

Received August 6, 2019, accepted August 30, 2019, date of publication September 5, 2019, date of current version October 15, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2939561

# Robust Nonlinear Multiple Unmanned Surface Vessels Control Against Probabilistic Faults and Time-Varying Delay

WEI YUE<sup>1,2</sup>, XIANHE GUAN<sup>1</sup>, AND LIYUAN WANG<sup>1,2</sup>

<sup>1</sup>College of Marine Electrical Engineering, Dalian Maritime University, Dalian 116026, China

<sup>2</sup>Key Laboratory of Intelligent Perception and Advanced Control of State Ethnic Affairs Commission, Dalian 116026, China

Corresponding author: Liyuan Wang (wangliyuanmu@163.com)

This work was supported in part by the Natural Science Foundation of China under Grant 61703072, in part by the Dalian Science and Technology Innovation Fund under Grant 2019J12GX040, in part by the Key Laboratory of Intelligent Perception and Advanced Control of State Ethnic Affairs Commission under Grant MD-IPAC-201901, in part by the Fundamental Research Funds for the Central Universities under Grant 3132019355, and in part by the Liaoning Natural Science Foundation Project under Grant 20180540064.

**ABSTRACT** This paper is concerned with robust nonlinear control for multiple unmanned surface vessel (MUSV) systems. Firstly, a mixed nonlinear model was built, in which the probabilistic actuator and fault time-varying communication delay was considered. On the basis of the new model, a robust nonlinear control approach by utilizing Lyapunov-Krasovskii functional is proposed, which can robustly stabilize the MUSV system with a given level of disturbance attenuation ensure the MUSV can track well. Furthermore, additional conditions were established for the designed controller to guaranteeing mesh stability. Eventually, the superiority and effectiveness of the proposed control strategies are verified by numerical simulation.

**INDEX TERMS** Actuator faults, mesh stability, multiple unmanned surface vessel, time-varying delay.

## I. INTRODUCTION

In the past decades, the MUSV system exemplifies the advance in maritime transportation and has been attracting more and more interest from scientific survey and military mission in virtue of their potential to substantially increase operational efficiency and fuel economy in various ocean engineering projects [1]. The key features of the MUSV are to maintain a certain geometric shape during the vessel moving without operation from the pilots, which, the other side of the shield, justifies an elaborated cooperative control of autonomous vessels via communication network.

Up to now, a lot of problems have been solved from different standpoints in this field [2]–[7]. Just to name a few, Fossen [2] used a classic linear control method for tracking control of MUSV, and ensure tracking performance with the desired trajectory is piecewise linear; in [3] and [4], the back-stepping technology is adopted to reject the nonlinear dynamics and kinematics of the MUSV and achieve good performance in tracking control; and in [5], a combined sliding model control and PID method was proposed which

is more sensitive to model uncertainties; in order to eliminate the drawback ‘chattering’ associated with sliding model control, an adaptive sliding model method is designed in [6]; in [7], the author proposed more practical method which considers the thrust limit and safe operating area.

Since the MUSVs are dynamically coupled via maritime communication network, which have the same properties as autonomous platoon [8] or multiple unmanned aerial vehicles [9], i.e. the state occurring in single individual may still affect those around it, a phenomenon known as the mesh instability or string instability. In this research, we want to design coordination controller for MUSV so that any shock-wave arising from disturbance propagation should restrict as it travels away from the source. There have many researches on multi-agent application control field, see, e.g., in [10] an  $H_\infty$  control method for a string of vehicles without leader information was proposed; in [11] the inclusion principle was applied to decouple the interconnected vehicles system, and proposed a decentralized optimal controller to hold string stability; The communication constraints and sensor faults was investigated in [12] and strict demands was put forward to failure rate, sampling period and controller gains to achieve string stability. Furthermore, in [13], a controller design

The associate editor coordinating the review of this manuscript and approving it for publication was Muhammad Alam<sup>1</sup>.

methodology was designed by extensively investigated the limitations on performance, and in [14] the authors provided a warning system to avoid the string unstable in mixed traffic.

Although significant progress has been made in the research of MUSVs, there are still many potential problems to be solved, waiting for the emergence of new technologies. Firstly, the state of vessel transmitted via maritime communication network brings time-varying delay inevitable [15], which make the MUSV is difficult to control, in the worst case, it can even lead to collision accidents [16]. Under such a situation, in [17], cooperative path-following controllers are developed for a USVs subject to constant disturbances and a time-varying delay. The composite actuator failures are the second part, which may increase restrictions because actuator failures can lead to thrust and steering errors. Preliminary work on actuator fault detection and fault tolerant control bound up with MUSVs control have been developed by [18], [19]. However, the proposed detection technique does not apply to MUSVs that we are interested in here. To the author's knowledge, there are no reports on the expected performance of MUSVs, communication time-varying delay and probabilistic sensor fault simultaneously.

The major contributions of this study, which distinguish from the related literatures, are summarized as follows.

1) We formulate a mixed nonlinear model (including time-delay and actuator faults) for MUSVs, which is more realistic than the traditional model extensively used in literature, see, e.g., [4] and [7].

2) We obtain a nonlinear MUSV controller, which can simultaneously achieve disturbance attenuation, robust stability and mesh stability with the effects of maritime communication constraints.

The organization of this research is as follows. A nonlinear MUSVs model with the effects of maritime communication delay and actuator faults is established in section II. In section III, a robust nonlinear controller design procedure is suggested and the stability conditions are obtained. In section IV, the constraints on controller gains with disturbance attenuation and mesh stability is further investigated by considering the objectives. In Section V, contains the numerical simulations. The conclusion and future work are given in section VI.

## II. PROBLEM FORMULATION

Consider a MUSV system composed by  $n$  vessels with a maritime communication network, which includes a leading vessel and  $n$  following vessels. Each vessel transmits its sway velocity, yaw velocity, heading angle, roll velocity, and roll angle to its neighboring vessels via wireless communication. In the following, we will give the details on the MUSV system model, the probability fault, communication delay and our objective, respectively.

### A. MUSVs MODELING WITH TIME-VARYING DELAY

The motion of the  $i$ th vessel can be precisely described by a kinematic equation and a kinetic equation in the following

form,

$$\begin{cases} \dot{\eta}_i = R_i(\phi_i)V_i \\ \dot{V}_i = e_i V_i + f_i(V_i) + g_i u_i + h_i d_{1i} \end{cases} \quad (1)$$

where  $\eta_i = [x_i \ y_i \ \phi_i]^T$  with  $(x_i, y_i)$  represents the position the  $i$ th vessel in the earth-fixed reference frame;  $V_i = [\mu_i \ v_i \ r_i]^T$  with  $\mu_i, v_i$  and  $r_i$  denotes the surge velocity, the sway velocity and the yaw velocity of the  $i$ th vessel in the body-fixed reference frame, and

$$R_i(\phi_i) = \begin{bmatrix} \cos \phi_i & -\sin \phi_i & 0 \\ \sin \phi_i & \cos \phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

with  $R_i^T(\phi_i) = R_i^{-1}(\phi_i)$ ,  $\phi_i$  represents the heading of the  $i$ th vessel in the earth-fixed reference frame.

$$f_i(V_i) = \begin{bmatrix} \frac{m_{vi}}{m_{\mu i}} v_i r_i & -\frac{m_{\mu i}}{m_{v_i}} \mu_i r_i & \frac{m_{\mu v_i}}{m_{r_i}} \mu_i v_i \end{bmatrix}^T,$$

$e_i = \text{diag}\{-\frac{d_{\mu i}}{m_{\mu i}}, -\frac{d_{v_i}}{m_{v_i}}, -\frac{d_{r_i}}{m_{r_i}}\}$ ,  $u_i = [u_{\mu i}, u_{r_i}]^T$  is the control input in the surge direction and yaw direction with  $g_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$ ;  $d_{1i} = [d_{\mu i} \ d_{v_i} \ d_{r_i}]^T$  denotes the disturbance from the wind waves and ocean currents with  $h_i = I_{3 \times 3}$ .

While with the relative position related to the surface vessel as shown in Fig. 1, the relative model can be written as:

$$\begin{cases} \dot{R}_i = -\mu_i \cos(\varphi_i) + \mu_{i-1} \cos(\theta_i - \varphi_i) \\ \dot{\varphi}_i = \frac{1}{R_i} \mu_i \sin(\varphi_i) + \frac{1}{R_i} \mu_{i-1} \sin(\theta_i - \varphi_i) - r_i \\ \dot{\theta}_i = -r_i + r_{i-1} \\ \dot{V}_i = e_i V_i + f_i(V_i) + g_i u_i + h_i d_{1i} \end{cases} \quad (3)$$

where  $R_i$  denotes the relative distance from the previous vessel;  $\varphi_i$  denotes the angle between the heading direction of a vessel and vessel-to-vessel connection line;  $\theta_i$  denotes the relative orientation.  $X_i = [R_i \ \varphi_i \ \theta_i \ \mu_i \ v_i \ r_i]^T$  as the state vectors, and then the state space equation for the entire MUSV can be described,

$$\begin{cases} \dot{X}_i(t) = A_i X_i(t) + B_i u_i(t) + H_i w_i(t) + f_i(X_i) + G_i d_i(t) \\ Y_i(t) = C_{1i} X_i(t) + C_{2i} w_i(t) \end{cases} \quad (4)$$

where  $Y_i(t)$  is the measurement output vectors with  $C_{1i} = I_{6 \times 6}$ ,  $C_{2i} = \text{diag}\{0, 0, 1, 0, 0, 0\}$ ,

$$A_i = \begin{bmatrix} \text{zero}(3) & A_{i1} \\ \text{zero}(3) & e_i \end{bmatrix}, \quad A_{i1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T,$$

$$H_i = \text{diag}\{0, 0, 1, 0, 0, 0\},$$

$$G_i = \begin{bmatrix} \text{zero}(3) & \text{zero}(3) \\ * & I_{3 \times 3} \end{bmatrix},$$

$$w_i(t) = [0 \ 0 \ r_{i-1} \ 0 \ 0 \ 0]^T,$$

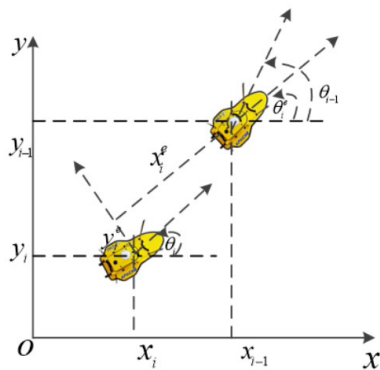


FIGURE 1. MUSV system with the relative coordinate.

$f_i(X_i) = [f_{1i}(X_i) f_{2i}(X_i) 0 f_i(V_i)]$  is the nonlinear term with  $f_{1i}(X_i) = -\mu_i \cos(\varphi_i) + \mu_{i-1} \cos(\theta_i - \varphi_i)$ ,

$$f_{2i}(X_i) = \frac{1}{R_i} \mu_i \sin(\varphi_i) + \frac{1}{R_i} \mu_{i-1} \sin(\theta_i - \varphi_i).$$

For vessels in the MUSV, the output feedback controller is designed as:

$$u_i(t) = K_i Y_i(t) \tag{5}$$

where  $K_i$  is the controller gain to be determined.

*Remark 1:* It's should be noting that in the MUSV system setup, the control method is based on the relative distance and the relative orientation between two adjacent vessels, the angle between the heading direction, the yaw velocity of vessel  $i - 1$ , and the surge velocity, the sway velocity and the yaw velocity of vessel  $i$ . The first fourth quantities are transmitted through a wireless communication channel, while the others can be measured by the onboard sensor on vessel  $i$ .

Considering the wireless communication delay, the controller (5) can be rewritten as

$$u_i(t) = K_i Y_i(t) = K_i \bar{C}_{1i} X_i(t) + K_i \bar{C}_{2i} X_i(t_k - \tau_i(t_k)) + K_i C_{2i} w_i(t_k - \tau_i(t_k)), \quad t \in [t_k, t_{k+1}) \tag{6}$$

where  $\bar{C}_{1i} = \begin{bmatrix} zero(3) & zero(3) \\ * & I_{3 \times 3} \end{bmatrix}$ ,  $\bar{C}_{2i} = \begin{bmatrix} I_{3 \times 3} & zero(3) \\ * & zero(3) \end{bmatrix}$ ,  $\tau_i(t_k)$  is the time-varying delay with  $t_k$  represents the updating instant of the ZOH, and satisfies  $0 \leq \tau_i(t_k) \leq \bar{\tau}_i$  and  $\dot{\tau}_i(t_k) = 1$ , and the sampling intervals are alterable with an upper bound  $h$ , i.e.,  $t_{k-1} - t_k \leq h$ .

**B. EFFECT OF PROBABILISTIC ON THE ACTUATOR**

In this research, the general actuator fault model in [20] was used to describe the probabilistic fault phenomenon, namely,  $u_{ui}^F = \sigma_{ui} u_{ui}$ ,  $u_{ri}^F = \sigma_{ri} u_{ri}$ , where  $\sigma_{ui}, \sigma_{ri} \in (0, 1]$  represents the effectiveness coefficient of  $i$ th vessel actuator, and the mathematical expectation of  $\sigma_{ui}, \sigma_{ri}$  are  $\varepsilon_{ui}, \varepsilon_{ri}$ , respectively. When  $\sigma_{ui} = \sigma_{ri} = 1$ , which means the  $i$ th actuator is in normal operation. When  $0 < \sigma_{ui}, \sigma_{ri} < 1$ , it corresponds to the case of certain fault happen.

Combing the actuator fault into consideration, the controller in (6) can be rewritten as

$$u_i(t) = \sigma_i K_i \bar{C}_{1i} X_i(t) + \sigma_i K_i \bar{C}_{2i} X_i(t_k - \tau_i(t_k)) + \sigma_i K_i C_{2i} w_i(t_k - \tau_i(t_k)),$$

where  $\sigma_i = diag\{\sigma_{ui}, \sigma_{ri}\}$ .

Based on the above analysis, the closed-loop MUSV system can be obtained,

$$\begin{aligned} \dot{X}_i(t) &= (A_i + B_i \sigma_i K_i \bar{C}_{1i}) X_i(t) + B_i \sigma_i K_i \bar{C}_{2i} X_i(t_k - \tau_i(t_k)) \\ &+ B_i \sigma_i K_i C_{2i} w_i(t_k - \tau_i(t_k)) + H_i w_i(t) + f_i(X_i) \\ &+ G_i d_i(t), \end{aligned} \tag{7}$$

Furthermore, define  $\tau_i(t) = t - t_k + \tau_i(t_k)$ ,  $t \in [t_k, t_{k+1})$ , then, we can rewrite (7) as

$$\begin{aligned} \dot{X}_i(t) &= (A_i + B_i \sigma_i K_i \bar{C}_{1i}) X_i(t) + B_i \sigma_i K_i \bar{C}_{2i} X_i(t - \tau_i(t)) \\ &+ B_i \sigma_i K_i C_{2i} w_i(t - \tau_i(t)) + H_i w_i(t) + f_i(X_i) + G_i d_i(t) \\ X_i(t) &= \Phi(t), \quad t \in [\bar{\tau}_i, 0) \end{aligned} \tag{8}$$

where  $\Phi(t)$  is the initial function of the MUSV system, and  $0 \leq \tau_i(t) \leq \bar{\tau}_i^*$ ,  $\bar{\tau}_i^* = h + \bar{\tau}_i$ .

**C. THE OBJECTIVE**

The objective in this research is to design a nonlinear robust control approach for the MUSV to achieve the objectives as follows,

(i) Robust stability: The state of all vessel in the MUSV system can be robustly stabilized to the origin, i.e., for all finite initial condition  $\Phi(t)$  with any initial status  $\Phi_i(s)$  ( $s \in [\bar{\tau}_i, 0)$ ), there exists a finite number  $\mu(s, \Phi(s)) > 0$  such that

$$\lim_{t \rightarrow \infty} E \left\{ \int_0^t \|X_i(t)\|^2 \right\} \leq \mu(s, \Phi(s))$$

where  $E\{\cdot\}$  is the mathematical expectation operator.

(ii) Mesh stability: The oscillations will not magnify with vessel index because of any maneuver of the lead vessel, that is,  $\|G(j\omega)\| \leq 1$  for  $\forall \omega$ , where  $G(s) = \begin{cases} a_{xi}(s)/a_{x(i-1)}(s) \\ a_{yi}(s)/a_{y(i-1)}(s) \end{cases}$  with  $a_{xi}(s)$  and  $a_{yi}(s)$  denotes the Laplace transforms of the acceleration of  $a_{xi}(t)$  and  $a_{yi}(t)$ , where  $a_{xi}(t) = \ddot{x}_i(t)$  and  $a_{yi}(t) = \ddot{y}_i(t)$ .

Note that the mesh stability discussed in here is similar as the string stability issues in [8]–[10], which caused by the dynamic coupling of the MUSV system.

We first give the following definitions and Lemmas, which will be used in the process of controller design.

*Definition 1* [21]: For a given function  $V(t)$ , defining the infinitesimal operator  $\Delta$  as

$$\Delta V(t) = \lim_{\delta \rightarrow 0^+} \frac{1}{\delta} [E\{V((t + \delta) | t)\} - V(t)].$$

*Lemma 1* [21]: For a given scalar  $\tau > 0$ , constant matrix  $R > 0$  and vector function  $\dot{e} : [-h, 0]$ , define the following

integration as,

$$-\tau \int_{t-\tau}^t \dot{X}^T(s)R\dot{X}(s)ds \leq -[X(t) - X(t - \tau)]^T R[X(t) - X(t - \tau)].$$

**Lemma 2 [21]:** Given appropriately dimensioned matrices  $\Sigma_1, \Sigma_2$  and  $\Sigma_3$ , with  $\Sigma_1^T = \Sigma_1$ . Then

$$\Sigma_1 + \Sigma_3 \Sigma_2 + \Sigma_2^T \Sigma_3^T < 0$$

holds if for some matrix  $\Sigma_0 > 0$

$$\Sigma_1 + \Sigma_3 \Sigma_0^{-1} \Sigma_3^T + \Sigma_2^T \Sigma_0 \Sigma_2 < 0.$$

### III. ROBUST CONTROLLER DESIGN

In this section, we will obtain sufficient condition on the robust stability of the MUSV system (8), and the controller is designed by using Lyapunov-Krosovskii method. We first give the following theorem to guarantee the stability conditions of the MUSV.

**Theorem 1:** MUSV system with  $w_i(t) = 0$  is robustly stable if there exist matrices  $P_i > 0, Q_i > 0, R_i > 0$  and appropriately dimensioned matrix  $U_i$  such that

$$\Sigma_i = \begin{bmatrix} \Sigma_{i1} & * & * \\ P_i & -I & * \\ \Sigma_{i2} & U_i & \Sigma_{i3} \\ R_i & 0 & 0 \\ 0 & 0 & 0 \\ (P_i B_i E_i K_i C_{2i})^T & 0 & (U_i B_i E_i K_i C_{2i})^T \\ 0 & 0 & -U_i^T \\ * & * & * \\ * & * & * \\ * & * & * \\ -Q_i - R_i & * & * \\ 0 & W_i & * \\ 0 & 0 & -W_i \\ 0 & 0 & 0 & (\bar{\tau}_i^*)^2 R_i \end{bmatrix} < 0, \quad (9)$$

where

$$\begin{aligned} \Sigma_{i1} &= (A_i + B_i E_i K_i \bar{C}_{1i})^T P_i + P_i (A_i + B_i E_i K_i \bar{C}_{1i}) \\ &\quad + Q_i - R_i + \lambda_{\max}(M_i), \\ \Sigma_{i2} &= (P_i B_i E_i K_i \bar{C}_{2i})^T + U_i (A_i + B_i E_i K_i \bar{C}_{1i}), \\ \Sigma_{i3} &= (U_i B_i E_i K_i \bar{C}_{2i})^T + U_i B_i E_i K_i \bar{C}_{2i}, \\ E_i &= \text{diag}\{\varepsilon_{ui}, \varepsilon_{ri}\}. \end{aligned}$$

*Proof:* Define a Lyapunov-Krosovskii function as

$$\begin{aligned} V_i(t) &= X_i^T(t)P_i X_i(t) + \int_{t-\bar{\tau}_i^*}^t X_i^T(s)Q_i X_i(s)ds \\ &\quad + \bar{\tau}_i^* \int_{-\bar{\tau}_i^*}^0 \int_{t+\alpha}^t \dot{X}_i^T(s)R_i \dot{X}_i(s)dsd\alpha \\ &\quad + \int_{t-\bar{\tau}_i^*}^t w_i^T(s)W_i w_i(s)ds \end{aligned}$$

where  $P_i, Q_i, R_i, W_i$  are positive-definite matrices with appropriate dimensions.

Using the infinitesimal operator in Definition 1 for  $V_i(t)$ , we get

$$\begin{aligned} \Delta V_i(t) &= \dot{X}_i^T(t)P_i X_i(t) + X_i^T(t)P_i \dot{X}_i(t) + X_i^T(t)Q_i X_i(t) - X_i^T \\ &\quad \times (t - \bar{\tau}_i^*)Q_i X_i(t - \bar{\tau}_i^*) + (\bar{\tau}_i^*)^2 \dot{X}_i^T(t)R_i \dot{X}_i(t) + \Upsilon_i \\ &\quad + w_i^T(t)W_i w_i(t) - w_i^T(t - \tau_i(t))W_i w_i(t - \tau_i(t)), \end{aligned}$$

where

$$\Upsilon_i = -\bar{\tau}_i \int_{t-\bar{\tau}_i^*}^t \dot{X}_i^T(s)R_i \dot{X}_i(s)ds. \quad (10)$$

According to Lemma 1, we get

$$\Upsilon_i \leq -[X_i(t) - X_i(t - \bar{\tau}_i^*)]^T R_i [X_i(t) - X_i(t - \bar{\tau}_i^*)]$$

From the equation (4), we have

$$\begin{aligned} f_i^T(t)f_i(t) &= v_i^2 \cos^2(\varphi_i) + v_{i-1}^2 \cos^2(\theta_i - \varphi_i) \\ &\quad - 2v_i v_{i-1} \cos(\varphi_i) \cos(\theta_i - \varphi_i) + \frac{1}{R_i^2} v_i^2 \sin^2(\varphi_i) \\ &\quad + \frac{1}{R_i^2} v_{i-1}^2 \sin^2(\theta_i - \varphi_i) + 2 \frac{1}{R_i^2} v_i v_{i-1} \sin(\varphi_i) \sin(\theta_i - \varphi_i) \\ &\quad + \left(\frac{m_{vi}}{m_{\mu i}}\right)^2 v_i^2 r_i^2 + \left(\frac{m_{\mu i}}{m_{vi}}\right)^2 \mu_i^2 r_i^2 + \left(\frac{m_{\mu vi}}{m_{ri}}\right)^2 \mu_i^2 v_i^2 \end{aligned}$$

Furthermore, by using the dynamic characteristics of vessel  $i$  in MUSV, we can get

$$\begin{aligned} f_i^T(t)f_i(t) &\leq 2v_i^2 + \left(\frac{m_{vi}}{m_{\mu i}}\right)^2 v_i^2 + \left(\frac{m_{\mu i}}{m_{vi}}\right)^2 r_i^2 + \left(\frac{m_{\mu vi}}{m_{ri}}\right)^2 v_i^2 \\ &= X_i^T M_i X_i \leq \lambda_{\max}(M_i) X_i^T M_i X_i \end{aligned} \quad (11)$$

where  $M_i = \begin{bmatrix} \text{zero}(4) & \mathbf{0} \\ * & M_{1i} \end{bmatrix}$ , with

$$M_{1i} = \begin{bmatrix} 2 + \left(\frac{m_{vi}}{m_{\mu i}}\right)^2 + \left(\frac{m_{\mu vi}}{m_{ri}}\right)^2 & 0 \\ * & \left(\frac{m_{\mu i}}{m_{vi}}\right)^2 \end{bmatrix}, \quad \lambda_{\max}(M_i)$$

denotes the maximum eigenvalue of matrix  $M_i$ . Define a zero equation as follows

$$\begin{aligned} \eta_{i1} &= X_i^T(t - \tau_i(t))U_i[-\dot{X}_i(t) + (A_i + B_i \bar{K}_{1i} \sigma_i)X_i(t) \\ &\quad + B_i \bar{K}_{2i} \sigma_i X_i(t - \tau_i(t)) + B_i K_{2i} \sigma_i w_i(t - \tau_i(t)) \\ &\quad + H_i w_i(t) + f_i(X_i)] \end{aligned}$$

Then taking the mathematical expectation on both sides of (10) and from (11), we have that

$$\begin{aligned} E\{\Delta V_i(t)\} &\leq \dot{X}_i^T(t)P_i X_i(t) + X_i^T(t)P_i \dot{X}_i(t) + X_i^T(t)Q_i X_i(t) \\ &\quad - X_i^T(t - \bar{\tau}_i^*)Q_i X_i(t - \bar{\tau}_i^*) + (\bar{\tau}_i^*)^2 \dot{X}_i^T(t)R_i \dot{X}_i(t) \\ &\quad - [X_i(t) - X_i(t - \bar{\tau}_i^*)]^T R_i [X_i(t) - X_i(t - \bar{\tau}_i^*)] \\ &\quad + w_i^T(t)W_i w_i(t) - w_i^T(t - \tau_i(t))W_i w_i(t - \tau_i(t)) \\ &\quad + \lambda_{\max}(M_i) X_i^T M_i X_i - f_i^T(t)f_i(t) + \eta_{i1}^T + \eta_{i1} \\ &= E\{\Psi_i^T \Sigma_i \Psi_i\} < 0 \end{aligned}$$

where

$$\Psi_{i1}(t) = [X_i^T(t) f_i^T(t) X_i^T(t - \tau_i(t)) X_i^T(t - \bar{\tau}_i^*) w_i^T(t) w_i^T(t - \tau_i(t)) \dot{X}_i^T(t)].$$

Then one has

$$E\{\Delta V_i(t)\} < -\beta_{i1} \|\Psi_i(t)\|^2 < 0,$$

where  $\beta_{i1} = \lambda_{\min}(-\Sigma_i) > 0$  denotes the minimum eigenvalue of matrix  $-\Sigma_i$ .

According to Dynkin's formula, one can get

$$\begin{aligned} E\{V_i(t)\} - V_i(0) &= E\left\{\int_0^t \Delta V_i(s) ds\right\} \\ &< -\beta_{i1} E\left\{\int_0^t \|\Psi_i(s)\|^2 ds\right\} \\ &< -\beta_{i1} E\left\{\int_0^t \|x_i(s)\|^2 ds\right\}. \end{aligned}$$

Meanwhile, we have

$$E\{V_i(t)\} > \beta_{i2} E\{\|X_i(t)\|^2\},$$

where  $\beta_{i2} = \lambda_{\min}(P_i) > 0$ . Combining the above two inequalities, it is obvious that

$$E\{\|X_i(t)\|^2\} < -\beta_{i1}\beta_{i2}^{-1} E\left\{\int_0^t \|X_i(s)\|^2 ds\right\} + \beta_{i2}^{-1} V_i(0, \Phi(0)).$$

By using Gronwall-Bellman lemma in [n], we get

$$E\{\|X_i(t)\|^2\} < -\beta_{i2}^{-1} e^{-\beta_{i1}\beta_{i2}^{-1}t} V_i(0),$$

which after integration equals to

$$E\left\{\int_0^t \|X_i(s)\|^2 ds\right\} < -\beta_{i1}^{-1} (1 - e^{-\beta_{i1}\beta_{i2}^{-1}t}) V_i(0, \Phi(0)).$$

as  $t \rightarrow \infty$ , we can get

$$\lim_{t \rightarrow \infty} E\left\{\int_0^t \|X_i(s)\|^2 ds\right\} < -\beta_{i1}^{-1} V_i(0, \Phi(0)).$$

Note that  $V_i(0, \Phi(0)) > 0$ , and according to Definition 1, we can prove that the MUSV system (8) is robustly stable.

Theorem 1 supplies a sufficient condition for the MUSV to achieve robustly stable, implying that each individual vessel can tracking well. Now we continue to give the design method of the controller.

*Theorem 2:* The MUSVs in (8) with  $d_i(t) = 0$  is robustly stabilized, if there exist matrices  $P_i > 0, Q_i > 0, R_i > 0, \Sigma_{i0} > 0$  and appropriately dimensioned matrix  $U_i, L_i$  such that the following linear matrix inequality

$$\begin{bmatrix} \Sigma_{i2} & * & * \\ \Sigma_{i3}^T & \Sigma_{i0} & * \\ \Sigma_{i4} & 0 & \Sigma_{i0} \end{bmatrix} < 0 \quad (12)$$

holds and the controller gains can be given by

$$K_i = \Sigma_{i0}^{-1} L_i.$$

where

$$\begin{aligned} \Sigma_{i1} &= (A_i + B_i E_i K_i \bar{C}_{1i})^T P_i + P_i (A_i + B_i E_i K_i \bar{C}_{1i}) \\ &\quad + Q_i - R_i + \lambda_{\max}(M_i), \end{aligned}$$

$$\Sigma_{i3} = [(P_i B_i)^T \ 0 \ (U_i B_i)^T \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Sigma_{i4} = [\bar{C}_{i1}^T L_i^T E_i^T \ 0 \ \bar{C}_{i2}^T L_i^T E_i^T \ 0 \ 0 \ C_{i2}^T L_i^T E_i^T \ 0],$$

$$E_i = \text{diag}\{\varepsilon_{ui}, \varepsilon_{ri}\},$$

$$\Sigma_{i2} = \begin{bmatrix} \Sigma_{i1} & * & * \\ P_i & -I & * \\ U_i A_i & U_i & 0 \\ R_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -U_i^T \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ -Q_i - R_i & * & * & * \\ 0 & W_i & * & * \\ 0 & 0 & -W_i & * \\ 0 & 0 & 0 & (\bar{\tau}_i^*)^2 R_i \end{bmatrix}$$

*Proof:* According to Theorem 1, the MUSV system (12) is robustly stable if

$$\Sigma_i = \Sigma_{i2} + \Sigma_{i3} \bar{\Sigma}_{i4} + \bar{\Sigma}_{i4}^T \Sigma_{i3}^T < 0,$$

where

$$\bar{\Sigma}_{i4} = [\bar{C}_{i1}^T K_i^T E_i^T \ 0 \ \bar{C}_{i2}^T K_i^T E_i^T \ 0 \ 0 \ C_{i2}^T K_i^T E_i^T \ 0].$$

According to Lemma 2 the above inequality holds if the following inequality holds for some matrix  $\Sigma_{i0} > 0$

$$\Sigma_{i2} + \Sigma_{i3} \Sigma_{i0}^{-1} \Sigma_{i3}^T + \bar{\Sigma}_{i4}^T \Sigma_{i0} \bar{\Sigma}_{i4} < 0,$$

which by Schur complement, is equivalent to (12) with  $K_i = \Sigma_{i0}^{-1} L_i$ . This completes the proof.

In the following, the robust controller design produce is investigated by considering a disturbance  $d_i(t) \in L_2[0, \infty)$  in (8).

*Theorem 3:* The closed-loop MUSV system with any disturbance  $d_i(t) \in L_2[0, \infty)$  is robustly stable with disturbance attention level  $\gamma_i > 0$ , if there exist matrices  $P_i > 0, Q_i > 0, R_i > 0, \Pi_{i0} > 0$  and appropriately dimensioned matrix  $U_i, L_i$  such that the following linear matrix inequality

$$\begin{bmatrix} \Pi_{i1} & * & * \\ \Pi_{i2}^T & \Pi_{i0} & * \\ \Pi_{i3} & 0 & \Pi_{i0} \end{bmatrix} < 0,$$

holds and the controller gains can be given by

$$K_i = \Pi_{i0}^{-1} L_i.$$

where, is obtained by the equation as shown at the top of next page.



**Theorem 4:** For the MUSV system (13),  $|a_i(jw)/a_{i-1}(jw)| \leq 1$  holds for any  $w > 0$ , if the following conditions are satisfied:

$$\sigma_{ui}k_R > 0 \tag{16a}$$

$$k_\mu \leq \left(1 + \frac{\sqrt{2} d_{ui}}{m_{ui}}\right) / \sigma_{ui} \tag{16b}$$

*Proof:* First,  $|a_i(jw)/a_{i-1}(jw)|$  can be written as:

$$G(jw) = \left| \frac{a_i(jw)}{a_{i-1}(jw)} \right| = \frac{\sigma_{ui}k_R}{\sqrt{\sigma_{ui}k_R + b}}$$

Since  $\sigma_{ui}k_R > 0$ , if  $b \geq 0$ , then  $|a_i(jw)/a_{i-1}(jw)| \leq 1$  holds true, i.e., the MUSV system is mesh stable, and from (16b), we have

$$b = \sqrt{2} \left(1 + \frac{\sqrt{2} d_{ui}}{m_{ui}} - \sigma_{ui}k_\mu\right) w^2.$$

According to the condition (16), one can get  $b \geq 0$ . This completes the proof.

Finally, the proposed method used in the MUSV control can be described as,

*Algorithm:* The sampled-data MUSV control algorithm

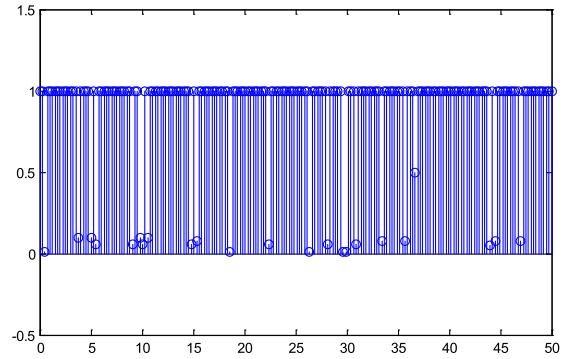
1). The controller gains can be obtained by utilizing standard linear matrix inequality (LMI) tool based on Theorem 2 or Theorem 3.

2). By using the Theorem 4 to constraint the derived controller gain  $k_R$  and  $k_\mu$ . If this is feasible, the obtained controller can be used for MUSV control. Otherwise, reset matrices the related parameters and return to step 1).

### V. SIMULATIONS

In this section, a simulation study is conducted to show how to apply the proposed robust controller to a MUSV (composed by five vessels), which is operated in virtual environment established by using MATLAB/Simulink. The proposed methods and controller in [15] are compared. The actuator faults state are simulated by a Bernoulli sequence between interval [0, 50s], as shown in Fig. 2, the faults status  $0 < \sigma_{ui} = \sigma_{ri} < 1$  with probability 0.02 and the normal operation status  $\sigma_{ui} = \sigma_{ri} = 1$  with probability 0.98.

Two cases are tested in here. The first case is validation performance when the five vessels are required to running on a straight lane. The second case is to show that the desired following angle, in which the angle between the heading



**FIGURE 2.** Actuator fault status.

direction of a vessel and the vessel-to-vessel connection line is not zero:

*Scenario 1 (Five Vessels Running on a Straight Line):* In the scenario, all the following vessels are tracking the lead vessel with the desired surge velocity and the yaw velocity given by  $\mu_1 = 2m/s$ ,  $r_1 = 0$ . The desired profile of

lead vessel is specified as  $\begin{cases} x_1(t) = t \cdot s \\ y_1(t) = 1 \cdot m/s \\ r_1(t) = 0 \cdot rad/s \end{cases}$ . The initial

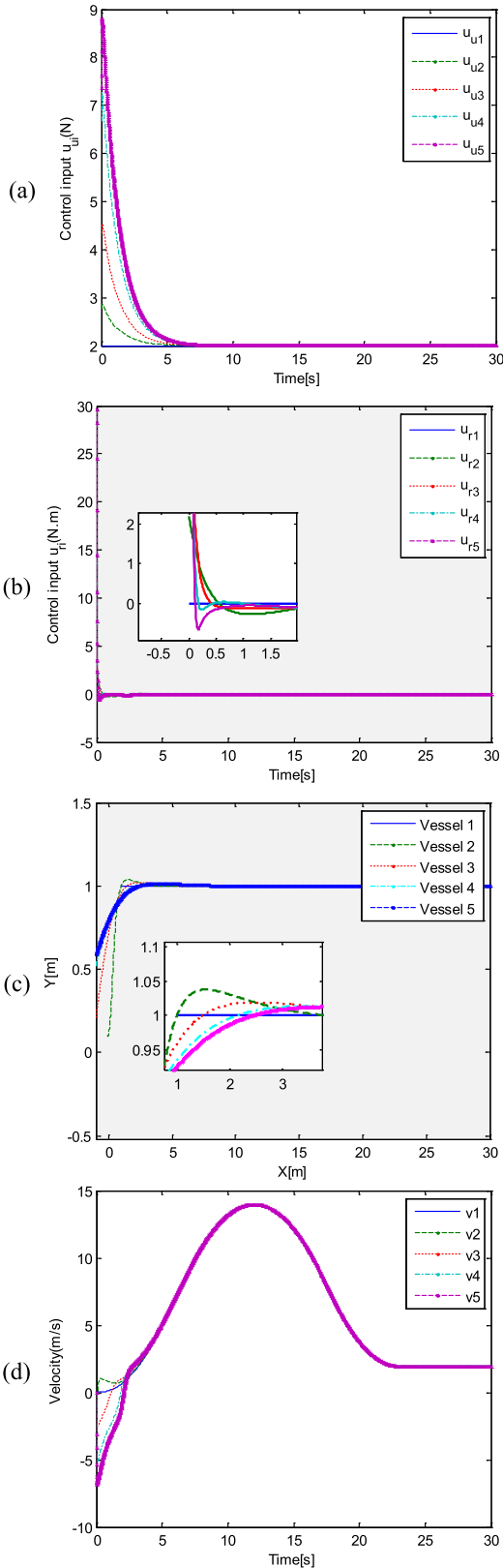
condition was set as

$$\begin{aligned} [R_1 \quad \varphi_1 \quad \theta_1 \quad \mu_1 \quad v_1 \quad r_1]^T &= [0.14 \quad 0.1 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ [R_2 \quad \varphi_2 \quad \theta_2 \quad \mu_2 \quad v_2 \quad r_2]^T &= [0.28 \quad 0.2 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ [R_3 \quad \varphi_3 \quad \theta_3 \quad \mu_3 \quad v_3 \quad r_3]^T &= [0.36 \quad 0.15 \quad 0 \quad 0 \quad 0 \quad 0]^T, \\ [R_4 \quad \varphi_4 \quad \theta_4 \quad \mu_4 \quad v_4 \quad r_4]^T &= [0.26 \quad -0.2 \quad 0 \quad 0 \quad 0 \quad 0]^T. \end{aligned}$$

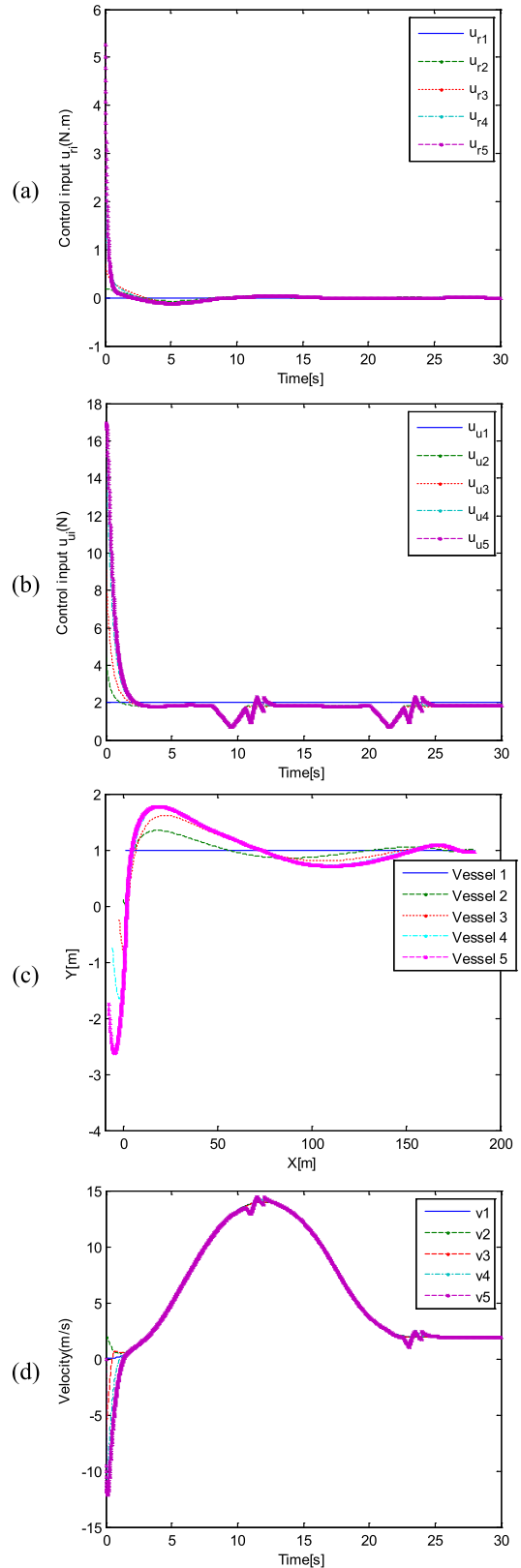
According to Theorem 2 and 4, the controller gains can be derived as  $k_R = 0.5$ ,  $k_\varphi = 2.3$ ,  $k_\theta = 0.82$ ,  $k_\mu = 1.56$ ,  $k_v = 0.85$ ,  $k_r = 1.35$ , respectively. Then, Fig. 3 and Fig. 4 are obtained, which shows the obvious advantages compared with that given in [15]. The maximum control input  $u_{ui}$  is  $8.9N$ , and  $u_{ri}$  is  $-0.5N.m$  as shown in Fig. 3, which is smaller than Fig. 4. The following vessel can track the desired trajectory with high accuracy, while the entire MUSV can achieve mesh stable.

*Scenario 2 (Five Vessels Running on a Curvilinear Lane):* By using Theorem 3 and 4, the controller gains can be derived

$$\Pi_i = \begin{bmatrix} \Pi_{i1} & * & * & * & * & * & * & * \\ P_i & -I & * & * & * & * & * & * \\ \Sigma_{i2} & U_i & \Sigma_{i3} & * & * & * & * & * \\ R_i & 0 & 0 & -Q_i - R_i & * & * & * & * \\ C_{i2}^T C_{i1} & 0 & 0 & 0 & W_i + C_{i2}^T C_{i2} & * & * & * \\ (P_i B_i E_i K_i C_{2i})^T & 0 & (U_i B_i E_i K_i C_{2i})^T & 0 & 0 & -W_i & * & * \\ 0 & 0 & -U_i^T & 0 & 0 & 0 & -(\bar{r}_i^*)^2 R_i & * \\ G_i^T P_i & 0 & U_i^T G_i^T & 0 & 0 & 0 & 0 & -\gamma_i^2 I \end{bmatrix}.$$

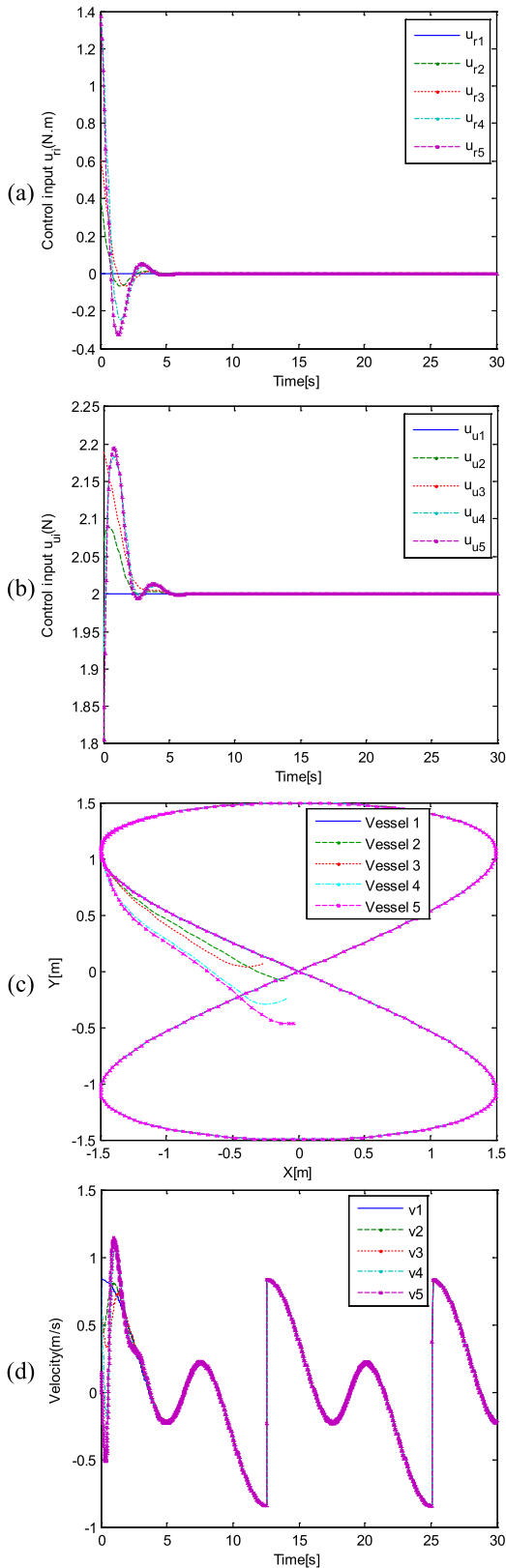


**FIGURE 3.** Five vessels running on the straight line with the proposed controller: (a) Surge direction  $u_{uj}$ ; (b) Yaw direction  $u_{rj}$ ; (c) The position in xoy plane; (d) The tracking velocities.

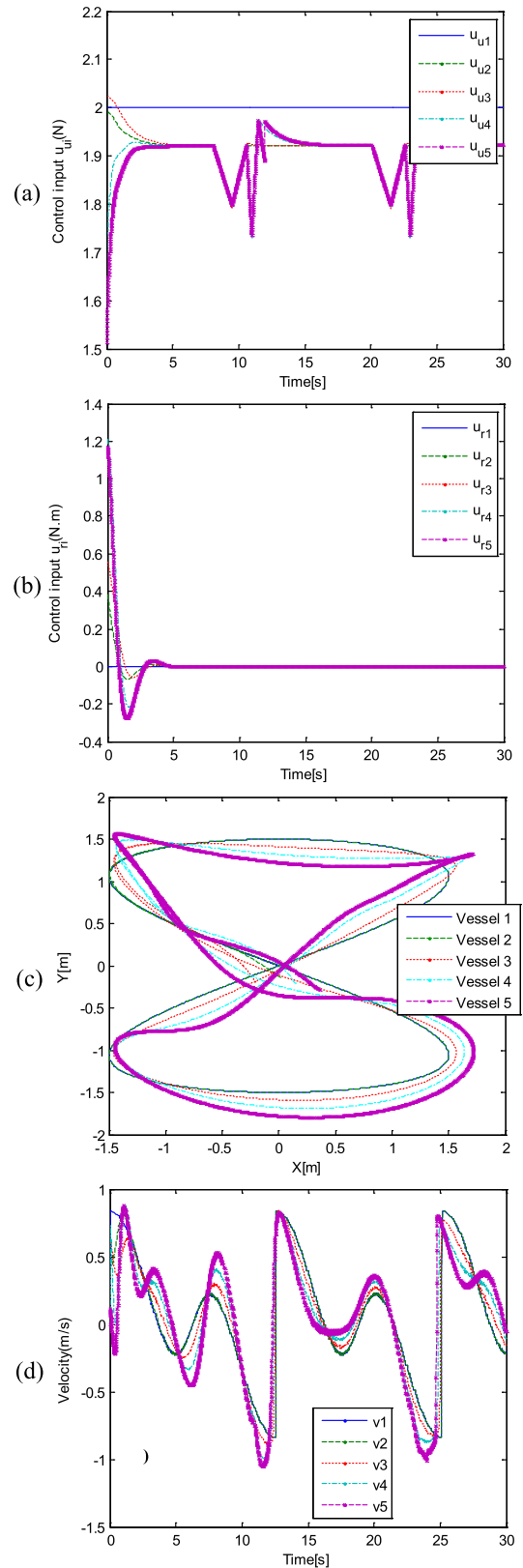


**FIGURE 4.** Five vessels running on the straight line under in [15]: (a) Surge direction  $u_{uj}$ ; (b) Yaw direction  $u_{rj}$ ; (c) The position in xoy plane; (d) The tracking velocities.





**FIGURE 5.** Five vessels running on an eight-shape trajectory with the proposed controller: (a) Surge direction  $u_{u_i}$ ; (b) Yaw direction  $u_{r_i}$ ; (c) The position in xoy plane; (d) The tracking velocities.



**FIGURE 6.** Five vessels running on an eight-shape trajectory under in [15]: (a) Surge direction  $u_{u_i}$ ; (b) Yaw direction  $u_{r_i}$ ; (c) The position in xoy plane; (d) The tracking velocities.

as  $k_R = 0.62$ ,  $k_\varphi = 1.86$ ,  $k_\theta = 0.62$ ,  $k_\mu = 1.35$ ,  $k_\nu = 0.58$ ,  $k_r = 0.96$ . The following vessels tracking the lead vessel with the desired surge velocity and the yaw velocity as  $\mu_1 = 2m/s$ ,  $r_1 = 1 \cdot rad/s$ . The desired profile of lead vessel is specified as

$$\begin{cases} x_1(t) = -1.5 \sin(0.5\pi t/15) \cdot m \\ y_1(t) = 1.5 \sin(0.25\pi t/15) \cdot m \\ r_1(t) = t \cdot rad/s \end{cases}$$

The proposed robust control approach shown better superiority than the controller in [15], as shown in Fig. 5 and 6, respectively. The maximum control input  $u_{ui}$  and  $u_{ri}$  is  $1.4N$ , and  $2.2Nm$ , respectively, which is bigger than [15], but the proposed control can well tracking the desired profile shown in Fig 5 (a) and (b).

## VI. CONCLUSION

In this research, a robust nonlinear control approach based on Lyapunov-Krossovskii functional is designed to meet the special performance requirements of MUSV with minimal negative communication delays and actuator fault effects. The simulations show that the proposed method is more superiority than the existing one.

In future research, the combination of packet loss, quantization and delay in MUSV system will be considered, which will lead to a variety of open issues worthy of investigation. One possibility is to apply the algorithm in Wang and Yue [23] and Qi [24] to MUSV control system.

## REFERENCES

- [1] C. Shen, Y. Shi, and B. Buckham, "Trajectory tracking control of an autonomous underwater vehicle using Lyapunov-based model predictive control," *IEEE Trans. Ind. Electron.*, vol. 65, no. 7, pp. 5796–5805, Jul. 2018.
- [2] T. Fossen, *Marine Control System-Guidance, Navigation and Control of Ships, Rigs and Underwater Vehicles*. Trondheim, Norway: Marine Cybernetics, 2002.
- [3] F. Repoulhas, and E. Papadopoulos, "Planar trajectory planning and tracking control design for underactuated AUVs," *Ocean Eng.*, vol. 34, nos. 11–12, pp. 1650–1667, Aug. 2007.
- [4] K. D. Do, Z. P. Jiang, J. Pan, and H. Nijmeijer, "A global output-feedback controller for stabilization and tracking of underactuated ODIN: A spherical underwater vehicle," *Automatica*, vol. 40, no. 1, pp. 117–124, Jan. 2004.
- [5] T. Salgado-Jimenez, J.-M. Spiewak, P. Fraisse, and B. Jouvencel, "A robust control algorithm for AUV: Based on a high order sliding mode," in *Proc. MTS/IEEE Techno-Oceans*, Kobe, Japan, Nov. 2004, pp. 276–281.
- [6] H. Zhang and J. Wang, "Adaptive sliding-mode observer design for a selective catalytic reduction system of ground-vehicle diesel engines," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 4, pp. 2027–2038, Aug. 2016.
- [7] H. Zhang, G. Zhang, and J. Wang, " $H_\infty$  observer design for LPV systems with uncertain measurements on scheduling variables: Application to an electric ground vehicle," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 3, pp. 1659–1670, Jun. 2016.
- [8] Y. Wei, W. Liyuan, and G. Ge, "Event-triggered platoon control of vehicles with time-varying delay and probabilistic faults," *Mech. Syst. Signal Process.*, vol. 87, no. 15, pp. 96–117, Mar. 2017.
- [9] C. Zou and H. Li, "Event-driven connected vehicular platoon control with mixed time-varying delay," *IEEE Access*, vol. 7, pp. 111477–111486, 2019. doi: [10.1109/ACCESS.2019.2927522](https://doi.org/10.1109/ACCESS.2019.2927522).
- [10] M. M. Tousi and K. Khorasani, "Optimal hybrid fault recovery in a team of unmanned aerial vehicles," *Automatica*, vol. 48, no. 2, pp. 410–418, Feb. 2012.
- [11] E. Kayacan, "Multiobjective  $H_\infty$  control for string stability of cooperative adaptive cruise control systems," *IEEE Trans. Intell. Veh.*, vol. 2, no. 1, pp. 52–61, Mar. 2017.
- [12] G. Guo and W. Yue, "Sampled-data cooperative adaptive cruise control of vehicles with sensor failures," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 6, pp. 2404–2418, Dec. 2014.
- [13] F. Li and Y. Wang, "Cooperative adaptive cruise control for string stable mixed traffic: Benchmark and human-centered design," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 12, pp. 3473–3485, Dec. 2017.
- [14] R. H. Middleton and J. H. Braslavsky, "String instability in classes of linear time invariant formation control with limited communication range," *IEEE Trans. Autom. Control*, vol. 55, no. 7, pp. 1519–1530, Jul. 2010.
- [15] A. Chakravarthy, K. Song, and E. Feron, "Preventing automotive pileup crashes in mixed-communication environments," *IEEE Trans. Intell. Transp. Syst.*, vol. 10, no. 2, pp. 211–225, Jun. 2009.
- [16] Y.-L. Wang and Q.-L. Han, "Network-based fault detection filter and controller coordinated design for unmanned surface vehicles in network environments," *IEEE Trans. Ind. Informat.*, vol. 12, no. 5, pp. 1753–1765, Oct. 2016.
- [17] G. E. Mullins and S. K. Gupta, "Adversarial blocking techniques for autonomous surface vehicles using model-predictive motion goal computation," in *Proc. Int. Conf. Intell. Robots Syst.*, Hamburg, Germany, Sep./Oct. 2015, pp. 2272–2278.
- [18] G.-H. Yang, H. Wang, and L. Xie, "Fault detection for output feedback control systems with actuator stuck faults: A steady-state-based approach," *Int. J. Robust Nonlinear Control*, vol. 20, no. 15, pp. 1739–1757, Oct. 2010.
- [19] H. Qin, C. Li, Y. Sun, X. Li, Y. Du, and Z. Deng, "Finite-time trajectory tracking control of unmanned surface vessel with error constraints and input saturations," *J. Franklin Inst.*, to be published. doi: [10.1016/j.jfranklin.2019.07.019](https://doi.org/10.1016/j.jfranklin.2019.07.019).
- [20] F. Liu, Y. Z. Hua, X. W. Dong, Q. Li, and Z. Ren, "Adaptive fault-tolerant time-varying formation tracking for multi-agent systems under actuator failure and input saturation," *ISA Trans.*, to be published. doi: [10.1016/j.isatra.2019.01.024](https://doi.org/10.1016/j.isatra.2019.01.024).
- [21] Y. Lin, J. Du, G. Zhu, and H. Fang, "Thruster fault-tolerant control for dynamic positioning of vessels," *Appl. Ocean Res.*, vol. 80, no. 2, pp. 118–124, Nov. 2018.
- [22] Q. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," *Automatica*, vol. 41, no. 12, pp. 2171–2176, Dec. 2005.
- [23] L. Wang and W. Yue, "A dwell time switching approach to channel assignment for stabilization of NCSs with medium access constraint," *IEEE Access*, vol. 7, pp. 107311–107319, 2019. doi: [10.1109/ACCESS.2019.2932437](https://doi.org/10.1109/ACCESS.2019.2932437).
- [24] W. H. Qi, H. Park, G. D. Zong, J. D. Cao, and J. Cheng, "A fuzzy Lyapunov function approach to positive  $L_1$  observer design for positive fuzzy semi-Markovian switching systems with its application," *IEEE Trans. Syst., Man, Cybern. Syst.*, to be published. doi: [10.1109/TSMC.2018.2882536](https://doi.org/10.1109/TSMC.2018.2882536).

• • •