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Generalized Belief Entropy and Its Application in Identifying Conflict Evidence

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ABSTRACT Dempster-Shafer evidence theory has wide applications in many fields. Recently, A new entropy called Deng entropy was proposed in evidence theory. Some scholars have pointed out that Deng Entropy does not satisfy the additivity in uncertain measurements. However, irreducibility may have a huge effect. The derived entropy from complex systems is often irreducible. Inspired by this, generalized belief entropy is proposed. The belief entropy implies the relationship between Deng entropy, Rényi entropy, Tsallis entropy. In addition, numerical examples demonstrate the flexibility of the proposed Rényi-Deng (R-D) entropy to measure the uncertainty of basic probability assignment (BPA). Finally, a method for identifying contradictory evidence based on Rényi-Deng (R-D) entropy is proposed. The experiment show the effectiveness of the proposed method.

INDEX TERMS Belief entropy, Deng entropy, Rényi entropy, Tsallis entropy, uncertaintity, Dempster Shafer evidence theory, conflict evidence.

I. INTRODUCTION

Dempster-Shafer theory of evidence [1], [2] was proposed by Dempster [1] and developed by Shafer *et al.* [2]. Evidence theory as a framework of uncertain reasoning is closely related to probability theory. It can be considered as a generalization of probability, assigning belief to power set of the propositions rather than single elements. This theory allows for the combination of evidence from different sources and draws a certain degree of conclusion, taking into account all available evidence. There are a lot of applications based on Dempster-Shafer theory and fuzzy sets theory, such as uncertainty [3]–[12], data fusion [13], [13]–[20], decision [16], [20]–[28].

How to measure uncertainty has always received widespread attention in evidence theory. Most of the measurements on uncertainty are related to Shannon entropy [29]. Yager [30] generalizes the entropy in probability theory to evidence theory. This entropy is based on the belief structure, which provides an indicator of the quality of the evidence. Maeda and Ichihashi [31] propose an uncertain measurement method. This uncertainty consists of two types, one containing the uncertainty of Shannon entropy determination and the other related to the cardinality of the set.

Recently, a new entropy named Deng entropy [32] has been proposed to solve uncertain measurement. This entropy is directly related to basic probability assignment. When the belief is assigned to the element of frame of discernment instead of the power set, this entropy is degenerated into the Shannon entropy. Deng entropy quickly attracts the attention of many scholars. Abellán [33] discusses the property of Deng entropy and points out that this entropy could quantify two types of uncertainty in evidence theory. There are other discussions and applications about Deng entropy such as decision, Uncertainty [34], [35], data fusion.

Entropy is diverse [36]–[39]. After Clausius [40] proposed the concept of entropy, various entropies were raised. Rényi *et al.* [41] proposed an entropy called Rényi entropy. Rényi entropy [41] has many applications in quantum information [42], information theory [43], and fractal theory [44]. Tsallis entropy as an extension of Boltzmann entropy is a non-extensive entropy [45]. Tsallis entropy has been controversial since it was proposed [46]. After many complex systems are derived from Tsallis entropy [47], Tsallis entropy has received a lot of attention.

It can be proved that Shannon entropy [29] is a special case of Tsallis entropy [45], Rényi entropy [41].

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So a natural question is what is the relationship between Deng entropy [32] and Rényi entropy [41] and Tsallis entropy [45] ? Therefore, in order to explore the relationship between Deng entropy [32] and these two entropies. In this paper, we propose generalized Deng entropies, which reveal the relationship with these entropies. With the discussion of the generalized Deng entropy, Rényi-Deng (R-D) entropy demonstrates the potential for uncertain measurements. Finally, a method based on Rényi-Deng (R-D) entropy to identify contradictory evidence is proposed.

The structure of this article is shown as follows. Section 2 introduces some basic knowledge. Section 3 proposes the generalized Deng entropy. In section 4, some examples are discussed. Section 5 shows how to identify contradictory evidence. Finally, conclusion is given.

II. PRELIMINARIES

In this section, Dempster shafer evidence theory [1], Deng entropy [32], Rényi entropy [41], Tsallis entropy [45] will be briefly introduced.

A. DEMPSTER SHAFER EVIDENCE THEORY

Compared to probability theory, Dempster shafer evidence theory [1], [2] has a greater advantage to deal with uncertainty [48]–[53]. First, Dempster shafer evidence theory [1], [2] can deal with more uncertainty in the real world. In Dempster shafer evidence theory [1], [2], belief is not only assigned to a single element but also to a multi-element set [54]. In addition, it does not require prior information before combining each individual evidence [55]. Some basic knowledge about evidence theory is introduced.

Suppose the power set of the frame of discernment $X = \{\theta_1, \theta_2, \ldots, \theta_N\}$ is $P(X)$. Where the elements of X are mutually exclusive and exhaustive. For a frame of discernment X, the mass function is defined as follows [2].

$$m : P(X) \mapsto [0, 1] \tag{1}$$

where $m(\phi) = 0$ and $\sum_{A_i \in P(X)} m(A_i) = 1$.

In evidence theory, mass function is also called basic probability assignment (BPA), indicating the degree of belief in $A_i \in P(X)$.

Dempster's combination rule is a simple method to combine different evidences. If there are two mass functions: m_1, m_2, when combine these two functions. The rule is presented as follows:

$$m(A) = \frac{\sum_{B \bigcap C = A \& B, C, A \in P(X)} m_1(B) \cdot m_2(C)}{1 - K} \tag{2}$$

with

$$K = \sum_{B \cap C = \Phi} m_1(B) \cdot m_2(C) \tag{3}$$

where the K is the degree of evidences, the Dempster's rule is only applicable to such two evidences, when $0 < K < 1$.

B. DENG ENTROPY

Deng entropy in evidence theory is defined as follows [32].

$$E_d = -\sum_i m(A_i) \ln \frac{m(A_i)}{2^{|A_i|} - 1} \tag{4}$$

where $A_i \in P(X)$ and $|A_i|$ is the cardinality A_i. the term $\sum_i m(A_i) \ln(2^{|A_i|} - 1)$ can be interpreted as a measure of total nonspecificity in the BPA m, and the term $-\sum_i m(A_i) \ln m(A_i)$ is the measure of discord of the mass function among various focal elements [32].

Note that the base of all log functions is taken as a natural number e.

C. RÉNYI ENTROPY

For a discrete random Y, its probability distribution is $P_Y = \{p_i | i = 1, 2, \ldots, N\}$. Rényi entropy is defined as follows [41].

$$H_\alpha = \frac{1}{1-\alpha} \ln(\sum_i p_i^\alpha) \tag{5}$$

where $\alpha \geq 0 \& \alpha \neq 1$. When $\alpha = 1$, the Rényi entropy is shown as follows.

$$H_\alpha = \sum_i p_i \ln p_i$$

D. TSALLIS ENTROPY

Given a discrete Z, its probability distribution is $P_Z = \{p_i | i = 1, 2, \ldots, N\}$. Tsallis entropy is defined as follows [45].

$$H_q = \frac{k}{q-1}(1 - \sum_i p_i^q) \tag{6}$$

where q and k are parameters. For analysis, k is set to 1, which means that Tsallis entropy can be expressed as follows.

$$H_q = \frac{1}{q-1}(1 - \sum_i p_i^q) \tag{7}$$

where $q \geq 0 \& q \neq 1$. When $q = 1$, the Tsallis entropy is shown as follows.

$$H_q = \sum_i p_i \ln p_i$$

III. GENERALIZED DENG ENTROPY

In evidence theory, Klir and Wierman define five types of uncertainty requirements: probability consistency, set consistency, range, subadditive, additivity [56]. Are all the uncertain measurements satisfying these five requirements? Abellán [33] points out that Deng entropy [32] does not satisfy additivity and sub-additiveness. In fact, Tsallis entropy [45] does not satisfy additivity [57]. Rényi pointed out that if the additivity of Rényi entropy [41] is strictly satisfied, then there are only two possible Kolmogorov-Nagumo functions [57]. For example, in some systems involving long range forces [57], this kind of nonlinear system has come to receive widespread attention [58].

Deng entropy has been proposed as an entropy in the field of information [32], although there is currently no physical explanation. However, this non-additive nature seems to imply a connection to more complex systems.

A. RÉNYI-DENG (R-D) ENTROPY

In order to bridge the relationship between Deng entropy [32] and Rényi entropy [41], a generalized Rényi-Deng (R-D) entropy is proposed as follows.

$$E_\alpha(m(A_i)) = \frac{1}{1-\alpha}\ln\left[\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)\right] \quad (8)$$

Theorem 1: When $\alpha \to 1$, Rényi-Deng (R-D) entropy degenerates into Deng entropy.

Proof:

$$\begin{aligned}\lim_{\alpha\to 1} E_\alpha(m(A_i)) &= \lim_{\alpha\to 1}\frac{\frac{\partial}{\partial\alpha}\left[\ln\left(\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)\right)\right]}{\frac{\partial}{\partial\alpha}(1-\alpha)}\\ &= \frac{\sum_i e^{\alpha\ln\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)}(2^{|A_i|}-1)\ln\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)}{-\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)}\\ &= -\sum_i m(A_i)\ln\frac{m(A_i)}{2^{|A_i|}-1}\end{aligned}$$

□

It can be easily proved that the Rényi-Deng (R-D) entropy degenerates into Rényi entropy when the belief is assigned to single elements. Naturally, when $\alpha \to 1$ and belief is assigned to single elements, the Rényi-Deng (R-D) entropy degenerates into Shannon entropy.

B. TSALLIS-DENG (T-D) ENTROPY

Tsallis-Deng (T-D) entropy is proposed as follows, which may expose the relationship between Deng entropy [32] and Tsallis entropy [45].

$$E_q(m(A_i)) = \frac{1}{q-1}\left[1-\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^q(2^{|A_i|}-1)\right] \quad (9)$$

Theorem 2: When $q \to 1$, Tsallis-Deng (T-D) entropy degenerates into Deng entropy.

Proof:

$$\begin{aligned}\lim_{q\to 1} E_q(m(A_i)) &= \lim_{q\to 1}\frac{\frac{\partial}{\partial q}\left[1-\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^q(2^{|A_i|}-1)\right]}{\frac{\partial}{\partial q}(q-1)}\\ &= -\sum_i e^{q\ln\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)}(2^{|A_i|}-1)\ln\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)\\ &= -\sum_i m(A_i)\ln\frac{m(A_i)}{2^{|A_i|}-1}\end{aligned}$$

□

Similarly, it can be proved that the Tsallis-Deng (T-D) entropy degenerates into Tsallis entropy when the belief is assigned to single elements. Naturally, when $q \to 1$ and belief is assigned to single elements, the Tsallis-Deng (T-D) entropy degenerates into Shannon entropy.

C. MAXIMUM TSALLIS-DENG (T-D) ENTROPY AND RÉNYI-DENG (R-D) ENTROPY

The principle of maximum entropy is also called the principle of maximum information. The maximum entropy distribution exists in nature. In probability theory, given the basic event, the maximum Shannon entropy corresponds to the same probability of all basic events. Then, in the evidence theory, given the power set of the basic event, how to find the corresponding maximum Tsallis-Deng (T-D) and Rényi-Deng (R-D) entropy? What is the BPA in the case of maximum confidence entropy?

Theorem 3: the maximum Rényi-Deng (R-D) entropy in power set if and only if: $m(A_i) = \frac{2^{|A_i|}-1}{\sum_j(2^{|A_j|}-1)}$.

Let

$$q_i^\alpha = \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1) \quad (10)$$

It can be found that

$$q_i = \frac{m(A_i)}{2^{|A_i|}-1}\sqrt[\alpha]{2^{|A_i|}-1} \geq 0 \quad (11)$$

Then, let

$$H_\alpha(q_i) = E_\alpha(m(A_i)) = \frac{1}{1-\alpha}\ln\left[\sum_i(q_i)^\alpha\right] \quad (12)$$

According to [59], the function $H_\alpha(q_i)$ is concavity.

Proof: Suppose

$$E_\alpha(m(A_i)) = \frac{1}{1-\alpha}\ln\left[\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)\right]$$

and

$$\sum_i m(A_i) = 1$$

Let

$$F_\alpha(m(A_i)) = \frac{1}{1-\alpha}\ln\left[\sum_i\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)\right] + \lambda\left(\sum_i m(A_i)-1\right)$$

In order to deduce the maximum Rényi-Deng (R-D) entropy in power set, let

$$\frac{\partial F_\alpha(m(A_i))}{\partial m(A_i)} = \frac{1}{1-\alpha}\frac{\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)}{\sum_i^N\left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^\alpha(2^{|A_i|}-1)} + \lambda$$

That is,

$$\frac{m(A_1)}{2^{|A_1|}-1} = \frac{m(A_2)}{2^{|A_2|}-1} = \cdots = \frac{m(A_N)}{2^{|A_N|}-1}$$

where $N = 2^{|X|}-1$ and λ is a parameter.

Therefore, when $m(A_i) = \frac{2^{|A_i|}-1}{\sum_j (2^{|A_j|}-1)}$, the Rényi-Deng (R-D) entropy has maximum entropy. □

The same proof process has

Theorem 4: the maximum Tsallis-Deng (T-D) entropy in power set if and only if: $m(A_i) = \frac{2^{|A_i|}-1}{\sum_j (2^{|A_j|}-1)}$.

From the process and conclusion of the proof, it can be found that the maximum Rényi-Deng (R-D) entropy and the maximum Tsallis-Deng (T-D) are not related to the probability of each basic event but to the size of the power subset. More importantly, the maximum entropy is not affected by the parameters α and q.

D. RÉNYI-TSALLIS-DENG (R-T-D)

It should be known that Deng entropy [32], Rényi entropy [41], Tsallis entropy [45] are three different entropies. Rényi entropy and Tsallis entropy respectively generalized to quasi-linear means and non-extensive systems [57], while Deng entropy comes from evidence theory based on power set. However, an entropy can be found so that it can degenerate to non-linear means, non-extensive sets, power set, respectively.

Masi [57] proposes a unified entropy that links Rényi entropy [41] and Tsallis entropy [45]. Inspired by him, a unified form of entropy is proposed which could link Rényi entropy [41], Tsallis entropy [45] and Deng entropy [32].

$$E_{t,r}(m(A_i)) = \frac{1}{1-r}\left[\left[\sum_i \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^t (2^{|A_i|}-1)\right]^{\frac{1-r}{1-t}} - 1\right] \tag{13}$$

It can be proved that when r tends to t, the Rényi-Tsallis-Deng (R-T-D) degenerates into Tsallis-Deng (T-D) entropy. when r tends to 1, the Rényi-Tsallis-Deng (R-T-D) entropy degenerates into Rényi-Deng (R-D) entropy *Eq*. 8.

Theorem 5: When $r \rightarrow 1$, *Tsallis-Deng (T-D) entropy degenerates into Rényi-Deng (R-D) entropy.*

Proof:

$$\begin{aligned}
&\lim_{r\to 1} E_{t,r}(m(A_i)) \\
&= \lim_{r\to 1} \frac{\frac{\partial}{\partial r}\left[\left[\sum_i \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^t (2^{|A_i|}-1)\right]^{\frac{1-r}{1-t}} - 1\right]}{\frac{\partial}{\partial r}(1-r)} \\
&= \lim_{r\to 1} \frac{1}{1-t}\left[\sum_i \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^t (2^{|A_i|}-1)\right]^{\frac{1-r}{1-t}} \\
&\quad \cdot \ln\left[\sum_i \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^t (2^{|A_i|}-1)\right] \\
&= \frac{1}{1-t} \ln\left[\sum_i \left(\frac{m(A_i)}{2^{|A_i|}-1}\right)^t (2^{|A_i|}-1)\right]
\end{aligned}$$

□

It can be proved that when r tends to t, the R-T-D entropy degenerates into Tsallis-Deng (T-D) entropy.

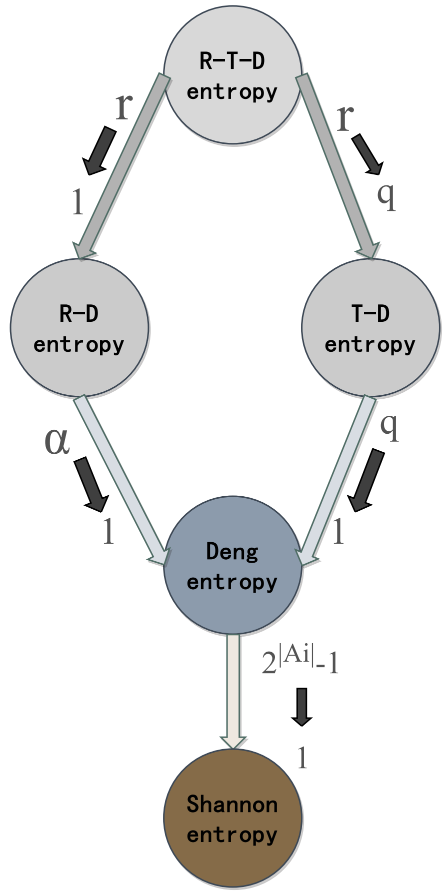


FIGURE 1. The relationship between entropies.

To summarize, the relationship between all entropy is shown in *Figure* 1.

IV. NUMERICAL EXAMPLES

A. EXAMPLE 1

How to measure uncertainty has always been an important issue [60], [61]. Since Deng Entropy was proposed, it has been considered as an effective means of measuring BPA uncertainty [32]. In order to compare the generalized Deng entropy with Deng entropy, some special cases are presented in this subsection.

Iris data set [62] is used to compare the uncertainty of various entropy measurements. Iris has a total of 150 data in three categories: Setosa (a), Versicolour (b), and Virginica (c). There are four attributes for each data: Sepal Length (SL), Sepal Width (SW), Petal Length (PL), and Petal Width (PW). There are many ways to generate BPA [63]. A method based on membership graph is used to generate BPA in this paper. The 60% data is randomly selected to construct a membership graph to generate BPA. The membership graph is shown in Figure 2.

Starting from a value of 0, the step size is 0.1 up to a value of 10. A total of 101 BPAs are generated, all of which are shown in Figure 3.

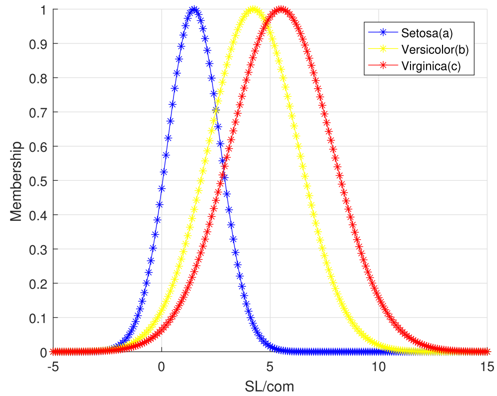


FIGURE 2. Membership graph.

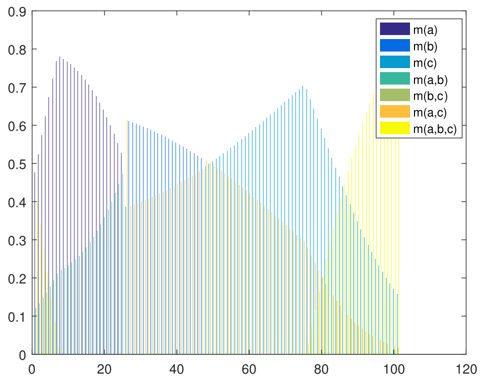


FIGURE 3. Distribution of different BPA.

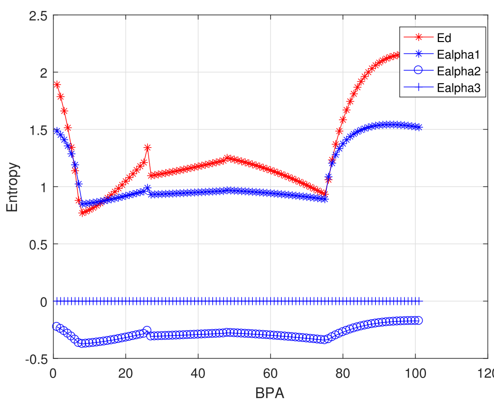
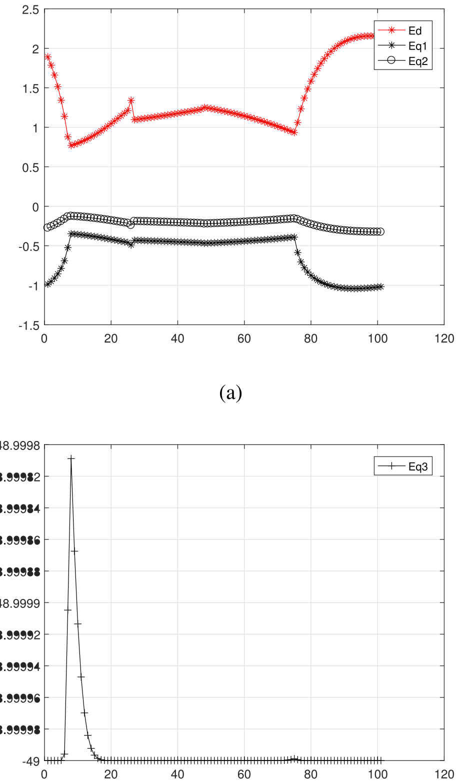


FIGURE 4. Rényi-Deng (R-D) entropy and Deng entropy.



(a)

(b)

FIGURE 5. Tsallis-Deng (T-D) entropy and Deng entropy.

First, in order to compare the effects of Rényi-Deng (R-D) entropy and Deng entropy to measure uncertainty of BPA. The parameter α is set to three cases. $0 < \alpha < 1$, $1 < \alpha$, $a >> 1$. The specific setting is $\alpha_1 = 0.5$, $\alpha_2 = 1.5$, $\alpha_3 = 50$.

In Figure 4, when $\alpha_1 = 0.5$, the Rényi-Deng (R-D) entropy is greater than 0, the change trend is the same as Deng entropy and the change is relatively flat. When $\alpha_2 = 1.5$, all Rényi-Deng (R-D) entropy are less than 0 and the trend of change is consistent with Deng entropy. When $\alpha_3 = 50$, all Rényi-Deng (R-D) entropy basically tends to zero.

The second is to compare the uncertainty of Tsallis-Deng (T-D) entropy and Deng entropy to measure BPA. Corresponding to the three cases of q, respectively, $0 < q < 1$, $q > 1$, $q >> 1$. Here q is set to $q_1 = 0.5$, $q_2 = 1.5$, $q_3 = 50$.

In Figure 5, the Tsallis-Deng (T-D) entropy of all cases is less than 0. When $q = 50$, the Tsallis-Deng (T-D) entropy basically approaches -49.

B. EXAMPLE 2

As can be seen from example 1, when $0 < \alpha < 1$, the change trend of R-D entropy is the same as that of Deng Entropy. Moreover, the trend of change is relatively flat. The influence of parameter α on Rényi-Deng (R-D) entropy is further discussed. Given a frame of discernment $X = \{1, 2, \ldots, 15\}$ with 15 elements. An BPA function is shown as follows. $m(\{3, 4, 5\}) = 0.05$, $m(\{6\}) = 0.05$, $m(\{A\}) = 0.8$, $m(\{X\}) = 0.1$. where the elements of set A are gradually increased from 1 to 14 [32].

In Example 2, α is set to $\alpha 1 = 0.1$, $\alpha 2 = 0.5$, $\alpha 3 = 0.9$, respectively. From *Figure* 6, as the size of A increases, the values of Rényi-Deng (R-D) entropy and Deng entropy increase. The value of α is close to 0, and the change of Rényi-Deng (R-D) entropy is more gentle.

The relative uncertainty between the Rényi-Deng (R-D) entropy measurement data has the same effect as the Rényi-Deng (R-D) entropy measurement. It is also adjustable. The Rényi-Deng (R-D) entropy can adjust the parameter α according to the actual situation. If the uncertainty value of

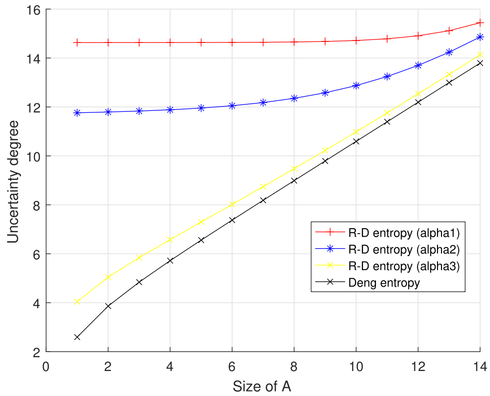


FIGURE 6. Comparison between Deng entropy and Rényi-Deng (R-D) entropy.

the processed data fluctuates greatly, the parameter a can be adjusted to be small.

V. IDENTIFY CONFLICT EVIDENCE

In data fusion, when evidence is highly conflicting [3], [7]–[9], [18], [19], [64], it often leads to counter-intuitive results. So some scholars have proposed some methods to solve this problem [65]–[75], which are mainly divided into two aspects: modifying Dempster's rule of combination and evidence [16], [17], [23], [25], [26], [76], [77].

However, there are few ways to identify contradictory evidence. Based on the graph model, a method for identifying contradictory evidence has been proposed.

A graph model was proposed to better handle basic probability assignment(BPA). This model not only exploits the potential relationship between BPAs, but also does not lose any valid information.

A. IDENTIFY MODEL

The graph is a mathematical structure that can be used to study the relationships between objects. In this section, a mapping is created, which links a set of BPAs to a graph.

Definition 1: The mapping f is definded as follow.

$$f : M = \{m_1, m_2, \ldots, m_N\} \mapsto G(V, E) \tag{14}$$

where M is the set of N BPAs. $G(V, E)$ is a graph, V and E are sets of all nodes and edges, respectively.

Note that $V = \{v_1, v_2, \ldots, v_N\}$, where $v_i = m_i$, $(i = 1, \ldots, N)$. and $E = \left\{e_{ij}^k | i, j \in N, k \in M\right\}$, where e_{ij} is the edge of node v_i and node v_j. The value of e_{ij}^k represents the k-th attribute of the relationship between two nodes, such as distance, similarity, et al.

The main idea is to use the theory and techniques of graph to study DST. The core issue is how to find a mapping f so that the graph could reflect some structure between the BPAs. In mathematical statistics, relative entropy is a tool to measure the difference between two probability distributions, but this relative entropy is asymmetrical. Inspired by relative entropy, a relative Rényi-Deng (R-D) entropy in DST is proposed to measure the relationship of BPA.

Definition 2: Given two BPAs: m_1 and m_2, the BPA relative Rényi-Deng (R-D) entropy is defined as follows.

$$D(m_1||m_2) = \frac{1}{1-\alpha} \ln \left[\frac{\sum_i (\frac{m_1(A_i)}{2^{|A_i|}-1})^{\alpha} (2^{|A_i|}-1)}{\sum_i (\frac{m_2(A_i)}{2^{|A_i|}-1})^{\alpha-1} (2^{|A_i|}-1)} \right] \tag{15}$$

It can be proved that when belief is assigned to a single element and α tends to 1, relative R-D entropy degenerates into relative entropy. In order to make the relative Rényi-Deng (R-D) entropy symmetric, Eq. (15) can be expressed as follows.

$$D(m_1||m_2) = \frac{1}{2(1-\alpha)} \ln \left[\frac{\sum_i (\frac{m_1(A_i)}{2^{|A_i|}-1})^{\alpha} (2^{|A_i|}-1)}{\sum_i (\frac{m_2(A_i)}{2^{|A_i|}-1})^{\alpha-1} (2^{|A_i|}-1)} \right] + \frac{1}{2(1-\alpha)} \ln \left[\frac{\sum_i (\frac{m_2(A_i)}{2^{|A_i|}-1})^{\alpha} (2^{|A_i|}-1)}{\sum_i (\frac{m_1(A_i)}{2^{|A_i|}-1})^{\alpha-1} (2^{|A_i|}-1)} \right] \tag{16}$$

The specific method can be seen in Algorithm 1.

Algorithm 1 Identify Conflict Evidence Algorithm

```
Input:
    The set of BPAs, M = {m_1, m_2, ..., m_N};
Output:
    Adjacency matrix of G(V, E), A;
for i = 1 to N do
    for j = 1 to N do
        e_ij = D(m_i||m_j);
    end for
end for
for i = 1 to N do
    for j = 1 to N do
        if 0 < e_ij < λ then
            a_ij = 1;
        else
            a_ij = 0;
        end if
    end for
end for
```

Where a_{ij} is the element of matrix A and λ is a threshold.

In **Algorithm** 1, the output is a Adjacency matrix A that represents the connection between the evidences. When the node i and node j are linked, the evidence m_i and m_j are considered to be non-conflicting, otherwise the conflicting.

In this model, the attribute assigned to the edge e_{ij} is the difference between the evidences (relative entropy of evidence). In relative entropy, if the relative entropy of the two probability distributions is smaller, the difference is considered to be smaller. Therefore, it can be reasonably considered that when e_{ij} is less than the threshold l, the evidences m_1 and m_2 are disconnected (conflicting).

In order to better verify **Algorithm** 1, an example is described.

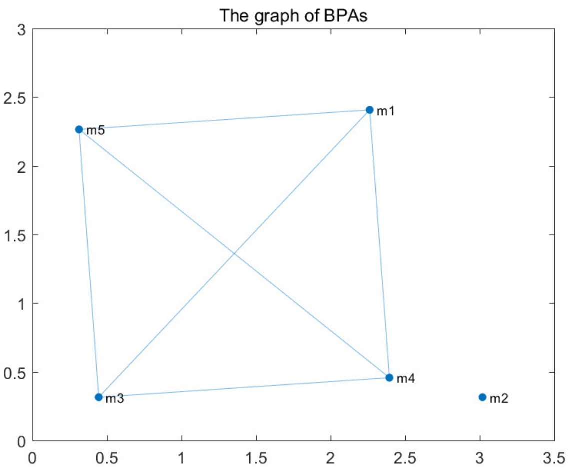


FIGURE 7. Connection of Evidence.

TABLE 1. Comparison with other methods.

Evidence	Method	m(a)	m(b)	m(c)	m(a,c)	Target
m_1, m_2, m_3, m_4, m_5	Dempster'method ([80])	0	0.1422	0.8578	0	C
m_1, m_2, m_3, m_4, m_5	Murphy's method ([81])	0.9620	0.0210	0.0138	0.0032	A
m_1, m_2, m_3, m_4, m_5	Deng *et al.*'s method ([82])	0.9820	0.0039	0.0107	0.0034	A
m_1, m_2, m_3, m_4, m_5	Jang *et al.*'s method ([79])	0.9837	0.0021	0.0110	0.0032	A
m_1, m_3, m_4, m_5	Proposed's method	0.9974	0.0026	0	0	A

Example 1: In a multi-target recognition system, it is known that there are a total of three targets: $X = \{a, b, c\}$*. Assuming that there are a total of five sensors, respectively obtaining five evidences [78]:*

$m_1 : m_1(a) = 0.41, m_1(b) = 0.29, m(c) = 0.30;$
$m_2 : m_2(a) = 0.00, m_2(b) = 0.90, m(c) = 0.10;$
$m_3 : m_3(a) = 0.58, m_2(b) = 0.07, m(a, c) = 0.35;$
$m_4 : m_4(a) = 0.55, m_4(b) = 0.10, m(a, c) = 0.35;$
$m_5 : m_5(a) = 0.60, m_5(b) = 0.10, m(a, c) = 0.30;$

In this example, λ is set $\sum_{i,j} e_{ij}/(N(N-1))$.

According to **Algorithm** 1, its final result is shown in *Figure* 7.

According to the results, it is shown that the evidences: m_1, m_3, m_4, and m_5 are not conflicting, but the evidence m_2 is in conflict with any other evidence. Therefore, there is reason to believe that the evidence m_2 is a ''problem'' evidence. Then the evidence m_2 is removed. After re-using the dempster's rule of combination, the comparison with other methods is shown in *Table* 1.

In *Table* 1, the evidence m_2 is the cause of the failure of the Dempster's rule of combination [79]. After the evidence m_2 is removed, the other evidences are merged one after the other. The judgment target obtained is consistent with the judgment result of the method proposed by Murphy [80], Yong *et al*. [81], and Jiang *et al*. [78]. Therefore, identifying conflicting evidence is also a feasible way to resolve the fusion of evidence, when evidences are highly conflicting.

VI. CONCLUSION

In this paper, we proposed generalized Deng entropies.

Our contributions are to propose generalized Deng entropies which reveal the relationship among Deng entropy, Rényi entropy and Tsallis entropy. Then, explain how the generalized Deng entropy degenerates into different entropies by simple proof. Further, we discuss the generalized Deng entropy and find that the parameters do not affect the distribution of the maximum entropy.

Finally, the numerical examples are used to compare the uncertainty of BPA with Deng entropy and Tsallis-Deng (T-D) and Rényi-Deng (R-D) entropy. Rényi-Deng (R-D) entropy has a great advantage in measuring the uncertainty of BPA. It not only has Deng Entropy to measure the same effect of BPA uncertainty but also has adjustability. Because the value of parameter α can be set according to the actual situation. In addition, an identify model was proposed based on Rényi-Deng (R-D) entropy to identify contradictory evidence, experiment showing the effectiveness of the method.

The shortcoming of our work is that we only unify these three kinds of entropy from the formula, not to derive a more unified one from the micro level, and we will work hard in this direction in the future.

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DATA AVAILABILITY STATEMENT

The authors confirm that the data sources in this paper are public.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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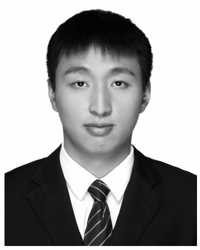

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