; 3) the receivers within AZ transmit ACK frames. Thus, the collision probability $p_{ix}(n)$ can be calculated through

$$\begin{aligned} p_{ix}(n) = 1 - \prod_{\substack{i\in IZ,j\in PZ,\\ k\in AZ}} &(1-\tau_{x_i}(i))(1-\tau'_{x_j}(j)) \\ &\times (1-\tau_{x_k}(k)p_{suc,x_k}(k)), \end{aligned} \tag{18}$$

where $p_{suc,xk}(k)$ is the probability that the transmitted packet of node k with power level x_k is successfully received. At subsequent slots, cases 2) and 3) always exist. Therefore, the collision probability $p_{sx}(n)$ is derived by

$$p_{sx}(n) = 1 - \prod_{i\in IZ,j\in PZ} (1-\tau_{x_i}(i))(1-\tau_{x_j}(j)p_{suc,x_j}(j)). \tag{19}$$

The packet can be successfully received only when no collision occurs during the whole transmission process. Hence, the successful probability of node n at power level x is

$$p_{suc,x}(n) = (1-p_{Ix}(n))(1-p_{sx}(n))^{D-1}. \tag{20}$$

Considering the successful situations of all the power levels, the successful probability of node n can be derived as

$$p_s(n) = \sum_{x=1}^{N} f_x p_{suc,x}(n). \tag{21}$$

D. FREEZING PROCESS

Based on the second and third equations of (2), the probability that the tagged node is in any state of the freezing process can be obtained as

$$p(1,j,k,l) = p_f(n)p(0,j,k,k) \quad 0\le j\le m,\ 1\le k\le W_j-1 \\ 0\le l\le M(n)-1. \tag{22}$$

By adopting the normalization condition, we have

$$\begin{aligned} &\sum_{j=0}^{m}\sum_{k=1}^{W_j-1} p(0,j,k,k) + \sum_{x=1}^{N}\sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x+1,j,0,l) \\ &\quad + \sum_{x=1}^{N}\sum_{j=0}^{m}\sum_{l=0}^{D-1} p(2x,j,0,l) + \sum_{j=0}^{m}\sum_{k=1}^{W_j-1}\sum_{l=0}^{M(n)-1} p(1,j,k,l) \\ &= 1. \end{aligned} \tag{23}$$

For convenience, we assume that

$$p_1 = \frac{(1+M(n)*p_f(n))}{2}, \tag{24}$$

$$p_2 = D\frac{1-(1-p_s(n))^{m+1}}{p_s(n)}. \tag{25}$$

It should be noted that $p_f(n)$ and $M(n)$ in the freezing process can be calculated iteratively based on the continuous-time Markov chain model proposed in [14].

Substituting (8), (9), (12), (14), and (22) into (23), we have, (26), as shown at the bottom of this page.

Finally, by substituting the expression of p(0,0,0,0) into the former equations, we can derive the per-flow throughput. In our work, we focus on the MAC layer saturation throughput of the multi-hop networks. Similar with [3], from the viewpoint of the MAC layer, the concept of "per-flow" in our work is actually "one of the hops in a multi-hop flow".

$$p(0,0,0,0) \begin{cases} \dfrac{1}{p_1\left[\frac{1-2^{m+1}(1-p_s(n))^{m+1}}{2p_s(n)-1}W_0 - \dfrac{1-(1-p_s(n))^{m+1}}{p_s(n)}\right]+p_2} & m\le m' \\[2ex] \dfrac{1}{p_1\left[\begin{array}{l}\dfrac{1-(2-2p_s(n))^{m'+1}}{2p_s(n)-1}W_0 - \dfrac{1-(1-p_s(n))^{m+1}}{p_s(n)} \\ +\frac{(W_{\max}-1)(1-p_s(n))^{m'+1}\left(1-(1-p_s(n))^{m-m'}\right)}{p_s(n)}\end{array}\right]+p_2} & m>m' \end{cases} \tag{26}$$

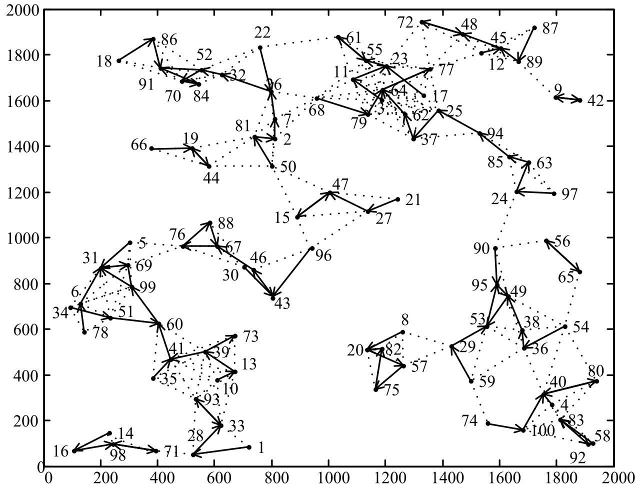


FIGURE 2. Topology of the simulated network with 100 randomly distributed flows.

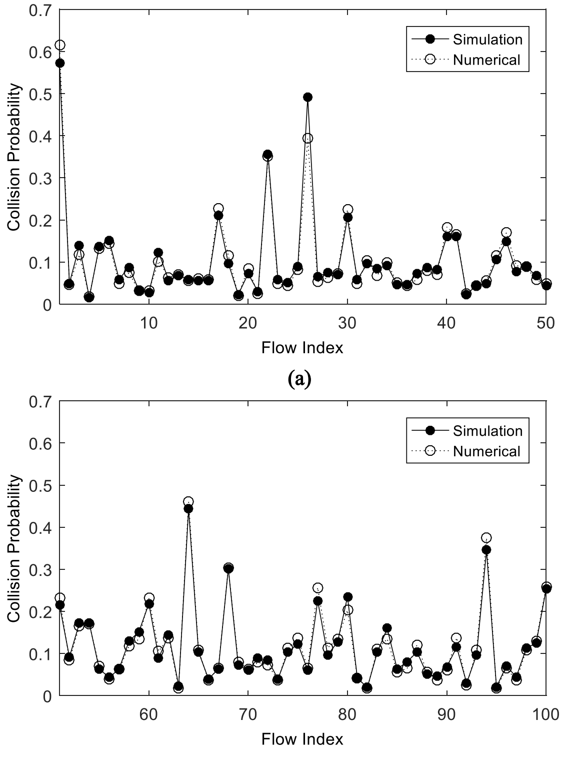


FIGURE 3. Validation of the collision probabilities in each flow: (a) results of flows 1-50; (b) results of flows 51-100.

Thus, the transmission powers are determined on a per-flow basis.

IV. MODEL VALIDATION AND NUMERICAL ANALYSIS

In this section, we evaluate the performance of the wireless ad hoc network with heterogeneous transmission powers. The performance evaluation consists of three parts, i.e., collision probability, transmission probability and per-flow throughput. In each part, we first validate the accuracy of the proposed four-dimensional Markov model. Next, we investigate the impact of the heterogeneous power levels on the performance of the multi-hop ad hoc networks.

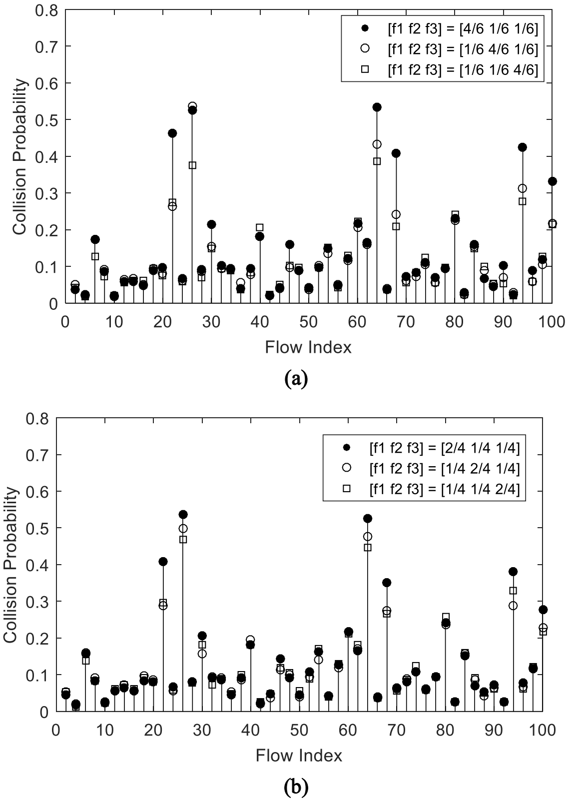


FIGURE 4. Collision probabilities with different settings of power level selection probabilities: (a) results of scenario S2; (b) results of scenario S3.

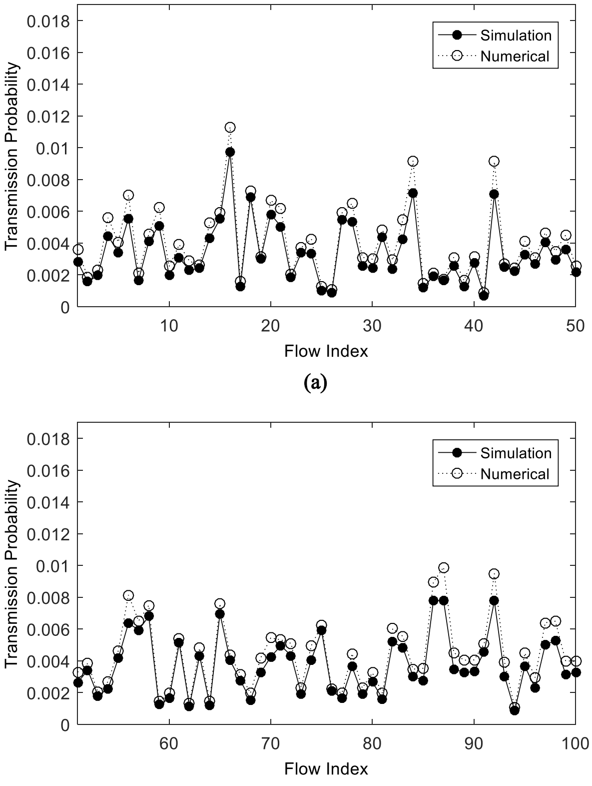
TABLE 2. Simulation parameters.

Parameter	Value	Parameter	Value
Propagation model	Two-ray	Bandwidth	11Mbps
Retransmission limit	4	Payload	256Bytes
PHY header size	192bits	MAC header	224bits
Slot duration	20μs	SIFS	10μs
Propagation limit	-87dBm	SINR Threshold	10dB
Distance between the transmitter and receiver	200m	Physical carrier sense range	530m

In our simulations, we operate in saturation conditions to derive the per-flow saturation throughput, i.e., the transmission queue of the flow transmitter is always nonempty. The simulations are conducted in EXata 2.1 simulator, and the parameters of this simulator are lists in Table 2.

A. NETWORK TOPOLOGY

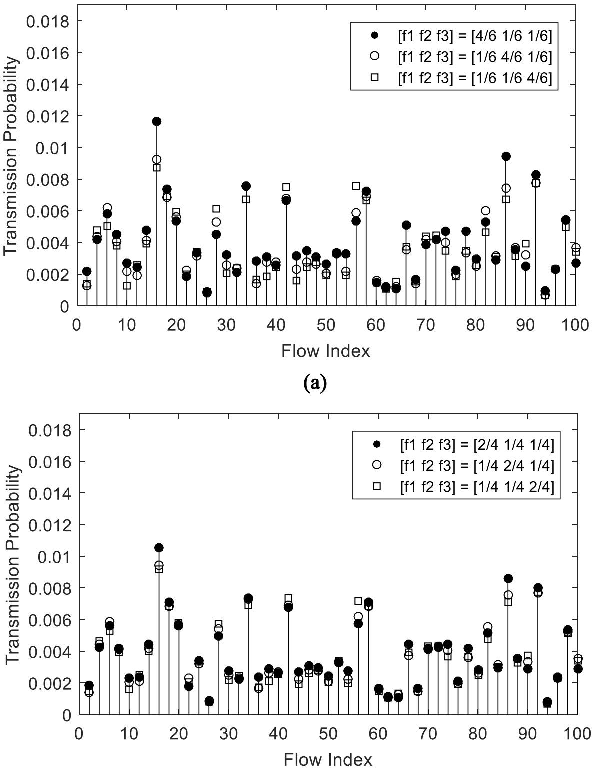
The topology of the simulated network is given in Fig. 2, where 100 flows are randomly distributed in a 2000m by 2000m square area. In the figure, the solid and dashed lines refer to the transmission pairs and the one-hop transmission



(a)

(b)

FIGURE 5. Validation of the transmission probabilities in each flow: (a) results of flows 1-50; (b) results of flows 51-100.



(a)

(b)

FIGURE 6. Transmission probabilities with different settings of power level selection probabilities: (a) results of scenario S2; (b) results of scenario S3.

range, respectively. Here, flow n means that the transmitter of the flow is node n. We assume that each node always has at least one packet to send.

In the simulated network, each node has 3 available power levels, i.e., 13dBm, 15dBm and 17dBm. At the beginning of a transmission process, the node randomly chooses one of the three power levels with probabilities f_1, f_2, and f_3, respectively. In our simulation, we consider the following three situations:

S1: $f_x \in \{1/3\}$, x $\in \{1, 2, 3\}$;
S2: $f_x \in \{1/6, 4/6\}$, x $\in \{1, 2, 3\}$;
S3: $f_x \in \{1/4, 2/4\}$, x $\in \{1, 2, 3\}$.

Note that $f_1 + f_2 + f_3 = 1$. In S1, we validate the accuracy of our proposed model by comparing the simulation and numerical values. In S2 and S3, we use the numerical results to analyze the impact of heterogeneous power levels.

B. PERFORMANCE EVALUATION

1) COLLISION PROBABILITY

Firstly, the collision probability of each flow in scenario S1 is demonstrated in Fig. 3, where Figs. 3(a) and 3(b) illustrate the results of flows 1-50 and 51-100, separately. As can be observed, the numerical values of our proposed model are close to the simulation results, which validates our proposed model.

Secondly, the numerical collision probabilities of each flow are given in Fig. 4. More specifically, Figs. 4(a) and 4(b) are obtained under scenarios S2 and S3, respectively. To clearly show the numerical results, we only illustrate the values of the flows with even indices. As can be observed, most of the flows have much higher collision probabilities when they choose the minimum power level with higher probabilities. The main reason is that the decrease of the transmission power enlarges the collision range and leads to the POINT problem.

In addition, Fig. 4 also demonstrate that the flows in the network incur quite different collisions from each other. Hence, it can be seen that either the aggregate or average throughputs of the multi-hop ad hoc networks appears to be a gross metric without due details. From the numerical results of Flows 1, 22, 26, 64, 68 and 94 in Fig. 4, we get another interesting finding, i.e., the persistent collisions occupy the most part of the collisions.

2) TRANSMISSION PROBABILITY

In the simulation, the transmission probability is measured as the average ratio of the number of the transmissions and the length of the slot. The numerical and simulation results are given in Fig. 5. From this figure, we know that the transmission probability of our model is very close to the simulation results.

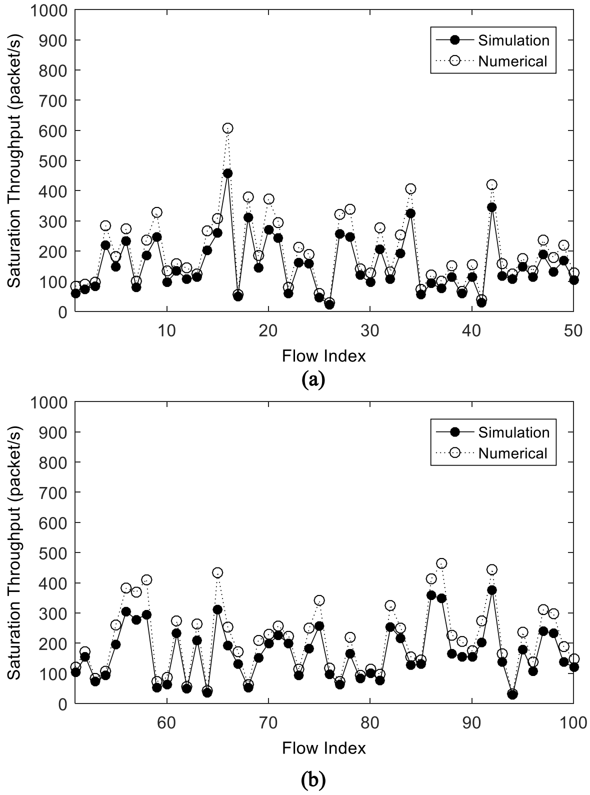


FIGURE 7. **Saturation throughputs of the flows, (a) results of flows 1-50; (b) results of flows 51-100.**

The numerical results of the transmission probabilities of the flows are given in Fig. 6. From this figure, we observe significant discrepancies among the flows. The transmission probabilities of some flows decrease when the flows choose the minimum power level with higher probability. From Fig. 6, we also observe that the transmission probabilities of most of the flows decrease with the transmission power. According to the BEB scheme, the contention window is doubled after each collision slot till the retransmission limit. When the transmission power reduces, the transmission probabilities of most of the flows decrease, such as flow 42.

However, there are also some flows get higher transmission probabilities when they transmit packet at the minimum power level with higher probabilities, e.g., flow 16. This is because the transmission power not only affects the collision range, but also determines the interference to other flows. Although the decreased transmission power may lead to more collisions, it causes less interference to other flows and improves the spatial reuse of the network. Therefore, the nodes in the network spend less time in the freezing process and thus increase the transmission probability.

3) PER-FLOW THROUGHPUT

The validation of the per-flow throughput of our proposed model is demonstrated in Fig. 7. As can be observed, the simulation and numerical results match well.

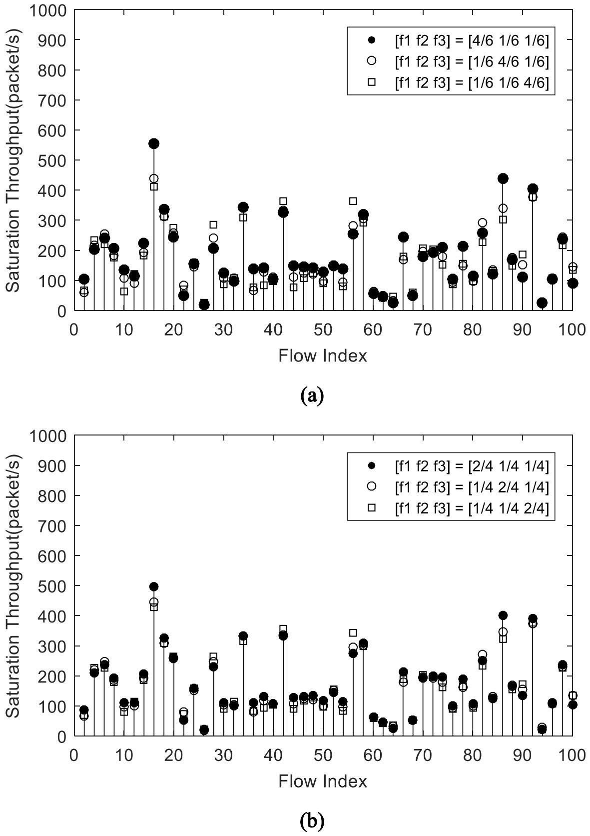


FIGURE 8. **Per-flow throughput with different settings of power level selection probabilities: (a) results of scenario S2; (b) results of scenario S3.**

Finally, Fig. 8 gives the throughputs of the flows with different power selection probabilities in situations S2 and S3. The per-flow saturation throughput changes with the transmission power levels in two aspects. Firstly, the transmissions at a low power level tend to be collided by others. As to the flows that experience serious persistent collisions, the increase of the transmission power can increase the throughputs significantly. Secondly, the transmissions at a lower power level could decrease the interference to others. Some flows may spend less time on the freezing progress and obtain more opportunities to transmit packets.

V. CONCLUSION

In this paper, we proposed a fixed-length slot-based Markov chain model to analyze the per-flow saturation throughput of the multi-hop ad hoc network with heterogeneous transmission power levels. In the model, we analyzed the behavior of each tagged node with a fixed length time slot method, and proposed a four dimensional Markov chain model to analyze this network. Next, we calculated the collision probability, transmission probability, and the probability of the node in the freezing process of the proposed model and obtained their closed-form expressions. With these probabilities, the per-flow saturation throughput is finally derived. We also validated the accuracy of the proposed model in a random grid multi-hop topology with 100 flows, and discussed the impact of heterogeneous power levels based on the simulation results.

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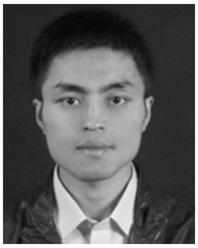

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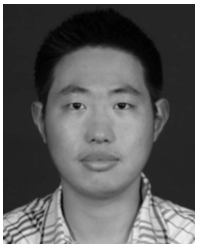

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