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A Universal Channel Estimation Algorithm Based on DFT Smoothing Filtering

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ABSTRACT Channel estimation is a virtual component to ensure the performance in the orthogonal frequency division multiplexing (OFDM). Smoothing filter is often used to suppress noise to improve the estimation performance. However, the conventional smoothing filtering methods have paid little attention on the virtual subcarriers, which is required in practical system but will introduce interference. Therefore, in this paper, we proposed a universal channel estimation algorithm in which the discrete Fourier transform (DFT) smoothing filtering is used when the virtual subcarriers is considered. What is more, the proposed method is applicable to the OFDM system with either block-type or comb-type pilot, and it can be easily extended to multiple input multiple output (MIMO) system. The proposed method employs windowing, extraction and adaptive filter modules one or more based on conventional DFT-based channel estimation for different pilot patterns. The simulation results show that the proposed method has better performance than the conventional DFT-based channel estimation under the same complexity.

INDEX TERMS Channel estimation, discrete Fourier transform (DFT), orthogonal frequency division multiplexing (OFDM), multiple input multiple output (MIMO).

I. INTRODUCTION

Channel estimation is a key component in orthogonal frequency division multiplexing (OFDM) system. It is well known that the accurate channel parameters are required for coherent detection and decoding, hence, the accuracy of channel estimation is very important to improve the performance of the system. Pilot-assisted channel estimation method is widely applied to OFDM system, because of its simple implementation as well as accurate and robust performance [1]–[3]. The most common patterns of pilot symbol arrangement are block-type and comb-type which are shown in Fig. 1, the former is often used in frequency selective channel and the latter is often used in fast fading channel.

On the other hand, the multiple input multiple output (MIMO) [4]–[6] technique is often simultaneous used with OFDM technique to achieve huge transmission capacity. As shown in Fig. 2, the pilot sequences in the different antennas are orthogonal in MIMO system to avoid interference between different antennas. Compared with Fig. 1 and Fig. 2, it can be easily derived that the channel smoothing filtering

FIGURE 1. Different pilot arrangement patterns in OFDM system.

problem of MIMO-OFDM system can be equivalent to its corresponding problem of OFDM system with comb-type pilot.

The channel smoothing filtering is an effective approach to improve the accuracy of channel estimation [7]–[10]. The core idea about the conventional channel smoothing filtering method is resorted to transforming the channel estimates in frequency domain by Discrete Fourier Transform (DFT) into

time domain, suppressing noise, and then changing back into frequency domain. However, the conventional DFT-based smoothing filtering methods mainly focus on OFDM systems with block-type pilot, and without virtual subcarriers. Nevertheless, the virtual subcarriers is essential in practical application because it can relaxes the implementation requirements on the receiver analog filters and DC offset. With this in mind, it is necessary to investigate the scenario with virtual subcarriers. The virtual subcarriers break the equidistance in pilot spacing, degrade the estimation performance, and cause the interference (called '*leakage*') because the orthogonality of Fourier matrix can not be hold true. In [11], the leakage caused by the virtual subcarriers is analyzed using the DFT-inverse DFT process, and the pilot subcarriers inside virtual subcarriers area are estimated by the inverse of the estimated leakage. In [12], an optimal linear estimator for leakage suppression is derived to minimize the mean square error (MSE). The optimal estimator requires information about the covariance matrix of the channel and the noise variance. In [13], the filling of zeros in the time domain channel estimate is used to reduce the leakage effect, which requires information on the bit error rate (BER) and the MSE evaluation performance to determine the optimal fill position. On the other hand, conventional channel smoothing filtering uses the guard interval as the truncation filtering constant. It only reduces the noise outside the guard interval, but the noise within the guard interval is not suppressed. In [14], the optimal threshold to reduce the MSE of the estimation is derived without requiring any channel statistical information. In [15], the noise within the guard interval is removed by using transform domain clustering and discriminant analysis.

In this paper, we intend to extend the DFT-based channel smoothing filtering method to more complicated OFDM systems in which the virtual subcarriers is considered. Moreover, it is not limited to OFDM systems with blocktype pilot, the OFDM systems with comb-type pilot can also be applicable. We firstly establish a universal model to represent the time and frequency relationship of the channel impulse response when the block-type pilot is employed in OFDM system, and discuss the impact of virtual subcarriers on the model. Then, we extend the model to the scenario

in which the comb-type pilot is used. With these considerations in mind, the channel estimates in frequency-domain is transformed into time-domain by using DFT operation with windowing module to decrease the negative effect caused by virtual subcarriers when the block-type pilot is used, and an additional extraction module is applied to decrease the complexity. Since the energy of channel impulse response (CIR) is concentrated on few samples in time domain, it is reasonable to select the principal samples of the estimated CIR with the help of adaptive filtering module and set the other samples to zero to suppress noise more thoroughly. Finally, the filtered CIR is transformed back to the frequency domain for following demodulation and decoding component.

The remainder of this paper is organized as follows. Section II introduces the system model and analyzes the conventional DFT-based channel estimation method in detail. In Section III, The impact of the virtual subcarriers on the conventional DFT-based channel estimation method is analyzed. In Section IV, a universal channel estimation algorithm is proposed and the implementation structure of algorithm is illustrated. In Section V, some simulations are conducted to illustrate the effectiveness of the proposed channel estimation method. In Section VI, the implementation complexity is compared with the existing methods. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL AND CHANNEL ESTIMATION

A. SYSTEM MODEL

Considering an OFDM system which has *N* subcarriers and *M* effective subcarriers. A cyclic prefix of length *P* is added before each OFDM symbol. $X(k)$ is the frequency domain symbol modulated on the *k*-th subcarrier. Therefore, the time domain sampling signal of the OFDM system $x(n)$ can be expressed as

$$
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{M-1} X(k) \exp\left(2\pi j \frac{nk}{N}\right),
$$
 (1)

where $n \in [-P, N-1]$ and *j* is $\sqrt{-1}$.

Without loss of generality, the channel impulse response of the multi-path fading channel can be expressed

$$
h(n) = \sum_{l=0}^{L-1} h_l \delta(n - \tau_l),
$$
\n(2)

where *L* is the number of paths, h_l and τ_l are complex gain and delay of the *l*-th path, and $\delta(n-\tau_l)$ is the unit impulse response function. Assuming that the guard interval is longer than the maximum channel delay and the synchronization is perfect, the received signal is $y(n) = x(n) * h(n)$, where $*$ represents a convolution operation. The frequency domain expression of the received signal can be expressed as

$$
Y(k) = H(k)X(k) + W(k),\tag{3}
$$

where $X(k)$ is the transmitted signals, $Y(k)$ is the received signals, $H(k)$ is the frequency response of multipath channel,

FIGURE 3. The conventional DFT-based channel estimation.

 $W(k)$ is the additive white Gaussian noise with zero mean and a variance σ^2 , and *k* is the index of subcarrier.

B. LS CHANNEL ESTIMATION

It can be seen from (3) that recovering the transmitted signals require the knowledge of the sampled channel frequency response, and the sampled channel frequency response can be obtained by least square (LS) estimation. The LS channel estimation on each subcarrier can be expressed as

$$
H_{LS} (k) = \frac{Y(k)}{X(k)},
$$
\n(4)

where $H_{LS}(k)$, $Y(k)$, $X(k)$ are LS channel estimates, received training symbol and pilot signal on the *k*-th subcarrier. The individual MSE of *k*-th subcarrier [16] is

$$
MSE_{LS} (k) = \frac{\beta}{SNR}, \qquad (5)
$$

where *SNR* is the average signal-to-noise ratio (SNR) and $\beta = \mathbb{E}[|X(k)|^2] \mathbb{E}[|X(k)|^{-2}]$ is a constant depending on signal constellation, and E[.] denotes expectation operation.

It can be seen from (5) that the MSE of LS estimation is inversely proportional to SNR, and the estimated performance of LS estimation is severely limited by noise.

C. DFT-BASED CHANNEL ESTIMATION

In order to improve the performance of LS estimation algorithm, a channel estimation algorithm based on DFT is proposed. The structure of DFT-based channel estimation algorithm is shown in Fig. 3. All DFT operations in the algorithm are implemented by fast Fourier transform (FFT) for its low complexity.

First, the channel frequency response (CFR) which obtained by the LS algorithm is transformed into the time domain by inverse fast Fourier transform (IFFT)

$$
h_{LS}(n) = IFFT[H_{LS}(k), N] = h(n) + w(n), \qquad (6)
$$

where $n = 0, 1, \dots, N - 1$ and *N* is the total number of subcarriers. It can be seen from (6) that the channel time domain

impulse response estimates $h_{LS}(n)$ is composed of channel time domain impulse response $h(n)$ and noise $w(n)$. Since the CIR is not longer than the guard interval in OFDM system, the channel coefficients that outside the guard interval *P* can set to zero to reduce noise. Define the channel coefficient as

$$
h_{DFT}(n) = \begin{cases} h_{LS}(n), & n = 0, 1, \dots, P - 1 \\ 0, & \text{otherwise.} \end{cases}
$$
 (7)

Finally, transform *hDFT* (*n*) into the frequency domain to complete DFT smoothing filtering

$$
H_{DFT} (k) = FFT [h_{DFT} (n), N]. \tag{8}
$$

The MSE of conventional channel smoothing filtering method can be obtained from [16], expressed as

$$
MSE_{DFT} (k) = \frac{P}{N} \frac{\beta}{SNR}.
$$
 (9)

It is worth noting that the channel estimation variance can get minimum when $P = L$, where L is the length of the channel time domain impulse response.

III. INTERFERENCE OF VIRTUAL SUBCARRIER

The time-frequency relationship of the channel estimates is the basis of the DFT-based smoothing filtering. Without loss of generality, taking OFDM system as an example, and the relevant conclusions can be extended to the MIMO-OFDM system.

The time-frequency relationship of the channel estimates can be expressed as

$$
\mathbf{h}^f = \mathbf{F}_N \mathbf{h}^t,\tag{10}
$$

where \mathbf{h}^f and \mathbf{h}^t are frequency domain channel estimates and time domain channel estimates, \mathbf{F}_N is the N-point DFT transformation matrix. The correlation function of channel frequency domain response can be calculated by (10)

$$
\mathbf{R}_f = \mathbf{F}_N \mathbf{R}_t \mathbf{F}_N^{\mathrm{H}},\tag{11}
$$

where \mathbf{R}_f and \mathbf{R}_t are the correlation functions of frequency domain estimates and time domain estimates, and $(.)^H$ is the conjugate transpose operation. Since the sparsity of multipath channels [17], the eigenvalues of the correlation function of channel frequency response are mostly zero. This is the basis for DFT to perform smoothing.

Actually, the virtual subcarriers always exist in practical OFDM system either with block-type or comb-type pilot to avoid DC offset and decrease adjacent channel interference, and the channel estimates of the location of the virtual subcarriers cannot be obtained. The time-frequency relationship of the channel estimates shown in (10) will degrade into

$$
\mathbf{h}_M^f = \mathbf{F}_{M \times N} \mathbf{h}^t,\tag{12}
$$

where *M* is the number of effective subcarriers and $M < N$. It is worth noting that $\mathbf{F}_{M \times N}$ is no longer an orthogonal matrix. Therefore, the smoothing filtering method requires to be redesigned.

Although the distribution position of effective subcarriers is discontinuous, the time-frequency relationship of channel estimates shown in (12) is still valid and it can be further derived as

$$
\mathbf{h}_{M}^{f} = \mathbf{F}_{M \times N} \mathbf{h}^{t}
$$

= $\mathbf{Q}_{M \times M} \mathbf{R}_{M \times N} \mathbf{h}^{t}$
= $\mathbf{Q}_{M \times M} \tilde{\mathbf{h}}^{t}$, (13)

where $Q_{M \times M}$ and $R_{M \times N}$ are an orthogonal matrix and a lower triangular matrix obtained by QR decomposition for matrix $\mathbf{F}_{M\times N}$, and $\tilde{\mathbf{h}}^t$ is the equivalent channel time domain impulse response. It is worth noting that since $\mathbf{R}_{M \times N}$ is a lower triangular matrix, the coefficient sparsity of **h** *t* is retained to \tilde{h}^t , which provides the basis of smoothing filtering.

Considering the influence of noise further, the timefrequency relationship shown in (13) can be expressed as

$$
\mathbf{h}_{M}^{f} = \mathbf{F}_{M \times N} \mathbf{h}^{t} + \mathbf{w}^{f}
$$

= $\mathbf{Q}_{M \times M} \tilde{\mathbf{h}}^{t} + \mathbf{w}^{f}$, (14)

where \mathbf{w}^f is the frequency domain distribution of noise. It can be seen that (14) and (10) are identical in form. Since $Q_{M \times M}$ is an orthogonal matrix, the noise distribution characteristics of the system are maintained and the effects of noise are not amplified during processing. Therefore, the conventional smoothing filtering method can also be used to implement. Firstly, the channel estimates of frequency response h'_j $\frac{1}{M}$ is multiplied by $Q_{M \times M}^{\rm H}$. Then, according to the sparsity of CIR in time domain, the channel estimates in time domain can also be reasonable truncated. Finally, the channel estimates in time domain is multiplied by the $Q_{M \times M}$ to obtain the channel frequency domain estimates. However, because the multiplication of the $Q_{M \times M}$ matrix can not be implemented in a fast form as like FFT, it will take up a lot of hardware resources. Therefore, an efficient and low latency smoothing filtering algorithm is required.

IV. PROPOSED CHANNEL ESTIMATION

A. BLOCK-TYPE PATTERN

Considering the OFDM system with block-type pilot, a windowing method is proposed to fill the gap caused by the virtual subcarriers. The windowing operation is shown in Fig. 4. First, the estimates of channel frequency domain response on the effective subcarrier position $H_{LS}(k)$, $1 \leq$ $k \leq M$ are obtained by LS algorithm. On this basis, the frequency response of the virtual subcarrier position is set to zero for constructing the complete frequency response $H(k)$, $1 \leq$ $k \leq N$. Then (10) can also be used to depict the timefrequency relationship of the channel estimates, and complete the channel smoothing filtering by resorting to the conventional DFT-based smoothing filtering method as shown in $(6-8)$.

B. COMB-TYPE PATTERN

Considering the OFDM systems with comb-type pilot, the pilot subcarriers are generally equally spaced by inserting

FIGURE 4. The diagram of the process of windowing.

data subcarriers as shown in Fig. 1. In order to have a deep insight about the representation model of the time-frequency relationship of the channel estimates, we firstly derived a lemma about Fourier transformation matrix as follow.

Lemma: Let **F***^N* denote Fourier transform matrix with dimension *N*, then

$$
\mathbf{F}_{N} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & F_{N}^{1} & F_{N}^{2} & \cdots & F_{N}^{N-1} \\ 1 & F_{N}^{2} & F_{N}^{4} & \cdots & F_{N}^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & F_{N}^{N-1} & F_{N}^{2(N-1)} & \cdots & F_{N}^{(N-1)(N-1)} \end{bmatrix} . \quad (15)
$$

If extracting rows from \mathbf{F}_N with interval *q* $1, 2, \cdots, N/2 - 1$ rows, then the extracted rows can be formed a new matrix $\mathbf{F}_{K \times N}$

$$
\mathbf{F}_{K\times N} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & F_N^{q+1} & F_N^{(q+1)2} & \cdots & F_N^{(q+1)(N-1)} \\ 1 & F_N^{2(q+1)} & F_N^{2(q+1)2} & \cdots & F_N^{2(q+1)(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & F_N^{K(q+1)} & F_N^{K(q+1)2} & \cdots & F_N^{K(q+1)(N-1)} \end{bmatrix},
$$
\n(16)

where $K = N/(q + 1)$. The matrix $\mathbf{F}_{K \times N}$ can be further equivalently expressed as

$$
\mathbf{F}_{K \times N} = \mathbf{F}_K \times [\mathbf{I}_K | \mathbf{R}_E],\tag{17}
$$

where \mathbf{F}_K is a Fourier transform matrix with dimension K and is an orthogonal matrix. \mathbf{I}_K is an identity matrix with dimension *K*. \mathbf{R}_E is the residual matrix with dimension *K* × $(N - K)$ and it can be expressed as

$$
\mathbf{R}_{\mathrm{E}} = \underbrace{\left[\mathbf{I}_{K}, \cdots, \mathbf{I}_{K}\right]}_{q}.
$$
 (18)

Here, we further investigate the smoothing filtering method when the comb-type pilot is used. Without loss of generality, it is assumed that *q* data subcarriers are inserted between the pilot subcarriers. In order to address the discontinuous distribution of pilot subcarriers caused by virtual subcarriers, a windowing and extracting process are proposed. Fig. 5 gives the diagram of the windowing and extracting processes, where the interval *q* is set as 4.

FIGURE 5. The diagram of the processes of windowing and extraction.

Firstly, implementing windowing operation as shown in Fig. 4 on the channel frequency domain response estimates of the effective subcarrier positions. Then, the time-frequency relationship can be expressed as

$$
\mathbf{h}_N^f = \mathbf{F}_{N \times N} \mathbf{h}^t. \tag{19}
$$

With the distribution pattern of pilot and data subcarriers in comb-type pilot, the channel frequency response is extracted with an interval of *q* as shown in Fig. 5. Therefore, (19) is converted to

$$
\mathbf{h}_K^f = \mathbf{F}_{K \times N} \mathbf{h}^t,\tag{20}
$$

where $K = N/(q + 1)$ and $h_K^f = [0, h_{\text{pilot}}, 0, \dots, 0, h_{\text{pilot}}]$, h_{pilot} is the channel frequency response of the pilot subcarriers position. With the help of the *Lemma* of Fourier transformation matrix derived above, (20) can be further simplified as

$$
\mathbf{h}_K^f = \mathbf{F}_K \mathbf{h}^t. \tag{21}
$$

Therefore, when OFDM system adopts the comb-type pilot, the proposed method can perform DFT smoothing filtering with a lower dimensional DFT matrix, and the computational complexity of smoothing filtering is greatly reducing.

It can be seen from the nature of the FFT that windowing and extracting process will result in the power expansion of the estimates of channel impulse response. Assuming that the back of the channel frequency domain response is a virtual subcarrier, then the channel estimates of CIR can be derived by using IFFT

$$
\hat{h}(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}kn}
$$
\n
$$
= \frac{1}{N} \sum_{k=0}^{M-1} \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi}{N}k\tau_l} \cdot e^{j\frac{2\pi}{N}kn}
$$
\n
$$
= \frac{M}{N} \sum_{l=0}^{L-1} h_l \frac{Sa\left(\frac{M}{N}\pi(n-\tau_l)\right)}{Sa\left(\frac{\pi}{N}(n-\tau_l)\right)} \cdot e^{j\frac{\pi}{N}(M-1)(n-\tau_l)}.
$$
\n(22)

FIGURE 6. CIR of energy leakage with $N = 512$, $M = 408$ and $SNR = 10$.

It can be seen from (22) that the non-ideal sampling of the *Sa* function leads to the expansion of CIR in the whole symbol, and the expansion factor is:

$$
\xi(m) = Sa\left(\frac{M\pi m}{N}\right)/Sa\left(\frac{\pi m}{N}\right),\tag{23}
$$

where *m* is the offset. As can be seen from (23), the power expansion caused by the virtual subcarriers will dacay rapidly to the vicinity of zero with the increase of offest. Fig. 6 is the CIR obtained by the CFR of the block-type pilot estimation after windowing, and the CIR obtained by the CFR of the comb-type pilot estimation after windowing and extraction is similar. It can be seen from Fig. 6 that when a virtual subcarrier exists and the CFR is transformed into the time domain by IFFT, the resulting CIR will spread over the entire symbol. The multipath energy is mainly concentrated at the head and tail of the symbol, and the multipath energy in the middle of the symbol is small. Generally, the noise energy is greater than the diffusion energy caused by the virtual subcarrier. Therefore, an adaptive filtering method can be used in the time domain to reduce the influence of noise on the channel estimation performance, and suppress the diffusion energy caused by the virtual subcarrier.

C. ADAPTIVE FILTER

The core idea of the adaptive smoothing filtering method is to use the threshold λ to adaptively choosing the significant channel estimates in time domain, which can be expressed as

$$
\hat{h}_{DFT}(n) = \begin{cases}\n\hat{h}_{LS}(n), & P_{LS}(n) > \lambda \\
0, & otherwise,\n\end{cases}
$$
\n(24)

where $P_{LS}(n)$ is the power of the estimate of the *n*-th tap of the channel impulse response and $P_{LS}(n) = |\hat{h}_{LS}(n)|^2$. As derived in [14], the MSE of the adaptive filtering method is

$$
MSE_{zero} (k) = \sigma_h^2 (n) + \frac{P-1}{N} \frac{\beta}{SNR}, \qquad (25)
$$

where $\sigma_h^2(n)$ is the average power of the estimates of the non-significant channel impulse response and

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 $\sigma_h^2(n) = E[|\hat{h}(n)|^2]$. β is a constant depending on signal constellation (*e.g.* $\beta = 1$ for QPSK, $\beta = 17/9$ for 16-QAM, $β = 1289/480$ for 64-QAM).

Assuming that the adaptive filtering method performs better than the conventional smoothing filtering, the variance of channel estimation using the conventional smoothing filtering method should be less than the variance of channel estimation using adaptive filtering. Thus, subtracting (9) from (25), it should be satisfied

$$
MSE_{zero} (k) - MSE_{DFT} (k) = \sigma_h^2 (n) - \frac{1}{N} \frac{\beta}{SNR} < 0. \quad (26)
$$

Note that $SNR = E[|X(k)|^2]/\sigma_{wf}^2$ is the average signal-tonoise ratio and $\sigma_{wf}^2 = N \sigma_{wt}^2$, hence, it can be derived that

$$
\frac{1}{N} \frac{\beta}{SNR} = \beta \sigma_{wt}^2 E[|X(k)|^{-2}].
$$
 (27)

Because the power of the estimates of channel impulse response $\hat{h}_{LS}(n)$ of (24) is composed of the power of noise and the power of channel impulse response, we add the power of noise to both side of (26), then we obtain

$$
\sigma_h^2(n) + \sigma_{wt}^2 < \left(1 + \beta E[|X(k)|^{-2}]\right) \sigma_{wt}^2 = \lambda. \tag{28}
$$

Without loss of generality, the training signal $X(k)$ is modulated signal with average power one. Therefore, the threshold λ can be obtained as

$$
\lambda = (1 + \beta) \sigma_{wt}^2,\tag{29}
$$

where σ_{wt}^2 is the variance of noise. Since the diffusion energy caused by the virtual subcarriers is mainly concentrated at the head and tail of the symbol, σ_{wt}^2 can be expressed as

$$
\sigma_{wt}^2 = \frac{1}{N - 2P} \sum_{P}^{N - P - 1} \left| \hat{h}(n) \right|^2.
$$
 (30)

D. IMPLEMENTATION MODEL

The implementation structure of the proposed DFT-based channel estimation methods for OFDM systems with blocktype and comb-type pilot are shown in Fig. 7 and Fig. 8, respectively. *N* is the total number of subcarriers of OFDM system, *M* is the number of used subcarriers, $K = N/(q + 1)$ where q is the interval between the adjacent pilot subcarriers when comb-type pilot is used, and *P* is the length of guard interval.

The proposed DFT-based channel estimation method in terms of block-type pilot are implemented as follows:

- 1) estimating the frequency response of channel according to LS method as shown (4) with the received signal and pilot signal;
- 2) performing the windowing operation shown in Fig. 4 according to the position of the virtual subcarriers;
- 3) transforming the channel estimates of frequency response into its time domain by IFFT with low complexity;
- 4) calculating the adaptive threshold λ according to (29) with the noise power estimator by using the noise

FIGURE 7. The DFT-based smoothing filtering with block-type pilot.

FIGURE 8. The DFT-based smoothing filtering with comb-type pilot.

samples located outside the guard interval as shown in (30), and selecting the significant channel taps of the estimated channel impulse response;

5) converting the estimates of channel impulse response to its corresponding frequency response by FFT.

The proposed DFT-based channel estimation method in terms of comb-type pilot are implemented as follows:

- 1) estimating the frequency response of channel according to LS method as shown (4) with the received signal and pilot signal;
- 2) performing the windowing operation shown in Fig. 5 according to the position of the virtual subcarriers;
- 3) extracting the channel estimates of frequency response according to the position distribution of pilots as shown in Fig. 5, and obtaining the simpler model (21) to represent the relationship between the time and frequency estimates of channel response;
- 4) transforming the channel estimates of frequency response into its time domain by IFFT;
- 5) calculating the adaptive threshold λ according to (29) with the noise power estimator by using the noise samples located outside the guard interval as shown in (30), and selecting the significant channel taps of the estimated channel impulse response;
- 6) converting the estimates of channel impulse response to its corresponding frequency response by FFT.

V. SIMULATION RESULTS

In this section, simulation experiments are conducted to evaluate the performance of the proposed DFT-based channel estimation method. In the simulations, the relevant

TABLE 1. Simulation parameters.

FIGURE 9. Comparison of MSE performance under block-type pilot condition.

parameters used in the OFDM systems are indicated in table 1 and the pilot is evenly distributed. The multi-path channel model is considered and 100 Monte Carlo simulations are conducted. The bit error ratio (BER) and mean square error (MSE) are the principal indexes to measure the performance. Unit delay of channel is assumed to be the same as sample period, thus, there is no power loss caused by non-sample spaced. The performance of the conventional DFT-based channel estimation [13] method and the estimation method based on minimum mean square error (MMSE) criteria are the baselines. In addition, the performance of the proposed method is also compared with the technique proposed in [16], in which extra pilot is transmitted to eliminate leakage.

Fig. 9 and Fig. 10 give the MSE and BER performance of the tested methods when the block-type pilot is used in OFDM system. In Fig. 9, it can be observed that the LS and MMSE channel estimation methods have the same trend that the MSE is decreasing with the increase of SNR, and the latter has much less MSE. Various methods based on DFT smoothing filtering have better performance than LS estimation method at low SNR, and there is still a gap in comparison with the MMSE estimation method. Note that the MSE of the DFT-based estimation methods is determined by SNR and the leakage caused by DFT smoothing filtering. When the SNR is high, the latter factor plays the principal role and the impact of SNR can be neglected. Hence, the MSE of the DFT-based estimation methods would be larger than the one of the LS estimation method when the SNR is high. Meanwhile, the performance of proposed DFT-based estimation method is also better than that of the conventional DFT-based estimation and the adaptive filtering method proposed in [16]. It can

FIGURE 10. Comparison of BER performance under block-type pilot condition.

FIGURE 11. Comparison of MSE performance under comb-type pilot condition.

also conclude from Fig. 10 that the BER performance of the proposed estimation method is better than other tested methods except the MMSE when the SNR is lower than 12dB. It is worth noting that wireless communication systems can work normally when the SNR is higher than 12dB when the error coding technique is applied.

Fig. 11 and Fig. 12 give the MSE and BER performance of the tested methods when the comb-type pilot is used in OFDM system. These results show that the performance trends of the tested estimation methods in the case of comb-type pilot is consistent with the trends showed in the case of block-type pilot, and the performance improvement of the comb-type pilot is slightly lower than that of the block-type pilot. The main reason is that the CIR aliasing and superposition caused by CFR extraction operation and noise.

VI. IMPLEMENTATION COMPLEXITY

Implementation complexity is an critical issue should be taking into account. In this paper, we use the number of complex multiplications as a measure of implementation complexity. As shown in Fig.7 and Fig. 8, the proposed DFT-based channel estimation method is composed of three major parts: the LS channel estimation, two complex FFT

FIGURE 12. Comparison of BER performance under comb-type pilot condition.

TABLE 2. Comparison of the hardware complexity of different techniques.

Methods	Complex Multiplications/Symbol
Conventional DFT-based	$M+2M^2$
Proposed DFT-based for block-type	$M + 2log_2 N + N - 2P$
Proposed DFT-based for comb-type	$M + 2log_2(N/q) + (N - 2P)/q$
Zhu [2015] DFT-based [16]	$M + 2log_2N + N - P$

operations and threshold calculation. Without loss of generality, the LS estimation method shown in (4) requires *M* times multiplication operations per OFDM symbol, the DFT/IDFT can be implemented with the radix-2 FFT algorithm that requires about *log*₂*N* complex multiplications for an N-point DFT when *N* is a power of 2 [18], and the threshold calculation requires $N - 2P$ times complex multiplications per OFDM symbol. Hence, The computational complexity of the proposed DFT-based channel estimation method is approximately $M + 2log_2N + N - 2P$ when the block-type pilot is employed and $M + 2log_2(N/q) + (N - 2P)/q$ when the comb-type pilot is used, respectively.

Since the existence of the virtual subcarrier destroys the continuity of the subcarrier distribution position, the conventional DFT estimation can only use the DFT algorithm for time-frequency transform, which greatly increases the hardware complexity. The noise range to be calculated by the proposed estimation algorithm is smaller than the adaptive filter in [16]. Therefore, the proposed estimation algorithm has a slightly smaller complexity than [16]. The complexity of the hardware implementation of different channel estimation techniques is given in table 2.

VII. CONCLUSION

In this paper, channel estimation method can be applied for the OFDM systems with either block-type or comb-type pilot under the condition that the virtual subcarriers is considered. Firstly, by analyzing the influence of virtual subcarriers on the relationship between the time and frequency response of channel impulse response, it can be proved that the windowing process is required to depress interference when DFT-based smoothing filter is used to suppress noise

for OFDM system with block-type pilot. As for the OFDM system with comb-type pilot, an additional extraction process is needed to reduce the complexity. What is more, in order to suppress the noise more effectively, the significant estimates of channel taps are selected by using an adaptive threshold which is obtained by the average power of noise in the time domain, and does not require any information about the channel statistics. The simulation results and analysis show that the performance of the proposed method is better than the conventional DFT-based methods and also require low implementation complexity.

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