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An Improved Two-Step Face Recognition Algorithm Based on Sparse Representation

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ABSTRACT Weighted score fusion is a widely used score fusion scheme, but the weights need to be set manually. The results generally vary greatly when the weights are different, so it is difficult to find the optimal weights. This is why it is necessary to constantly set different weights for experimental comparisons to find the optimal weights. In this paper, an improved fusion method is proposed for above shortcoming, that is, multiplication fusion applicable to sparse representation. The fusion scheme not only is easy to use but also does not need to be artificially set weights. Moreover, it is consistent with the correlation between the classification error and the score obtained by the experimental analysis. In the field of face recognition, it has been shown that the two-step face recognition (TSFR) based on representation using the original training samples and the generated ''symmetric face'' training samples can achieve excellent face recognition performance. Face recognition based on multiplication fusion and TSFR proposed in this paper can further improve the recognition accuracy.

INDEX TERMS Sparse representation, symmetrical face, face recognition, multiplication fusion.

I. INTRODUCTION

Face recognition is a kind of biometric recognition technology based on human facial feature information, so the field of face recognition has been concerned by people [1], [3]–[5]. Many papers have proposed corresponding face recognition algorithms and face databases. But there are still great difficulties in face recognition, such as different illumination [6], facial expression [7], occlusion face [8] and so on, which hinder the face accuracy recognition rate. Other fields have put forward relevant methods to solve the existing problems [9], [10], but still need to be solved in the field of face recognition. Therefore, it is important to find more face training samples to express these scenes and design a good face recognition algorithm. More training samples can reflect facial morphology better, which will improve the accuracy of face recognition [11], [12].

Not enough face training samples is the biggest problem that exist at present. The well-known face databases released include AR database, FERET database, ORL database, YaleA database and so on. However, these face databases are still unable to meet the needs of fully expressing the face morphology, so the efficiency of correctly recognizing the face does not meet our needs. How to improve the recognition accuracy under the limited training samples is a problem to be considered. In order to solve this problem, researchers have proposed the concept of virtual sample, which is to synthesize a certain number of samples by using limited real face samples [13]–[17]. These generated virtual samples may be somewhat different from the original samples, but in general they can be approximately equivalent to the different forms represented by the original samples. However, the generated virtual sample set is no more than the real training sample set and still could not meet the actual needs. In [18], a new training sample of ''symmetrical face'' is generated by using the original training samples. Through this method, an original training sample can generate two ''symmetrical face'' training samples, which the training sample set is expanded twice, thereby solving the problem of the small number of face recognition samples, and the candidate faces can be

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quickly and accurately selected to improve the recognition rate.

At present, many face recognition algorithms have been proposed. For example, Rahimzadeh Arashloo *et al.* [19] proposed a new method for single sample face recognition problem based on local dual-tree complex wavelet transform (DT-CWT) representation which offers invariance to moderate real world image variations, such as illumination, expression, head pose, shift and in-plane rotation. Mehta *et al.* [20] proposed a novel approach to face recognition problem using directional and texture information from face images. Yang *et al.* [6] proposed an adaptive Weber-face method to deal with varying lighting and extract illumination-insensitive representation for face recognition. And the local binary patterns are extracted from the Weber-face images which further alleviate the effect of varying illumination. Feng *et al.* [21] presented a novel face recognition method based on direct discriminant Volterra kernels and effective feature classification (DD-VK). This method can simultaneously maximize inter-class distances and minimize intra-class distances in the feature space. Gao *et al.* [22] to tackle this problem that 2-DPCA is not robust to outliers. They presented an efficient robust method, namely R_1 -2-DPCA for feature extraction which is not only robust to outliers but also helps encode discriminant information. These classic algorithms have made a huge contribution to the accurate recognition of faces. Wright [23] proposed sparse representation algorithm has been widely concerned and studied by researchers because of its fast speed and high recognition rate. Wang *et al.* [24] present a unified framework based on kernel collaborative representation for linear and non-linear schemes. The framework provides insights of the relationships among several effective representation schemes, and facilitates the designing of new algorithms by choosing kernel functions, regularizations, and/or additional constraints. Zhang *et al.* [25] made a survey of sparse representation, and analyzed the rationales of different algorithms in each category and summarized a wide range of sparse representation applications. Sparse representation algorithm increases recognition speed and accuracy, but there's still a lot of research space in improving performance. So, Xu *et al.* [26] propose a two-phase test sample representation method for face recognition. The first phase of the proposed method seeks to represent the test sample as a linear combination of all the training samples and exploits the representation ability of each training sample to determine M "nearest neighbors" for the test sample. The second phase represents the test sample as a linear combination of the determined M nearest neighbors and uses the representation result to perform classification. Based on "two-phase", some improved algorithms are proposed, and excellent results are obtained [17], [27], [28].

In this paper, the shortcoming of the paper [17] that need to continuously experiment to find the optimal weights is improved, that is, the final weighted score fusion is transformed into multiplication fusion. The steps of the algorithm are as follows: the first step generates "symmetric face" training samples; the second step and the third step respectively use the original training samples and the "symmetric face" samples to perform two-step face recognition; the last step is the weighted score fusion of the original algorithm is transformed into multiplication fusion by using the scores obtained in the second and third steps.

FIGURE 1. Some original training samples of the ORL database and two corresponding "symmetrical face" samples. The first line is the original training samples, the second line is the "symmetrical face" samples obtained from the left half of the original samples. and the third line is the "symmetrical face" samples obtained from the right half of the original samples.

II. ALGORITHM INTRODUCTION

In this section, we will mainly introduce the main algorithms of the paper and improved method of the paper. Suppose there are C classes, each of which has r training samples, denoted by $x_1, \ldots, x_r$ respectively. There is a total of $M = Cr$ training samples, each sample is converted into a one-dimensional column vector.

A. ABBREVIATIONS AND ACRONYMS

In the algorithm, it is first necessary to generate corresponding two "symmetric face" training samples. For each original training sample x_i, two "symmetric face" training samples can be generated, expressed as s_i^1, s_i^2 respectively. The left half of s_i^1 is the same as the left half of x_i, while the right half is the mirror of the left half of s_i^1. On the contrary, the right half of s_i^2 is the same as the right half of x_i, while the left half is the mirror of the right half of s_i^2. Figure 1 shows some original training samples in the ORL face database and the corresponding two "symmetrical face" training samples, in which the first line is the original training samples, the second line is the first "symmetrical face" sample s_i^1, and the third line is the second "symmetrical face" sample s_i^2. Figure 2, Figure 3 and Figure 4 show some of the original training samples of FERET, AR and YaleA databases and two corresponding "symmetrical face" training samples, respectively.

Secondly, two-step face recognition is performed on the original training samples and the "symmetric face" training samples, respectively. The first step of face recognition is that the linear combination of all training samples is

FIGURE 2. Some original training samples of the FERET database and two corresponding "symmetrical face" samples. The first line is the original training samples, the second line is the "symmetrical face" samples obtained from the left half of the original samples. and the third line is the "symmetrical face" samples obtained from the right half of the original samples.

FIGURE 3. Some original training samples of the AR database and two corresponding "symmetrical face" samples. The first line is the original training samples, the second line is the "symmetrical face" samples obtained from the left half of the original samples. and the third line is the "symmetrical face" samples obtained from the right half of the original samples.

approximately equal to the test sample. The formula is:

$$y = a_1 x_1 + \ldots + a_M x_M \tag{1}$$

a_i is the corresponding coefficient of x_i, y is the test sample, and $x_i (i = 1, 2, \ldots, M)$ denote the training samples of all these classes. Assuming that $A = [a_1 \ldots a_M]^T, X = [x_1 \ldots x_M]$, then equation (1) can be changed to:

$$y = XA \tag{2}$$

A is calculated by $A^* = (X^T X + \mu I)^{-1} X^T y$, where μ is a small positive constant and I is the identity matrix.

Next, calculate the representation error of the k-th class and the test sample, using the formula:

$$d_k = \left\| y - \sum_{i=j}^{l} a_i x_i \right\|_2 \tag{3}$$

Among them, $x_j, \ldots, x_l$ denote all the training samples of the k-th class, d_k denotes the representation error, and

FIGURE 4. Some original training samples of the YaleA database and two corresponding "symmetrical face" samples. The first line is the original training samples, the second line is the "symmetrical face" samples obtained from the left half of the original samples. and the third line is the "symmetrical face" samples obtained from the right half of the original samples.

the smaller the d_k, the closer it is to the test sample. Suppose$d_{m1}, \ldots, d_{mm} (d_{m1} \leq d_{m2} \ldots \leq d_{mm})$ indicate that the smallest $m(m = 0.1C \sim 0.3C)$ selected in all representation errors, which means that corresponding classes $c_{m1}, \ldots, c_{mm}$ are closer to the test sample, $c_{m1}, \ldots, c_{mm}$ are candidate classes for the test sample. These training samples of m candidate classes are retained, while training samples of other classes should be discarded.

The above is the first step of face recognition, also known as coarse classification. The second step of face recognition is to operate on the remaining m candidate classes, also known as fine classification.

In the fine classification, the test sample is represented by a linear combination of the training samples of the remaining m candidate classes, as follows:

$$y = \beta_1 b_1 + \ldots + \beta_n b_n \tag{4}$$

where$n$ is the number of all the training samples from the first m candidate classes.y is the test sample, β_i is the coefficient of b_i, and $b_i (i = 1, 2, \ldots, n)$ represent the all training samples of m candidate classes. Equation (4) can be written as:

$$y = B\beta \tag{5}$$

where $B = [b_1, \ldots b_n]$, $\beta = [\beta_1 \ldots \beta_n]^T$. β is calculated by using $\beta^* = (B^T B + \gamma I)^{-1} B^T y$. γ is a small positive constant and I is the identity matrix.

Suppose $b_e, \ldots, b_h$ denote the training samples of the candidate class r $(r \in c_{m1}, \ldots, c_{mm})$, $\beta_e, \ldots, \beta_h$ denote the coefficients of $b_e, \ldots, b_h$,respectively. The final effect of class r can be evaluated by the representation error σ_r.

$$\sigma_r = \left\| y - \sum_{i=e}^{h} \beta_i b_i \right\|_2 \tag{6}$$

If σ_p is the smallest representation error, the test sample y is finally assigned to the p-th class. In other words, the test sample belongs to the p-th class, thus completing the fine classification. Through the above method, we can get more accurate classification results and reduce the error rates.

Both the original training samples and the "symmetric face" training samples can use the two-step sparse representation with the smallest 2-norm method to calculate the scores of the test samples and each class. In the sparse representation algorithm, the score fusion has the following advantages: the algorithm is classified based on the representation error. When the representation error of a class is smaller, the greater the probability of the test sample belongs to this class. The corresponding scores are calculated through the above method, and then the final score is obtained by weighted score fusion.

g_i^1 represents the final score of the original training samples, and g_i^2 represents the final score of the "symmetrical face" training samples. Finally, using the formula (7) for weighted score fusion:

$$g_i = w_1 g_i^1 + w_2 g_i^2 \tag{7}$$

w_1 and w_2 represent the weights of the original training samples and the "symmetrical face" training samples, respectively, and $w_1 + w_2 = 1$.

B. IMPROVED ALGORITHM

Weighted score fusion is generally better than feature fusion and decision fusion. In the algorithm of [17], the scores of the original training samples and the "symmetric face" training samples are calculated by the two-step sparse representation method with the smallest 2-norm. In the end, the scores are fused by weighted score fusion. However, there is also a defect: the weights of weighted score fusion need to be set manually, and different weights will produce different results. This requires constant experiments to select the optimal weights, which is not suitable for practical applications. Moreover, in practical applications, it is almost impossible to find the optimal weights. To solve this problem, an improved method is proposed in which the weighted score fusion is replaced by multiplication fusion. The formula is:

$$f_i = g_i^{1*} g_i^2 \tag{8}$$

It is clear from the formula that multiplication fusion does not require setting parameters. An algorithm that does not require parameters in actual applications is a very practical algorithm, even reduce a parameter for existing algorithms is a great progress. Even under certain conditions, the "optimal" parameters can be roughly obtained by some experimental means, but compared with the weighted score fusion, the proposed multiplication fusion scheme can well preserve the case that one of the algorithms has a small representation error (the probability of belonging to a certain class is very large). The average effect of weighted score fusion is especially obvious, which will "smooth" out the information of very small representation error obtained by an algorithm. Here can be illustrated by an example: If the representation error of a class obtained from the original training samples is 0.1 for the test sample and that of the same class of the "symmetrical face" training samples is 0.8 for the test sample. The weighted score fusion (two samples allocate the equal weight coefficient 0.5) shows that the classification error of the test sample is 0.45, which is likely to be judged as not belonging to the class and will be classified as a mistake. And the representation error of this class to test samples obtained by multiplication fusion is 0.08, which reflects the fact that the probability of test sample belongings to this class is very high. Therefore, it is easy to get better classification results.

C. NUMERICAL ANALYSIS OF THE RATIONALITY OF THE ALGORITHM

The relationship between the final score and the original score is linear in the weighted score fusion scheme $f_i = w_1 g_i^1 + w_2 g_i^2$, which does not necessarily conform to the actual situation. Here, using the FERET database to illustrate why multiplication fusion is advantageous. There are 200 classes in the FERET database, and each class has 7 samples. Assuming that each class has 2 training samples, the remaining 5 samples are test samples. There will be a total of 1000 test samples. As mentioned above, the class labels used in the second step of face recognition are the first m classes selected in the first step of face recognition with the smallest representation errors. Here only aim at the original training samples, select the smallest representation errors for 1000 test samples, denoted by $e_1, \ldots, e_{1000}$, respectively. According to the experimental results, $e_1, \ldots, e_{1000}$ is divided into five intervals of [0, 0.6), [0.6, 0.7), [0.7, 0.8), [0.8, 0.9), [0.9, 1] and the number of these intervals is statistically divided. Among them, 136 of them are within the range of [0,0.6], and 136 are classified correctly, that is, the correct classification rate is 100%, and the classification error rate is 0. There are 175 in the interval [0.6,0.7), and the correct number is 165. 336 are in the interval [0.7,0.8), the number of correctly classification is 213. There are 347 in the interval [0.8, 0.9), the correct classification number is 111. And there are 6 in interval [0.9,1], the correct number is 0. The histogram of Figure 5 shows the classification error rate for each interval, the horizontal axis represents the minimum representation error e_i, and the vertical axis represents the classification error rate. The histogram is represented by a curve, as shown in Figure 6. It can be seen from the figure that the classification error rate has a nonlinear relationship with the original minimum representation error, that is, with the increase of the minimum representation error, the classification error rate shows a nonlinear rapid increase. This shows that for the sparse representation method, if the relationship between the final score and the original score is nonlinear, the classification error rate will be very small. Roughly speaking, the final score proposed in this paper has a certain nonlinear relationship with the original score, so it is reasonable.

Further formal analysis is as follows: Since g_i^2 is the score of the "symmetrical face" training samples which is generated from the original training samples, it can be assumed that

TABLE 1. Comparison of classification error rates in first stage of the FERET database.

Numbers of training samples per class	1	2	3	4
Original training sample classification error rate	0.5567	0.4160	0.5563	0.4467
"symmetric face" training sample classification error rate	0.5833	0.3920	0.4138	0.4583
Classification error rate through weighted score fusion	0.5567	0.4020	0.5225	0.4250
Classification error rate through multiplication fusion	0.5475	0.4000	0.4725	0.4133

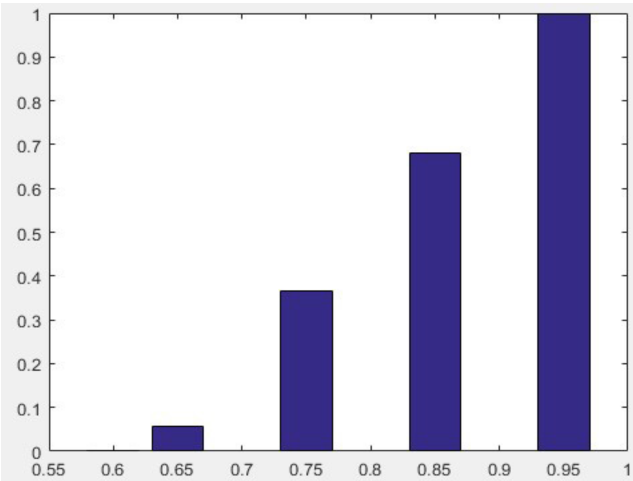


FIGURE 5. Error rates for each minimum representation error interval classification.

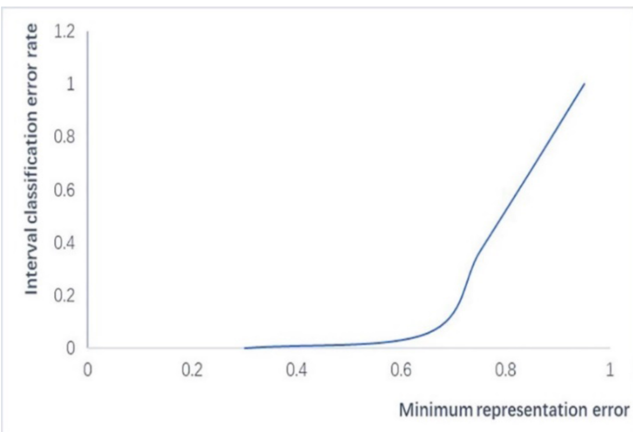


FIGURE 6. Classification error rate and minimum representation error relationship.

$g_i^1 = g_i^2 + \epsilon$, then for formula (8) there is $f_i = g_i^2\left(g_i^2 + \epsilon\right) = \left(g_i^2\right)^2 + \epsilon g_i^2$. It can be seen that the result of multiplication fusion is nonlinear with the original minimum representation error, which will be more conducive to classification.

III. EXPERIMENTAL RESULTS

In the experiment, we used four databases of FERET database [30], ORL database [35], AR database [36] and YaleA database [37] to carry out experiments. The following will analyse the results of each database by our algorithm. Through the analysis of the results of the four databases, we can get the improved algorithm is reasonable and effective.

A. RESULT ANALYSIS OF FERET DATABASE

In the FERET database, there are 200 classes, and each class has 7 samples. When the training samples of each class are m(m<7), the test samples are (7-m). In the coarse classification stage (the first step), the algorithm is cooperative representation classification (CRC) [29]. The original training samples of each class are 1, 2 and 3, 4 respectively, and the corresponding ''symmetrical face'' training samples are 2, 4 and 6, 8 respectively. In the first step, the classification error rates of the original training samples and the virtual training samples are compared, and the classification error rates of the original training samples and the virtual training samples are combined by the weighted score fusion($w_1 = 0.8, w_2 = 0.2$) and multiplication fusion are compared, as shown in Table 1. From the table, we can see that the classified results may be less than ideal in the coarse classification stage. At this time, a method is needed to improve it. What we propose is to enter the fine classification stage. That is to say, it is necessary to discard the classes that are far away from the test samples, and retain the relatively close classes for fine classification and then perform face recognition by the proposed algorithm. In the fine classification stage, the classes which are closest to the test samples are selected, that is, the class labels which are finally determined from the coarse classification stage. Those classes which have been discarded need not be considered any more. At this stage, the improved multiplication fusion method is used for score fusion to reduce the final classification error rates.

Table 2 shows the comparison of the original classification error rates and the improved classification error rates with different weights. Different weight values are taken in the experiment, and the average classification error rates are calculated. From the results of Table 2, on the one hand, the classification error rates of the different weights selected by the original algorithm are generally higher than that of the improved algorithm, which shows that the improved algorithm is useful. On the other hand, the classification error rates of the improved algorithm are lower than the

TABLE 2. **Comparison of the classification error rate between the original algorithm of the FERET database and the improved algorithm.**

Numbers of training samples per class	1	2	3	4
$w_1 = 0.10$ $w_2 = 0.90$	0.4758	0.3030	0.3550	0.3433
$w\_1 = 0.30$ $w_2 = 0.70$	0.4658	0.2960	0.3600	0.3250
$w_1 = 0.50$ $w_2 = 0.50$	0.4650	0.3100	0.3600	0.3083
$w_1 = 0.70$ $w_2 = 0.30$	0.4850	0.3350	0.3925	0.3217
$w_1 = 0.90$ $w_2 = 0.10$	0.5083	0.3580	0.4263	0.3383
Average classification error rate of the original algorithm	0.4799	0.3204	0.3788	0.3273
Improved classification error rate	0.4650	0.3100	0.3625	0.3050

TABLE 3. **Comparison of the classification error rate between the original algorithm of the ORL database and the improved algorithm.**

Numbers of training samples per class	2	3	4	5
$w_1 = 0.10$ $w_2 = 0.90$	0.1469	0.1000	0.0917	0.1100
$w_1 = 0.30$ $w_2 = 0.70$	0.1219	0.0857	0.0792	0.1050
$w_1 = 0.50$ $w_2 = 0.50$	0.1031	0.0857	0.0750	0.0850
$w_1 = 0.70$ $w_2 = 0.30$	0.1063	0.0893	0.0792	0.0900
$w_1 = 0.90$ $w_2 = 0.10$	0.1031	0.0964	0.0750	0.0950
Average classification error rate of the original algorithm	0.1163	0.0914	0.0800	0.0970
Improved classification error rate	0.1031	0.0857	0.0750	0.0850

average classification error rates of the original algorithm, which further ensures that the improved algorithm is effective. In the experiment, the classification error rates of the original algorithm may be lower than the improved algorithm, but the original algorithm setting and selecting a reasonable weight is a difficult problem. In practice, the weights set randomly may lead to relatively poor results. A typical feature of practical applications is that they do not know which type of score can produce better results, thus setting a larger weighting factor. Although the test set can be repeatedly tested under laboratory conditions, and give a better weight coefficient on the test set according to the classification results, it has no guiding significance for practical application. Therefore, the proposed multiplication fusion scheme is very meaningful. It does not need to set parameters, only needs the original training samples scores and the ''symmetrical face'' training samples scores for multiplication fusion.

B. RESULT ANALYSIS OF ORL DATABASE

There are 40 classes in the ORL database. Each class has 10 samples. Table 3 shows the comparisons between the classification error rates of the original algorithm under different weights and the improved classification error rates. Different weight values are taken in the experiment, and the average classification error rates are calculated. It can be seen from table 3 that the classification error rates of the improved algorithm are lower than the average classification error rates of the original algorithm. The most important thing is that the improved algorithm does not need to manually set the parameters for multiple experiments, and the results can also be optimized, which proves the rationality of the improved algorithm.

C. RESULT ANALYSIS OF AR DATABASE

The AR database is also a face database that is often used for face recognition. It consists of 120 classes, each with 14 samples. Table 4 gives the comparisons of the classification error rates obtained by different training samples corresponding to different weights in the original algorithm and the classification error rates corresponding to the improved algorithm, and compares the average classification error rates of the original algorithm. It can be seen that the improved classification error rates are lower than the average classification error rates of the original algorithm. Although some classification error rates in the original algorithm are lower than the improved classification error rates, in practice, selecting the weights will be difficult, which does not have favourable conditions

TABLE 4. Comparison of the classification error rate between the original algorithm of the AR database and the improved algorithm.

Numbers of training samples per class	6	7	8	9
$w_1 = 0.10$ $w_2 = 0.90$	0.1938	0.2357	0.0681	0.0500
$w_1 = 0.30$ $w_2 = 0.70$	0.1875	0.2179	0.0514	0.0317
$w_1 = 0.50$ $w_2 = 0.50$	0.1865	0.2083	0.0389	0.0233
$w_1 = 0.70$ $w_2 = 0.30$	0.1885	0.2167	0.0319	0.0200
$w_1 = 0.90$ $w_2 = 0.10$	0.2000	0.2286	0.0319	0.0167
Average classification error rate of the original algorithm	0.1913	0.2214	0.0444	0.0283
Improved classification error rate	0.1844	0.2095	0.0389	0.0217

TABLE 5. Comparison of the classification error rate between the original algorithm of the YaleA database and the improved algorithm.

Numbers of training samples per class	2	3	4	5
$w_1 = 0.10$ $w_2 = 0.90$	0.2000	0.1250	0.1333	0.0667
$w_1 = 0.30$ $w_2 = 0.70$	0.2074	0.1500	0.1429	0.0556
$w_1 = 0.50$ $w_2 = 0.50$	0.1926	0.1417	0.1333	0.0556
$w_1 = 0.70$ $w_2 = 0.30$	0.2222	0.1583	0.1333	0.0667
$w_1 = 0.90$ $w_2 = 0.10$	0.2148	0.2000	0.1524	0.0667
Average classification error rate of the original algorithm	0.2074	0.1550	0.1390	0.0623
Improved classification error rate	0.1778	0.1417	0.1238	0.0556

for practical application. In contrast, the improved algorithm does not require weights, which creates an advantageous condition for the actual application.

D. RESULT ANALYSIS OF YALEA DATABASE

YaleA database possesses 165 images of 15 volunteers created by the Center for Computing Vision and Control at Yale University, which each volunteer has 11 images. In the experiment, the training samples for each class were selected as 2, 3, 4 and 5. The results of the original algorithm at different weights are compared and the average error rate is calculated and compared with the algorithm of this paper. From Table 5, it can be seen that the error rates of multiplication fusion proposed in this paper are lower than the average error rates of different weights of the original algorithm, and the proposed fusion method does not need to set weights manually, which reduces manpower and time, which is very meaningful for practical application.

IV. CONCLUSION

In this paper, an algorithm is proposed for improving the algorithm of the paper [17]. The weighted score fusion proposed previously has many inconveniences in selecting weighting coefficients. It is necessary to manually set various weights, and different weights will have different results. Finding the optimal weights is a difficult process. In this paper, a score fusion based on multiplication fusion is proposed without any parameters, which is a great improvement. The experimental results give proof and analysis to show that the multiplication fusion proposed in this paper is effective. By using the two-step sparse representation with the smallest 2-norm method, the scores of the original training samples and the "symmetrical face" training samples are calculated, and then the final classification results are obtained by multiplication fusion. Multiplication fusion improves the accuracy of classification. The algorithm of this paper is meaningful in the following two aspects: first, the classification error rates of the algorithm are lower than the average classification error rates of the original algorithm, which ensures that the algorithm can get better results. Second, the fusion step of the algorithm does not require any parameters. In the future, people's life is becoming more and more intelligent. Researchers have also carried out researches in various fields for intelligence [31]–[34]. And the intelligence of face recognition has been gradually applied in real life. However, practical problems such as face occlusion, illumination and expression change need to be further studied.

REFERENCES

[1] P. Viola and M. J. Jones, "Robust real-time face detection," *Int. J. Comput. Vis.*, vol. 57, no. 2, pp. 137–154, May 2004.

[2] J. Yang, D. Zhang, A. F. Frangi, and J.-Y. Yang, "Two-dimensional PCA: A new approach to appearance-based face representation and recognition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, no. 1, pp. 131–137, Jan. 2004.

[3] F. W. Olufade and A. J. Kolawole, "Biometric authentication with face recognition using principal component analysis and feature based technique," *Int. J. Comput. Appl.*, vol. 41, no. 1, pp. 13–20, Mar. 2012.

[4] G. Gao, Y. Yu, M. Yang, H. Chang, P. Huang, and D. Yue, "Cross-resolution face recognition with pose variations via multilayer locality-constrained structural orthogonal procrustes regression," *Inf. Sci.*, vol. 506, pp. 19–36, Jan. 2020.

[5] A. A. M. Al-Shiha, "Biometric face recognition using multilinear projection and artificial intelligence," Univ. Newcastle upon Tyne, Newcastle upon Tyne, U.K., Tech. Rep., 2013.

[6] C. Yang, S. Wu, H. Fang, and M. Joo, Er., "Adaptive Weber-face for robust illumination face recognition," *Computing*, vol. 101, no. 6, pp. 605–619, Jun. 2019.

[7] C. Cui and V. K. Asari, "Adaptive weighted local textural features for illumination, expression, and occlusion invariant face recognition," *Proc. SPIE*, vol. 9027, Mar. 2014, Art. no. 90270L.

[8] G. Gao, J. Yang, X.-Y. Jing, W. Yang, D. Yue, and F. Shen, "Learning robust and discriminative low-rank representations for face recognition with occlusion," *Pattern Recognit.*, vol. 66, pp. 129–143, Jun. 2017.

[9] S. Serikawa and H. Lu, "Underwater image dehazing using joint trilateral filter," *Comput. Elect. Eng.*, vol. 40, no. 1, pp. 41–50, 2014.

[10] H. Lu, Y. Li, T. Uemura, H. Kim, and S. Serikawa, "Low illumination underwater light field images reconstruction using deep convolutional neural networks," *Future Gener. Comput. Syst.*, vol. 82, pp. 142–148, May 2018.

[11] W. Jadoon, L. Zhang, and Y. Zhang, "Extended collaborative neighbor representation for robust single-sample face recognition," *Neural Comput. Appl.*, vol. 26, no. 8, pp. 1991–2000, Nov. 2015.

[12] C. Shao, X. Song, X. Yang, and X. Wu, "Extended minimum-squared error algorithm for robust face recognition via auxiliary mirror samples," *Soft Comput.*, vol. 20, no. 8, pp. 3177–3187, Aug. 2016.

[13] Y. Du and Y. Wang, "Generating virtual training samples for sparse representation of face images and face recognition," *J. Mod. Opt.*, vol. 63, no. 6, pp. 536–544, Sep. 2015.

[14] L. Li, Y. Peng, G. Qiu, Z. Sun, and S. Liu, "A survey of virtual sample generation technology for face recognition," *Artif. Intell. Rev.*, vol. 50, no. 1, pp. 1–20, Jun. 2018.

[15] Y. Xu, Z. Li, B. Zhang, J. Yang, and J. You, "Sample diversity, representation effectiveness and robust dictionary learning for face recognition," *Inf. Sci.*, vol. 375, pp. 171–182, Jan. 2017.

[16] L. Fu, D. Chen, K. Lin, and A. Li, "An improved SRC method based on virtual samples for face recognition," *J. Mod. Opt.*, vol. 65, no. 13, pp. 1565–1576, Jul. 2018.

[17] Y. Xu, X. Fang, X. Li, J. Yang, J. You, H. Liu, and S. Teng, "Data uncertainty in face recognition," *IEEE Trans. Cybern.*, vol. 44, no. 10, pp. 1950–1961, Oct. 2014.

[18] Y. Xu, X. Zhu, Z. Li, G. Liu, Y. Lu, and H. Liu, "Using the original and 'symmetrical face' training samples to perform representation based two-step face recognition," *Pattern Recognit.*, vol. 46, no. 4, pp. 1151–1158, Apr. 2013.

[19] H. Hamidi, M. C. Amirani, and S. R. Arashloo, "Local selected features of dual-tree complex wavelet transform for single sample face recognition," *IET Image Process.*, vol. 9, no. 8, pp. 716–723, Aug. 2015.

[20] R. Mehta, J. Yuan, and K. Egiazarian, "Face recognition using scale-adaptive directional and textural features," *Pattern Recognit.*, vol. 47, no. 5, pp. 1846–1858, May 2014.

[21] G. Feng, H. Li, J. Dong, and J. Zhang, "Face recognition based on Volterra kernels direct discriminant analysis and effective feature classification," *Inf. Sci.*, vol. 441, pp. 187–197, May 2018.

[22] Q. Gao, S. Xu, F. Chen, C. Ding, X. Gao, and Y. Li, "R_1 -2-DPCA and face recognition," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1212–1223, Apr. 2019.

[23] J. Wright, A. Y. Yang, A. Ganesh, S. S. Sastry, and Y. Ma, "Robust face recognition via sparse representation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 31, no. 2, pp. 210–227, Feb. 2009.

[24] D. Wang, H. Lu, and M.-H. Yang, "Kernel collaborative face recognition," *Pattern Recognit.*, vol. 48, no. 10, pp. 3025–3037, Oct. 2015.

[25] Z. Zhang, Y. Xu, J. Yang, X. Li, and D. Zhang, "A survey of sparse representation: Algorithms and applications," *IEEE Access*, vol. 3, pp. 490–530, 2015.

[26] Y. Xu, D. Zhang, J. Yang, and J.-Y. Yang, "A two-phase test sample sparse representation method for use with face recognition," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 21, no. 9, pp. 1255–1262, Sep. 2011.

[27] Y. Xu, Q. Zhu, Z. Fan, D. Zhang, J. Mi, and Z. Lai, "Using the idea of the sparse representation to perform coarse-to-fine face recognition," *Inf. Sci.*, vol. 238, no. 20, pp. 138–148, Jun. 2013.

[28] S. Zeng, X. Yang, and J. Gou, "Using kernel sparse representation to perform coarse-to-fine recognition of face images," *Optik*, vol. 140, pp. 528–535, Jul. 2017.

[29] L. Zhang, M. Yang, and X. Feng, "Sparse representation or collaborative representation: Which helps face recognition?" in *Proc. IEEE Int. Conf. Comput. Vis.*, Barcelona, Spain, Nov. 2011, pp. 471–478.

[30] P. J. Phillips, H. Wechsler, J. Huang, and P. J. Rauss, "The FERET database and evaluation procedure for face-recognition algorithms," *Image Vis. Comput.*, vol. 16, no. 5, pp. 295–306, Apr. 1998.

[31] H. Lu, Y. Li, S. Mu, D. Wang, H. Kim, and S. Serikawa, "Motor Anomaly Detection for Unmanned Aerial Vehicles Using Reinforcement Learning," *IEEE Internet Things J.*, vol. 5, no. 4, pp. 2315–2322, Aug. 2018.

[32] H. Lu, Y. Li, M. Chen, H. Kim, and S. Serikawa, "Brain intelligence: Go beyond artificial intelligence," *Mobile Netw. Appl.*, vol. 23, no. 2, pp. 368–375, Apr. 2018.

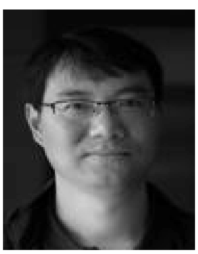
[33] Y. Yu, S. Tang, K. Aizawa, and A. Aizawa, "Category-based deep CCA for fine-grained venue discovery from multimodal data," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 4, pp. 1250–1258, Apr. 2019.

[34] H. Lu, D. Wang, Y. Li, J. Li, X. Li, H. Kim, S. Serikawa, and I. Humar, "CONet: A cognitive ocean network," *IEEE Wireless Commun.*, vol. 26, no. 3, pp. 90–96, 2019.

[35] M. Aleix and R. Benavente, "The AR face database," CVC Tech. Rep., Jun. 1998.

[36] *ORL Face*. Accessed: 2002. [Online]. Available: https://www.cl.cam.ac.uk/research/dtg/attarchive/facedatabase.html

[37] *YaleA Face*. Accessed: 1997. [Online]. Available: http://cvc.cs.yale.edu/cvc/projects/yalefaces/yalefaces.html



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