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Finite-Time H_∞ Fault-Tolerant Synchronization Control for Complex Dynamical Networks With Actuator Faults

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ABSTRACT This paper focuses on the problem of finite-time H_∞ fault tolerant synchronization control for complex dynamical network with actuator faults. By using the prior knowledge of the actuator faults, the complex dynamical network with actuator faults is described as a complex dynamical network with parameter uncertainties. By using some tools to deal with the Laypunov functional, a sufficient condition is obtained. Finally, an example is given to demonstrate the validity of the main results.

INDEX TERMS Complex dynamical networks, fault-tolerant control, actuator faults, finite-time H_∞ synchronization.

I. INTRODUCTION

Complex dynamical networks are composed of a large of nodes and interactions each other. Many systems in nature and society can be described by complex networks, including biological and society, the Internet, the world-wide web, computer networks, metabolic networks, power grids, neural networks, and so on [1]–[5]. Over the past few years, lots of attention has been attracted to the research of complex dynamical networks(CDNs) due to the successful application. In particular, the synchronization of CDNs is an interesting research direction since it is a typical collective behavior in nature. And, as various unexpected factors will disturb the systems in practical applications, many efforts have been made to obtain the synchronization of systems in the cases of time-delays [6]–[10], nonlinear couplings [11]–[15], uncertainties and disturbances [16]–[22], etc. therefore, the research on the synchronization control and CDNs has theoretical significance and practical prospect, which motivates our work. Recently, Faults as an unexpected factor inevitably in a complex dynamical network due to a large scale of population and complicated interaction. The unexpected interconnection faults have been considered and fault-tolerant controller is designed. The problem of fault-tolerant is studied for Markovian Jump system in [23]–[26], for singular system in [27]–[29], for T-S fuzzy system in [30]–[32] and for complex network system in [33]–[36]. The added controller is needed synchronize fault-tolerantly a complex network. According to this literature, we can found in articles [33], [34], the authors concerned with the problem of fault-tolerant synchronization control of a class of complex dynamical networks with actuator faults. And the author in article [35] investigated the problem of fault-tolerant synchronization control for complex dynamical networks with semi-Markov jump topology. From these references, we can know that all fault tolerance control is imposed on the external input of the system. Inspired by this, we will think of how to adjust the internal structure of the network without synchronizing the external controller. Just as paper [36], the complex network with actuator faults is described as a complex network with parameter uncertainties. The reason that can be done is that it depends on the information interconnections between subsystems to synchronize a complex network. In fact, the actuators of complex network synchronization control are located on the side of the subsystem, which are presented as the inner coupling, just as the present in [36]. In this view,

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we need not to add the controller. And the proposal of this view has greatly reduced the cost and saved time in practical applications. Therefore, it is necessary to further research in complex dynamical network with actuator faults.

In practical application, the behavior of systems during a fixed time interval may be received more attention. Up to now, most of the existing results relating to stability or synchronization focus on Lyapunov asymptotic stability or synchronization are defined over an infinite time interval. In finite-time interval, finite-time stability is investigated to address these transient performances of the control system. There are many results on finite time stability for linear system and nonlinear systems [6], [8], [15], [26], [37]–[40]. To the best of our knowledge, complex dynamical networks model receive few research in finite-time H_∞ synchronization with actuator faults. Inspired by above analysis, in this paper we considered a class of complex dynamical networks with actuator faults.

Motivated by the aforementioned discussion, the inner coupling links is the characteristic of the complex networks. In this paper, we built artificial complex dynamical networks, the inner coupling links is not known. We treat the established nonlinear system as a complex network system. By establishing a complex network system, the finite time H_∞ synchronization can be achieved, and the characteristics of the internal structure of the system can be obtained. The main contributions of this paper are summarized as follows:

1. This is the first attempt to consider the problem of finite-time H_∞ fault-tolerant synchronization control for complex dynamical networks with actuator faults.
2. In this paper, we the first attempt to adjust the internal structure of the network without synchronizing the external controller. And the complex network with actuator faults is described as a complex network with parameter.
3. Differentiating $V_3(t)$ using Lemma 1, Lemma 2 and Lemma 3 to deal with cross terms, which can reduce conservative.

The rest of this paper is arranged as follows: Some preliminaries of complex dynamical networks with actuator fault are introduced in Section 2. The finite-time H_∞ fault-tolerant synchronization control criteria for complex dynamical networks are obtained in Section 3. The numerical simulations are combined to prove the effectiveness of the theoretical results in Section 4.

Notation: In this paper, R^n denotes the n dimensional Euclidean space and $R^{n\times n}$ represents the $n\times n$ real matrices; $M > 0$ (< 0) means M is a symmetric positive (negative) define matrix, and M^{-1} stand for the inverse of matrix M; M^T represents the transpose of matrix M; $\lambda_{\max}(M)$ (respectively, $\lambda_{\min}(M)$) means the largest (respectively, smallest) eigenvalue of the matrix M; I is the identity matrix with appropriate dimension. The notation $A\otimes B$ stand for the Kronecker product of the matrices A and B; The symbol "$*$" is used to represent a term that is induced by symmetry. Matrix, if not explicitly stated, are assumed to have compatible dimensions.

II. PRELIMINARIES AND PROBLEM STATEMENT

Considering the following complex dynamical networks (CDNs), consisting of N identical coupled nodes, in which each node is an n-dimensional system:

$$\begin{cases} \dot{x}_k(t) = f(x_k(t)) + \sigma \sum_{j=1}^{N} g_{kj}\Gamma^F x_j(t-\tau(t)) + w_k(t) \\ z_k(t) = Dx_k(t), \quad k = 1, 2, \ldots, N. \end{cases} \tag{1}$$

where $x_k(t) = (x_{k1}(t), x_{k2}(t), \ldots, x_{kn})^T \in R^n$ represents the state vector of the i-th nodes at time t; the constant $\sigma > 0$ denotes the coupling strength; $w_k(t) \in R^n$ is the disturbance, which belongs to $l_2[0,\infty)$; $z_k(t) \in R^n$ is the output of the node i; $D = (d_{ij})_{n\times n} \in R^{n\times n}$is constant matrix with appropriate dimension; $G = (g_{kj})_{N\times N}$ are the coupling configuration matrices representing the the topological structure for delayed one at time t, in which g_{kj} is defined as follows: if there is a connection between node k and node j ($j \neq k$), then $g_{kj} > 0$, otherwise $g_{kj} = 0$ ($j \neq k$), and the diagonal elements of matrices G are defined by

$$g_{kk} = -\sum_{j=1,k\neq j}^{N} g_{kj}, \quad i = 1, 2, \ldots, N.$$

The scalar function $\tau(t)$ stand for the time-varying delay function and satisfies:

$$0 \le \tau(t) \le \tau < 1, \quad \dot{\tau}(t) \le \mu < 1$$

where $\tau > 0$, μ are given known constants.

Let $e_k(t) = x_k(t) - s(t)$ be the synchronization error system, where $s(t) \in R^n$ is can be either an equilibrium point, or a (quasi-) periodic orbit, or an orbit of a chaotic attractor, which satisfied that $\dot{s}(t) = f(s(t))$. Thus, the synchronization error system of CDN (1) can be easily inferred as follows:

$$\begin{cases} \dot{e}_k(t) = g(e_i(t)) + \sigma \sum_{j=1}^{N} g_{kj}\Gamma^F e_j(t-\tau(t)) + w_k(t) \\ z_k(t) = Dx_k(t) \end{cases} \tag{2}$$

where $f(e_k(t)) = f(x_k(t)) - f(s(t))$.

When the actuator failure occurs for the node k, the synchronization control inputs are modeled by $\sum_{j=1}^{N} g_{kj}\Gamma^F x_j(t-\tau(t)) = \sum_{j=1}^{N} g_{kj}M\Gamma x_j(t-\tau(t))$, where the $\sum_{j=1}^{N} g_{kj}\Gamma x_j(t-\tau(t))$ is the reliable synchronization controller in which the gain matrix Γ is to be designed, where $M = diag\{m_1, m_2, \ldots, m_n\} \in R^{n\times n}$ is the failure matrix with the following properties [41].

$$0 \le \underline{m_i} \le m_i \le \overline{m_i} \le 1, \quad i = 1, 2, \ldots, n.$$

where $\underline{m}_i$ and $\overline{m}_i$ represent the minimum and maximum of the matrix M, respectively.

$$\begin{cases} \underline{m}_i = \overline{m}_i = 1, \quad i = 1, 2, \ldots, n. \\ \quad \Rightarrow ith\ actuator\ has\ no\ failure. \\ \underline{m}_i = \overline{m}_i = 0, \quad i = 1, 2, \ldots, n. \\ \quad \Rightarrow ith\ actuator\ is\ outage. \\ 0 < m_i < 1, \quad i = 1, 2, \ldots, n. \\ \quad \Rightarrow\ ith\ actuator\ has\ partial\ failure. \end{cases}$$

Define the following notations:

$$\begin{aligned} M_0 &= diag\left\{m_{0,1},\ m_{0,2},\ \ldots\ m_{0,n}\right\}, \\ H &= diag\left\{h_1,\ h_2,\ \ldots, h_n\right\}, \\ C &= diag\left\{c_1,\ c_2,\ \ldots,\ c_n\right\}, \\ |C| &= diag\left\{|c_1|\,,\ |c_2|\,,\ \ldots,\ |c_n|\right\}. \end{aligned}$$

where

$$m_{0,i} = \frac{\underline{m}_i + \overline{m}_i}{2}, \quad h_i = \frac{\overline{m}_i - \underline{m}_i}{\underline{m}_i + \overline{m}_i}$$
$$c_i = \frac{m_i - m_{0,i}}{m_{0,i}}, \quad i = 1, 2, \ldots, n$$

So, we can get

$$M = M_0 (I + C)\,, \quad \|C\| \leq \|H\| \leq I \tag{3}$$

Obviously, the complex dynamical network system (1) can be described in the following form

$$\begin{cases} \dot{e}(t) = g(e(t)) + \sigma (G \otimes \Gamma)\, \overline{M}_0 \left(I + \overline{C}\right) e(t - \tau(t)) \\ \qquad + w(t) \\ z(t) = (I \otimes D)\, e(t) \end{cases} \tag{4}$$

where

$$\begin{aligned} e(t) &= \left[e_1(t)\,,\ e_2(t)\,,\ \ldots,\ e_N(t)\right]^T \\ g(e(t)) &= \left[g(e_1(t))\,,\ g(e_2(t))\,,\ \ldots,\ g(e_N(t))\right]^T \\ w(t) &= diag\left\{w_1(t)\,, w_2(t)\,, \ldots, w_N(t)\right\} \\ \overline{M}_0 &= diag\left\{\underbrace{M_0, M_0, \ldots, M_0}_{N}\right\} \\ \overline{C} &= diag\left\{\underbrace{C, C, \ldots, C}_{N}\right\} \end{aligned}$$

Remark 1: From (3), the actuator failure M is not well know in advance exactly, because actuator failure occurring is unknown in advance (1). However, H and M_0 are known in advance. Therefore, the synchronization controller matrix Γ can be designed based on the knowledge of H and M_0.

Remark 2: For a complex network system, the internal coupling structure is certain, that is to say, the internal coupling matrix is a fixed matrix. But in this paper, we study that the internal coupling structure of a complex network system is unknown, and deduce the internal structure of artificial complex network by satisfying certain performance. The purpose of this paper is to apply this type of complex network system to engineering technology research. It makes such an artificial complex network meet certain performance indicators, and makes theoretical research into practical application.

Before presenting our main result, we will introduce one assumption, definitions and some basic lemmas as follows, which are necessary for the subsequent development.

Assumption 1 [35]: Where $f : R^n \rightarrow R^n$ is a continuous vector-valued function and satisfies the following sector-bounded condition:

$$\begin{aligned} &\left[f(x) - f(y) - U_1(x - y)\right]^T \\ &\quad * \left[f(x) - f(y) - U_2(x - y)\right] \leq 0, \quad \forall x, y \in R^n \end{aligned}$$

where U_1 and U_2 are known constant matrices of appropriate dimensions.

Assumption 2 [26]: For a given positive parameter b, the external disturbance input $w(t)$ is time-varying and satisfies

$$\int_0^T w^T(t) w(t) dt \leq b, \quad b \geq 0$$

Lemma 1 [42]: For any constant matrix $Z \in R^{n\times n}, Z = Z^T > 0$, scalars $h > 0$ such that the following integrations are well defined, then:

$$\begin{aligned} &-h \int_{t-h}^{t} \varpi^T(s)\, Z \varpi(s) ds \\ &\leq -\left(\int_{t-h}^{t} \varpi(s)\, ds\right)^T Z \left(\int_{t-h}^{t} \varpi(s)\, ds\right) \\ &-\frac{h^2}{2} \int_{-h}^{0} \int_{t+\theta}^{t} \varpi^T(s)\, Z \varpi(s)\, ds d\theta \\ &\leq -\left(\int_{-h}^{0} \int_{t+\theta}^{t} \varpi(s)\, ds d\theta\right)^T Z \left(\int_{-h}^{0} \int_{t+\theta}^{t} \varpi(s)\, ds d\theta\right) \end{aligned}$$

Lemma 2 [43]: For a given symmetric matrix $Z > 0$,and any differentiable function ϖ in $[a, b] \rightarrow R^n$, then the following inequality holds:

$$\begin{aligned} \int_a^b \dot{\varpi}^T(s) Z \dot{\varpi}(s) ds \geq &\frac{1}{b-a} (\varpi(b) - \varpi(a))^T Z (\varpi(b) - \varpi(a)) \\ &+ \frac{3}{b-a} \Theta^T Z \Theta \end{aligned}$$

Lemma 3 [26]: Let $f_i : R^m \rightarrow R\ (i = 1, 2, \ldots, N)$ have positive values in an open subset D of R^m. Then, the reciprocally convex combination of f_i over D satisfies

$$\min_{\{\beta_i | \beta_i > 0, \sum_i \beta_i = 1\}} \sum_i \frac{1}{\beta_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$
$$\text{Subject to } \left\{ g_{i,j} : R^m \rightarrow R,\ g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}$$

Lemma 4 [44]: For symmetric matrix $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix}$, the following inequalities are equivalent as follows:

$$\begin{aligned} \Omega &< 0 \\ \Omega_{11} &< 0, \quad \Omega_{22} - \Omega_{12}^T \Omega_{11}^{-1} \Omega_{12} < 0 \\ \Omega_{22} &< 0, \quad \Omega_{11} - \Omega_{12} \Omega_{22}^{-1} \Omega_{12}^T < 0 \end{aligned}$$

Lemma 5 [41]: Given matrices S, H, E and R of appropriate dimensions and with S and R symmetrical and $R > 0$, then

$$S + HFE + E^T F^T H^T < 0$$

For all F satisfying $F^T F \le R$, if only if there exists some $\varepsilon > 0$ such that

$$S + \varepsilon^2 H H^T + \varepsilon^{-2} E^T R E < 0$$

Definition 1: Given the time constant $t_p > 0$, the complex dynamical networks (4) is finite-time synchronization with respect to (a_1, a_2, t_p, R_c, w), if there exist a positive matrix $R_c > 0$ and two scalar $a_1 > 0$ and $a_2 > a_1$ such that the following condition holds for any $t \in [0, t_p]$

$$\sup_{-\tau \le \theta \le 0} \left\{ e^T(\theta) R_c e(\theta), \dot{e}^T(\theta) R_c \dot{e}(\theta) \right\} \le a_1 \Rightarrow e^T(t) R_c e(t) \le a_2$$

Definition 2: Given two scalar $\gamma > 0$,$t_p > 0$, he complex dynamical networks (4) is finite-time H_∞ synchronized, if the system (4) is said finite-time synchronization in Definition 1, and under zero initial state, the following condition is satisfied:

$$\int_0^{t_p} z^T(s) z(s) - \gamma^2 w^T(s) w(s) ds < 0$$

III. MAIN RESULTS

A sufficient condition is given to ensure the finite time H_∞ synchronization of complex dynamical error system (4) in this section.

Theorem 1: Suppose that Assumption 1-2 hold. For given positive constant a_1 , b, T_c, δ, ε_0, matrix Λ_1, Λ_2 with appropriate dimension and positive definite matrix R_c, complex networks dynamical system (4) is finite time H_∞ synchronization subject to $\left(a_1 \;\; a_2 \;\; b \;\; T_c \;\; R_c \right)$, if there exists a constant $a_2 > a_1$, symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0, X_1 > 0, X_2 > 0$, Such that the following constraints holds:

$$\begin{bmatrix} \frac{X_1}{\tau} & W \\ * & \frac{X_1}{\tau} \end{bmatrix} > 0 \tag{5}$$

$$\begin{bmatrix} \bar{\Theta} & \Theta_{12} & \Theta_{13} \\ * & \bar{\Theta}_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} \bar{\Theta} & \Theta_{12} & \Theta_{13} \\ * & \hat{\Theta}_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0 \tag{7}$$

$$e^{\delta T_p} \left[\Delta a_1 + b \left(1 - e^{-\delta T_p} \right) \right] < a_2 \lambda_{\min} \left(\tilde{P} \right) \tag{8}$$

where

$$\bar{\Theta} = \begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & \frac{1}{2}\tau W - X_1 \\ * & \bar{\Theta}_{22} & \frac{1}{2}(X_1 - \tau W) \\ * & * & -\frac{5}{2} X_1 \end{bmatrix}$$

$$\Theta_{12} = \begin{bmatrix} \frac{3}{\tau} X_1 & 2X_2 & 2X_2 \\ 0 & 0 & 0 \\ \frac{3}{\tau} X_1 & 0 & 0 \end{bmatrix}$$

$$\Theta_{13} = \begin{bmatrix} -\Lambda_1^T + P & -\varepsilon \bar{S} + \Lambda_1^T & \Lambda_1^T \\ \Theta_{13}^0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Theta}_{22} = \begin{bmatrix} -\frac{6}{\tau^2} X_1 & 0 & 0 \\ * & -2X_2 & 0 \\ * & * & -\tau Q_2 - 2X_2 \end{bmatrix}$$

$$\hat{\Theta}_{22} = \begin{bmatrix} -\frac{6}{\tau^2} X_1 & 0 & 0 \\ * & -(2+\tau) X_2 & 0 \\ * & * & -2X_2 \end{bmatrix}$$

$$\Theta_{23} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Theta_{33} = \begin{bmatrix} \tau^2 X_1 + \frac{\tau^2}{2} X_2 - 2\Lambda_2^T & \Lambda_2^T & \Lambda_2^T \\ * & -\varepsilon_0 I & 0 \\ * & * & -\delta I \end{bmatrix}$$

$$\begin{aligned} \bar{\Theta}_{11} &= Q_1 + \tau Q_2 - \frac{5}{2} X_1 - 4X_2 - \varepsilon_0 \bar{R} \\ &\quad + (I \otimes D)^T (I \otimes D) - \delta P \\ \bar{\Theta}_{12} &= \frac{1}{2}(X_1 - \tau W) + \Lambda_1^T \sigma (G \otimes \Gamma) \overline{M}_0 \left(I + \overline{C} \right) \\ \bar{\Theta}_{22} &= -(1-\mu) Q_1 - X_1 + \tau W \\ \Delta &= \lambda_{\max}\left(\tilde{P}\right) + \tau \lambda_{\max}\left(\tilde{Q}_1\right) + \frac{\tau^2}{2} \lambda_{\max}\left(\tilde{Q}_2\right) \\ &\quad + \frac{\tau^3}{2} \lambda_{\max}\left(\tilde{X}_1\right) + \frac{\tau^3}{6} \lambda_{\max}\left(\tilde{X}_2\right) \\ \Theta_{13}^0 &= \sigma \left(I + \overline{C} \right)^T \overline{M}_0^T (G \otimes \Gamma)^T \Lambda_2^T \\ \tilde{P} &= R_c^{-1/2} P R_c^{-1/2}, \quad \tilde{Q}_1 = R_c^{-1/2} Q_1 R_c^{-1/2} \\ \tilde{Q}_2 &= R_c^{-1/2} Q_2 R_c^{-1/2}, \quad \tilde{X}_1 = R_c^{-1/2} X_1 R_c^{-1/2} \\ \tilde{X}_2 &= R_c^{-1/2} X_2 R_c^{-1/2} \end{aligned}$$

Proof: Choose the Lyapunov-Krasovskii function candidate as follows:

$$V(t) = \sum_{i=1}^{3} V_i(t) \tag{9}$$

where

$$\begin{aligned} V_1(t) &= e^T(t) P e(t) \\ V_2(t) &= \int_{t-\tau(t)}^{t} e^T(s) Q_1 e(s) ds \\ &\quad + \int_{-\tau}^{0} \int_{t+\theta}^{t} e^T(s) Q_2 e(s) ds d\theta \\ V_3(t) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{e}^T(s) X_1 \dot{e}(s) ds d\theta \\ &\quad + \int_{-\tau}^{0} \int_{\upsilon}^{0} \int_{t+\theta}^{t} \dot{e}^T(s) X_2 \dot{e}(s) ds d\theta d\upsilon \end{aligned}$$

Then, by calculating the first derivatives of $V(t)$ along the trajectories of error system (4), we can easily obtain

$$\dot{V}_1(t) = 2e^T(t)P\dot{e}(t) \tag{10}$$

$$\begin{aligned}\dot{V}_2(t) \le\ & e^T(t)(Q_1+\tau Q_2)e(t) \\ & -(1-\mu)e^T(t-\tau(t))Q_1e(t-\tau(t)) \\ & -\int_{t-\tau}^{t} e^T(s)Q_2e(s)\,ds \end{aligned}\tag{11}$$

$$\begin{aligned}\dot{V}_3(t) =\ & \dot{e}^T(t)\left(\tau^2X_1+\frac{\tau^2}{2}X_2\right)\dot{e}(t) \\ & -\tau\int_{t-\tau}^{t}\dot{e}^T(s)X_1\dot{e}(s)\,ds \\ & -\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^T(t)X_2\dot{e}(t)\,dsd\theta \end{aligned}\tag{12}$$

The following inequalities hold according to Lemma 1:

$$\begin{aligned}-\int_{t-\tau}^{t}e^T(s)Q_2e(s)\,ds =\ & -\int_{t-\tau(t)}^{t}e^T(s)Q_2e(s)\,ds \\ & -\int_{t-\tau}^{t-\tau(t)}e^T(s)Q_2e(s)\,ds \\ \le\ & -\tau(t)\xi_1^T(t)Q_2\xi_1(t) \\ & -(\tau-\tau(t))\xi_2^T(t)Q_2\xi_2(t) \end{aligned}\tag{13}$$

where

$$\xi_1(t) = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}e(s)\,ds$$

$$\xi_2(t) = \frac{1}{\tau-\tau(t)}\int_{t-\tau}^{t-\tau(t)}e(s)\,ds$$

$$\lim_{\tau(t)\to 0}\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}e(s)\,ds = e(t)$$

$$\lim_{\tau(t)\to \tau}\frac{1}{h-\tau(t)}\int_{t-\tau}^{t-\tau(t)}e(s)\,ds = e(t-h)$$

By Lemma 1, Lemma 2 and Lemma 3, it yield that

$$\begin{aligned} & -\tau\int_{t-\tau}^{t}\dot{e}^T(t)X_1\dot{e}(t)\,ds \\ =\ & -\frac{1}{2}\tau\int_{t-\tau}^{t}\dot{e}^T(t)X_1\dot{e}(t)\,ds \\ & -\frac{1}{2}\tau\int_{t-\tau}^{t}\dot{e}^T(t)X_1\dot{e}(t)\,ds \\ & -\frac{1}{2}\tau\int_{t-\tau}^{t}\dot{e}^T(s)X_1\dot{e}(s)\,ds \\ \le\ & -\frac{1}{2}(e(t)-e(t-\tau))^TX_1(e(t)-e(t-\tau)) \\ & -\frac{3}{2}\left(e(t)+e(t-\tau)-\frac{2}{\tau}\int_{t-\tau}^{t}e(s)\,ds\right)^T \\ & \times X_1\left(e(t)+e(t-\tau)-\frac{2}{\tau}\int_{t-\tau}^{t}e(s)\,ds\right) \\ & -\frac{1}{2}\tau\int_{t-\tau}^{t}\dot{e}^T(s)X_1\dot{e}(s)\,ds \end{aligned}\tag{14, 15}$$

$$\begin{aligned} =\ & -\frac{1}{2}\tau\int_{t-\tau(t)}^{t}\dot{e}^T(s)X_1\dot{e}(s)\,ds \\ & -\frac{1}{2}\tau\int_{t-\tau}^{t-\tau(t)}\dot{e}^T(s)X_1\dot{e}(s)\,ds \\ \le\ & -\frac{1}{2}\tau\left(\frac{\tau}{\tau(t)}\right)\left(\int_{t-\tau(t)}^{t}\dot{e}(s)\,ds\right)^T\frac{X_1}{\tau}\left(\int_{t-\tau(t)}^{t}\dot{e}(s)\,ds\right) \\ & -\frac{1}{2}\tau\left(\frac{\tau}{\tau-\tau(t)}\right)\left(\int_{t-\tau}^{t-\tau(t)}\dot{e}(s)\,ds\right)^T\frac{X_1}{\tau} \\ & \times\left(\int_{t-\tau}^{t-\tau(t)}\dot{e}(s)\,ds\right) \\ \le\ & -\frac{\tau}{2}\begin{pmatrix} e(t)-e(t-\tau(t)) \\ e(t-\tau(t))-e(t-\tau)\end{pmatrix}^T\begin{pmatrix}\frac{X_1}{\tau} & W \\ * & \frac{X_1}{\tau}\end{pmatrix} \\ & \times\begin{pmatrix} e(t)-e(t-\tau(t)) \\ e(t-\tau(t))-e(t-\tau)\end{pmatrix} \end{aligned}\tag{16}$$

By Lemma 1, it yield that

$$\begin{aligned} & -\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)X_2\dot{e}(s)\,dsd\theta \\ =\ & -\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)X_2\dot{e}(s)\,dsd\theta \\ & -\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t}\dot{e}^T(s)X_2\dot{e}(s)\,dsd\theta \\ \le\ & -\frac{2}{\tau^2(t)}\left(\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)\,dsd\theta\right)^T \\ & \times X_2\left(\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)\,dsd\theta\right) \\ & -\frac{2}{(\tau-\tau(t))^2}\left(\int_{-h}^{-\tau(t)}\int_{t+\theta}^{t}\dot{e}^T(s)\,dsd\theta\right)^T X_2 \\ & \times\left(\int_{-h}^{-\tau(t)}\int_{t+\theta}^{t}\dot{e}^T(s)\,dsd\theta\right) \\ =\ & -\frac{2}{\tau^2(t)}\left(\tau(t)e(t)-\int_{t-\tau(t)}^{t}e(s)\,ds\right)^TX_2 \\ & \times\left(\tau(t)e(t)-\int_{t-\tau(t)}^{t}e(s)\,ds\right) \\ & -\frac{2}{(\tau-\tau(t))^2}\left((\tau-\tau(t))e(t)-\int_{t-h}^{t-\tau(t)}e(s)\,ds\right)^TX_2 \\ & \times\left((\tau-\tau(t))e(t)-\int_{t-h}^{t-\tau(t)}e(s)\,ds\right) \\ =\ & -2(e(t)-\xi_1(t))^TX_2(e(t)-\xi_1(t)) \\ & -2(e(t)-\xi_2(t))^TX_2(e(t)-\xi_2(t)) \end{aligned}\tag{17}$$

Moreover, the following inequality can be easily acquired for any $\varepsilon > 0$

$$y(t) = \varepsilon_0\begin{bmatrix} e(t) \\ g(e(t))\end{bmatrix}^T\begin{bmatrix}\bar{R} & \bar{S} \\ * & I\end{bmatrix}\begin{bmatrix} e(t) \\ g(e(t))\end{bmatrix} \le 0 \tag{18}$$

where

$$\bar{R} = \frac{(I \otimes U_1)^T (I \otimes U_2)}{2} + \frac{(I \otimes U_2)^T (I \otimes U_1)}{2}$$
$$\bar{S} = -\frac{(I \otimes U_1)^T + (I \otimes U_2)^T}{2}$$

For any appropriate dimension matrixes Λ_1^T and Λ_2^T, it is clear to see that the following equation holds:

$$\begin{aligned} 0 = 2&\left[e^T(t)\Lambda_1^T + \dot{e}^T(t)\Lambda_2^T\right] \\ &\times[-\dot{e}(t) + \bar{g}(e(t))w(t) \\ &+\sigma(G\otimes\Gamma)\overline{M}_0\left(I+\overline{C}\right)e(t-\tau(t))] \end{aligned} \tag{19}$$

By considering (9)-(19) together, we eventually have

$$\begin{aligned} &\dot{V}(t) + z^T(t)z(t) - \delta w^T(t)w(t) \\ &\quad\le \dot{V}(t) + z^T(t)z(t) - \delta w^T(t)w(t) + \delta e^T(t)Pe(t) \\ &\qquad - \delta e^T(t)Pe(t) - y(t) \\ &\quad\le \xi^T(t)\Theta\xi(t) + \delta e^T(t)Pe(t) \\ &\quad< \delta e^T(t)Pe(t) \end{aligned} \tag{20}$$

where

$$\begin{aligned} \xi^T(t) = &\Big[e(t),\ e(t-\tau(t)),\ e(t-\tau),\ \int_{t-\tau}^{t} e(s)\,ds, \\ &\xi_1(t), \xi_2(t),\ \dot{e}(t),\ g(e(t))\ w(t)\Big] \end{aligned}$$

$$\Theta = \begin{bmatrix} \bar{\Theta} & \Theta_{12} & \Theta_{13} \\ * & \tilde{\Theta}_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix}$$

$$\tilde{\Theta}_{22} = \begin{bmatrix} -\frac{6}{\tau^2} & 0 & 0 \\ * & -\tau(t)Q_2 - 2X_2 & 0 \\ * & * & \tilde{\Theta}_{22}^0 \end{bmatrix}$$

$$\tilde{\Theta}_{22}^0 = -(\tau - \tau(t))Q_2 - 2X_2$$

The inequality (6) and (7) lead for $\tau(t) \to 0$ and for $\tau(t) \to \tau$, respectively. Thus by lemma 4, it yields that

$$\begin{aligned} \dot{V}(t) &\le \delta e^T(t)Pe(t) - z^T(t)z(t) + \delta w^T(t)w(t) \\ &\le \delta V(t) - z^T(t)z(t) + \delta w^T(t)w(t) \end{aligned} \tag{21}$$

Pre-and post-multiplying (21) by $e^{-\delta t}$ we have

$$\begin{aligned} \frac{d}{dt}\left(e^{-\delta t}V(t)\right) &\le -e^{-\delta t}z^T(t)z(t) + \delta e^{-\delta t}w^T(t)w(t) \\ &\le \delta e^{-\delta t}w^T(t)w(t) \end{aligned} \tag{22}$$

and integrating (22) from 0 to t_p, $\forall t \in [0, t_p]$, it follows that

$$V(t) < e^{\delta t}\left[V(0) + \delta\int_0^{t_p} e^{-\delta s}w^T(s)w(s)\,ds\right] \tag{23}$$

Among them

$$\begin{aligned} &V(0) \\ &= e^T(0)Pe(0) + \int_{\tau(0)}^{0} e^T(s)Q_1e(s)\,ds \\ &\quad + \int_{-\tau}^{0}\int_{\theta}^{0} e^T(s)Q_2e(s)\,ds\,d\theta \\ &\quad + \tau\int_{-\tau}^{0}\int_{\theta}^{0} \dot{e}^T(s)X_1\dot{e}(s)\,ds\,d\theta \\ &\quad + \int_{-\tau}^{0}\int_{\upsilon}^{0}\int_{\theta}^{0} \dot{e}^T(s)X_2\dot{e}(s)\,ds\,d\theta\,d\upsilon \\ &\le \lambda_{\max}\left(\tilde{P}\right)\left\{e^T(0)R_ce(0)\right\} \\ &\quad + \tau\lambda_{\max}\left(\tilde{Q}_1\right)\left\{e^T(0)R_ce(0)\right\} \\ &\quad + \frac{\tau^2}{2}\lambda_{\max}\left(\tilde{Q}_2\right)\sup_{-\tau\le\theta\le0}\left\{\dot{e}^T(0)R_c\dot{e}(0)\right\} \\ &\quad + \frac{\tau^3}{2}\lambda_{\max}\left(\tilde{X}_1\right)\sup_{-\tau\le\theta\le0}\left\{\dot{e}^T(0)R_c\dot{e}(0)\right\} \\ &\quad + \frac{\tau^3}{6}\lambda_{\max}\left(\tilde{X}_2\right)\sup_{-\tau\le\theta\le0}\left\{\dot{e}^T(0)R_c\dot{e}(0)\right\} \\ &\le \left(\lambda_{\max}\left(\tilde{P}\right) + \tau\lambda_{\max}\left(\tilde{Q}_1\right)\right)\left\{e^T(0)R_ce(0)\right\} \\ &\quad + \left(\frac{\tau^2}{2}\lambda_{\max}\left(\tilde{Q}_2\right) + \frac{\tau^3}{2}\lambda_{\max}\left(\tilde{X}_1\right) + \frac{\tau^3}{6}\lambda_{\max}\left(\tilde{X}_2\right)\right) \\ &\quad \times \sup_{-\tau\le\theta\le0}\left\{\dot{e}^T(0)R_c\dot{e}(0)\right\} \\ &\le \Delta a_1 \end{aligned} \tag{24}$$

then,

$$V(t) \le e^{\delta T_p}\left[\Delta a_1 + b\left(1 - e^{-\delta T_p}\right)\right] \tag{25}$$

On the other hand

$$V(t) \ge \lambda_{\min}\left(\tilde{P}\right)e^T(t)R_ce(t) \tag{26}$$

From (25)(26) we can obtain that

$$e^T(t)R_ce(t) \le \frac{e^{\delta T_p}\left[\Delta a_1 + b\left(1 - e^{-\delta T_p}\right)\right]}{\lambda_{\min}\left(\tilde{P}\right)} < a_2 \tag{27}$$

Moreover integrating (22) from 0 to t_p, and under the zero initial condition, we can obtain

$$0 < e^{-\delta t}V(t) \le \int_0^{t_p} e^{-\delta t}\left(\delta w^T(t)w(t) - z^T(t)z(t)\right)dt \tag{28}$$

That is to say

$$\int_0^{t_p} z^T(t)z(t) - \delta w^T(t)w(t)\,dt < 0,\ \gamma = \sqrt{\delta} \tag{29}$$

And the condition in Definition 2 is satisfied. Hence the complex dynamical networks (4) is finite time H_∞ synchronization. According to Definition 2, the proof of this theorem is now complete.

Remark 3: During the process, it is worth to point out when we are processing differential terms $\dot{V}_3(t)$. Among them, $\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)X_2\dot{e}(s)\,ds\,d\theta$ and $\int_{-\tau}^{-\tau(t)}\int_{t+\theta}^{t}\dot{e}^T(s)X_2\dot{e}(s)$ $ds\,d\theta$ are bounded and satisfied the follows $-2(e(t) - \xi_1(t))^T$

$$\begin{bmatrix} \bar{\Theta}_{11} & \Pi_{12} & \Pi_{13} & \frac{3}{\tau}X_1 & 2X_2 & 2X_2 & \Pi_{17} & \Pi_{18} & \Lambda_1^T & 2\sqrt{\sigma}\Lambda_1^T K & 0 \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{5}{2}X_1 & \frac{3}{\tau}X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55}^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66}^1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & \Lambda_2^T & \Lambda_2^T & 0 & 2\sqrt{\sigma}\Lambda_2 K \\ * & * & * & * & * & * & * & -\varepsilon_0 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -(\varepsilon_1+\varepsilon_2)I & 0 \\ * & * & * & * & * & * & * & * & * & * & -(\varepsilon_3+\varepsilon_4)I \end{bmatrix} < 0 \quad (32)$$

$$\begin{bmatrix} \bar{\Theta}_{11} & \Pi_{12} & \Pi_{13} & \frac{3}{\tau}X_1 & 2X_2 & 2X_2 & \Pi_{17} & \Pi_{18} & \Lambda_1^T & 2\sqrt{\sigma}\Lambda_1^T K & 0 \\ * & \Pi_{22} & \Pi_{23} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{5}{2}X_1 & \frac{3}{\tau}X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66}^2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & \Lambda_2^T & \Lambda_2^T & 0 & 2\sqrt{\sigma}\Lambda_2 K \\ * & * & * & * & * & * & * & -\varepsilon_0 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -(\varepsilon_1+\varepsilon_2)I & 0 \\ * & * & * & * & * & * & * & * & * & * & -(\varepsilon_3+\varepsilon_4)I \end{bmatrix} < 0 \quad (33)$$

$X_2\left(e(t)-\xi_1(t)\right)$ and $-2(e(t)-\xi_2(t))^T X_2\left(e(t)-\xi_2(t)\right)$, respectively, which can be reduced the conservatism.

Now, we are ready to present a method to design the inner coupling matrix for satisfying the aforementioned requirement.

Theorem 2: Suppose that Assumption 1-2 hold. For given positive constant a_1 , b, T_c, δ, $\varepsilon_i(i = 1, 2, \ldots, 4)$, matrix Λ_1, Λ_2 with appropriate dimension and positive definite matrix R_c, complex networks dynamical system (4) is finite time H_∞ synchronization subject to $\left(a_1 \;\; a_2 \;\; b \;\; T_c \;\; R_c\right)$, if there exists a constant $a_2 > a_1$, symmetric matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0, X_1 > 0, X_2 > 0, K > 0$ Such that the following constraints holds:

$$\begin{bmatrix} \frac{X_1}{\tau} & W \\ * & \frac{X_1}{\tau} \end{bmatrix} > 0 \quad (30)$$

$$e^{\delta T_p}\left[\Delta a_1 + b\left(1-e^{-\delta T_p}\right)\right] < a_2 \lambda_{\min}(P) \quad (31)$$

where

$$\begin{aligned}
\Pi_{12} &= \frac{1}{2}\left(X_1-\tau W\right), \quad \Pi_{17} = -\Lambda_1^T + P \\
\Pi_{18} &= -\varepsilon\bar{S} + \Lambda_1^T \\
\Pi_{22} &= -(1-\mu)Q_1 - X_1 + \tau W \\
&\quad + \sigma\left(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2+\varepsilon_4^2\right)\bar{M}_0^T\bar{M}_0 \\
\Pi_{23} &= \frac{1}{2}\left(X_1-\tau W\right), \quad \Pi_{44} = -\frac{6}{\tau^2}X_1 \\
\Pi_{55}^1 &= -2X_2, \quad \Pi_{55}^2 = -(2+\tau)X_2 \\
\Pi_{66}^1 &= -\tau Q_2 - 2X_1, \quad \Pi_{66}^2 = -2X_2 \\
\Pi_{77} &= \tau^2 X_1 + \frac{\tau^2}{2}X_2 - 2\Lambda_1^T
\end{aligned}$$

With other parameters described in Theorem 1, then the complex dynamical network (1) with actuator failure is finite time H_∞ synchronization, and the reliable gain matrix $(G\otimes\Gamma) = K$. Through simple calculations, Γ is easy to get.

Proof: From Equation(19), the following inequality is right.

$$\begin{aligned}
&\xi^T(t)\Theta\xi(t) \\
&\quad = \xi^T(t)\widehat{\Theta}\xi(t) \\
&\qquad + \sigma e^T(t)\Lambda_1^T(G\otimes\Gamma)M_0(I+C)e(t-\tau(t)) \\
&\qquad + \sigma e^T(t-\tau(t))(I+C)^T M_0^T(G\otimes\Gamma)^T\Lambda_1 e(t) \\
&\qquad + \sigma e^T(t-\tau(t))(I+C)^T M_0^T(G\otimes\Gamma)^T\Lambda_2^T\dot{e}(t) \\
&\qquad + \sigma\dot{e}^T(t)\Lambda_2(G\otimes\Gamma)M_0(I+C)e(t-\tau(t)) \\
&\quad = \sigma e^T(t)\Lambda_1^T(G\otimes\Gamma)M_0 e(t-\tau(t)) \\
&\qquad + \sigma e^T(t)\Lambda_1^T(G\otimes\Gamma)M_0 C e(t-\tau(t)) \\
&\qquad + \sigma e^T(t-\tau(t))M_0^T(G\otimes\Gamma)^T\Lambda_1 e(t) \\
&\qquad + \sigma e^T(t-\tau(t))C^T M_0^T(G\otimes\Gamma)^T\Lambda_1 e(t) \\
&\qquad + \sigma e^T(t-\tau(t))C^T M_0^T(G\otimes\Gamma)^T\Lambda_2^T\dot{e}(t) \\
&\qquad + \sigma e^T(t-\tau(t))M_0^T(G\otimes\Gamma)^T\Lambda_2^T\dot{e}(t) \\
&\qquad + \sigma\dot{e}^T(t)\Lambda_2(G\otimes\Gamma)M_0 e(t-\tau(t)) \\
&\qquad + \sigma\dot{e}^T(t)\Lambda_2(G\otimes\Gamma)M_0 C e(t-\tau(t)) \qquad (34)
\end{aligned}$$

where

$$\widehat{\Theta} = \begin{bmatrix} \bar{\Theta}_0 & \Theta_{12} & \Theta_{13}^0 \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix}$$

$$P = \begin{bmatrix} 78.5889 & 4.8968 & -0.8986 & 0.2725 & -1.7768 & -0.2187 \\ 4.8968 & 71.2108 & 0.3660 & 0.5605 & -0.1723 & -0.2898 \\ -0.8986 & 0.3660 & 76.9356 & 4.6620 & 1.0273 & 0.1653 \\ 0.2725 & 0.5605 & 4.6620 & 71.3168 & -0.0504 & -0.0075 \\ -1.7768 & -0.1723 & 1.0273 & -0.0504 & 77.9478 & 4.5901 \\ -0.2187 & -0.2898 & 0.1653 & -0.0075 & 4.5901 & 71.7349 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 28.4848 & -0.2842 & 0.1760 & -0.0272 & 0.5188 & 0.2058 \\ -0.2842 & 28.9060 & 0.2807 & 0.1923 & 0.1640 & -0.0407 \\ 0.1760 & 0.2807 & 28.6006 & -0.0297 & -0.0262 & -0.0897 \\ -0.0272 & 0.1923 & -0.0297 & 29.0365 & -0.0608 & 0.0398 \\ 0.5188 & 0.1640 & -0.0262 & -0.0608 & 28.3195 & -0.4456 \\ 0.2058 & -0.0407 & -0.0897 & 0.0398 & -0.4456 & 29.2730 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 31.6301 & -0.2888 & 0.1579 & 0.0476 & 0.1704 & 0.0872 \\ -0.2888 & 32.1809 & 0.1371 & 0.0688 & 0.0416 & 0.0097 \\ 0.1579 & 0.1371 & 31.7988 & -0.2190 & 0.0120 & -0.0404 \\ 0.0476 & 0.0688 & -0.2190 & 32.1729 & 0.0848 & -0.0398 \\ 0.1704 & 0.0416 & 0.0120 & 0.0848 & 31.6196 & -0.3112 \\ 0.0872 & 0.0097 & -0.0404 & -0.0398 & -0.3112 & 32.0574 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1.7762 & -0.2302 & -0.1483 & 0.0158 & -0.5705 & -0.2837 \\ -0.2302 & 2.1624 & -0.4678 & -0.3646 & -0.2280 & 0.1018 \\ -0.1483 & -0.4678 & 1.8189 & -0.5913 & -0.0858 & 0.1165 \\ 0.0158 & -0.3646 & -0.5913 & 1.9473 & 0.1107 & -0.0668 \\ -0.5705 & -0.2280 & -0.0858 & 0.1107 & 2.1056 & 0.0612 \\ -0.2837 & 0.1018 & 0.1165 & -0.0668 & 0.0612 & 1.5020 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 2.7696 & -0.5039 & -0.3099 & 0.0633 & -1.3187 & -0.6371 \\ -0.5039 & 3.6772 & -1.0358 & -0.8162 & -0.5243 & 0.2365 \\ -0.3099 & -1.0358 & 2.8946 & -1.3315 & -0.1714 & 0.2619 \\ 0.0633 & -0.8162 & -1.3315 & 3.1718 & 0.2932 & -0.1745 \\ -1.3187 & -0.5243 & -0.1714 & 0.2932 & 3.5222 & 0.1649 \\ -0.6371 & 0.2365 & 0.2619 & -0.1745 & 0.1649 & 2.0881 \end{bmatrix}$$

$$K = \begin{bmatrix} -0.6542 & -2.4640 & 0.3271 & 1.2320 & 0.3271 & 1.2320 \\ 0.6250 & -0.4944 & -0.3125 & 0.2472 & -0.3125 & 0.2472 \\ 0.3271 & 1.2320 & -0.3271 & -1.2320 & 0 & 0 \\ -0.3125 & 0.2472 & 0.3125 & -0.2472 & 0 & 0 \\ 0.3271 & 1.2320 & 0 & 0 & -0.3271 & -1.2320 \\ -0.3125 & 0.2472 & 0 & 0 & 0.3125 & -0.2472 \end{bmatrix}$$

$$\bar{\Theta}_0 = \begin{bmatrix} \bar{\Theta}_{11} & \frac{1}{2}(X_1 - \tau W) & \frac{1}{2}\tau W - X_1 \\ * & -(1-\mu) Q_1 - X_1 + \tau W & \frac{1}{2}(X_1 - \tau W) \\ * & * & -\frac{5}{2} X_1 \end{bmatrix}$$

$$\Theta_{13}^0 = \begin{bmatrix} -\Lambda_1^T + P & -\varepsilon \bar{S} + \Lambda_1^T & \Lambda_1^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Based on Lemma 4 and Lemma 5, and $C^T C \leq I$ in equation (3)

$$\begin{aligned} &\sigma e^T(t) \Lambda_1^T (G \otimes \Gamma) M_0 e(t-\tau(t)) \\ &\quad + \sigma e^T(t-\tau(t)) M_0^T (G \otimes \Gamma)^T \Lambda_1 e(t) \\ &\leq \sigma \varepsilon_1^2 e^T(t-\tau(t)) M_0^T M_0 e(t-\tau(t)) \\ &\quad + \sigma \varepsilon_1^{-2} e^T(t) \Lambda_1^T (G \otimes \Gamma)(G \otimes \Gamma)^T \Lambda_1 e(t) \end{aligned} \tag{35}$$

$$\begin{aligned} &\sigma e^T(t) \Lambda_1^T (G \otimes \Gamma) M_0 C e(t-\tau(t)) \\ &\quad + \sigma e^T(t-\tau(t)) C^T M_0^T (G \otimes \Gamma)^T \Lambda_1 e(t) \\ &\leq \sigma \varepsilon_2^2 e^T(t-\tau(t)) C^T C M_0^T M_0 e(t-\tau(t)) \\ &\quad + \sigma \varepsilon_2^{-2} e^T(t) \Lambda_1^T (G \otimes \Gamma)(G \otimes \Gamma)^T \Lambda_1 e(t) \\ &\leq \sigma \varepsilon_2^2 e^T(t-\tau(t)) M_0^T M_0 e(t-\tau(t)) \\ &\quad + \sigma \varepsilon_2^{-2} e^T(t) \Lambda_1^T (G \otimes \Gamma)(G \otimes \Gamma)^T \Lambda_1 e(t) \end{aligned} \tag{36}$$

$$\begin{aligned} &\sigma e^T(t-\tau(t)) C^T M_0^T (G \otimes \Gamma)^T \Lambda_2^T \dot{e}(t) \\ &\quad + \sigma \dot{e}^T(t) \Lambda_2 (G \otimes \Gamma) C M_0 e(t-\tau(t)) \\ &\leq \sigma \varepsilon_3^{-2} \dot{e}^T(t) \Lambda_2 (G \otimes \Gamma)(G \otimes \Gamma)^T \Lambda_2^T \dot{e}(t) \\ &\quad + \sigma \varepsilon_3^2 e^T(t-\tau(t)) C^T C M_0^T M_0 e(t-\tau(t)) \\ &\leq \sigma \varepsilon_3^{-2} \dot{e}^T(t) \Lambda_2 (G \otimes \Gamma)(G \otimes \Gamma)^T \Lambda_2^T \dot{e}(t) \end{aligned}$$

$$
\begin{aligned}
&+\sigma\varepsilon_3^2 e^T(t-\tau(t))\, M_0^T M_0 e(t-\tau(t)) \qquad (37)\\
&\sigma e^T(t-\tau(t))\, M_0^T (G\otimes\Gamma)^T \Lambda_2^T \dot{e}(t)\\
&\quad +\sigma \dot{e}^T(t)\,\Lambda_2 (G\otimes\Gamma) M_0 e(t-\tau(t))\\
&\le \sigma\varepsilon_4^2 e^T(t-\tau(t))\, M_0^T M_0 e(t-\tau(t))\\
&\quad +\sigma\varepsilon_4^{-2}\dot{e}^T(t)\,\Lambda_2 (G\otimes\Gamma)(G\otimes\Gamma)^T \Lambda_2^T \dot{e}(t) \qquad (38)
\end{aligned}
$$

we defined $(G\otimes\Gamma) = K$. And, according to Schur complement lemma, matrix inequality (6)-(7) can be transformed into linear matrix inequality (32)-(33), as shown at the top of 7th page. Based on Lyapuniv stability theorem, synchronization error system (1) is finite time H_∞ synchronization, that is, complex network (1) with the actuator failure is finite-time H_∞ synchronized to the manifold (4) the proof is completed.

IV. EXAMPLE

In this section, a numerical example is provided to demonstrate that the proposed method in this paper is effective.

Example 1: Consider the complex dynamical network (1) with three nodes and the follows parameters:

$\tau(t) = 0.2 + 0.3\sin t$, it can be calculated that $\tau = 0.5$, $\mu = 0.3$..

$f(x_i(t)) = \begin{bmatrix} 0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}$, it can be found that f satisfies the Assumption 1 with

$$U_1 = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}$$

The output matrix $B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$.

Consider the actuator fault matrix

$$M = diag(m_1, m_2), \quad 0.4 \le m_1 \le 1,\ 0 \le m_2 \le 1$$

Then we have $M_0 = diag\{0.6 \;\; 0.7\}$.

The topology structure has the following form

$$G = (g_{kj})_{3\times 3} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

We choose $\varepsilon_0 = 2$, $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.4$, $\varepsilon_4 = 0.1$, $T_p = 5$, $a_1 = 0.3$, $\sigma = 0.2$, $\delta = 1$.

The disturbance is $w(t) = \frac{0.8}{\sqrt{6(t+1)}}$, and based on Assumption 2, we can know $b = 0.64$.

By using the Matlab toolbox solving the LMIs, we can obtain $a_2 = 1.5757$, and the matrices as follows:

Based on $(G\otimes\Gamma) = K$, inner coupling matrix Γ can be calculated.

$$\Gamma = \begin{bmatrix} 0.3271 & 1.232 \\ -0.3125 & 0.2472 \end{bmatrix}$$

This means that the complex network (1) is finite-time H_∞ synchronization. Based on the obtain inner coupling matrix Γ and the given parameters, some simulations results are obtained. Assume that the initial states are $X_1(0) = \begin{bmatrix} 5 & -3 \end{bmatrix}^T$, $X_2(0) = \begin{bmatrix} 3 & -1.5 \end{bmatrix}^T$, $X_3(0) = \begin{bmatrix} -9 & 4 \end{bmatrix}^T$.The state trajectories of the error system (4) are shown in Fig.1, Fig.2 and Fig.3, which shows that the complex dynamical networks system is synchronization. Therefore, the approach method is effective.

Remark 4: Through the simulation of Example 1, we can clearly see that the complex network system can be synchronized in finite time by adjusting the internal coupling matrix without external control input. And we know that the dimension of numerical simulation is determined by the number of actual state variables. The increase of dimension will only increase the difficulty of calculation, but will not change the validity. Through the research of this paper, we can apply such artificial complex network to actual production. This is also the focus of our next study.

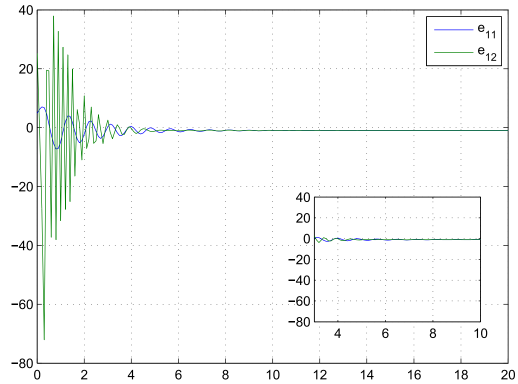

FIGURE 1. State trajectory of the synchronization error $e_{1k}(t)$, $k = 1, 2$.

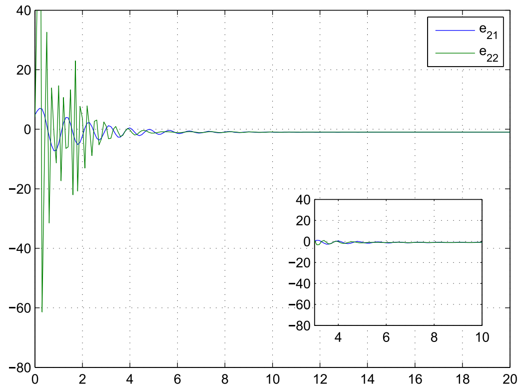

FIGURE 2. State trajectory of the synchronization error $e_{2k}(t)$, $k = 1, 2$.

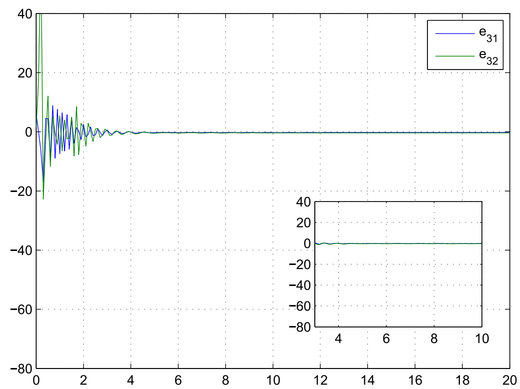

FIGURE 3. State trajectory of the synchronization error $e_{3k}(t)$, $k = 1, 2$.

V. CONCLUSION

In this paper, we research the finite-time H_∞ synchronization for complex dynamical network with actuator faults. The complex dynamical network with actuator faults is described as a complex dynamical network with parameter uncertainties. Conditions are obtained, by using appropriate lemmas and Assumptions. Finally, a numerical example has been provided to illustrate the effectiveness of the proposed approach.

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