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Low-Complexity PTS Scheme for Improving PAPR Performance of OFDM Systems

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ABSTRACT Orthogonal frequency division multiplexing (OFDM) has been one of the mainstream technologies in the fields of modern wireless communications. However, its own high peak to average power ratio (PAPR) problem can impair the performance of system, which has greatly restricted its wide applications. Partial transmit sequence (PTS) was proposed for improving the PAPR performance of OFDM systems. But its introduction greatly increases the computational complexity of the system. In this paper, a low-complexity PTS scheme is proposed based on the dominant time-domain samples, and two new metrics for choosing these samples are introduced. In addition, to further reduce the computational complexity, grouping method has been involved in proposed scheme. Simulation results indicate that the proposed low-complexity PTS scheme can provide a perfect PAPR reduction performance with more computational complexity savings.

INDEX TERMS OFDM, PAPR, PTS, computational complexity.

I. INTRODUCTION

In the development of mobile communications, the key technologies mainly include analog technology, digital technology, multimedia technology, wireless broadband technology and the new fifth generation digital mobile communication. Orthogonal frequency division multiplexing (OFDM) technology [1] has high spectral efficiency, simple frequency domain equalization method and strong anti-symbol interference (ISI) capability and so on [2]. Thus, it is widely used in digital subscriber line standards and wireless standards, and it is also one of the important alternative technologies for 5G [3].

In addition, OFDM technology has the advantages of supporting multiple business demand and easily combining with other technologies. But it also has some certain disadvantages, such as high synchronization requirement and high peak to average power ratio (PAPR), where high PAPR is a serious shortcoming carried by the OFDM system itself [4]. For an OFDM system with a high PAPR, it is easily to cause nonlinear distortion of the amplitude and phase of the output symbols, which causes distortion of the constellation points of the received signals and inter carrier interference (ICI) [5]. In view of the PAPR problem, many scholars have conducted in-depth analysis and discussions [6]. Currently, many effective strategies have been proposed to mitigate the high PAPR in the OFDM system, such as active constellation extension (ACE) [7], [8], partial transmit sequence (PTS) [9]–[11], selective mapping (SLM) [12]–[14], clipping and filtering [15], block coding [16], [17] and so on. Despite the wide variety of technologies that reduce the PAPR of system, a perfect PAPR suppression method has not been found, which leads to further research on PAPR suppression technologies.

Among them, the PTS technology is one of the most important methods for reducing PAPR in OFDM systems. But PTS algorithm has the disadvantage of high computational complexity when searching for optimal phase combination [18]. For this issue, some researchers combine intelligent algorithms with PTS technology, such as genetic algorithm (GA) [19], particle swarm optimization (PSO) [20], simulated annealing (SA) [21], ant colony algorithm [22] and so on. But those methods are more suitable for the case where the number of subcarriers is very large. In addition, researchers have introduced clipping technique into the PTS scheme [23]. Although this combination can improve the PAPR performance, it increases bit error rate of the system and causes signal distortion. PTS combined with companding or active

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constellation extension were given in [24], [25], which can gain good PAPR performance with increasing the complexity of system.

Recently, a reduced-complexity PTS (RC-PTS) algorithm [26] was proposed to select some time-domain samples for estimating the PAPRs of candidate signals, but its complexity was still high. In this paper, grouping method and two new metrics for selecting the ideal time-domain samples are proposed for obtaining more reduction in computational complexity without degrading the PAPR performance. Simulation results show that the proposed PTS scheme can further reduce the computational complexity of OFDM systems with similar PAPR performance compared with the original PTS (OPTS) and RC-PTS [26].

The other parts of this paper are organized as follows: Section II describes the PAPR problem in the OFDM system and Section III describes the existing PTS schemes. Section IV introduces the proposed low-complexity PTS scheme. Then, the simulation results are analyzed in Section V. Finally, Section VI summarizes the results.

II. PAPR PROBLEM IN OFDM

In an OFDM system with N subcarriers [27], the discrete OFDM signal can be expressed as:

$$s_l(k) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} S_{i,l} e^{j2\pi f_i k}, \quad 0 \le k \le N-1$$
(1)

where $S_{i,l}$ represents the symbol modulated on the *i*-th subcarrier in the *l*-th OFDM symbol, *N* is the number of subcarriers, f_i is the carrier frequency of the *i*-th subcarrier.

In addition, the PAPR is the ratio of the maximum instantaneous power to the average power of the signal [27]. Therefore, the PAPR of the discrete time signal $s_l(k)$ is defined as

PAPR (dB) =
$$10 \log_{10} \frac{\max_{0 \le k \le N-1} \{|s_l(k)|^2\}}{E\{|s_l(k)|^2\}} dB$$
 (2)

where max $\{.\}$ indicates the maximum value and $E\{.\}$ represents the mathematical expectation.

The real and imaginary parts of the OFDM signal are independently and identically distributed. According to the central limit theorem [28], as the increase of the number of subcarriers N, the real and imaginary parts of the OFDM signal gradually obey the Gaussian distribution with a zero mean and a variance σ^2 . According to the above analysis, complementary cumulative distribution function (CCDF) is used to analyze the distribution of PAPR performance of OFDM systems, which indicates the probability that the PAPR value of an OFDM signal exceeds a certain PAPR threshold, as follow:

$$CCDF = 1 - \left(1 - \exp\left(-z^2\right)\right)^N \tag{3}$$

where z^2 denotes a given PAPR threshold.

In addition, in the case of the oversampling, the distribution of each sampling point is no longer independent statistically



FIGURE 1. The block diagram of OPTS.

and the correlation between sampling points is enhanced with the increase of oversampling factor L [29]. Moreover, the oversampling is usually used for better approximating the PAPR of real OFDM signals. It is known in [30] that an oversampling factor L = 4 is sufficient to reach the real PAPR results. Accordingly, the CCDF of PAPR of the oversampled signals can be approximated as:

$$CCDF = 1 - \left(1 - \exp\left(-z^2\right)\right)^{\alpha N} \tag{4}$$

where the curve based on Eq. (4) is closest to the actual one when $\alpha = 2.8$ [29].

III. PTS SCHEMES

A. ORIGINAL PTS

The block diagram of OPTS is shown in Figure 1. First, the frequency domain signal $X = [X_0, X_1, \dots, X_{N-1}]$ is equivalently converted into the sum of *V* mutually disjoint subblocks X_v , $1 \le v \le V$ and the entered data appears only in one data block, as follow:

$$X = \sum_{\nu=1}^{V} X_{\nu} \tag{5}$$

$$X_n = \sum_{\nu=1}^{V} X_{\nu,n}, \quad 0 \le n \le N - 1$$
 (6)

where

$$X_{\nu,n} = \begin{cases} X_n, & \text{Data appears in the } \nu\text{-th subblock} \\ 0, & \text{Data appears in other subblocks} \end{cases}$$

After that, V subblocks are passed to IFFT blocks to be multiplied by phase factors, and then superimposed together to produce an OFDM candidate signal. Setting the number of elements in the phase factor set is $W, U = W^{V-1}$ OFDM candidate signals can be obtained. The *u*-th candidate signal $\mathbf{x}^{u} = [x_{0}^{u}, x_{1}^{u}, \cdots, x_{N-1}^{u}]$ is represented as:

$$\boldsymbol{x}^{u} = \sum_{\nu=1}^{V} b_{\nu}^{u} IFFT[\boldsymbol{X}_{\nu}] = \sum_{\nu=1}^{V} b_{\nu}^{u} \cdot \boldsymbol{x}_{\nu}, \quad 1 \le u \le U \quad (7)$$

where $b_{v}^{u} = e^{j\phi_{v,u}}, \phi_{v,u} \in [0, 2\pi).$

Finally, the candidate signal \hat{x} with the lowest PAPR is chosen for transmission.

At the receiver, in order to perform the detection correctly, the system needs to transmit $\lceil \log_2 U \rceil$ -bit side information to represent the selected phase factor combination [31].

B. REDUCED-COMPLEXITY PTS

Recently, a RC-PTS algorithm [26] has been proposed to reduce the computational complexity of system. For each candidate signal, it used the sum of sample powers of OFDM signals as a criterion to select dominant time-domain samples for PAPR calculation and the selection of optimal OFDM candidate signal. The RC-PTS scheme is introduced in detail as follow.

In [26], the power of the *m*-th sample of OFDM signal is given by:

$$|x_{m}|^{2} = \left|\sum_{\nu=1}^{V} b_{\nu} \cdot x_{\nu,m}\right|^{2}$$

= $\sum_{\nu=1}^{V} |x_{\nu,m}|^{2} + \sum_{\nu_{1}=1}^{V} \sum_{\substack{\nu_{2}=1\\\nu_{2}\neq\nu_{1}}}^{V} (b_{\nu_{1}} \cdot x_{\nu_{1},m}) (b_{\nu_{2}} \cdot x_{\nu_{2},m})^{*}$
= $Q_{m} + H_{m}$ (8)

where $Q_m = \sum_{\nu=1}^{V} |x_{\nu,m}|^2$, $H_m = \sum_{\nu_1=1}^{V} \sum_{\substack{\nu_2=1\\\nu_2\neq\nu_1}}^{V} (b_{\nu_1} \cdot x_{\nu_1,m})$

 $(b_{v_2} \cdot x_{v_2,m})^*$, (.)* represents the complex conjugate. Next, let S be the set of the indices of some samples of the OFDM signal, defined as $S = \{m | Q_m \ge Td_Q, 0 \le m \le N - 1\}$, where Td_Q is the threshold decided by pondering the computational complexity and the PAPR performance of system. Besides, only the samples whose indices in the set S need be optimized by phase factors and utilized to estimate the PAPRs of all the OFDM candidate signals. And then the optimal combination of phase factors can be found for generating the transmitted OFDM signal.

Because only a part of the time-domain samples are selected for optimization, the computational complexity could be reduced. Moreover, the choice of threshold Td_Q is very critical for reducing computational complexity and getting an ideal PAPR reduction performance.

IV. THE PROPOSED PTS SCHEME

A. TWO NEW METRICS FOR SELECTING DOMINANT TIME-DOMAIN SAMPLES

In RC-PTS scheme [26], more precise PAPR values were implemented by selecting a smaller threshold Td_Q because of more samples selected for estimating the PAPRs of candidate signals, which increases the computational complexity of the system. In this section, two new metrics have been proposed for analyzing the distribution of amplitudes of timedomain samples, which have enabled the proposed scheme to get more precise PAPR estimations with less computational complexity compared with RC-PTS. Next, two new metrics are given as follow.

a: THE FIRST METRIC

Theorem: For any numbers $a_i (1 \le i \le m)$, there exists $(|a_1|+|a_2|+\cdots+|a_m|)^2 \le m (|a_1|^2+|a_2|^2+\cdots+|a_m|^2).$

(

Since
$$(|a| - |b|)^2 = |a|^2 + |b|^2 - 2|a||b| \ge 0$$
 (9)
Then, $|a|^2 + |b|^2 \ge 2|a||b|$ (10)

Thus,

$$|a| + |b|)^{2} = |a|^{2} + |b|^{2} + 2|a||b|$$

$$\leq 2\left(|a|^{2} + |b|^{2}\right)$$
(11)

So,

$$(|a_{1}| + |a_{2}| + \dots + |a_{m}|)^{2}$$

$$= |a_{1}|^{2} + |a_{2}|^{2} + \dots + |a_{m}|^{2} + 2|a_{1}||a_{2}|$$

$$+ 2|a_{1}||a_{3}| + \dots + 2|a_{1}||a_{m}| + 2|a_{2}||a_{3}|$$

$$+ \dots + 2|a_{m-1}||a_{m}|$$

$$\leq |a_{1}|^{2} + |a_{2}|^{2} + \dots + |a_{m}|^{2} + |a_{1}|^{2} + |a_{2}|^{2} + |a_{1}|^{2}$$

$$+ |a_{3}|^{2} + \dots + |a_{1}|^{2} + |a_{m}|^{2} + |a_{2}|^{2} + |a_{3}|^{2}$$

$$+ \dots + |a_{m-1}|^{2} + |a_{m}|^{2}$$

$$\leq m \left(|a_{1}|^{2} + |a_{2}|^{2} + \dots + |a_{m}|^{2} \right)$$
(12)

From the above descriptions, the *n*-th sample of the *u*-th OFDM candidate signal can be written as:

$$x_n^u = \sum_{\nu=1}^V x_{\nu,n}^u, \quad 0 \le n \le N - 1$$
(13)

Thus,

$$\begin{aligned} |x_n^u|^2 &= \left| \sum_{\nu=1}^V x_{\nu,n}^u \right|^2 = |x_{1,n}^u + x_{2,n}^u + \dots + x_{V,n}^u|^2 \\ &\leq \left(|x_{1,n}^u| + |x_{2,n}^u| + \dots + |x_{V,n}^u| \right)^2 \\ &\leq V \times \left(|x_{1,n}^u|^2 + |x_{2,n}^u|^2 + \dots + |x_{V,n}^u|^2 \right) \\ &= V \sum_{\nu=1}^V |x_{\nu,n}^u|^2 \end{aligned}$$
(14)

From Eq. (7), we can get:

$$x_{\nu,n}^{u} = b_{\nu}^{u} x_{\nu,n}, \quad 0 \le n \le N - 1$$

$$b_{\nu}^{u} = e^{j\phi_{\nu,u}} = \cos\phi_{\nu,u} + j\sin\phi_{\nu,u}$$
(15)

Substituting Eq. (15) into Eq. (14), we can get:

$$\begin{aligned} x_n^u \Big|^2 &\leq V \sum_{\nu=1}^V |x_{\nu,n}^u|^2 \\ &= V \sum_{\nu=1}^V |b_{\nu}^u \cdot x_{\nu,n}|^2 \\ &= V \sum_{\nu=1}^V |(\cos \phi_{\nu,u} + j \sin \phi_{\nu,u}) \cdot x_{\nu,n}|^2 \\ &= V \sum_{\nu=1}^V |x_{\nu,n} \cos \phi_{\nu,u} + j x_{\nu,n} \sin \phi_{\nu,u}|^2 \end{aligned}$$

$$\leq V \sum_{\nu=1}^{V} \left(\left| x_{\nu,n} \cos \phi_{\nu,u} \right| + \left| j x_{\nu,n} \sin \phi_{\nu,u} \right| \right)^{2}$$

$$\leq 2V \sum_{\nu=1}^{V} \left(\left| x_{\nu,n} \cos \phi_{\nu,u} \right|^{2} + \left| j x_{\nu,n} \sin \phi_{\nu,u} \right|^{2} \right)$$

$$= 2V \sum_{\nu=1}^{V} \left(\left| x_{\nu,n} \right|^{2} \left| \cos \phi_{\nu,u} \right|^{2} + \left| x_{\nu,n} \right|^{2} \left| j \sin \phi_{\nu,u} \right|^{2} \right)$$

$$= 2V \sum_{\nu=1}^{V} \left| x_{\nu,n} \right|^{2} = 2VQ_{n}$$
(16)

where $Q_n = \sum_{\nu=1}^{V} |x_{\nu,n}|^2$ is the sum of power of the samples at time *n* in all the *V* subblocks.

In terms of Eq. (16), there exists

$$E\left\{\left|x_{n}^{u}\right|^{2}\right\} \leq 2VQ_{n} \tag{17}$$

That is:

$$\frac{\max_{\substack{0 \le n \le N-1}} |x_n^u|^2}{2V} \le Q_n$$

$$E\left\{ |x_n^u|^2 \right\} \le \max_{\substack{0 \le n \le N-1}} |x_n^u|^2$$
(18)

Thus, the threshold for selecting the dominant time-domain samples can be given by:

$$Td_Q = \frac{E\left\{\left|x_n^u\right|^2\right\}}{2V} = \frac{\chi}{2V}$$
(19)

where $\chi = E\left\{\left|x_n^u\right|^2\right\}$ denotes the even power of the *u*-th candidate signal.

In addition, to reduce computational complexity further, grouping method is introduced into the proposed PTS scheme, which divides all the time-domain subblocks into G groups before the dominant time-domain samples are selected. Thus, for each group, its own threshold for selecting dominant time-domain samples can be given by

$$Td_Q^g = \frac{\chi_g}{2M_g}, \quad 1 \le g \le G \tag{20}$$

where M_g and χ_g represent the number of subblocks and the average power of a candidate signal in g-th group respectively.

Let Q_n^g , $1 \le g \le G$, $0 \le n \le N - 1$ be the sum of power of the samples at time *n* in the *g*-th group. Based on the above analysis, if $Q_n^g < Td_Q^g$, the corresponding samples can be directly subjected to PAPR calculation without phase factor weighting. That is to say, the phase factors are used only for weighting these samples with $Q_n^g \ge Td_Q^g$. Thus, each group has its own process of selecting the samples for phase factor weighting.

According to Eq.(20), the first metric of proposed scheme is only affected by two factors: the average power of candidate signals χ_g and the number of subblocks M_g in each group. But for the metric of RC-PTS [26], besides the average power of candidate signals, there are the other three decisive factors: the number of subcarriers N, the total number of subblocks V and the minimum probability of capturing the peak of OFDM signal γ . Therefore, compared to the metric of RC-PTS [26], the first metric of proposed scheme reduces the number of decisive factors, which results in complexity reduction when the threshold for selecting the dominant timedomain samples is calculated. It is more conducive to the stability and availability for obtaining good PAPR performance of OFDM systems.

b: THE SECOND METRIC

In [26], the threshold Td_Q for selecting the samples in all the candidate signals is given by:

$$Td_{Q} = -\frac{\chi}{V} \ln \left[\frac{\ln [1 - \gamma]}{-N \left(\frac{\pi}{3} \ln N \right)^{0.5}} \right]$$
$$= -\frac{\chi}{V} \varphi$$
(21)

where $\varphi = \ln \left[\frac{\ln[1-\gamma]}{-N(\frac{\pi}{3} \ln N)^{0.5}} \right]$, χ is the average power of an OFDM signal, γ represents the minimum probability that captures the peak of the OFDM signal and *V* is the number of time-domain subblocks.

Because grouping method is adopted in the proposed scheme and each group contains part of the subblocks. In this way, the energy of an OFDM signal is distributed into every group. Thus, based on Eq. (21), the second new metric Td_Q^g for each group can be given by

$$Td_Q^g = -\frac{\chi_g}{GV}\varphi, \quad 1 \le g \le G$$
 (22)

where G denotes the number of groups. Compared with the metric of RC-PTS given by Eq. (21), the number of groups G is involved in the proposed second metric, which is because the grouping method is adopted.

According to the above two metrics, only the samples with $Q_n^g \ge Td_Q^g(1 \le g \le G, 0 \le n \le N - 1)$ in each group need be weighted by phase factors for estimating the PAPRs of candidate signals, where Q_n^g denotes the sum of power of the samples at time *n* in the *g*-th group.

Now, the sets of indices of samples selected by the two new metrics in all the groups are given by:

$$\begin{cases} \boldsymbol{S} = \{\boldsymbol{S}_1, \boldsymbol{S}_2, \dots, \boldsymbol{S}_G\} \\ \boldsymbol{S}_g = \left\{ n \left| \boldsymbol{Q}_n^g \ge T \boldsymbol{d}_{\boldsymbol{Q}}^g \right\}, \quad 1 \le g \le G, \ 0 \le n \le N-1 \end{cases}$$
(23)

where S_g is the set of indices of selected samples in the *g*-th group.

B. THE PROCESS OF PROPOSED PTS SCHEME

The block diagram of proposed PTS scheme is given in Figure 2.

The specific steps of proposed PTS scheme are given as follows.

TABLE 1. Computational complexity of OPTS, RC-PTS and proposed PTS.

Complexity	OPTS	RC-PTS	Proposed PTS
Complex Multiplications	LN(V+1)U	$LNV + p_{\gamma}LN(V+1)U + LN$	$VLN + \beta_1 M_1 W^{M_1 - 1} + \sum_{g=2}^G \beta_g M_g W^{M_g} $ $+ U \min_{g=1}^G \beta_g + LN$
Real Additions	$2LN \cdot (V-1)U + LNU - 1$	$LN(V-1) + LN +2Up_{\gamma}LN(V-1) +U(p_{\gamma}LN-1) +U-1 +2LN(V-1)$	$(V-G)LN + GLN + 2\beta_{1}W^{M_{1}-1}(M_{1}-1) + \sum_{g=2}^{G} 2\beta_{g}W^{M_{g}}(M_{g}-1) + 2U\left(\sum_{g=1}^{G}\beta_{g} - \max_{k=1}^{G}\beta_{k}\right) + U(\min_{g=1}^{G}\beta_{g}-1) + U - 1 + 2LN(V-1)$



FIGURE 2. The block diagram of proposed PTS scheme.

Step 1: The input modulated data X is divided into V subblocks X_v , $1 \le v \le V$. Then, the time-domain subblocks x_v , $1 \le v \le V$ are obtained by IFFTs.

Step 2: The *V* time-domain subblocks $\mathbf{x}_v (1 \le v \le V)$ are divided into *G* groups, and each group contains two or more subblocks.

Step 3: Calculate $Q^g = [Q_0^g, Q_1^g, \dots, Q_{N-1}^g], 1 \le g \le G$ of all the groups.

Step 4: Find the samples with $Q_n^g \ge Td_Q^g$, $0 \le n \le N-1$ in each group and generate the sets of indices S_g , $1 \le g \le G$.

Step 5: Based on the sets of indices in each group, the corresponding samples are weighted by phase factors, and then perform PAPR calculation.

Step 6: Find the candidate signal with the lowest PAPR and its corresponding combination of phase factors $\hat{b} = \{\hat{b}_1, \hat{b}_2, \dots, \hat{b}_V\}.$

Step 7: Use the optimal combination of phase factors \hat{b} obtained in Step 6 to weight all the time-domain subblocks in Step 1 to achieve the OFDM signal for transmission.

It is worthy of note that in proposed scheme, since the grouping method is involved, each group has its own processes of selecting dominant time-domain samples and phase factor weighting. Because there are part of the subblocks in each group, both the number of phase weighting sequences and the number of elements in each sequence in proposed scheme are much less than those in OPTS and RC-PTS when the total number of OFDM candidate signals is same. For example, the number of subblocks V = 4, the set of phase weighting factors is $\{\pm 1, \pm j\}$ (i.e. W = 4), the number of groups G = 2 and there are two subblocks in each group. In proposed scheme, 20 phase weighting sequences can be generated and there are only 2 elements in each sequence, but for OPTS and RC-PTS, 64 phase weighting sequences are generated and there are 4 elements in each sequence. For these three schemes in this example, the total number of OFDM candidate signals is same, i.e. 64 OFDM candidate signals. Thus, compared with OPTS and RC-PTS, proposed scheme can reduce computational complexity clearly.

C. COMPUTATIONAL COMPLEXITY ANALYSIS

TABLE I gives the computational complexity of proposed PTS scheme, RC-PTS [26] and OPTS in terms of complex multiplication and real addition, where the optimization process for obtaining the optimal candidate signal is only taken into account because these three schemes have the same number of IFFTs.

In TABLE I, L and W represent the oversampling factor and the number of elements in phase factor set respectively, $U = W^{V-1}$ denotes the number of all the candidate signals, M_g and β_g represent the number of subblocks and the average number of the selected samples in the g-th group of proposed PTS, p_{γ} is the average number of the selected samples in RC-PTS.

Here, in order to compare the computational complexity of these three schemes conveniently, it can be regarded that a square operation and a square root operation are both equivalent to a complex multiplication; a comparison and a complex addition are equivalent to one real addition and two real additions respectively [26].

In TABLE I, for the proposed scheme, in order to obtain Q_n^g ($1 \le g \le G$) of all the groups in Step 3 of Section III, it needs *VLN* complex multiplications and (*V*-*G*)*LN* real additions. Then, selecting the dominant time-domain samples of all the groups needs *GLN* real additions. In Step 5 of Section III, the computational complexity for obtaining all OFDM candidate sequences is given as follows:

The number of complex multiplications =

$$\beta_1 M_1 W^{M_1 - 1} + \sum_{g=2}^G \beta_g M_g W^{M_g}$$

The number of real additions =

$$2\beta_1 W^{M_1 - 1}(M_1 - 1) + \sum_{g=2}^G 2\beta_g W^{M_g}(M_g - 1) + 2U\left(\sum_{g=1}^G \beta_g - \max_{k=1}^G \beta_k\right)$$

Next, the computational complexity for estimating the PAPRs of all the candidates is given by

The number of complex multiplications = $U \min_{g=1}^{G} \beta_g$

The number of real additions = $U(\min_{g=1}^{o} \beta_g - 1)$

At last, there are LN complex multiplications and U - 1 + 2LN (V - 1) real additions in the generation of the transmitted OFDM signal.

It is worth mentioning that when L, W, V and N are fixed, the computational complexity of RC-PTS [26] is decided by p_{γ} , and the computational complexity of the proposed PTS is mainly dominated by G, M_g and β_g . Moreover, the low-complexity of RC-PTS is mainly due to the introduction of the metric for selecting the dominant time-domain samples. These selected samples are used to estimate the PAPR value of each candidate signal, which results in avoiding the full process of generating each candidate signal. But for the proposed scheme, besides introducing two new metrics for reducing computational complexity, the grouping scheme is adopted for reducing computational complexity further. Therefore, the low-complexity of the proposed scheme is mainly affected by the selected metric and the number of groups G.

V. SIMULATION RESULTS

In this section, PAPR performances of proposed PTS scheme, RC-PTS [26] and OPTS are shown by simulations. The basic parameters of OFDM system are the number of subcarriers N = 128, QPSK modulation and the oversampling factor L = 4. For the proposed PTS scheme, Prop.PTS1 and Prop.PTS2 represent the proposed scheme using two new metrics respectively.

Figure 3 shows the PAPR performances of proposed PTS, RC-PTS and OPTS, where the set of allowed phase factors $\{\pm 1, \pm j\}$ (i.e. W = 4), the number of subblocks V = 4, the number of groups G = 2 and each group includes two subblocks.

In Figure 3, the simulation results show that compared with OPTS and RC-PTS with $\gamma = 0.9999$, proposed PTS employing the first metric can achieve the same PAPR performance, but the proposed scheme employing the second metric with



FIGURE 3. PAPR performances of proposed PTS, RC-PTS and OPTS when W = 4, V = 4, N = 128 and G = 2.



FIGURE 4. The comparison of computational complexity among these three schemes when W = 4, V = 4, N = 128, G = 2 and $\gamma = 0.9999$.

 $\gamma = 0.9999$ gives a slightly worse PAPR performance. For example, the PAPRs of OPTS, RC-PTS with $\gamma = 0.9999$, Prop.PTS1 and Prop.PTS2 with $\gamma = 0.9999$ are 8.00dB, 8.00dB and 8.20dB respectively when CCDF = 10^{-3} . Compared to the results with $\gamma = 0.9999$, it can be seen that when $\gamma = 0.1000$, PAPR performances of proposed scheme with the second metric and RC-PTS both degrade a little, but they are still similar. Meanwhile, the comparison of computational complexity among these three schemes are given in Figure 4, where the computational complexities of the proposed scheme with the first metric (i.e. Prop.PTS1), the proposed scheme employing the second metric with $\gamma =$ 0.9999 (i.e. Prop.PTS2), RC-PTS with $\gamma = 0.9999$ and OPTS are shown.

It is clearly seen in Figure 4 that when the three schemes show the similar PAPR performance, proposed PTS scheme can obtain more reduction in computational complexity compared with RC-PTS [26]. For instance, compared with OPTS, RC-PTS needs 56.84% complex multiplications and 57.51% real additions, but Prop.PTS1 only needs 31.17% complex multiplications and 49.42% real additions, and Prop.PTS2 only needs 27.24% complex multiplications and 43.14% real additions. Thus, the proposed scheme achieves lower computational complexity than RC-PTS, which is mainly thanks to the grouping method. In view of the results in Figure 3 and Figure 4, it is clearly known that compared



FIGURE 5. PAPR performances of proposed PTS, RC-PTS and OPTS when W = 4, V = 6, N = 128, and G = 2.



FIGURE 6. The comparison of computational complexity of among these three methods when W = 4, V = 6, N = 128, G = 2 and $\gamma = 0.9999$.

with OPTS and RC-PTS, the proposed scheme can gain the similar PAPR performance with lower computational complexity.

In order to further demonstrate the performance of the proposed PTS scheme, the simulations in Figure 3 is repeated except V = 6, shown in Figure 5, where each group includes three subblocks.

In Figure 5, with the given parameters, the OFDM system based on proposed PTS scheme with the first metric has the same PAPR performance compared with OPTS and RC-PTS with $\gamma = 0.9999$, but the proposed scheme makes a slight loss of PAPR performance when the second metric with $\gamma = 0.9999$ is employed. For example, the PAPRs of OPTS and Prop.PTS1 are both 7.00dB, the PAPRs of RC-PTS and Prop.PTS2 with $\gamma = 0.9999$ are 7.00dB and 7.34dB when CCDF = 10^{-3} . In addition, for proposed scheme with the second metric and RC-PTS, they also have the similar PAPR performance when $\gamma = 0.1000$. Then, the corresponding computational complexity of these schemes are given in Figure 6, where the computational complexities of proposed scheme with the first metric (i.e. Prop.PTS1), proposed scheme using the second metric with $\gamma = 0.9999$ (i.e. Prop.PTS2), RC-PTS with $\gamma = 0.9999$ and OPTS are given.

It can be seen from Figure 6 that the proposed PTS scheme can achieve more computational complexity reduction compared with RC-PTS when those schemes get the same PAPR



FIGURE 7. PAPR performances of proposed PTS, RC-PTS and OPTS when W = 2, V = 6, N = 128 and G = 3.

reduction. In Figure 6, compared with OPTS, RC-PTS with $\gamma = 0.9999$ requires 87.11% complex multiplications and 87.15% real additions, but Prop.PTS1 only requires 17.49% complex multiplications and 29.84% real additions, and Prop.PTS2 with $\gamma = 0.9999$ only requires 16.78% complex multiplications and 28.11% real additions. Therefore, compared with OPTS and RC-PTS, low-complexity of the proposed scheme is checked again. Besides, based on the above analysis, in the case of ensuring that the other parameters of OFDM system are unchanged, as the number of subblocks *V* increasing, proposed PTS scheme can save more computational complexity compared with RC-PTS.

Last, in order to further verify the superiority of proposed scheme, the simulations are done with G = 3 and the set of phase factors $\{1, -1\}$, shown in Figure 7, where each group contains two subblocks.

In Figure 7, it is clearly found that for the proposed PTS scheme using the first metric and the second metric with $\gamma = 0.9999$, RC-PTS with $\gamma = 0.9999$ and OPTS, they can gain the similar PAPR performance. For instance, the PAPRs of OPTS and RC-PTS with $\gamma = 0.9999$ are both 7.80dB, and the PAPRs of Prop.PTS1 and Prop.PTS2 with $\gamma = 0.9999$ are 8.00dB and 7.90dB when CCDF = 10^{-3} . Besides, when $\gamma = 0.1000$, the proposed scheme with the second metric also has the similar PAPR performance compared to RC-PTS. At the same time, the corresponding computational complexity of these three schemes are given in Figure 8, where computational complexities of proposed scheme with the first metric (i.e. Prop.PTS1), proposed scheme employing the second metric with $\gamma = 0.9999$ (i.e. Prop.PTS2), RC-PTS with $\gamma = 0.9999$ and OPTS are shown.

In Figure 8, it is obvious that the proposed PTS scheme can achieve the purpose of getting more reduction in computational complexity compared with RC-PTS when those schemes have the similar PAPR performance. For instance, compared with OPTS, RC-PTS with $\gamma = 0.9999$ needs



FIGURE 8. The comparison of computational complexity among these three methods when W = 2, V = 6, N = 128, G = 3 and $\gamma = 0.9999$.

90.20% complex multiplications and 91.60% real additions, but Prop.PTS1 only requires 24.32% complex multiplications and 51.24% real additions, and Prop.PTS2 with $\gamma = 0.9999$ only requires 25.07% complex multiplications and 52.89% real additions. Thus, it is clearly seen that the proposed scheme can obtain more reduction in computational complexity compared with RC-PTS.

In a word, simulation results demonstrate that compared with OPTS and RC-PTS, the proposed scheme can achieve dramatic computational complexity reduction with the similar PAPR performance.

In fact, for OPTS, its PAPR performance is totally determined by the number of candidate signals. But for RC-PTS scheme and the proposed scheme, besides the number of candidate signals, their PAPR performances are affected by the selected metric. That is to say, even if RC-PTS scheme and the proposed scheme have the same number of candidate signals, they may obtain different CCDF curves because they employ different metrics. For example, in Fig. 3 and Fig. 5, when $\gamma = 0.1000$, the CCDF curves of RC-PTS scheme and Prop.PTS2 are slightly different, and there exists the intersection of CCDF curves between RC-PTS and Prop.PTS2. The intersection means that under the given parameters, the probabilities that the PAPR values of OFDM signals in the OFDM system employing RC-PTS and Prop.PTS2 are greater than the given PAPR threshold are same. Based on the above analysis, for the proposed scheme, both the PAPR performance and the low-complexity are affected by the selected metric. But for either the PAPR performance or the low-complexity, the selected metric is not the only determinant. Thus, it is difficult to clearly identify the relationship between PAPR performance and low-complexity in the proposed scheme.

VI. CONCLUSION

In this paper, a low-complexity PTS scheme is proposed for PAPR reduction of OFDM systems. In proposed PTS scheme, two new metrics for selecting the ideal time-domain samples and grouping method are involved for achieving more reduction in computational complexity without degrading the PAPR performance. For one thing, the first metric of proposed scheme is dominated only by two factors: the average power of candidate signals and the number of subblocks in each group. But for the metric of RC-PTS [26], besides the average power of candidate signals, there are the total number of subblocks and the minimum probability of capturing the peak of OFDM signal. Hence, compared with RC-PTS, the proposed scheme can gain the threshold for selecting the dominant time-domain samplings easily and is more conducive to achieve the stability and availability of good PAPR performance of OFDM systems. For another, grouping method is introduced before selecting the dominant time-domain samples. All the time-domain subblocks are divided into several groups, and each group has its own processes of selecting the dominant time-domain samples and phase factor weighting. Compared with OPTS and RC-PTS, the proposed scheme reduces computational complexity clearly when the total number of OFDM candidate signals generated in these three schemes is same. As a result, the introduction of grouping method can achieve the purpose of reducing more computational complexity with no loss of PAPR performance. Thus, compared with RC-PTS [26], the proposed PTS scheme can obtain similar PAPR performance with more reduction in computational complexity.

the other three decisive factors: the number of subcarriers,

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