An Improved Teaching-Learning Based Optimization Algorithm and Its Application to Aero-Engine Start Model Adaptation

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ABSTRACT The modeling of the engine starting process is vital to ensure the successful start of the engine. However, the engine starting process is very complicated and challenging to model. To optimize the start model performance, an improved teaching-learning based optimization (ITLBO) algorithm is proposed. In ITLBO, a collective lesson preparation phase is increased to enhance the teaching ability of the teacher. The random learning phase is replaced by S-shape group learning, and students learn from the top students of their groups. Also the deterministic sampling selection phase is introduced to ITLBO, and the students with higher evaluation have more possibility to advance in class. The improved algorithm is tested on 18 benchmark functions. The results indicate that the proposed ITLBO algorithm performs much better in terms of convergence speed and accuracy than standard TLBO. When applied to the model adaptation of the turbofan engine starting process, ITLBO is used to optimize the speed line of the rotation components gradually from the lower speed line to the idle speed line. The weighted sum of relative errors between the model outputs and the start test data is taken as the fitness function. After adaptation, the maximum relative errors of model outputs to start test data are significantly decreased, which shows the effectiveness of the ITLBO in model adaption.

INDEX TERMS Model adaptation, teaching-learning based optimization, collective lesson preparation, turbofan engine.

I. INTRODUCTION

The aero-engine starting process is a very complex non-equilibrium and non-linear aero-thermodynamic process. It involves the co-operation of starter, fuel supply system, ignition system and all other engine components. It is difficult to establish an accurate starting process model [1]. An effective engine start model can provide information for engine start controller design and improve the success rate of start significantly. Therefore, the modeling of the engine starting process is essential [2], [3].

The widely used start models are component-level models based on component performance maps. However, the characteristics of turbomachinery at low speed are difficult to be obtained by experimental method. Also the accuracy of component performance map obtained by computational fluid dynamics method cannot meet the engineering requirements.

The associate editor coordinating the review of this manuscript and approving it for publication was Chaoyong Li.

Therefore, the commonly used method is to extrapolate the sub-idle characteristics of the start model based on the known above-idle characteristics or to construct the whole component characteristics by the stage-stacking method [4], [5]. Even so, the obtained performance map can hardly represent the real starting process. So the accuracy of the start model still needs to be enhanced. The commonly used method to enhance the accuracy is performance map adaptation based on the start test data.

The performance adaptations use a series of scaling factors optimized by least square [6], [7], genetic algorithm [8]–[10], particle swarm optimization [11] to scale and shift the performance maps. These studies of performance adaptations only involve the performance adaptation of steady state points from idle to maximum power, but ignore the adaptation of transient performance of the starting process. For the starting is a continuous dynamic process, the performance adaptations method used in steady state optimization cannot be applied directly to start model optimization.

The performance adaptation problem of the start model can also be viewed as a parameter identification problem [12]. The scaling factors are the parameters of the start model to be identified. With the development of evolutionary algorithms, many parameter identification problems are solved by intelligent optimizations such as particle swarm optimization [13], bat algorithm [14], differential evolution algorithm [15] and other heuristic optimization algorithms [16].

Different from all these evolutionary algorithms, Rao proposed an efficient optimization method called Teaching-Learning-Based Optimization (TLBO) in 2011 [17]. The principle of the TLBO method is the philosophy of the teaching-learning process and it is based on the effect of the influence of a teacher on the output of learners in a class. Similar to other nature-inspired algorithms, TLBO is a population-based heuristic stochastic algorithm for global optimization. While other nature-inspired algorithms need to be set proper algorithm parameters to work efficiently, TLBO is nearly parameter-free. Due to TLBO algorithm is easy to implement and has fast convergence ability, it has been widely used in various engineering problems [18], [19]. So we adopt TLBO to optimize the scaling factors of the start model to achieve model performance adaption.

The optimization ability of TLBO will directly affect model performance. So an improved TLBO (ITLBO) algorithm is proposed to enhance its searching ability and the model accuracy. The improvements include: 1) Increasing a preparing lesson collectively phase. 2) Adopting the S-shape group method in the learning phase. 3) Increasing a Deterministic Sampling selection phase. The results of simulations reveal that ITLBO performs better in terms of convergence speed and accuracy, and a more accurate start model can be obtained by ITLBO, which can satisfy the simulation requirement of the start model.

The rest of the paper is organized as follows. The related work about TLBO is introduced in Section 2. Section 3 presents the proposed ITLBO algorithm. The verification of the ITLBO algorithm is given in Section 4. Section 5 shows the application of ITLBO in start model performance adaptation. Finally, the paper is concluded in Section 6.

II. RELATED WORK ABOUT TLBO

As a swarm intelligence algorithm, the standard TLBO algorithm may get into local optimum when solving complex global optimization problems, so researchers have made some improvements to enhance its searching ability.

One commonly used method is modifying the learning process of TLBO to improve its efficiency. An elitist concept was introduced into TLBO (ETLBO) by Rao, and the worst individuals are replaced by the elite ones [20]. An opposition-based learning method was introduced into TLBO by Roy et al., and the independent variables of combined heat and power dispatch systems were generated by oppositional initialization, and the opposition-based jumping was used based on jumping probability [21]. Satapathy et al. employed an adaptive weight to simulate the forgetting process of

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Algorithm 1 ITLBO

Begin
  Initialize the population, set control parameters and termination criterion;
  Evaluate individuals and select the teacher;
  while stop condition is not met:
      Generate the teaching group according to Eq.(1)
      Prepare lesson collectively according to Eq.(2) and Eq.(3)
      for all students (not including the teacher)
          Take part in the teaching phase according to Eq.(4)
      end for
      for all students
          Divide into several groups in S-shape.
      end for
    for students in each group
          Take part in group learning phase according to Eq.(7) and Eq.(8)
    end for
    for all students
          Take part in the selection phase according to Eq.(9) and Eq.(10).
    end for
      Evaluate the new generation and select the teacher
    end while
end

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the human brain in the learning phase, the new algorithm (Weighted-TLBO) can do faster in simulation time [22]. Large population size may not always be helpful [23], so Chen et al. introduced a new TLBO algorithm with a variable-population scheme to increase the convergence accuracy [24]. Learning from oneself historical positions phase and a mutation and crossover phase were added into TLBO by Ji et al. to enhance its exploration ability [19]. Zou et al. proposed a modified TLBO with differential calculation in teacher phase and repulsion in the self-learning phase to improve its optimization performance [25].

To improve the overall computational efficiency, Camp et al. modified the TLBO algorithm by using a fitness-based weighted mean in the teaching phase and a refined learner updating process [26]. A hybrid TLBO is presented by Xie et al. to deal with the permutation flow shop scheduling problem [27]. An autonomous TLBO is presented by Ge et al. to deal with the global optimization problems on the continuous space [28]. Except for modifying the learning way of TLBO, researchers also improved TLBO by combining it with other evolutionary algorithms. Artificial bee colony algorithm (ABC) is combined with TLBO to solve solar photovoltaic parameter estimation problems [29]. The gravitational search algorithm (GSA) is combined with TLBO to estimate the energy demand of Turkey in Ref [30]. Ghasemi et al. developed a hybrid TLBO variant with double differential evolution (DDE) to handle the optimal reactive

power dispatch problem [31]. Lim et al. presented a bidirectional teaching and peer-learning PSO (BTPLPSO) to improve the search accuracy and efficiency of PSO [32].

As a nature-inspired algorithm, many beneficial strategies in teaching process are not included in standard TLBO. In the real teaching process, especially in middle and primary school, teachers play a dominant role in class, so the improvement of teaching ability is significant for teachers. The teaching ability can be improved by preparing lessons collectively. During this process, teachers can learn from each other and get better preparations for their class. Also in the real teaching process, teachers tend to group their class in S-shape according to the student's performance. Therefore, there are top students in each group, and the performance of each group is similar. Then in the learner phase, learners learn knowledge from the top students of each group which can increase the diversity of the population, and the top students learn from the teacher or the randomly selected student in his group. Also the selection strategy in the genetic algorithm is considered to imitate keeping failing students from advancing in class, which can be viewed as an elitist strategy. We learned from the above teaching strategies and improved TLBO algorithm by increasing a preparing lesson collectively phase and a Deterministic Sampling selection phase and taking an S-shape group learning method in the learning phase. All these modifications will benefit the algorithm in convergence speed and accuracy.

III. IMPROVED TEACHING-LEARNING BASED OPTIMIZATION ALGORITHM

TLBO algorithm seeks the optimal solution of the problem by simulating the process of ''teaching'' and ''learning'' in class, which typically includes two phases: the teaching phase and the learning phase. Each learner improves his or her score by teaching or learning from other learners. The best learner is considered as a teacher. The goal of the teacher's teaching process is to improve the average grade of the class, and the goal of the learner's learning process is to improve the grade of the individual learner, to promote the performance of the whole class. In this part, the improvements are advanced to enhance the searching ability of the algorithm.

A. COLLECTIVE LESSON PREPARATION PHASE
In the typical TLBO algorithm, the role of teachers is weak and the study of teachers is ignored. A capable and experienced teacher can improve the performance of the class more quickly. It has already been proved by facts that a great teacher produces a brilliant student. Teachers can enhance their teaching skills by taking part in training or preparing lessons collectively. In training, teachers can learn from more talent experts. In collective lesson preparation, teachers can learn from each other to make up their deficiencies. Because the experts cannot be predicted in the algorithm, the collective lesson preparation process is introduced into the algorithm to enhance the teacher's performance.

First, a teaching group should be generated around the current teacher, who is the best individual of the current iteration. The jth dimension of the ith teacher in the teaching group $(X_{G,i,j})$ is generated within a circle of radius r_j whose center is the jth element of the current teacher (X_j).

$$X_{G,i,j} = X_j + 2(\gamma - 0.5) X_j r_j \qquad (1)$$

where γ is a random number in the range [0,1].

During the collective lesson preparation phase, if other teachers' performance is better than the current teacher, the current teacher should adopt other teachers' teaching methods according to Eq(2).

$$X = X_{G,i} \; if \; f(X_{G,i}) < f(X) \qquad (2)$$

Other teachers in the teaching group learn from the best teacher and explore new teaching methods according to Eq. (3).

$$X_{G,i} = X + R_D \odot (X_i - X_{G,i}) \qquad (3)$$

where R_D is D dimension random vector in the range [0,1], $\odot$ is Schur-Hadamard product of two vectors.

The performance of all the teachers in the teaching group will be sorted, the one with the best fitness will be signed to the current teacher X.

B. TEACHING PHASE
In the teaching phase, the teacher imparts knowledge to learners to enhance the average performance of the class. For an n-dimensional optimization problem $f(X)$, assume the position of the ith learner is $Y_i = \{y_{i,1}, y_{i,2},..., y_{i,n}\}$, the mean position of the current class is noted as Y_{Mean}. Then we have:

$$Y_{new,i} = Y_{old,i} + R_D \odot (X - T_F Y_{Mean}) \qquad (4)$$

where $Y_{new,i}$ and $Y_{old,i}$ are the new and old position of ith learner. T_F is the teaching factor, its value is heuristically set to either 1 or 2 by the following equation:

$$T_F = round[1 + \gamma] \qquad (5)$$

The greedy selection is carried out. That is if $f(Y_{new,i}) < f(Y_{old,i})$, $Y_{old,i} = Y_{new,i}$. $Y_{old,i}$ will be appeared in the following learning phase.

It can be seen that we adopt the typical teaching phase unchanged.

C. LEARNING PHASE
During the learning phase, learners increase their knowledge by interacting among themselves. In the typical learning phase, the ith learner learns new knowledge from the jth learner, who is randomly selected from the class. This phenomenon is expressed as below.

$$Y_{new,i}$$
$$= \begin{cases} Y_{old,i} + R_D \odot (Y_{old,i} - Y_{old,j}) & if \; f(Y_{old,i}) < f(Y_{old,j}) \\ Y_{old,i} + R_D \odot (Y_{old,j} - Y_{old,i}) & else \end{cases}$$
$$(6)$$

The randomly selected individual may not be better than the current one. So the searching is aimless in a way. In the real class, teachers tend to divide the students into groups and the S-shape group is preferred. There are top students in each group in the S-shape group, and the students in one group learn from the best student of their group. So the searching is towards a better position. Also the different group has a different target, which can increase the diversity of the population.

$$Y_{G,new,i} = Y_{G,old,i} + R_D \odot (Y_{G,best} - Y_{G,old,i}) \quad (7)$$

where the subscript G represents the Gth group in the population.

For the best student in the Gth group, he learns from the teacher with the probability P_G, otherwise he will learn from the randomly selected students in his group.

$$Y_{G,best,new}$$
$$= \begin{cases} Y_{G,best} + R_D \odot (X - Y_{G,best}), & if\ \gamma < P_G \\ Y_{G,best} - R_D \odot (Y_{G,old,j} - Y_{G,best}) & else \end{cases} \quad (8)$$

D. DETERMINISTIC SAMPLING SELECTION PHASE
As the selection operation in the genetic algorithm, the Deterministic Sampling selection phase is introduced into the TLBO algorithm. The deterministic selection shows the ability to reach better fitness with lower computational time and a smaller number of generations.

Take N_a as the number of students advancing in class. N_p is the population size. The expected number of student i advancing in class is

$$N_i = N_p J(Y_i) / \sum_{j=1}^{N_p} J(Y_j) \quad (9)$$

where $J(Y)$ is the adaptive function of the maximum problem.

For minimum problem min $f(Y)$, $J(Y)$ can be set as -$f(Y)$ or the reciprocal of $f(Y)$.

The number of students advancing in class is $\sum_{j=1}^{N_p} \lfloor N_i \rfloor$, then the number of students that fail to advance in class is

$$N_F = N_p - \sum_{j=1}^{N_p} \lfloor N_i \rfloor \quad (10)$$

where $\lfloor N_i \rfloor$ is the integer part of N_i.

So N_F students need to be added to the class to keep the population size unchanged. The N_F students are selected according to the sorting of the fractional part of N_i. The N_F students with bigger fractional parts are duplicated and add to the class.

E. PSEUDO-CODE OF ITLBO
From the above description, the improved TLBO (ITLBO) including four phases. Compared with TLBO, ITLBO has four additional control parameters: 1) The number of teachers in a teaching group. 2) The number of learning groups that

a class divided into. 3) The radius r_j of the teaching group generated. 4) The probability (P_G) of the top student in each learning group learns from the teacher. They all can be set constant. So the pseudo-code of ITLBO can be summarized as follows.

IV. VERIFICATION OF ITLBO
Eighteen benchmark functions listed in Table 1 are selected to verify the performance of ITLBO. In Table 1, f_1-f_5 are multimodal functions, f_6-f_{10} are unimodal functions, f_{11}-f_{18} are rotated models. For fairness, we compare ITLBO with TLBO [17], ETLBO [20] and CLP-TLBO (TLBO with the Collective lesson preparation phase). The "Range" in Table 1 is the lower and upper bounds of the variables searching space. "f_{min}" is the theoretical global minimum solution. All algorithms were coded in Matlab (Matlab 9.0) and all executions were made on a computer of E5-1620 v3 CPU, 16GB RAM.

50 times independent experiments were carried out on these test functions. Each test function is set to 30-dimension. The mean results of the experiments were used to reduce the statistical errors. For all the algorithms, we set the number of population to 50. The number of ETLBO's elitist is set to 4. In ITLBO, the number of teachers in a teaching group is set to 9, the number of learning group in a class is set to 5, and r_i is 0.2, P_G is 0.8.

A. COMPARISON OF CONVERGENCE SPEED
The stop criterion is set to FEs=10000 to evaluate the convergence speed of the algorithm. To save space, we only give the changes of mean values of function f_3, f_4, f_8, f_{10} and f_{12} versus the number of function evaluation times (FEs). The results are shown in Figures 1 to Figure 5.
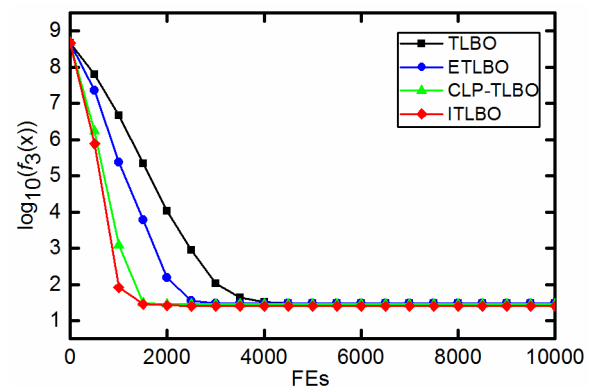

FIGURE 1. Convergence performance of different algorithm on 30-dimensional function f_3 (Rosenbrock).

From Figure 1 to Figure 5, we can find that ITLBO has faster convergence speed than other algorithms except for the Rotated Zakharov function, on which it is slightly slower than CLP-TLBO, but its accuracy is better than CLP-TLBO. Its convergence speed is the best in the remainder unshown functions. The results show that ITLBO performs

TABLE 1. Eighteen benchmark functions.

Function	Formula	Range	f_{min}
f_1(Weierstrass)	$f_1(x) = \sum\limits_{i=1}^{D}\left(\sum\limits_{k=0}^{k\max}\left[a^k\cos\left(2\pi b^k\left(x_i+0.5\right)\right)\right]\right) - D\sum\limits_{k=0}^{k\max}\left[a^k\cos\left(2\pi b^k\times 0.5\right)\right]$ $a=0.5 \quad b=3 \quad k\max=20$	[-0.5, 0.5]	0
f_2(Rastrigin)	$f_2(x) = \sum\limits_{i=1}^{D}[x_i^2 - 10\cos(2\pi x_i)+10]$	[-5.12, 5.12]	0
f_3(Rosen-brock)	$f_3(x) = \sum\limits_{i=1}^{D-1}100(x_i^2 - x_{i+1})^2 + (1-x_i)^2$	[-30, 30]	0
f_4(Griewank)	$f_4(x) = \sum\limits_{i=1}^{D}\dfrac{x_i^2}{4000} - \prod\limits_{i=1}^{D}\cos\dfrac{x_i}{\sqrt{i}}+1$	[-600, 600]	0
f_5(Ackley)	$f_5(x) = 20 - 20\exp\left(-\dfrac{1}{5}\sqrt{\dfrac{1}{D}\sum\limits_{i=1}^{D}x_i^2}\right) - \exp\left(\dfrac{1}{5}\sum\limits_{i=1}^{D}\cos\left(2\pi x_i\right)\right)+e$	[-32.768, 32.768]	0
f_6(Sum Square)	$f_6(x) = \sum\limits_{i=1}^{D}ix_i^2$	[-100, 100]	0
f_7(Quadric)	$f_7(x) = \sum\limits_{i=1}^{D}\left(\sum\limits_{j=1}^{i}x_j\right)^2$	[-100, 100]	0
f_8(Zakharov)	$f_8(x) = \sum\limits_{i=1}^{D}x_i^2 + \left(\sum\limits_{i=1}^{D}0.5ix_i\right)^2 + \left(\sum\limits_{i=1}^{D}0.5ix_i\right)^4$	[-10, 10]	0
f_9(Schwefel's p2.22)	$f_9(x) = \sum\limits_{i=1}^{D}|x_i| - \prod\limits_{i=1}^{D}|x_i|$	[-10, 10]	0
f_{10}(Sphere)	$f_{10}(x) = \sum\limits_{i=1}^{D}x_i^2$	[-100, 100]	0
f_{11}(Rotated Quadric)	$f_{11}(x) = \sum\limits_{i=1}^{D}\left(\sum\limits_{j=1}^{i}x_j\right)^2 \quad y_i = M\times x_i$	[-100, 100]	0
f_{12}(Rotated Quadric)	$f_{12}(x) = \sum\limits_{i=1}^{D}x_i^2 + \left(\sum\limits_{i=1}^{D}0.5ix_i\right)^2 + \left(\sum\limits_{i=1}^{D}0.5ix_i\right)^4 \quad y_i = M\times x_i$	[-10, 10]	0
f_{13}(Rotated Schwefel's p2.22)	$f_{13}(x) = \sum\limits_{i=1}^{D}|x_i| - \prod\limits_{i=1}^{D}|x_i| \quad y_i = M\times x_i$	[-10, 10]	0
f_{14}(Rotated Rosen-brock)	$f_{14}(x) = \sum\limits_{i=1}^{D-1}100(x_i^2 - x_{i+1})^2 + (1-x_i)^2 \quad y_i = M\times x_i$	[-2.048, 2.048]	0
f_{15}(Rotated Ackley)	$f_{15}(x) = 20 - 20\exp\left(-\dfrac{1}{5}\sqrt{\dfrac{1}{D}\sum\limits_{i=1}^{D}x_i^2}\right) - \exp\left(\dfrac{1}{5}\sum\limits_{i=1}^{D}\cos\left(2\pi x_i\right)\right)+e \quad y_i = M\times x_i$	[-32.768, 32.768]	0
f_{16}(Rotated Rastrigin)	$f_{16}(x) = \sum\limits_{i=1}^{D}[x_i^2 - 10\cos(2\pi x_i)+10] \quad y_i = M\times x_i$	[-5.12, 5.12]	0
f_{17}(Rotated Weierstrass)	$f_{17}(x) = \sum\limits_{i=1}^{D}\left(\sum\limits_{k=0}^{k\max}\left[a^k\cos\left(2\pi b^k\left(x_i+0.5\right)\right)\right]\right) - D\sum\limits_{k=0}^{k\max}\left[a^k\cos\left(2\pi b^k\times 0.5\right)\right]$ $a=0.5 \quad b=3 \quad k\max=20 \quad y_i = M\times x_i$	[-10, 10]	0
f_{18}(Rotated Griewank)	$f_{18}(x) = \sum\limits_{i=1}^{D}\dfrac{x_i^2}{4000} - \prod\limits_{i=1}^{D}\cos\dfrac{x_i}{\sqrt{i}}+1 \quad y_i = M\times x_i$	[-600, 600]	0

well in unimodal and multimodal functions optimization problems.

B. COMPARISON OF CONVERGENCE ACCURACY

To show the convergence accuracy, we set the maximum number of fitness evaluations (FEs) to 20000. The optimum results got by the above algorithms are shown in Table 2. From Table 2, we can see that ITLBO got minimum values on 15 problems and CLP-TLBO got minimum value on 12 problems. The optimum values got by ITLBO and CLP-TLBO are much smaller than that of TLBO and ETLBO on functions f_6, f_7, f_9, f_{10}, f_{12}, f_{13}, which shows that the

increasing of CLP phase enhances the convergence accuracy greatly. The process of group learning and the deterministic sampling selection phase can enhance the accuracy of the TLBO slightly.

V. TURBOFAN ENGINE START MODEL ADAPTATION

In this paper, a small twin spool mixed exhaust turbofan engine was taken as an example to illustrate the start model performance adaption process using ITLBO. To enhance the optimization accuracy, the starting process was divided into several periods according to the corrected rotor speed. For each period, we optimize a series of scaling factors and

TABLE 2. The mean best values and standard deviations of the functions.

| Function	TLBO		ETLBO		CLP-TLBO		ITLBO	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
f_1	3.17E+01	4.92E+00	3.00E+01	0.00E+00	**3.00E+01**	4.97E-03	**3.00E+01**	7.05E-03
f_2	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
f_3	2.99E+01	4.09E-02	2.99E+01	3.75E-02	**2.80E+01**	6.89E+00	**2.80E+01**	6.88E+00
f_4	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
f_5	4.58E-15	7.03E-16	1.31E-15	1.17E-15	**8.88E-16**	0.00E+00	**8.88E-16**	0.00E+00
f_6	4.81E-33	8.58E-33	8.27E-67	3.09E-66	**3.74E-167**	0.00E+00	8.45E-152	5.97E-151
f_7	1.45E-33	1.74E-33	1.48E-62	1.03E-61	1.64E-65	8.79E-65	**3.47E-71**	2.35E-70
f_8	8.87E-36	1.34E-35	2.52E-66	1.44E-65	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
f_9	1.35E-17	1.01E-17	2.11E-34	1.12E-33	6.42E-323	0.00E+00	**5.43E-323**	0.00E+00
f_{10}	2.13E-34	4.57E-34	2.28E-65	1.48E-64	1.87E-158	1.32E-157	**8.59E-165**	0.00E+00
f_{11}	3.23E-33	9.70E-33	6.36E-66	3.68E-65	6.02E-66	4.25E-65	**4.02E-74**	2.07E-73
f_{12}	5.01E-36	1.09E-35	8.99E-68	5.90E-67	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
f_{13}	6.60E-18	6.41E-18	5.85E-35	2.21E-34	3.95E-323	0.00E+00	**2.96E-323**	0.00E+00
f_{14}	1.56E+01	2.65E-02	1.57E+01	2.37E-02	**9.67E+00**	7.15E+00	1.17E+01	6.41E+00
f_{15}	**1.06E+00**	6.73E-16	**1.06E+00**	1.53E-15	**1.06E+00**	6.73E-16	**1.06E+00**	6.73E-16
f_{16}	3.55E-16	2.51E-15	**0.00E+00**	0.00E+00	1.07E-15	5.27E-15	5.33E-16	3.77E-15
f_{17}	3.51E+01	2.44E-01	3.53E+01	2.40E-01	**3.42E+01**	3.66E-06	**3.42E+01**	5.95E-06
f_{18}	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00	**0.00E+00**	0.00E+00
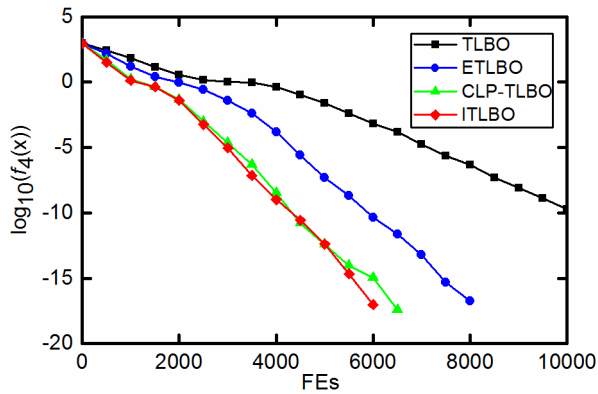
FIGURE 2. Convergence performance of different algorithm on 30-dimensional function f_4 (Griewank).
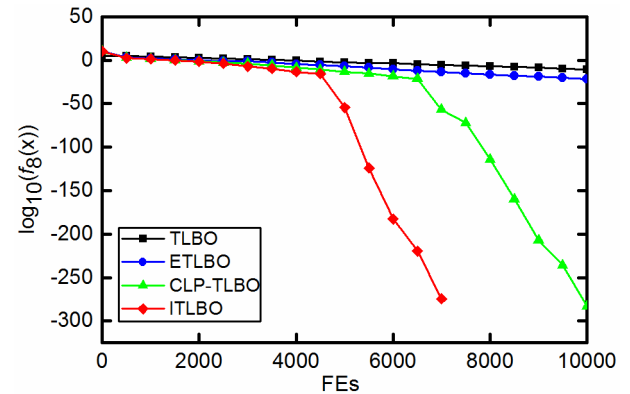

FIGURE 3. Convergence performance of different algorithm on 30-dimensional function f_8 (Zakharov).

correct the corresponding rotor speed line. The key measured parameters were weighted to evaluate the scaling factors. The layout of the engine is shown in Figure 6. There are five rotating components in the engine. The high-pressure compressor (HPC) and the high-pressure turbine (HPT) are on the high-pressure shaft. The fan, the low-pressure compressor (LPC) and the low-pressure turbine (HPT) are on the low-pressure shaft.

The performance maps of the five rotating components will be modified to adapt the model outputs to the test data. For each component, there are three coefficients about pressure ratio, flow rate and efficiency that need to be optimized. They are listed in Table 3.

In Table 3, subscripts π, w and η respectively indicate the pressure ratio, corrected flow rate and efficiency of components. Subscripts F, LC, HC, HT, LT denote Fan, LPC, HPC, HPT and LPT.

TABLE 3. Coefficients of components to be optimized.

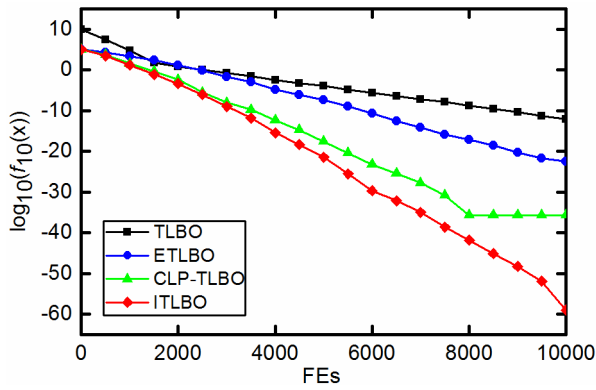| components	Coefficients		
Fan	$x_{F\pi}$	x_{Fw}	$x_{F\eta}$
LPC	$x_{LC\pi}$	x_{LCw}	$x_{LC\eta}$
HPC	$x_{HC\pi}$	x_{HCw}	$x_{HC\eta}$
HPT	$x_{HT\pi}$	x_{HTw}	$x_{HT\eta}$
LPT	$x_{LT\pi}$	x_{LTw}	$x_{LT\eta}$

FIGURE 4. Convergence performance of different algorithm on 30-dimensional function f_{10} (Sphere).
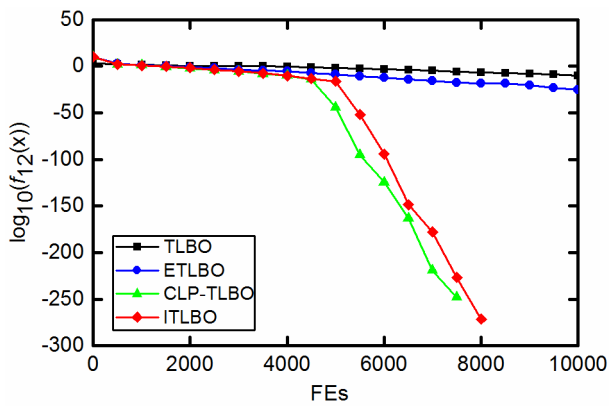

FIGURE 5. Convergence performance of different algorithm on 30-dimensional function f_{12} (Rotated Zakharov).
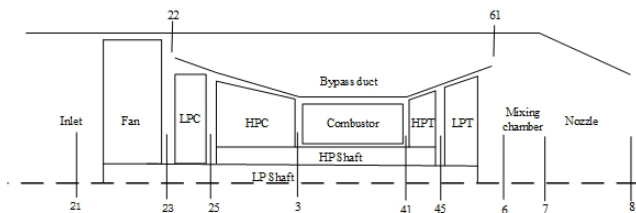

FIGURE 6. The layout of the small twin-spool turbofan engine.

There are 15 parameters to be modified together. Taking the fan map as an example. If the operating point got from the performance map is denoted by π_F, w_F and η_F, the modified operating point is $\pi_F x_{F\pi}$, $w_F x_{Fw}$ and $\eta_F x_{F\eta}$. Once these coefficients are gotten, they are used to modify the component performance maps as shown in Figure 7.

For the complex of engine starting process, the adaption cannot achieve satisfying accuracy by only one set of coefficients. So the starting process is divided into six segments evenly according to the time as shown in Figure 8. Also new speed lines are inserted to the performance maps to separate the acceleration process as shown in Figure 9, where the solid lines n_1, n_2, n_3, n_4 are the existing speed lines and the rest dash lines are the inserted lines.
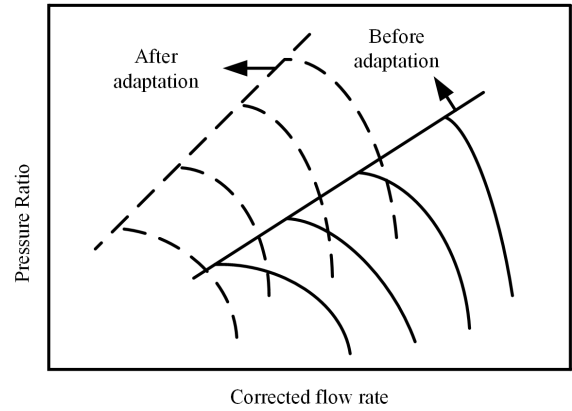

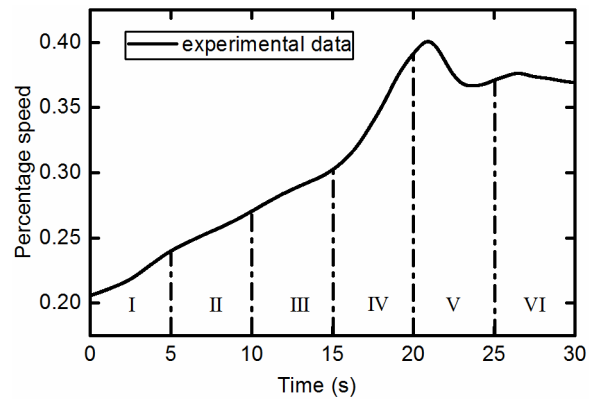**FIGURE 7.** Scaling of the fan performance map.

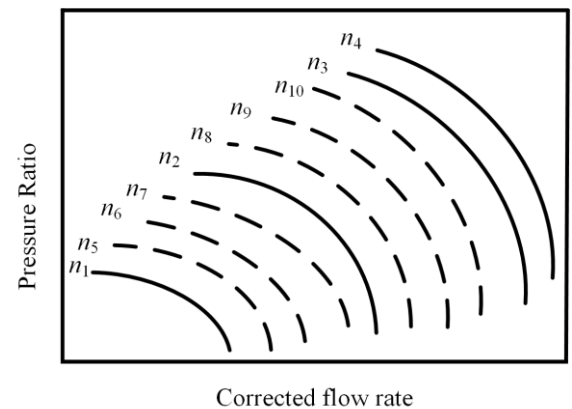**FIGURE 8.** Percentage speed of the low-pressure shaft.

FIGURE 9. Scaling of the fan performance map.

The performance map adaptations are carried out from the lower speed line to the idle speed gradually. Firstly, we modify the speed lines n_1 and n_5 according to the correction coefficients obtained in the segment I, then we keep the modified n_1 and n_5 unchanged, and modify the speed lines n_6 and n_7 according to the correction coefficients obtained in segment II. After that, we keep the four speed lines unchanged and modify the speed lines n_2 and n_8 according to

the correction coefficients obtained in segment III, and so on. The relative errors between the outputs of the model and the start test data of the engine are taken as the objective function to evaluate the coefficients.

$$O = \sum_{j=1}^{q} (\sum_{i=1}^{m} a_i \left| \frac{X_{iT} - X_{iS}}{X_{iT}} \right|)_j \qquad (11)$$

where q is the number of data points participating in the evaluation, m is the number of output variables participating in the evaluation, X_{iT} is the ith test data, X_{iS} is the ith model output data, a_i is a weighting factor.

In order to limit the maximum relative error, a penalty factor is introduced to the objective function.

$$\text{if} \left(\left(\left| \frac{N_{1T} - N_{1S}}{N_{1T}} \right| > 10\% \right) \right\| \left(\left| \frac{N_{2T} - N_{2S}}{N_{2T}} \right| > 10\% \right) \right\|$$
$$\times \left(\left| \frac{P_{3T} - P_{3S}}{P_{3T}} \right| > 10\% \right) \right)$$
$$\Downarrow$$
$$f(x) = 1.2O \qquad (12)$$

where N_1, N_2 and P_3 represent low-pressure rotor speed, high-pressure rotor speed and total pressure at the outlet of high-pressure compressor respectively. Subscripts T and S represent test data and model output data respectively.

When the relative error exceeds 10%, the fitness of the individual is multiplied by 1.2 to reduce the probability of the individual to be selected.

Because there are fifteen coefficients to be optimized, $f(x)$ is a fifteen-dimensional function. All fifteen coefficients are ranged from 0.95 to 1.05 to ensure that the modified lines are not changed much. The rest of the optimization control parameters are the same as those in section 4.

The changes of fitness function of model performance adaption in the segment I of Figure 8 are shown in Figure 10 as an example, and the proposed ITLBO method also shows advantages in convergence speed and accuracy, which means the model modified by ITLBO is more accurate.
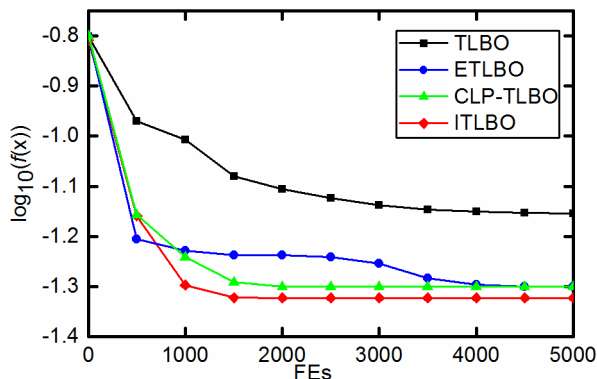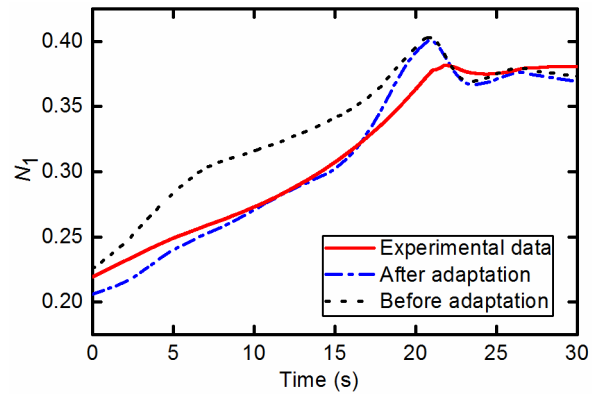
After finishing all the speed line modification by using ITLBO, a new set of component maps for rotating components are got, and the start model performance adaption is achieved. The comparison of start model output N_1 to the start test data before and after adaptation is given in Figure 11. Figure 12-14 show the relative error of the start model outputs to the start test data before and after adaptation. It can be seen that the output of the start model after adaption is more
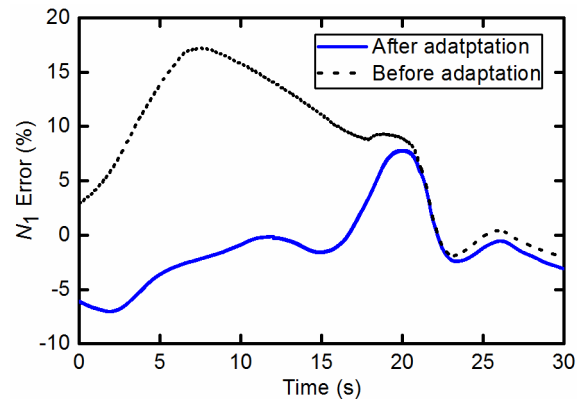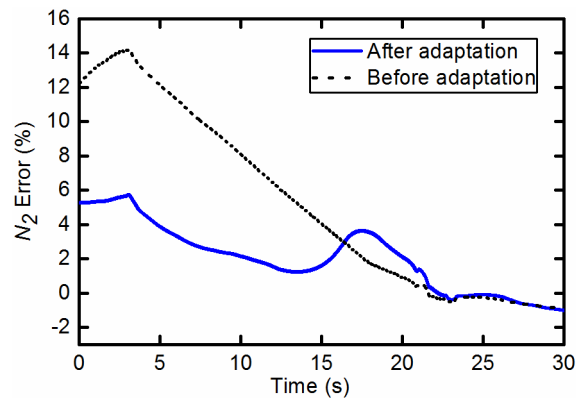

FIGURE 11. Adaptation results of N_1.

FIGURE 12. Adaptation results of N_1 error.

FIGURE 10. Convergence performance of different algorithm on engine start model adaptation.
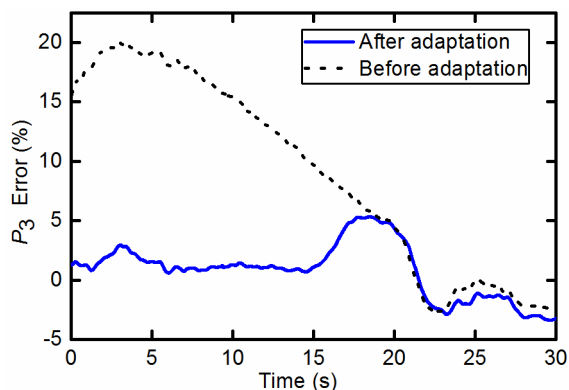

FIGURE 13. Adaptation results of N_2 error.

FIGURE 14. Adaptation results of P_3 error.

consistent with the test data and the maximum relative error of each output reduces significantly. The maximum relative error of N_1 decreases from 17.2% to 7.8%. The maximum relative error of N_2 decreases from 14.2% to 5.75%. The maximum relative error of P_3 decreases from 20% to 5.34%. So the capability of ITLBO to adapt the performance of start model performance maps is validated.

VI. CONCLUSION

An improved TLBO algorithm is proposed to realize the performance adaption of aero-engine start model. Inspired by the teaching process of daily life, a collective lesson preparation phase is introduced to the TLBO algorithm, and the dominant role of the teacher in class is manifested. Inspired by the practical teaching environment, the random learning phase is replaced by an S-shape group learning phase, and a deterministic sampling selection phase was introduced to the TLBO algorithm. The innovations mentioned above effectively improve the convergence speed and accuracy of the TLBO algorithm on Benchmark functions tests, and benefit the start model adaption by shorting the optimization time and providing more accurate component maps.

The start process is segmented to several periods to enhance the adaption accuracy. The speed lines relative to each period are optimized separately and the start model performance adaption is achieved from lower speed to idle speed gradually. The maximum relative error of N_1 decreases from 17.2% to 7.8%. The maximum relative error of N_2 decreases from 14.2% to 5.75%. The maximum relative error of P_3 decreases from 20% to 5.34%. The proposed ITLBO can decrease the modeling error effectively.

ACKNOWLEDGMENT

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