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SETQR Propagation Model for Social Networks

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ABSTRACT The study of the laws and influencing factors of information dissemination in social networks is critical to analyzing the spread of public opinion, preventing the spread of rumors, and guiding information transmission. This paper addresses the shortcomings of the traditional SEIR model. We establish the SETQR model and use the probability theorem to derive the law of information propagation. Furthermore, the equilibrium point and the basic regeneration number of the SETQR model are obtained using differential dynamics and the regenerative matrix method. The stability of the SETQR model at the equilibrium point is derived theoretically. Finally, experimental verification is conducted. The simulation results indicate that the SETQR model achieves local stability at the equilibrium point, which is consistent with the results of the theoretical analysis. Through further simulation, the effects of time lag, containment, and forgetting mechanisms on the speed of information dissemination and the time required for the network to reach equilibrium are analyzed.

INDEX TERMS Basic reproductive number, complex network, infectious disease model, information dissemination, stability.

I. INTRODUCTION

Complex networks can be used to represent complex systems in nature and the real world. They have the characteristics of complexity, scale free [1], [2], and small world [3] and they have come to become an important method to study social networks gradually. As one of the focus areas in complex network research, the infectious disease model [4]–[6] aims to simulate the process of information dissemination in social networks through mathematical models and to analyze the mechanism of information dissemination in the network [7]. When information is transmitted through social networks, only a small portion is spread widely while most information is spread in a small area or is overshadowed by a large amount of information flow without being spread. Exploring the mechanism of information dissemination in social networks, establishing an information dissemination model and analyzing its stability can greatly aid in accurately predicting the trend of network information dissemination, preventing the spread of rumors, and guiding information transmission. Therefore, it is practical to develop an information dissemination model.

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The established SETQR information propagation model is based on the time lag, containment, and forgetting mechanisms. During information dissemination, the infected person is divided into two states—trusted state and questioned state.

In summary, the main contributions of this paper are as follows:

(1) In order to ensure that the network model closely resembles the actual situation, the SETQR model considers the following factors: i) Infected nodes can selectively disseminate information; ii) There are nodes in the network that are offline and the processing of information has a time lag; and iii) The immune node may be activated again after a long interval and resume receiving information.

(2) The equilibrium point and basic regeneration number of the SETQR model are calculated. The stability of the two equilibrium points is theoretically derived. The simulation results indicate that the SETQR model has local asymptotic stability in both the $R_0 < 1$ and $R_0 > 1$ cases, which are consistent with the theoretical derivation.

(3) Through the simulation experiment of the actual network, it is proved that the SETQR model is integrated with the actual network.

(4) Furthermore, the effects of the time lag, containment, and forgetting mechanisms on the information dissemination

process and network equilibrium state are simulated, and the simulation results are analyzed.

The rest of this article is organized as follows. In section II, the related work is introduced. In section III, we detail our SETQR model. Section IV analyzes the balance point and the basic regeneration number. In section V, the formula proves the stability of information dissemination. The experimental results are given in section VI. Finally, we provide conclusions in section VII.

II. RELATED WORK

Mathematical modeling methods are widely used in the study of the spread and evolution of infectious diseases [8], [9]. In 1760, Bernoulli analyzed the propagation process of the variola virus through mathematical modeling [10], [11], which marked the establishment of the first mathematical model of epidemiology. In 1927, Kermack and McKendrick established a classical SIR infectious model by dividing the population into three different categories: Susceptible S, Infected I, and Removed R [12]. However, immunity from diseases such as avian flu and hand, foot, and mouth is temporary, and people who are removed from susceptible populations can be transferred back again. For such diseases, Kermack and McKendrick proposed the SIS model in 1932 [13]. Based on this, Zhao *et al.* [6] considered the existence of disease latency, added an exposed population E and proposed the SEIR model under the social network. Bentaleb and Amine [14] proposed a multi-strain SEIR epidemic model with bilinear and non-monotonic event functions. Wu *et al.* [15] studied the existence and nonexistence of the traveling wave of the diffusion-diffusion equation and proposed a nonlocal scattering SEIR model. Liu *et al.* [16] proposed a new SEIR rumor propagation model with a hesitation mechanism.

Domestic and foreign research scholars are not satisfied with the establishment and improvement of the model and wish to achieve a deeper understanding. Research on the balance point of models and the stability of information dissemination is gradually emerging. Kuniya and Wang [17] studied the global asymptotic stability of a spatially diffusive SIR epidemic model with homogeneous Neumann boundary conditions. Kuniya and Wang [18] demonstrated the stability of the non-locally diffused SIR epidemic model at the equilibrium point in an in-depth study of the SIR model. Cao *et al.* [19]. deduced the threshold of the classical SIS model and verified it experimentally. Hsu and Lin [20] examined the stability of the spatially discrete SIS epidemic model. Wang *et al.* [21] proposed an improved SEIR model and proved its stability. Bentaleb and Amine [14] studied the Lyapunov function and global stability of a two-strain SEIR model with bilinear and non-monotone incidence. Wu *et al.* [22] classified individuals into susceptibility, trust, infection, immunity, or recoverability and proposed a new STCIR model to study the dynamic propagation of rumors. Liu and Li [23] discussed a new epidemic SEIR model with a discrete delay in the complex networks. In Zheng et al.'s study [24], based on the degree of different nodes in the network, designed a new state transition function for each node and proposed a new rumor propagation ILSR model.

The abovementioned models are consistent with the trend of network information dissemination and conform to certain characteristics and laws of the communication process. However, there are still limitations in describing the process of information dissemination. For example, the infected node defaults to the terminating node for information propagation, and it can only passively receive information; however, it cannot judge this information. Moreover, the information dissemination process is too idealized, and the nodes in the network can receive information in real time. Simultaneously, the immune node is removed from the information dissemination process, ignoring the possibility that the immune node is re-infected. These shortcomings cause the constructed model to slightly differ from real-world scenarios. Our model is therefore more adaptable to the real situation.

III. SETQR INFORMATION PROPAGATION MODEL

A. DIVISION OF NODE SETS

Taking the incident of ''Restrictions of new energy vehicles' travel and purchase is not allowed in all parts of China'' as an example, the information dissemination data on Sina Weibo was analyzed. ''People's Daily,'' ''National Business Daily,'' ''Weibo Auto,'' ''People's Network,'' ''China Daily,'' ''Xinhua Net,'' and other official Weibo released news of ''Restrictions of new energy vehicles' travel and purchase is not allowed in all parts of China'' on June 6, 2019. This news received a high degree of attention and was commented and forwarded by many Weibo users. As of July 6, 2019, Weibo users' comments on the official Weibo of ''People's Daily'' was 300+, and their forwarding volume was 100+. Weibo users' comments on the official Weibo of ''National Business Daily'' was 300+, and their forwarding volume was 100+. Weibo users' comments on the official Weibo of ''Weibo Auto'' was 100+, and their forwarding volume was 30+. Similarly, Weibo users' comments on the official Weibo of ''People's Network,'' ''China Daily,'' ''Xinhua Net,'' and others were all 50+, and their forwarding volumes were all 20+. After this period, a small number of Weibo users still commented on and forwarded the news; however, the numbers of such additional comments and forwards were relatively small. Obviously, Weibo's forwarding volumes were far less than those of the comments. According to the statistical results, users who trusted this information were most likely to forward the Weibo while commenting on it, whereas most users who questioned the information chose not to forward the Weibo. Therefore, the users' attitudes towards public sentiment in the network affect the diffusion of information. The node is thus divided into two types: trusted node T and questioned node Q.

In the SETQR model, the node set comprises five categories of nodes, namely S(Susceptible), E (Exposed), T(Trusted), Q(Questioned), R(Recovered). The specific meanings of the nodes are shown in Table 1.

TABLE 1. Specific meaning of different types of nodes.

Status	Name	Meaning
S	Susceptible node	Initial user who did not receive the message
E	Exposed node	User who received the message but did not determine whether it was propagated
T	Trusted node	user who believes in the information received and propagates the message
Q	Questioned node	Users who do not believe the information received but propagate the message
R	Recovered node	Users who no longer transmit information or lose interest in the information they transmit

B. PROPAGATION MECHANISM

The conversion relationship among the SETQR model nodes is shown in TABLE 1. The symbolic meaning of the initial parameters used is shown in TABLE 2.

TABLE 2. Symbolic meaning of the initial parameters.

Symbol	Meaning
p_1	Probability that the susceptible node S is converted into the questioned node Q
p_2	Probability that the susceptible node S is converted into the exposed node E
p_3	Probability that the susceptible node S is converted into the trusted node T
p_4	Probability that the exposed node E is converted into the questioned node Q
p_5	Probability that the exposed node E is converted into the trusted node T
p_6	Probability that the questioned node Q is converted into the recovered node R
p_7	Probability that the recovered node R is converted into the susceptible node S
p_8	Probability that the trusted node T is converted into the recovered node R
$1/w$	Latency of exposed node E

The propagation mechanism of the SETQR model follows the time lag, containment, and the forgetting mechanisms:

(1) Time lag mechanism

The susceptible nodes S in the network are divided into online and offline. When online, the susceptible node S quickly completes the transition of the node state after receiving the information. When offline, the susceptible node S cannot receive the information in real time; it first transforms into the exposed node E and lurks for a period of time. Then, it transforms into a questioned node Q or a trusted node T with a probability of w^*p_4 and w^*p_5, respectively. The transition of the node state is then completed.

(2) Containment mechanism

The trusted node T and the questioned node Q are not termination states of information propagation; thus, they will be transformed into the immune node R with a certain probability to curb the spread of information.

(3) Forgetting mechanism

The recovered node R forgets the received information over time and re-transforms from the recovered state to the susceptible state, which is the forgetting mechanism.

The transition probabilities of states between all nodes in the network are in the interval $[0, 1]$, that is $0 \leq p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8 \leq 1$. The latency of the exposed node is in the interval $(0, +\infty]$, that is, $w > 0$. The propagation of information in the SETQR network is described as follows:

(1) S→ Q→R→S

The online susceptible node S in the network quickly makes a decision after receiving the information and turns into the questioned node Q with a probability of p_1; the questioned node Q changes into the recovered node R with a probability of p_6; finally, the recovered node R changes into the susceptible node S again with a probability of p_7.

(2) S→T→R→S

The online susceptible node S in the network quickly makes a decision after receiving the information and converts into the trusted node T with a probability of p_3; the trusted node T changes into the recovered node R with a probability of p_8; finally, the recovered node R changes into the susceptible node S again with a probability of p_7.

(3) S→E→Q→R→S

The offline susceptible node S in the network is first converted into the exposed node E with a probability of p_2. After a latency of $1/w$, the information is successfully received, and the exposed node E is transformed into the questioned node Q with a probability of w^*p_4; the questioned node Q changes into the recovered node R with a probability of p_6; the recovered node R changes into the susceptible node S again with a probability of p_7.

(4) S→E→T→R→S

The offline susceptible node S in the network is first converted into the exposed node E with a probability of p_2. After a latency of $1/w$, the information is successfully received, and the exposed node E is transformed into the trusted node T with a probability of w^*p_5; the trusted node T changes into the recovered node R with a probability of p_8; the recovered node R changes into a susceptible node S again with a probability of p_7.

At time t, the proportions of the susceptible, exposed, trusted, questioned, and recovered nodes in the network are $S(t)$, $E(t)$, $T(t)$, $Q(t)$, and $R(t)$, respectively. It is assumed that the total number of users in the network remains unchanged, and (2) is obtained.

$$S(t) + E(t) + T(t) + Q(t) + R(t) = 1 \tag{1}$$

The differential form of the SETQR model is shown in (2).

$$\begin{cases} \dfrac{d(S)}{d(t)} = -(p_1 + p_2 + p_3)\,ST + p_7R \\ \dfrac{d(E)}{d(t)} = p_2ST - (wp_4 + wp_5)\,E \\ \dfrac{d(T)}{d(t)} = p_3ST + wp_5E - p_8T \\ \dfrac{d(Q)}{d(t)} = p_1ST + wp_4E - p_6Q \\ \dfrac{d(R)}{d(t)} = p_6Q + p_8T - p_7R \end{cases} \tag{2}$$

At $t = 0$, that is, the initial moment of information dissemination, the proportion of each node in the network is as shown in (3).

$$\begin{cases} S(0) \approx 1 \\ E(0) = 0 \\ T(0) \approx 0 \\ Q(0) = 0 \\ R(0) = 0 \end{cases} \tag{3}$$

IV. BALANCE POINT AND BASIC REGENERATION NUMBER

A. BALANCE POINT

As the information in the network spreads, $S(t)$, $E(t)$, $T(t)$, $Q(t)$ and $R(t)$ constantly change over time. When the proportion of these five types of nodes no longer changes, the network is in equilibrium. At this time, (4) is obtained.

$$\begin{cases} \frac{d(S)}{d(t)} = -(p_1 + p_2 + p_3) ST + p_7 R = 0 \\ \frac{d(E)}{d(t)} = p_2 ST - (wp_4 + wp_5) E = 0 \\ \frac{d(T)}{d(t)} = p_3 ST + wp_5 E - p_8 T = 0 \\ \frac{d(Q)}{d(t)} = p_1 ST + wp_4 E - p_6 Q = 0 \\ \frac{d(R)}{d(t)} = p_6 Q + p_8 T - p_7 R = 0 \end{cases} \tag{4}$$

By solving (4), two balance points $X_1(S_1, E_1, T_1, Q_1, R_1)$ and $X_2(S_2, E_2, T_2, Q_2, R_2)$ are obtained, as shown in (5) and (6), respectively.

$$\begin{cases} S_1 = 1 \\ E_1 = 0 \\ T_1 = 0 \\ C_1 = 0 \\ R_1 = 0 \end{cases} \tag{5}$$

$$\begin{cases} S_2 = \frac{p_8 (p_4 + p_5)}{p_2 p_5 + p_3 p_4 + p_3 p_5} \\ E_2 = \frac{p_2 [p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{(p_2 p_5 + p_3 p_4 + p_3 p_5)[p_2 + w(p_4 + p_5)(p_1 + p_2 + p_3)]} \\ T_2 = \frac{w[p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{p_8 [p_2 + w(p_4 + p_5)(p_1 + p_2 + p_3)]} \\ Q_2 = \frac{w(p_1 p_4 + p_1 p_5 + p_2 p_4)[p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{p_6 (p_2 p_5 + p_3 p_4 + p_3 p_5)[p_2 + w(p_4 + p_5)(p_1 + p_2 + p_3)]} \\ R_2 = \frac{w(p_1 + p_2 + p_3)(p_4 + p_5)[p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{p_7 (p_2 p_5 + p_3 p_4 + p_3 p_5)[p_2 + w(p_4 + p_5)(p_1 + p_2 + p_3)]} \end{cases} \tag{6}$$

B. BASIC REGENERATION NUMBER

The basic regeneration number R_0 is an important parameter for characterizing the initial stage of information dissemination. It indicates the expectation of introducing an infected person into the susceptible population and the number of people who can be infected during the average disease period [25], [26]. $R_0 = 1$ is the critical value for the continuous transmission of information in the network [25], [27]. When $R_0 > 1$, users who receive information continue to grow until they reach a certain value. When $R_0 < 1$, a decreasing number of users receive this information in the network until the information disappears from the network.

In this paper, according to the local stability of the disease-free equilibrium point, the basic regeneration number is obtained by the regeneration matrix method.

N is used to represent the total number of node types, $n \in N$, and $n = 1, 2, 3, 4, 5$. ρ is used to represent the node density, and ρ_1, ρ_2, ρ_3, ρ_4, and ρ_5 represent the node densities of states S, E, T, Q, and R, respectively, $\rho_1 = S(t)$, $\rho_2 = \mathrm{E}(t)$, $\rho_3 = \mathrm{T}(t)$, $\rho_4 = \mathrm{Q}(t)$, and $\rho_5 = \mathrm{R}(t)$.

$$\rho = (\rho_1, \rho_2, \rho_3, \rho_4, \rho_5)^T \tag{7}$$

Let the M class node be a special class node whose attributes are received information and potential to spread information. In the above five types, the E, T, and Q type nodes conform to the attributes of the M-type node, and the M-type node contains three types. Thus, M represents the total number of M-type nodes, $M = 3$, and $m \in M$.

Definition 1: Function $g_n(\rho)$ represents the rate of increase of infected nodes in the state node N.

Let

$$g_N(\rho) = (g_{N1} \cdots g_{Nn} \cdots g_{NN})^T = (g_1, g_2, g_3, g_4, g_5)^T$$

where g_{Nn} represents the new rate of the nth type in the SETQR model node.

Equation (8) is obtained from (2).

$$g_n(\rho) = \begin{bmatrix} 0 \\ p_2 SI \\ p_3 SI \\ p_1 SI \\ 0 \end{bmatrix} \tag{8}$$

Definition 2: Function $v_n(\rho)$ represents the transfer rate of the state node N.

Let

$$v_N(\rho) = (v_{N1} \cdots v_{Nn} \cdots v_{NN})^T = (v_1, v_2, v_3, v_4, v_5)^T$$

where v_{Nn} represents the transfer rate of the nth type in the SETQR model node.

Equation (9) is obtained from (2).

$$v_n(\rho) = \begin{bmatrix} (p_1+p_2+p_3)ST - p_7R \\ (wp_4+wp_5)E \\ -wp_5E + p_8T \\ -wp_4E + p_6Q \\ -p_6Q - p_8T + p_7R \end{bmatrix} \tag{9}$$

Definition 3: Function $g_M(\rho)$ represents the rate of increase of infected nodes in the M-type node, which can be expressed as $g_M(\rho) = (g_{M1} \cdots g_{Mm} \cdots g_{MM})^T$, where g_{Mm} represents the new rate of the mth node in the M-type nodes.

Definition 4: Function $v_M(\rho)$ represents the transfer rate of infected nodes in the M-type nodes, which can be expressed as $v_M(\rho) = (v_{M1} \cdots v_{Mm} \cdots v_{MM})^T$ where v_{Mm} represents the transfer rate of the mth node in the M-type node.

Equation (10) is obtained at the disease-free balance point $X_1(S_1, E_1, T_1, Q_1, R_1)$.

$$\begin{cases} G = \left[\dfrac{\partial g_n}{\partial \rho_j}(X_1)\right] \\ V = \left[\dfrac{\partial v_n}{\partial \rho_j}(X_1)\right], \end{cases} \quad (1 \le i, j \le m) \tag{10}$$

Equations (5), (8), and (9) are substituted into (10) to obtain (11).

$$\begin{cases} G = \begin{bmatrix} 0 & p_2 & 0 \\ 0 & p_3 & 0 \\ 0 & p_1 & 0 \end{bmatrix} \\ V = \begin{bmatrix} wp_4+wp_5 & 0 & 0 \\ -wp_5 & p_8 & 0 \\ -wp_4 & 0 & p_6 \end{bmatrix} \end{cases} \tag{11}$$

According to (11), the regeneration matrix GV^{-1} is obtained, as shown in (12).

$$GV^{-1} = \begin{bmatrix} \dfrac{p_2p_5}{p_8(p_4+p_5)} & \dfrac{p_2}{p_8} & 0 \\ \dfrac{p_3p_5}{p_8(p_4+p_5)} & \dfrac{p_3}{p_8} & 0 \\ \dfrac{p_1p_5}{p_8(p_4+p_5)} & \dfrac{p_1}{p_8} & 0 \end{bmatrix} \tag{12}$$

The basic reproduction number R_0 is equal to the spectral radius $r\left(GV^{-1}\right)$ of the regeneration matrix [28], [29]. Therefore, the basic reproduction number of the SETQR model is as shown in (13).

$$R_0 = r\left(GV^{-1}\right) = \frac{p_2p_5 + p_3p_4 + p_3p_5}{p_8(p_4+p_5)} \tag{13}$$

V. STABILITY ANALYSIS OF INFORMATION DISSEMINATION

Theorem 1: When $R_0 < 1$, the equilibrium point $X_1(S_1, E_1, T_1, Q_1, R_1)$ is locally progressively stable.

The following is the proof process for Theorem 1.

The Jacobian matrix [30], [31] of the SETQR model is constructed according to (2), as shown in (14) in the next page.

Substituting (5) into (14), we obtain the Jacobian matrix J_1 at equilibrium point X_1, as shown in (15) in the next page.

The characteristic polynomial of (15) is (16).

$$|\lambda E - J_1| = m_0\lambda^5 + m_1\lambda^4 + m_2\lambda^3 + m_3\lambda^2 + m_4\lambda^1 + m_5 \tag{16}$$

Solving (16) yields (17), as shown in the next page.

Because $R_0 = \frac{p_2p_5+p_3p_4+p_3p_5}{p_8(p_4+p_5)} < 1$, we obtain (18).

$$p_2p_5 + p_3p_4 + p_3p_5 < p_4p_8 + p_5p_8 \tag{18}$$

Deforming (18) yields (19).

$$p_3p_4 + p_3p_5 < p_4p_8 + p_5p_8 \tag{19}$$

Simplification of this yields (20).

$$p_3 < p_8 \tag{20}$$

We substitute (20) into (17) to obtain (21).

$$m_1 > 0,\ m_2 > 0,\ m_3 > 0,\ m_4 > 0 \tag{21}$$

For the sake of convenience, when calculating $\Delta_1, \Delta_2, \Delta_3$, let $A = wp_4 + wp_5 > 0$, $B = p_6 + p_7 > 0$, $C = p_8 - p_3 > 0$, and $D = p_6p_7 > 0$ to obtain (22), (23), and (24), as shown in the next page.

$$\Delta_1 = m_1 > 0 \tag{22}$$

Equations (21), (22), (23), and (24) satisfy the requirements of the Routh-Hurwitz stability discriminant theorem [32]–[34]. Therefore, when $R_0 < 1$, the equilibrium point $X_1(S_1, E_1, T_1, Q_1, R_1)$ is locally progressively stable.

Theorem 2: When $R_0 > 1$, the equilibrium point $X_2(S_2, E_2, T_2, Q_2, R_2)$ is locally progressively stable.

The following is the proof process for Theorem 2.

We substitute (6) into (14) to obtain the Jacobian matrix J_2 at equilibrium point X_2, as shown in (25) in the next page.

The characteristic polynomial of (25) is (26), as shown in the next page.

Substituting (25) into (26) yields $m_0' = 1$. It is obvious that $m_1', m_2', m_3', m_4', m_5'$ contains only one minus sign. Thus, (27) is obtained.

$$p_8 - \frac{p_3p_8(p_4+p_5)}{p_2p_5 + p_3p_4 + p_3p_5} \tag{27}$$

Therefore, to prove $m_1', m_2', m_3', m_4', m_5' > 0$, we only need to prove $p_8 - \frac{p_3p_8(p_4+p_5)}{p_2p_5+p_3p_4+p_3p_5} > 0$.

Because $R_0 = -\frac{p_2p_5+p_3p_4+p_3p_5}{p_8(p_4+p_5)} > 1$:

$$p_2p_5 + p_3p_4 + p_3p_5 > p_4p_8 + p_5p_8 \tag{28}$$

Deforming (28) yields (29).

$$p_2p_4 + p_2p_5 + p_3p_4 + p_3p_5 > p_4p_8 + p_5p_8 \tag{29}$$

$$J = \begin{bmatrix} -(p_1+p_2+p_3)\,T & 0 & -(p_1+p_2+p_3)\,S & 0 & p_7 \\ p_2T & -w\,(p_4+p_5) & p_2S & 0 & 0 \\ p_3T & wp_4 & p_3S-p_8 & 0 & 0 \\ p_1T & wp_5 & p_1S & -p_6 & 0 \\ 0 & 0 & p_8 & p_6 & -p_7 \end{bmatrix} \tag{14}$$

$$J_1 = J\,(X_1) = \begin{bmatrix} 0 & 0 & -(p_1+p_2+p_3) & 0 & p_7 \\ 0 & -w\,(p_4+p_5) & p_2 & 0 & 0 \\ 0 & wp_4 & p_3-p_8 & 0 & 0 \\ 0 & wp_5 & p_1 & -p_6 & 0 \\ 0 & 0 & p_8 & p_6 & -p_7 \end{bmatrix} \tag{15}$$

$$\begin{cases} m_0 = 1 \\ m_1 = wp_4 + wp_5 + p_6 + p_7 + p_8 - p_3 \\ m_2 = p_6p_7 + (wp_4 + wp_5)\,(p_8 - p_3) + (wp_4 + wp_5 + p_8 - p_3)\,(p_6 + p_7) \\ m_3 = (wp_4 + wp_5)\,(p_8 - p_3)\,(p_6 + p_7) + p_6p_7\,(wp_4 + wp_5 + p_8 - p_3) \\ m_4 = p_6p_7\,(wp_4 + wp_5)\,(p_8 - p_3) \\ m_5 = 0 \end{cases} \tag{17}$$

$$\begin{aligned} \Delta_2 &= \begin{vmatrix} m_1 & m_0 \\ m_3 & m_2 \end{vmatrix} = m_1m_2 - m_3 \\ &= (A+B+C)\,[D+AC+B\,(A+C)] - [ABC + D\,(A+C)] \\ &= AD + BD + CD + A^2C + ABC + AC^2 + B\,(A+C)\,(A+B+C) - ABC - AD - CD \\ &= BD + A^2C + AC^2 + B\,(A+C)\,(A+B+C) > 0 \end{aligned} \tag{23}$$

$$\begin{aligned} \Delta_3 &= \begin{vmatrix} m_1 & m_0 & 0 \\ m_3 & m_2 & m_1 \\ m_5 & m_4 & m_3 \end{vmatrix} = m_1m_2m_3 - m_1^2m_4 - m_3^2 \\ &= (A+B+C)\,[D+AC+B\,(A+C)]\,[ABC+D\,(A+C)] - ACD\,(A+B+C)^2 - [ABC+D\,(A+C)]^2 \\ &= [ABC+D\,(A+C)]\,[(A+B+C)\,(D+AC+AB+BC) - ABC - AD - CD] - ACD\,(A+B+C)^2 \\ &= [ABC+D\,(A+C)]\,[BD + 2ABC + AC\,(A+C) + AB\,(A+B) + BC\,(B+C)] - ACD\,(A+B+C)^2 \\ &> [AD+CD]\,[BD + 2ABC + AC\,(A+C) + AB\,(A+B) + BC\,(B+C)] - ACD\,(A+B+C)^2 \\ &= ABD^2 + A^3BD + A^2B^2D + ABC^2D + BCD^2 + AB^2CD + B^2C^2D + BC^3D > 0 \end{aligned} \tag{24}$$

$$J_2 = J\,(X_2) = \begin{bmatrix} -\dfrac{w\,(p_1+p_2+p_3)\,[p_2p_5 + (p_4+p_5)\,(p_3-p_8)]}{p_8\,[p_2 + w\,(p_4+p_5)\,(p_1+p_2+p_3)]} & 0 & -\dfrac{p_8\,(p_1+p_2+p_3)\,(p_4+p_5)}{p_2p_5+p_3p_4+p_3p_5} & 0 & p_7 \\ \dfrac{wp_2\,[p_2p_5 + (p_4+p_5)\,(p_3-p_8)]}{p_8\,[p_2 + w\,(p_4+p_5)\,(p_1+p_2+p_3)]} & -w\,(p_4+p_5) & \dfrac{p_2p_8\,(p_4+p_5)}{p_2p_5+p_3p_4+p_3p_5} & 0 & 0 \\ \dfrac{wp_3\,[p_2p_5 + (p_4+p_5)\,(p_3-p_8)]}{p_8\,[p_2 + w\,(p_4+p_5)\,(p_1+p_2+p_3)]} & wp_4 & \dfrac{p_3p_8\,(p_4+p_5)}{p_2p_5+p_3p_4+p_3p_5} - p_8 & 0 & 0 \\ \dfrac{wp_1\,[p_2p_5 + (p_4+p_5)\,(p_3-p_8)]}{p_8\,[p_2 + w\,(p_4+p_5)\,(p_1+p_2+p_3)]} & wp_5 & \dfrac{p_1p_8\,(p_4+p_5)}{p_2p_5+p_3p_4+p_3p_5} & -p_6 & 0 \\ 0 & 0 & p_8 & p_6 & -p_7 \end{bmatrix} \tag{25}$$

$$|\lambda E - J_2| = m_0'\lambda^5 + m_1'\lambda^4 + m_2'\lambda^3 + m_3'\lambda^2 + m_4'\lambda^1 + m_5' \tag{26}$$

Simplification yields (30).

$$p_3 > p_8 - p_2 \tag{30}$$

By substituting (28) and (30) into (27) , we obtain (31).

$$\begin{aligned} p_8 - \frac{p_3 p_8 (p_4 + p_5)}{p_2 p_5 + p_3 p_4 + p_3 p_5} &> p_8 - \frac{p_3 p_8 (p_4 + p_5)}{p_4 p_8 + p_5 p_8} \\ &= p_8 - p_3 \\ &> p_8 - (p_8 - p_2) \\ &= p_2 > 0 \end{aligned} \tag{31}$$

Therefore, (32) is obtained.

$$m_1' > 0, m_2' > 0, m_3' > 0, m_4' > 0, m_5' > 0 \tag{32}$$

For the sake of convenience, when calculating $\Delta_1', \Delta_2', \Delta_3'$, deformation in (33) is performed.

$$\begin{cases} a = \dfrac{w (p_1 + p_2 + p_3) [p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{p_8 [p_2 + w (p_4 + p_5)(p_1 + p_2 + p_3)]} > 0 \\ b = w (p_4 + p_5) > 0 \\ c = p_8 - \dfrac{p_3 p_8 (p_4 + p_5)}{p_2 p_5 + p_3 p_4 + p_3 p_5} > 0 \\ d = \dfrac{w p_2 [p_2 p_5 + (p_4 + p_5)(p_3 - p_8)]}{p_8 [p_2 + w (p_4 + p_5)(p_1 + p_2 + p_3)]} > 0 \\ e = \dfrac{p_1 p_8 (p_4 + p_5)}{p_2 p_5 + p_3 p_4 + p_3 p_5} > 0 \end{cases} \tag{33}$$

Substituting (33) into (26), we obtain (34).

$$\begin{cases} m_0' = 1 \\ m_1' = a + b + c + p_6 + p_7 \\ m_2' = ab + ac + bc + (a + b + c + 1)(p_6 + p_7) \\ m_3' = abc + p_6 p_7 (a + b + c) + (p_6 + p_7)(ab + ac + bc) \\ m_4' = abc (p_6 + p_7) + p_6 p_7 (ab + ac + bc) \\ m_5' = abc p_6 p_7 + wde p_4 p_6 p_7 \end{cases} \tag{34}$$

According to (34), we obtain (35).

$$\begin{cases} \Delta_1' = m_1' > 0 \\ \Delta_2' = \begin{vmatrix} m_1' & m_0' \\ m_3' & m_2' \end{vmatrix} = m_1' m_2' - m_3' > 0 \\ \Delta_3' = \begin{vmatrix} m_1' & m_0' & 0 \\ m_3' & m_2' & m_1' \\ m_5' & m_4' & m_3' \end{vmatrix} \\ = m_1' m_2' m_3' + m_1' m_5' - m_1'^2 m_4' - m_3'^2 > 0 \end{cases} \tag{35}$$

Equations (32) and (35) satisfy the requirements of the Routh–Hurwitz stability discriminant theorem. Therefore, when $R_0 > 1$, the equilibrium point $X_2(S_2, E_2, T_2, Q_2, R_2)$ is locally asymptotically stable.

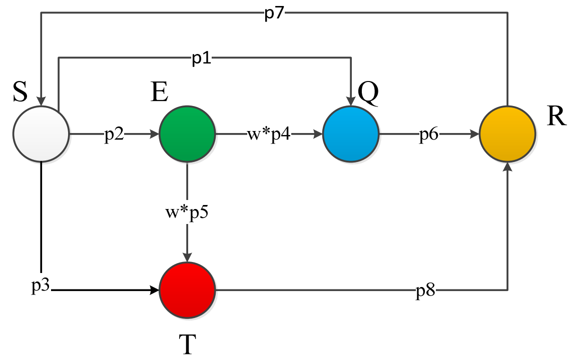

FIGURE 1. Conversion relationship between nodes in the SETQR model.

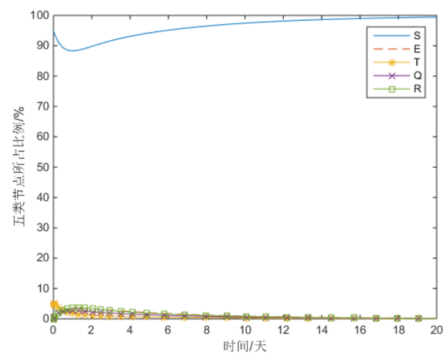

FIGURE 2. Verification diagram of information propagation stability when R0 < 1.

VI. SIMULATION RESULTS AND ANALYSIS

In this study, the stability of information dissemination and the impact of communication mechanisms on information dissemination are simulated and verified respectively. The specific simulation results and analyses are as follows.

A. VERIFICATION OF INFORMATION DISSEMINATION STABILITY

In this paper, $R_0 < 1$, $R_0 > 1$, and a real case are verified experimentally and the trends in the proportion of various types of nodes in the SETQR network with time are analyzed.

1) SITUATION ONE: $R_0 < 1$

The transition probability is randomly selected and multiple values are obtained for the simulation, wherein one such set of values is $p_1 = 0.01$, $p_2 = 0.01$, $p_3 = 0.001$, $p_4 = 0.2$, $p_5 = 0.2$, $p_6 = 0.8$, $p_7 = 0.9$, $p_8 = 0.8$,and $w = 0.99$. At this time, $R_0 \approx 0.02$ is obtained and the condition of $R_0 < 1$ is satisfied. When the transition probabilities are different, the experimental results are almost identical. The simulation results of the above values are selected, and the result is shown in FIGURE 2.

In FIGURE 2, the abscissa indicates the time of information propagation in days, and the ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates the change in the proportion of the susceptible node S. The dotted line indicates that of the exposed node E. The curve with $*$ indicates that of the trusted node T. The curve with $\times$ indicates that of the questioned node Q. Finally, the curve with $\square$ indicates that of the recovered node R.

It is observed that as time goes by, the proportion of susceptible node S in the network gradually approaches 1, while the numbers of exposed, trusted, questioned and recovered nodes gradually approach zero. At this point, the system has stabilized. The propagation node disappears, the information loses its ability to spread, and it does not form widespread communication. Therefore, it is verified that the equilibrium point is locally asymptotically stable, which is consistent with the results of the theoretical analysis.

2) SITUATION TWO: $R_0 > 1$

The transition probability is randomly selected and multiple values are obtained for simulation, wherein one such set of values is $p_1 = 0.1$, $p_2 = 0.25$, $p_3 = 0.001$, $p_4 = 0.2$, $p_5 = 0.01$, $p_6 = 0.3$, $p_7 = 0.3$, $p_8 = 0.01$, $w = 0.5$. At this time, $R_0 \approx 1.3$ is obtained and the condition of $R_0 > 1$ is satisfied. When the transition probabilities differ, the experimental results are almost identical. The simulation results of the above values are selected, and the result is shown in FIGURE 3.

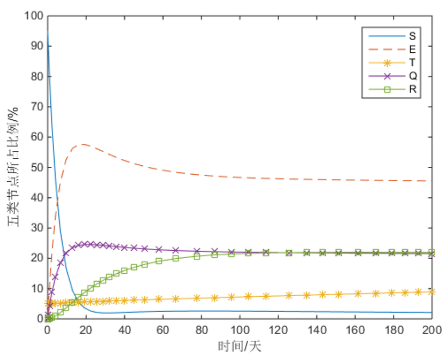

FIGURE 3. Verification diagram of information propagation stability when R0 > 1.

In FIGURE 3, the abscissa indicates the time of information propagation in days, and the ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates the change in the proportion of the susceptible node S. The dotted line indicates that of the exposed node E. The curve with $*$ indicates that of the trusted node T. The curve with $\times$ indicates that of the questioned node Q. The curve with $\square$ indicates that of the recovered node R.

It is observed that the proportions of exposed, trusted, questioned, and recovered nodes tend to be stable over time after a period of change, and the information will continue to spread stably in the system. Therefore, it is verified that the equilibrium point is locally asymptotically stable, which is consistent with the results of the theoretical analysis.

3) ACTUAL CASE

Data simulation and analysis were carried out as examples for "Restrictions of new energy vehicles' travel and purchase is not allowed in all parts of China". In the simulations, the professional crawler software "Octopus" was used for statistical analysis of Weibo users who were concerned about this news item in order to ensure accuracy of the experimental data. From this analysis, we observed that the Weibo users on the official Weibo of the "People's Daily," "National Business Daily," and "Weibo Auto" networks generated more comments and forwarding volumes than those on the other official Weibo mentioned above. Therefore, in the simulations, we selected the data of these three networks for separate evaluations. The simulation results of these three networks were almost identical. Owing to space limitations, only the results People's Daily have been shown in this paper. The results indicate that the number of times the information was forwarded on the microblog was approximately 100, and the number of comments was approximately 300. Users who trust the information will generally choose to forward it while commenting, whereas users who question the information do not choose to forward it. From the statistical analysis results, we can see that $p_1 \approx 2\,p_3, p_4 \approx 2\,p_5, p_6 \approx 2\,p_8, p_3 = p_5 = p_8 = 0.2$, and $p_1 = p_4 = p_6 = 0.4$, i.e., the number of people replying to Weibo on June 6 was approximately half of the total number, and therefore, $p_2 = p_1 + p_3 = 0.6$. The last reply to the Weibo was June 19, which took 13 days to complete; therefore, $1/w = 13$ or $w = 1/13$. After a period of time, there were very few users who paid attention to this Weibo again and therefore $p_7 \approx 0.01$. The above values are thus added to (13) to obtain $R_0 = 2$ and to meet the condition of $R_0 > 1$. The result is shown in FIGURE 4.

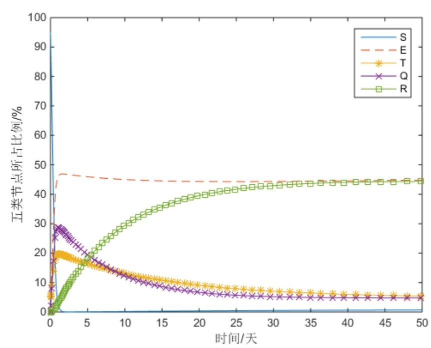

FIGURE 4. Information dissemination stability verification diagram for actual cases.

In FIGURE 4, the abscissa indicates the time of information propagation in days, and the ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates a change in the proportion of the susceptible node S. The dotted line indicates the exposed node E. The curve with $*$ indicates the trusted node T. The curve with $\times$ indicates that of the questioned node Q. The curve with $\square$ indicates that of the recovered node R.

The proportions of exposed, trusted, questioned, and recovered nodes tend to be stable over time after a period of change, and the information will continue to spread stably in the system. Therefore, it is verified by an actual case that the equilibrium point is locally asymptotically stable, which is consistent with the theoretical analysis results.

B. IMPACT OF COMMUNICATION MECHANISMS ON INFORMATION DISSEMINATION

1) IMPACT OF TIME LAG MECHANISMS ON INFORMATION DISSEMINATION

In the experiment to analyze the influence of the time lag mechanism on information transmission, the initial parameters $p_1 = 0.1, p_2 = 0.2, p_3 = 0.1, p_4 = 0.1, p_5 = 0.2, p_6 = 0.7, p_7 = 0.5$, and $p_8 = 0.2$ are maintained in the experiment. w is randomly selected such that $w \geq 0$. The random values taken in the experiment are $w_1 = 1/2$, $w_2 = 1/4$, $w_3 = 1/8$, $w_4 = 1/16$, and $w_5 = 1/32$, in sequence. FIGURE 5 shows the simulation results of the effect of the time lag mechanism on information dissemination.

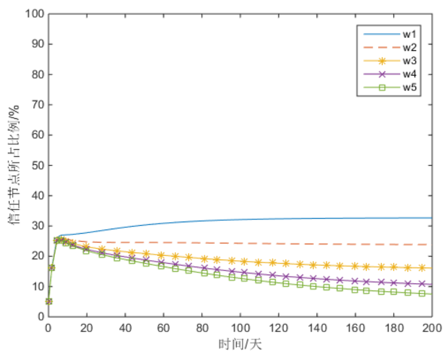

FIGURE 5. Impact of time lag mechanism on information dissemination.

In FIGURE 5, the abscissa indicates the time of information propagation in days, and the ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates the change in the proportion of the trusted node T when $w_1 = 1/2$. The dotted line indicates that of the trusted node T when $w_2 = 1/4$. The curve with $*$ indicates that of the trusted node T when $w_3 = 1/8$. The curve with $\times$ indicates that of the trusted node T when $w_4 = 1/16$. The curve with $\square$ indicates of the trusted node T when $w_5 = 1/32$.

As shown in FIGURE 5, the smaller w is, the longer the latency is, the longer the network requires to reach equilibrium, and the smaller the proportion of trust nodes when the network reaches equilibrium. Therefore, the offline nodes in the network will delay information propagation and reduce information transmission speed. Additionally, the longer the latency, the greater the delay effect.

2) IMPACT OF CONTAINMENT MECHANISMS ON INFORMATION DISSEMINATION

In the experiment to analyze the influence of the containment mechanism on information transmission, the initial parameters $p_1 = 0.1, p_2 = 0.2, p_3 = 0.1, p_4 = 0.1, p_5 = 0.2, p_6 = 0.7, p_7 = 0.5$, and $w = 0.6$ are maintained in the experiment. p_8 is randomly selected such that $0 \leq p_8 \leq 1$. The random values taken in the experiment are $p_{81} = 0.1$, $p_{82} = 0.3$, $p_{83} = 0.5$, $p_{84} = 0.7$, are $p_{85} = 0.9$, in sequence. FIGURE 6 shows the simulation results of the effect of the containment mechanism on information dissemination.

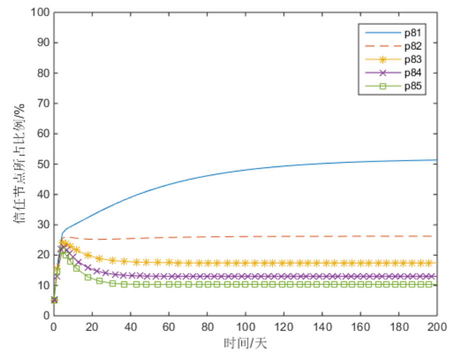

FIGURE 6. Impact of containment mechanism on information dissemination.

In FIGURE 6, the abscissa indicates the time of information propagation in days. The ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates the change in the proportion of the trusted node T when $p_{81} = 0.1$. The dotted line indicates that of the trusted node T when $p_{82} = 0.3$. The curve with $*$ indicates that of the trusted node T when $p_{83} = 0.5$. The curve with $\times$ indicates that of the trusted node T when $p_{84} = 0.7$. The curve with $\square$ indicates that of the trusted node T when $p_{85} = 0.9$.

FIGURE 6 shows that the larger p8 is, the smaller the proportion of trusted nodes is when the network reaches equilibrium. This is because p8 represents the intensity of containment. When the intensity of containment increases, the probability that the propagating node will become the recovered node increases. The speed of information dissemination can be accelerated so that the network reaches equilibrium faster and the proportion of trusted nodes decreases.

3) IMPACT OF FORGETTING MECHANISMS ON INFORMATION DISSEMINATION

In the experiment to analyze the influence of forgetting mechanism on information transmission, the initial parameters $p_1 = 0.1, p_2 = 0.2, p_3 = 0.1, p_4 = 0.1, p_5 = 0.2, p_6 = 0.7$, $p_8 = 0.2$, and $w = 0.6$ are maintained in the experiment. p_7 is randomly selected such that $0 \leq p_7 \leq 1$. The random values taken in the experiment are $p_{71} = 0.1$, $p_{72} = 0.3$, $p_{73} = 0.5$, $p_{74} = 0.7$, and $p_{75} = 0.9$, in sequence. FIGURE 7 shows the simulation result of the effect of the containment mechanism on information dissemination.

In FIGURE 7, the abscissa indicates the time of information propagation in days, and the ordinate indicates the proportions (%) of various nodes in the network. The lines in the figure indicate proportion changes during information propagation. The solid line in the figure indicates the change in the proportion of the trusted node T when $p_{71} = 0.1$. The dotted line indicates that of the trusted node T when $p_{72} = 0.3$. The curve with $*$ indicates that of the trusted node T when $p_{73} = 0.5$. The curve with $\times$ indicates that of the trusted node T when $p_{74} = 0.7$. The curve with $\square$ indicates that of the trusted node T when $p_{75} = 0.9$.

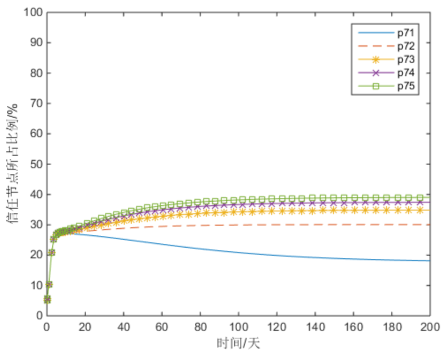

FIGURE 7. **Impact of forgetting mechanism on information dissemination.**

FIGURE 7 indicates that the larger p7 is, the larger the proportion of trusted nodes when the network reaches equilibrium. This is because the larger p7 is, the easier it is for the recovered node to forget that it has accepted the information, become a susceptible node again, and start spreading information.

VII. CONCLUSION

This study establishes the SETQR model based on the influence of the time lag, containment, and forgetting mechanisms on the propagation process during information dissemination. We derive the equilibrium point and basic regeneration number of the SETQR model and prove the stability of the SETQR model at the equilibrium point. The simulation experiment verifies the theoretical analysis of the information propagation stability of the SETQR model and proves that the time lag, containment, and forgetting mechanisms are all important factors affecting the information dissemination process. The research work in this paper explains the reasons why information in the network will have different propagation ranges, which helps provide understanding of the information dissemination behavior in complex networks.

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