

Structure Regulator for the Perturbations Attenuation in a Quadrotor

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ABSTRACT In this work, we study the structure regulator for the perturbations attenuation which is based on the infinite structure regulator. The structure regulator is able to attenuate the perturbations if the transfer function of the departures and perturbations has a numerical value almost equal to zero, and it does not require the perturbations to attenuate them. We apply the structure regulator and the infinite structure regulator to a quadrotor which maintains the horizontal position with respect to the earth for the step and sine perturbations.

INDEX TERMS Quadrotor, perturbations attenuation, model, roll and pitch angles.

I. INTRODUCTION

The unmanned aerial vehicles have multiple applications, among them is to facilitate access to difficult or dangerous places for a person, they can also operate in unknown environments giving the possibility to recognize the environment, in order to facilitate the work to the human. Due to their nature, they may present perturbations which are unwanted entries that alter their operation.

The quadrotors are aerial vehicles which have different functions such as vertical takeoff, horizontal movement or being suspended in the air. In other words, the quadrotor is an aerial vehicle with four arms forming four straight angles in whose ends are equal number of engines and propellers that allow the takeoff, orientation and landing. These quadrotors are exposed to perturbations when they are in flight, especially if they are used in external environments, where we need to take into account many difficult events. To deal with these complex events, authors have worked with different regulators.

We found some works about perturbations attenuation in quadrotors. In [1]–[6], authors focused the sliding mode regulation. In [7]–[10], authors addressed the active disturbance regulation. In [11]–[16], authors took into account the adaptive regulation. In [17]–[20], authors discussed the altitude regulation. In [21]–[26], authors detailed the

fuzzy regulation. These works show that the perturbations attenuation in quadrotors is a recent issue.

Most of the mentioned works use the sliding mode or the adaptive and fuzzy regulations, but they have two inconveniences. The adaptive and fuzzy regulators produce the undesired unmodeled error due to the usage of the estimated perturbations in the regulation function, and the sliding model regulators produce the undesired chattering due to the usage of a non-continuous regulation function. Hence, It would be interesting to find a strategy which does not have any of the mentioned inconveniences.

Two interesting perturbations attenuation strategies are the infinite structure regulator and structure regulator. In [27], [28], authors focus the infinite structure regulator, in which the infinite structure regulator allows the perturbations attenuation if the transfer function of the regulated model has a numerical value equal to the transfer function of the perturbations attenuation model, and it requires the perturbations to attenuate them. Later, in [29] and [30] authors study structure regulator as a variant of the infinite structure regulator, in which the structure regulator allows the perturbations attenuation if the transfer function of the departures and perturbations has a numerical value almost equal to zero, and it does not require the perturbations to attenuate them. In this work, we study the infinite structure regulator and the structure regulator for the perturbations attenuation in a quadrotor.

According to the infinite structure regulator, the perturbations attenuation has been carried out only to models in

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which it must be an indispensable condition that the transfer function of the regulated model has a numerical value equal to the transfer function of the perturbations attenuation model. Otherwise, we study the structure regulator for the perturbations attenuation due to air gusts that can affect the performance of a quadrotor, in which it is not necessary to know the perturbations in order to attenuate them. The vector of states will be fed back so that together with the attenuation function, the transfer function of the departures and perturbations will have a numerical value almost equal to zero. To observe the performance, we will compare the structure regulator with the infinite structure regulator strategy for the perturbations attenuation in a quadrotor for the step and sine perturbations.

In other words, the main contributions of the structure regulator for the perturbations attenuation in a quadrotor over other strategies are described as:

- 1) The sliding model regulators produce the undesired chattering due to the usage of a non-continuous regulation function, while the structure regulator does not produce the undesired chattering due to the usage of a continuous regulation function.
- 2) The adaptive and fuzzy regulators produce the unmodeled error due to the usage of the estimated perturbations in the regulation function, while the structure regulator does not produce the unmodeled error due to the non-usage of the estimated perturbations in the regulation function.
- 3) The infinite structure regulator requires the perturbations in the regulation function in order to attenuate them, while the structure regulator does not require the perturbations in the regulation function in order to attenuate them.

The work is organized in the next sentence. In Section II, we present the infinite structure regulator for the perturbations attenuation. We study the structure regulator for the perturbations attenuation which is based on the infinite structure regulator in Section III. In Section IV we compare the infinite structure regulator and structure regulator for the step and sine perturbations in a quadrotor. Finally, we detail the conclusions of the forthcoming work in Section V.

II. INFINITE STRUCTURE REGULATOR FOR THE PERTURBATIONS ATTENUATION

In this section, we express the infinite structure regulator. The perturbed model is:

$$\begin{aligned} \dot{X}_i &= A_i X_i + B_i U_i + E_i P_i \\ Y_i &= C_i X_i \end{aligned} \quad (1)$$

the variables are $X_i \in \mathcal{X}_i \subseteq \mathfrak{R}^n$, the entries are $U_i \in \mathcal{U}_i \subseteq \mathfrak{R}^m$, the departures are $Y_i \in \mathcal{Y}_i \subseteq \mathfrak{R}^p$. A_i , B_i , and C_i are matrices of $A_i : \mathcal{X}_i \rightarrow \mathcal{X}_i$, $B_i : \mathcal{U}_i \rightarrow \mathcal{X}_i$ and $C_i : \mathcal{X}_i \rightarrow \mathcal{Y}_i$, the perturbations are $P_i \in \mathcal{D}_i \subseteq \mathfrak{R}^q$, and E_i is the matrix of $E_i : \mathcal{D}_i \rightarrow \mathcal{X}_i$.

In this case, for the perturbations attenuation, the transfer function of the entries model without perturbations must

have a numerical value equal to the transfer function of the perturbations model without entries.

We define the transfer function of the entries model without perturbations $T_U(s)$ as:

$$T_U(s) = C_i(sI - A_i)^{-1}B_i \quad (2)$$

We define transfer function of the perturbations model without entries $T_P(s)$ as:

$$T_P(s) = C_i(sI - A_i)^{-1}E_i \quad (3)$$

From [27], [28], and [31], we present the next Theorem.

Theorem 1: The perturbations attenuation issue has a solution if $T_U(s)$ in (2) has a numerical value equal to $T_P(s)$ in (3) as:

$$T_U(s) = T_P(s) \quad (4)$$

We define the infinite structure regulator U_i as:

$$U_i = F_i X_i + G_i P_i \quad (5)$$

We employ the next equality to calculate F_i and G_i as:

$$TF_i(s) = C_i(sI - A_i - B_i F_i)^{-1}(B_i G_i + E_i) = 0 \quad (6)$$

$TF_i(s)$ is the transfer function of departures and perturbations.

Fig. 1 shows the design procedure for the infinite structure regulator of the Theorem 1.

III. STRUCTURE REGULATOR FOR THE PERTURBATIONS ATTENUATION

In this section we express the structure regulator as the contribution of this article. The perturbed model A_c , B_c , C_c , E_c is:

$$\begin{aligned} \dot{X}_c &= A_c X_c + B_c U_c + E_c P_c \\ Y_c &= C_c X_c \end{aligned} \quad (7)$$

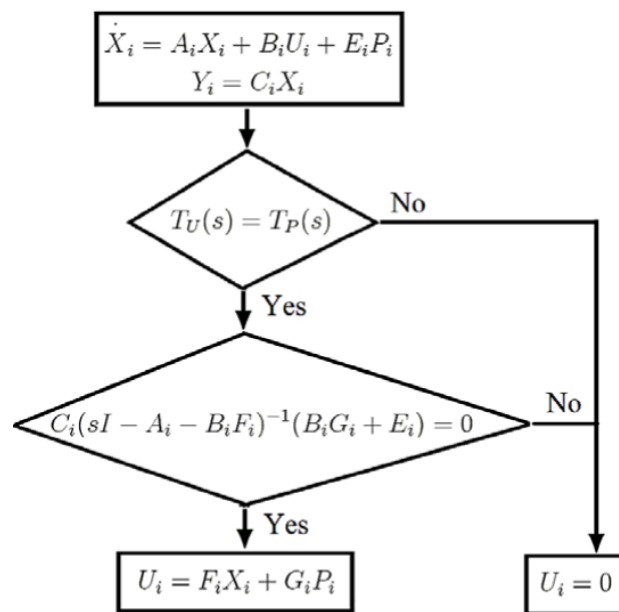


FIGURE 1. Design procedure for the infinite structure regulator.

the variables are $X_c \in \mathcal{X}_c \subseteq \mathbb{R}^n$, the entries are $U_c \in \mathcal{U}_c \subseteq \mathbb{R}^m$, the perturbations are $P_c \in \mathcal{D}_c \subseteq \mathbb{R}^q$. A_c, B_c, C_c and E_c are matrices of $A_c : \mathcal{X}_c \rightarrow \mathcal{X}_c, B_c : \mathcal{U}_c \rightarrow \mathcal{X}_c, C_c : \mathcal{X}_c \rightarrow \mathcal{Y}_c$ and $E_c : \mathcal{D}_c \rightarrow \mathcal{X}_c$.

We show the contribution of this work in the next Theorem.

Theorem 2: *The perturbations attenuation of the model (7) has solution if there exists a structure regulator such as the transfer function of departures and perturbations $TF_c(s)$ has a numerical value almost equal to zero, this structure regulator U_c is:*

$$U_c = F_c X_c \tag{8}$$

F_c is a matrix of $F_c : \mathcal{X}_c \rightarrow \mathcal{U}_c$, and the transfer function of departures and perturbations $TF_c(s)$ is:

$$TF_c(s) = C_c(sI - A_c - B_c F_c)^{-1} E_c = 0 \tag{9}$$

Proof: We substitute $U_c = F_c X_c$ of (8) in the perturbed model A_c, B_c, C_c, E_c in (7) as:

$$\begin{aligned} \dot{X}_c &= (A_c + B_c F_c) X_c + E_c P_c \\ Y_c &= C_c X_c \end{aligned} \tag{10}$$

We apply the Laplace transform as:

$$\begin{aligned} X_c(s) &= (sI - A_c - B_c F_c)^{-1} E_c P_c(s) \\ Y_c(s) &= C_c X_c(s) \end{aligned} \tag{11}$$

We substitute the first equation in the second in (11) as:

$$Y_c(s) = C_c(sI - A_c - B_c F_c)^{-1} E_c P_c(s) \tag{12}$$

Consequently, the transfer function of the departures and perturbations $TF_c(s)$ is:

$$TF_c(s) = \frac{Y_c(s)}{P_c(s)} = C_c(sI - A_c - B_c F_c)^{-1} E_c \tag{13}$$

We observe that equation (13) is equal to equation (9). \square

Fig. 2 shows the design procedure for the structure regulator of the Theorem 2.

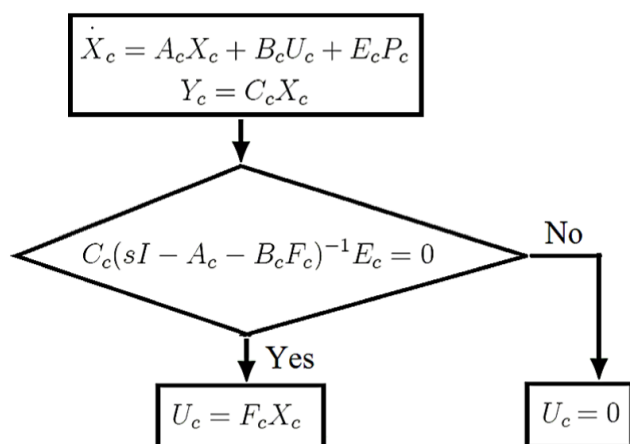


FIGURE 2. Design procedure for the structure regulator.

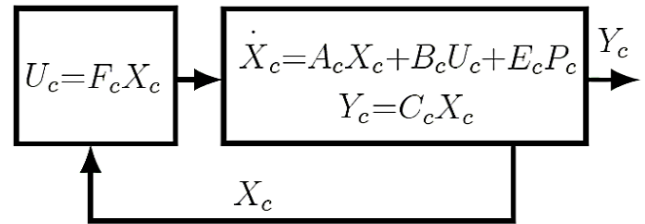


FIGURE 3. The structure regulator.

In the Fig. 3 we see the blocks diagram of the structure regulator we study for the perturbations attenuation, We do not need to know the perturbations for their attenuation, i.e., we consider this as a perturbations attenuation issue for unknown perturbations.

Remark 1: *If the transfer function of the departures and perturbations obtains a numerical value almost equal to zero (9), the structure regulator does not require the measure of the perturbations in the regulation function (8) in order to attenuate them.*

Remark 2: *Our structure regulator differs with the infinite structure regulator of [27], [28]. The first difference is that we apply our structure regulator to a quadrotor while they apply their infinite structure regulator to other plants. The second difference is that the infinite structure regulator requires the fulfillment of Theorem 1, which needs that the equation (2) is nearly to equation (3), while in our structure regulator this condition is not necessary since the equation (8) only needs to comply with the equation (9). And the third difference is that our structure regulator (8) do not need to know the perturbations to attenuate them, while the infinite structure regulator (5) needs to know the perturbations to attenuate them.*

Remark 3: *Our work differs with the works of [30], [32]. The first difference is that we apply our regulation strategy to a quadrotor while they apply their regulation strategy to other plants. The second difference is that our regulation strategy is applied to models like the one shown in the equation (7). And the third difference is that we do not need the perturbations in the structure regulator of equation (8) to attenuate them.*

In the next sections, we discuss the application of both regulators and their implementation in a quadrotor and its subsequent comparison of results. For the comparison of results, we employ the root of the mean square error *Error* as:

$$Error = \left(\frac{1}{T} \int_0^T e^2 dt \right)^{\frac{1}{2}} \tag{14}$$

$e^2 = Y_s^2 - 0 = Y_s^2$ is the error equivalent to the departure of the infinite structure regulator and $e^2 = Y_c^2 - 0 = Y_c^2$ is the error equivalent to the departure of the structure regulator.

Remark 4: *We take into account the regulation case where the goal of the structure regulator is to track constant trajectories with values of zero. Consequently, the errors are equivalent to the departures as was discussed in equation (14).*

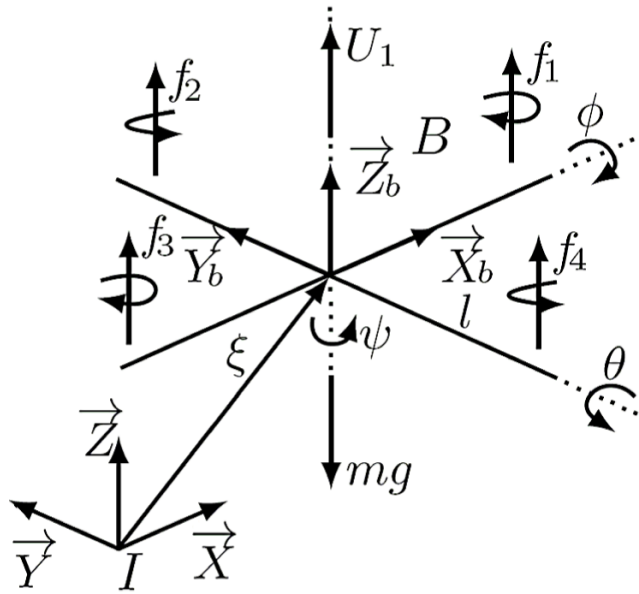


FIGURE 4. The quadrotor.

IV. THE QUADROTOR

A quadrotor like that of Fig. 4 is an aerial vehicle consisting of four propellers and an equal number of engines arranged at right angles to each other, the thrust force of the four engines allow the quadrotor can have upward, downward and lateral movements. For the movement of ascent and descent the same force is applied to the four engines whether it is decreased or increased, so that we follow a front trajectory (pitch) it decreases f_1 and increases f_3 , and vice versa if the movement is towards behind. Similarly, we make a movement to the right (roll), decrease f_4 and increase f_2 and vice versa for an opposite movement. Finally, we reach the movement on the Z axis (yaw) by increasing (reducing) the speed in the motors of f_1 and f_3 , and increasing (reducing) the speed in the motors of f_2 and f_4 . We should note that the motors in f_2 and f_4 rotate clockwise, while the motors in f_1 and f_3 rotate counterclockwise.

A quadrotor is divided into two subsystems, rotation and translation. In the present work we study the perturbations attenuation in the rotation subsystem of the Fig. 5, which correspond to the orientation of the quadrotor with respect to the earth in the reference system I .

In this section we define the model of the quadrotor using the Euler-Lagrange strategy, for this model we consider the quadrotor floating constantly and maintained which is a movement known as a small angle where: $\cos\phi \approx \cos\theta \approx \cos\psi \approx 1$, $\sin\phi \approx 0$, $\sin\theta \approx 0$ and $\sin\psi \approx 0$, from there we have the state variables as: $X_1 = \phi$, $X_2 = \dot{X}_1$, $X_3 = \theta$, $X_4 = \dot{X}_3$, $X_5 = \psi$, and $X_6 = \dot{X}_5$, the entries are: $U_2 = \tau_\phi$ where $\tau_\phi = l(f_2 - f_4) = lb(\omega_2^2 - \omega_4^2)$, $U_3 = \tau_\theta$ where $\tau_\theta = l(f_3 - f_1) = lb(\omega_3^2 - \omega_1^2)$, and $U_4 = \tau_\psi$ where $\tau_\psi = d(f_1 + f_3 - f_2 - f_4) = d(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2)$, the departures are $Y_1 = \phi$ for the roll, $Y_2 = \theta$ for the pitch, and $Y_3 = \psi$ for the yaw.

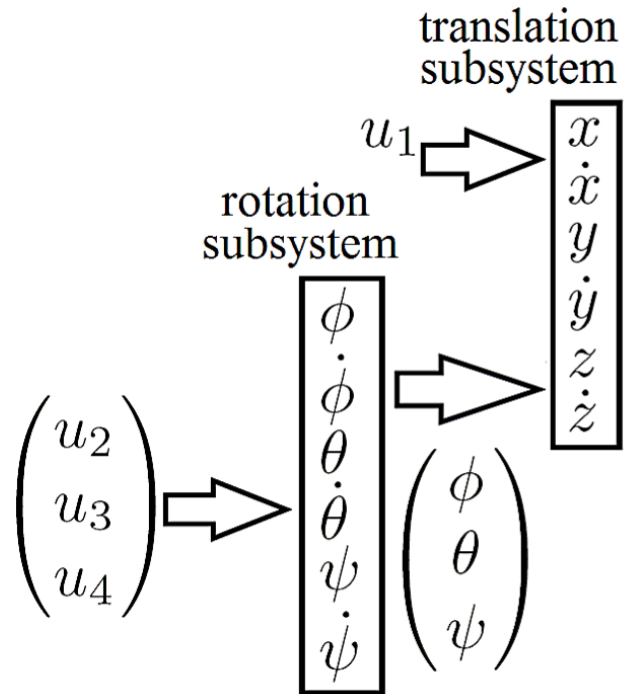


FIGURE 5. The subsystems.

We employ the next model of the rotation subsystem as:

$$\begin{aligned}
 \dot{X}_1 &= X_2 \\
 \dot{X}_2 &= \frac{(I_{YY} - I_{ZZ})}{I_{XX}} X_4 X_6 + \frac{l}{I_{XX}} \tau_\phi + \frac{P_\phi}{I_{XX}} \\
 \dot{X}_3 &= X_4 \\
 \dot{X}_4 &= \frac{(I_{ZZ} - I_{XX})}{I_{YY}} X_2 X_6 + \frac{l}{I_{YY}} \tau_\theta + \frac{P_\theta}{I_{YY}} \\
 \dot{X}_5 &= X_6 \\
 \dot{X}_6 &= \frac{(I_{XX} - I_{YY})}{I_{ZZ}} X_2 X_4 + \frac{1}{I_{ZZ}} \tau_\psi + \frac{P_\psi}{I_{ZZ}} \\
 Y_1 &= \phi \\
 Y_2 &= \theta \\
 Y_3 &= \psi
 \end{aligned} \tag{15}$$

τ_ϕ , τ_θ , τ_ψ are torques of motors that allow the movements of roll, pitch and yaw, $U_2 = \tau_\phi$, $U_3 = \tau_\theta$, $U_4 = \tau_\psi$ are the entries of the quadrotor, l is the distance between the motors in m and the center of gravity of the quadrotor, the thrust coefficient of the motors is b in Ns^2 , the angular velocity of the motors is ω_i , the drag coefficient is d in Nms^2 , I_{XX} , I_{YY} , and I_{ZZ} are the moments of inertia in the X, Y and Z axes in $\text{Kg}m^2$, ϕ is the angular position of the roll movement, θ is the angular position of the pitch movement, ψ is the angular position of the movement of yaw, $Y_1 = \phi$, $Y_2 = \theta$, $Y_3 = \psi$ are the departures of the quadrotor, P_ϕ , P_θ , P_ψ are the forces and aerodynamic pairs acting on the quadrotor considered as perturbations and are inversely proportional to the moments of inertia in the rotation subsystem, P_ϕ , P_θ , P_ψ are the perturbations of the quadrotor, they are calculated from

TABLE 1. Terms of the quadrotor.

Terms	Values
m	0.650kg
I_{XX}	$7.5 \times 10^{-3} \text{kgm}^2$
I_{YY}	$7.5 \times 10^{-3} \text{kgm}^2$
I_{ZZ}	$1.3 \times 10^{-2} \text{kgm}^2$
b	$3.13 \times 10^{-5} \text{Ns}^2$
l	0.23m
g	9.88m/s^2
ρ_{air}	1.225kg/m^3
V	1m/s
C_p	54.7687×10^{-3}

$P_k = \frac{1}{2} \rho_{air} C_p V^2$, where $k = \phi, k = \theta$, or $k = \psi$, ρ_{air} is the air density in Kg/m^2 , the velocity of the quadrotor is V in m/s , and C_p is a dimensionless moment that we can calculate in three ways, directly measured in a wind tunnel, using the characteristics of the quadrotor or using the derivative of the aerodynamic stability, for this case, in the second way we apply $C_p = 0.0375(6.35V)$.

In the Table I we show the terms of the quadrotor [32].

In order to have the matrices we employ the model in (1), we take into account the variables of (15) and the terms of the Table I. The matrices are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.733 & 0 & -0.733 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.733 & 0 & 0 & 0 & 0.733 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = [0 \quad 30.666 \quad 0 \quad 30.666 \quad 0 \quad 76.923]^T$$

$$C = [7.2 \times 10^{-6} \quad 0 \quad 7.2 \times 10^{-6} \quad 0 \quad 7.5 \times 10^{-7} \quad 0]$$

$$E = [0 \quad 4.47 \quad 0 \quad 4.47 \quad 0 \quad 2.58]^T \quad (16)$$

A. PERTURBATIONS ATTENUATION WITH THE INFINITE STRUCTURE REGULATOR

From Fig. 1 and equalities (2) and (3) we have $T_U(s)$ and $T_P(s)$ as:

$$T_U(s) = \frac{1.1789 \times 10^{27} s^2 - 1.3320 \times 10^{27}}{2.3611 \times 10^{30} s^4 + 1.2686 \times 10^{30} s^2} \quad (17)$$

$$T_P(s) = \frac{7.8276 \times 10^{24} s^2 - 2.2338 \times 10^{24}}{1.1806 \times 10^{29} s^4 + 6.3431 \times 10^{28} s^2} \quad (18)$$

As equalities (17) and (18) are different, we do not comply the equation (4) in the Theorem 1, we cannot apply the equality (6) to find F_i and G_i , the infinite structure regulator of (5) is:

$$U_i = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] X_i \quad (19)$$

B. PERTURBATIONS ATTENUATION WITH THE STRUCTURE REGULATOR

From Fig. 2, we calculate the regulator $U_c = F_c X_c$ in (8) than comply the equation (9):

$$TF_c(s) = C_c(sI - A_c - B_c F_c)^{-1} E_c = 0 \quad (20)$$

the function $F_c = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6]$, we substitute F_c in the equation (20) as:

$$\left\{ \begin{bmatrix} 7.2 \times 10^{-6} & 0 & 7.2 \times 10^{-6} & 0 & 7.5 \times 10^{-7} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.733 & 0 & -0.733 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.733 & 0 & 0 & 0 & 0.733 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} sI - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.733 & 0 & -0.733 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.733 & 0 & 0 & 0 & 0.733 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0 \\ 30.666 \\ 0 \\ 30.666 \\ 0 \\ 76.923 \end{bmatrix} [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6] \begin{bmatrix} 0 \\ 4.47 \\ 0 \\ 4.47 \\ 0 \\ 2.58 \end{bmatrix} = 0 \quad (21)$$

In order to calculate F_c we find that the product between C_c and E_c is nearly to zero in this particular case, otherwise it could be more difficult, Making the calculus we have:

$$\begin{aligned} f_1 &= -7.2 \times 10^{-6} \\ f_2 &= 0 \\ f_3 &= -7.2 \times 10^{-6} \\ f_4 &= 0 \\ f_5 &= -7.5 \times 10^{-7} \\ f_6 &= 0 \end{aligned} \quad (22)$$

Consequently, we find the structure regulator as:

$$U_c = [-7.2 \times 10^{-6} \quad 0 \quad -7.2 \times 10^{-6} \quad 0 \quad -7.5 \times 10^{-7} \quad 0] X_c \quad (23)$$

V. SIMULATIONS

In this subsection we compare the infinite structure regulator in (2), (3), (5) and (6) with the structure regulator in (8), (9), (11) and (13) for the perturbations attenuation. We express the model of the quadrotor in the equation (15) and the terms in Table I. We use Matlab as the software for the simulations. The main goal of the regulators is to attenuate the perturbations at the departures of the quadrotor. These perturbations are forces and aerodynamic pairs external to the quadrotor but which directly affect their performance, for this reason it is important that the departure angles of the

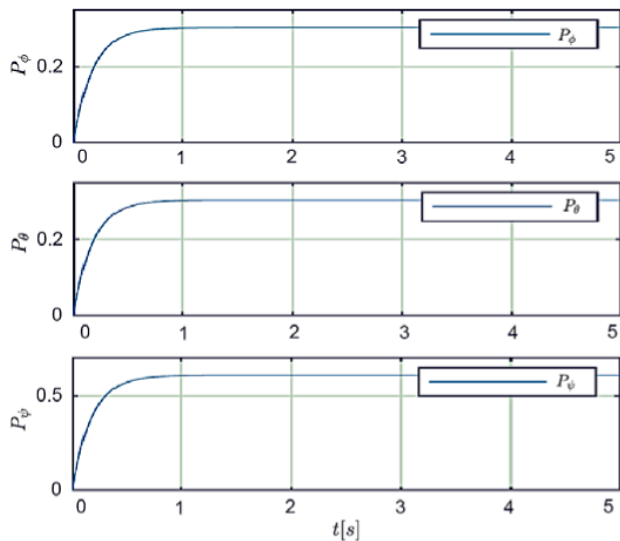


FIGURE 6. The step perturbations.

quadrotor remain near to zero even if an undesired force entry and aerodynamic torque. We define the perturbations in (15) as a constant inversely proportional to the moments of inertia of each of the axes such as: $\frac{P_\phi}{I_{xx}}$, $\frac{P_\theta}{I_{yy}}$, and $\frac{P_\psi}{I_{zz}}$.

We employ the infinite structure in (2), (3), and (6) with the regulator (19) to attenuate the perturbations of the quadrotor in (1) and (16).

We employ the structure in (11), (13) and the regulator (23) to attenuate the perturbations in (1) and (16).

A. SIMULATION 1: STEP PERTURBATIONS

The goal of the regulators is that the effect of step perturbations is not presented in the departures of the quadrotor, i.e. the departures of the quadrotor must have a numerical value almost equal to zero. In the Fig. 6, Fig. 7, and Fig. 8 we show the perturbations, departures, as well as the errors of the infinite structure regulator and the structure regulator for the perturbations attenuation in the quadrotor in the case of step perturbations, we denote the perturbations of (15), (16) as P_ϕ , P_θ , P_ψ , we denote the departures of (15), (16) as ϕ , θ , ψ , we denote the root mean squared errors of (14) as e_ϕ , e_θ , e_ψ , we denote the structure regulator as SR, and we denote the infinite structure regulator as ISR. In the Table II we show the root mean squared errors of (14).

TABLE 2. Errors in the quadrotor: Step perturbations.

Strategy	e_ϕ	e_θ	e_ψ
Infinite structure	0.925	1.591	253.3
Structure	0	0	0

In the Fig. 6 we observe the same step perturbations which are employed by both the infinite structure regulator and the structure regulator for their attenuation in the quadrotor. In the Fig. 7 we see that the infinite structure regulator obtains numerical values bigger than zero in the departures while the

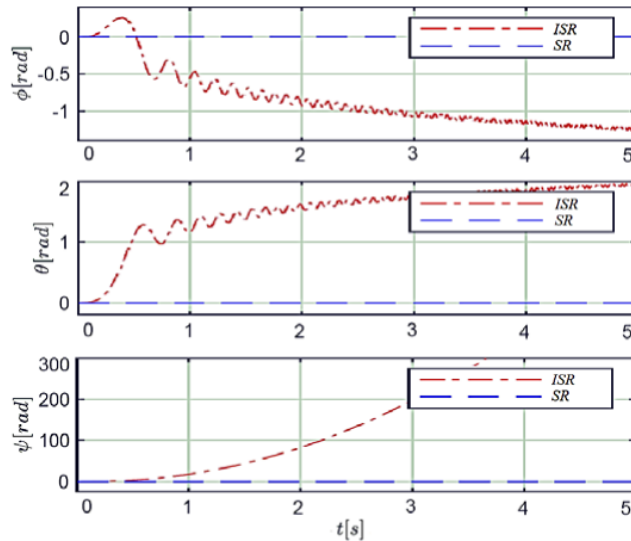


FIGURE 7. The departures for step perturbations.

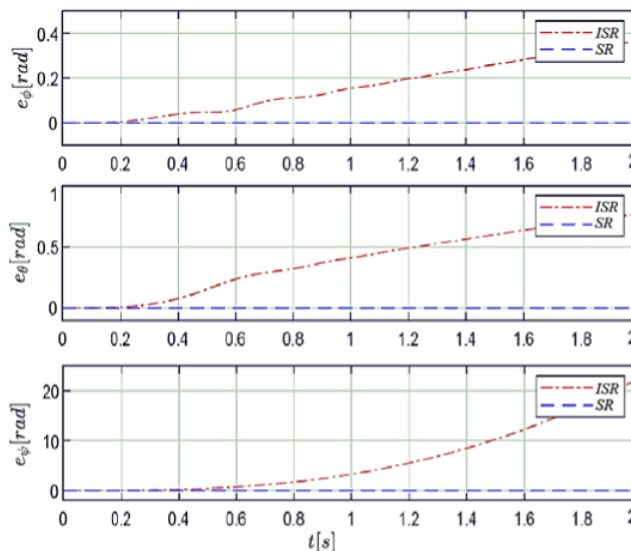


FIGURE 8. The errors for step perturbations.

structure regulator obtains numerical values equal to zero in the departures which is the goal of this work. In the Fig. 8 and in the Table II we observe that the infinite structure regulator obtains numerical values bigger than zero in the root mean squared errors while the structure regulator obtains numerical values equal to zero in the root mean squared errors which is the goal of this work.

B. SIMULATION 2: SINE PERTURBATIONS

The goal of the regulators is that the effect of sine perturbations is not presented in the departures of the quadrotor, i.e. the departures of the quadrotor must have a numerical value almost equal to zero. In the Fig. 9, Fig. 10, and Fig. 11 we show the perturbations, departures, as well as the errors of the infinite structure regulator and the structure regulator for

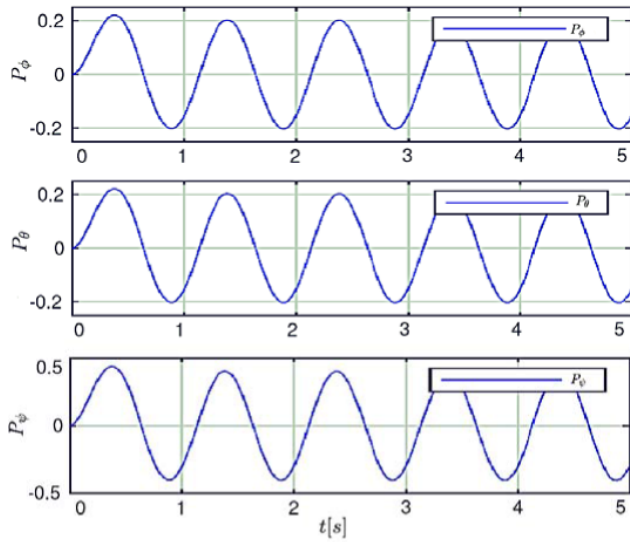


FIGURE 9. The sine perturbations.

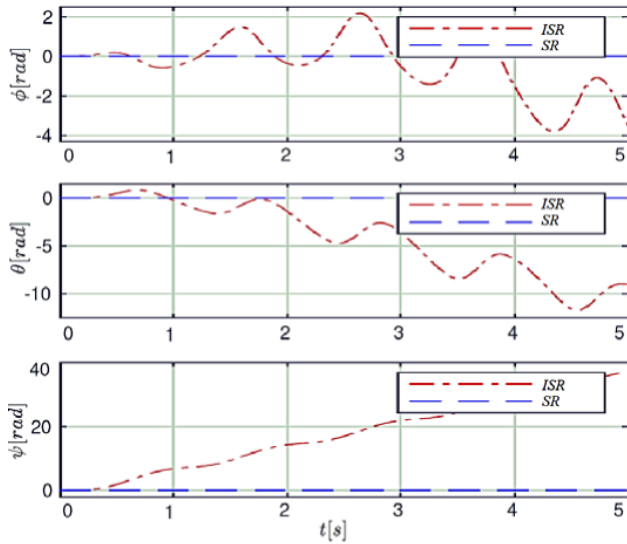


FIGURE 10. The departures for sine perturbations.

TABLE 3. Errors in the quadrotor: Sine perturbations.

Strategy	e_ϕ	e_θ	e_ψ
Infinite structure	1.441	5.532	20.75
Structure	0	0	0

the perturbations attenuation in the quadrotor in the case of sine perturbations, we denote the perturbations of (15), (16) as P_ϕ , P_θ , P_ψ , we denote the departures of (15), (16) as ϕ , θ , ψ , and we denote the root mean squared errors of (14) as e_ϕ , e_θ , e_ψ , we denote the structure regulator as SR, and we denote the infinite structure regulator as ISR. In the Table III we show the root mean squared errors of (14).

In the Fig. 9 we observe the same sine perturbations which are employed by both the infinite structure regulator and the structure regulator for their attenuation in the quadrotor. In the Fig. 10 we see that the infinite structure regulator obtains numerical values bigger than zero in the departures while the

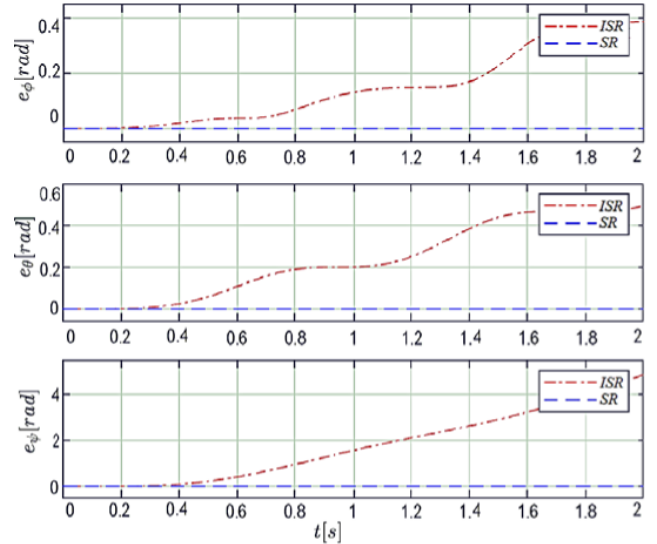


FIGURE 11. The errors for sine perturbations.

structure regulator obtains numerical values equal to zero in the departures which is the goal of this work. In the Fig. 11 and in the Table III we observe that the infinite structure regulator obtains numerical values bigger than zero in the root mean squared errors while the structure regulator obtains numerical values equal to zero in the root mean squared errors which is the goal of this work.

Remark 5: In this work, we apply the structure regulator for the perturbations attenuation of step and sine perturbations. Acceptable perturbations should have low frequency and bounded behaviors in order to reach a good performance in the structure regulator; some examples of the acceptable perturbations are step, sine, cosine, square, sawtooth, hyperbolic tangent, or sign. Not acceptable perturbations should have high frequency or unbounded behaviors in order to reach a bad performance in the structure regulator; some examples of not acceptable perturbations are tangent, noise, or cotangent.

Remark 6: In this work, the structure regulator is able to attenuate the perturbations if the transfer function of the departures and perturbations has a numerical value almost equal to zero. In practice, if the transfer function of the departures and perturbations has a numerical value equal to zero, then the structure regulator obtains the best performance in perturbations attenuation; but if the transfer function of the departures and perturbations has a numerical value equal to infinity, then the structure regulator obtains the worst performance in perturbations attenuation. The intermediate numerical values between zero and infinity in the transfer function of the departures and perturbations yield that the structure regulator obtains a better or worse performance in perturbations attenuation.

VI. CONCLUSION

In this work, we design the structure regulator in order to attenuate the perturbations in a quadrotor, which was

compared with the infinite structure regulator for step and sine perturbations. The goal of the regulators was that the effect of perturbations was not presented in the departures of the quadrotor. The results showed that our structure regulator obtained a better performance due to the structure regulator obtained numerical values equal to zero in the departures of the quadrotor. The regulation strategy can be applied to different models as electrical, mechanical or hydraulic. In the forthcoming work, we desire to design a regulator to track time-varying trajectories in the quadrotor, which includes greater complexity due to the nonlinear elements that it contains.

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