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Optimal Decisions of a Supply Chain With a Risk-Averse Retailer and Portfolio Contracts

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ABSTRACT In this paper, we investigate a supply chain involving one risk-neutral supplier and one risk-averse retailer, where the retailer adopts the conditional value-at-risk (CVaR) criterion as his performance measure. To hedge against high risk, the retailer purchases call options from the supplier to adjust his firm orders. We derive the optimal order and production policies, with and without a call option contract and demonstrate that the call option contract can benefit both the retailer and the supplier. In addition, we also generate insights regarding how the contract parameters, level of risk aversion and shortage cost impact the retailer's optimal policy, highlighting the importance of considering the risk aversion and shortage cost simultaneously. Finally, we derive the condition for the supply chain to be coordinated and show that compared to non-coordinating contracts, the wholesale price and call option portfolio contracts proposed in this paper can achieve Pareto optimality. Numerical experiments are conducted to demonstrate theoretical results and observations.

INDEX TERMS Supply chain coordination, call option contract, risk-aversion, conditional value-at-risk, shortage cost.

I. INTRODUCTION

Products characterized by long lead times, a short sales season and high demand uncertainty (''seasonal products'') are both well-researched and common in practice [1]. In recent years, technology has experienced swift advancement and competition has increased leading to rapidly changing consumer preferences. These market conditions have led to a rise in the number of products that exhibit these characteristics [2], [3]. This increase is of interest considering the problems that plague seasonal products such as airline tickets, fashion apparel and fresh food. Seasonal products often have low salvage values which, when coupled with high demand uncertainty and rapidly changing preferences, creates a large risk of over- and under-stocking which is problematic for supply chains. To hedge against these risks, a retailer usually lean order policies such as delayed ordering or small-volume multi-batch ordering [4]–[6]. However, for such an order policy to be feasible, the supplier has to adopt a more flexible production policy to fulfill irregular orders with a sharp increase in supply cost. Because the incentives of the retailer and supplier in such situations are not aligned, supply chain inefficiencies are inevitably created.

Research on supply chains has proposed numerous contracts that resolve these inefficiencies. One effective contract that allows for flexible ordering policies and is growing in popularity is a call option contract [7], [8]. In a basic contract, such as a wholesale price contract, the retailer places an order with a supplier based on a per-unit cost. In this type of contract, the retailer must purchase all units ordered (i.e., this is a firm order). A call option contract operates by making use of two additional parameters, an option price and an exercise

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price. The option price is a per-unit amount paid by the retailer to the supplier in order to reserve the right to purchase additional units of a product at some future time (i.e., this is an option order). The exercise price is the per-unit amount paid by the retailer to the supplier should the retailer choose to exercise any of these options. This call option, when included in a contract, provides obvious benefits for the retailer in terms of managing uncertain demand. It allows the retailer to pay some small amount to ensure additional available supply while also allowing him to wait until more demand information is available to determine whether or not those additional units are needed. Moreover, the call option contract can be attractive to the supplier. It provides the supplier with a small amount to invest in additional capacity and material planning early on with the opportunity to sell more items to the retailer in the future than he would have otherwise.

The benefits of call options have fueled increasing research interest in practice. In a report on product risk management, Hewlett-Packard noted that the uncertainties surrounding their products necessitated risk-management instruments such as call and put options [9]. The report goes on to describe the development of contracts with their suppliers that include both fixed and flexible quantities at varying prices. It was reported that approximately 35% of HP's procurement value has been achieved through call options [10] and that this move towards more complex contracts that include call options saved HP nearly $425 million in cumulative costs from 2000-2006 [9]. Similarly, China Telecom reported using both firm orders and call options to procure over 100 billion RMB worth of products from his suppliers annually [11]. These examples demonstrate the value and applicability of a call option contract.

In addition to evidence in practice, the potential benefits of call options have been studied in academic research [12]. However, the focus of this stream of literature has been on the call option's ability to effectively manage the risk of over- or under-stocking assuming that (1) the retailer is risk-neutral and (2) shortage costs are trivial. In a practical setting with seasonal products, it may be the case that retailer's have risk preferences (i.e., are not risk neutral) and that the cost of failing to meet demand is non-trivial. A large stream of literature within supply chain contracts has questioned whether or not decision makers are always concerned with expected profit maximization. This body of work demonstrates that, oftentimes, decision makers are concerned with risk containment or loss minimization, i.e., exhibit risk aversion [13]–[15]. Furthermore, with increased competition and rapidly changing consumer preferences, stock-outs may have large effects on future sales, profitability and a firm's reputation, creating a large shortage cost for retailers.

The presence of risk aversion, regardless of the magnitude of shortage costs, can have severe implications for supply chain and contract performance [10]. When retailer's are risk averse, they may order less than supply chain optimal quantities in an effort to minimize losses in the event that demand is low. This behavior occurs even under traditionally contracts, leading to decreased profits [16]. The shortage costs are difficult to quantify and, when retailers are risk-neutral, the assumption of negligible shortage costs does not alter overall insights. However, when the retailer is risk-averse, a non-trivial shortage cost magnifies the risk of under-ordering relative to over-ordering and may actually lead to over-ordering. In this scenario, a contract that allows for both firm and option orders may present a risk-averse retailer with the necessary flexibility to balance this trade-off while maintaining high levels of supply chain performance [17]–[19].

In this paper, we propose portfolio contracts by introducing the call option contract into a traditional wholesale price contract, and investigate the benefits when the retailer is risk-averse and faces a non-trivial shortage cost. We model a single-period supply chain with one risk-averse retailer and one risk-neutral supplier. Specifically, we assume that the retailer is willing to balance lower expected profit for downside protection against potential losses, which is in accordance with the conditional value at risk (CVaR) criterion[1]. Under the assumptions of this setting, the follows questions are addressed:

(1) What is the risk-averse retailer's optimal ordering policy and the risk-neutral supplier's optimal production policy under the portfolio contracts when shortage costs are non-trivial?
(2) What is the impact of the retailer's level of risk aversion, the shortage cost and the portfolio contracts' parameters on the retailer's optimal order policy?
(3) What specific benefit does the addition of the call option contract provide relative to the wholesale price contract?
(4) What is the condition under which the portfolio contracts are able to coordinate the supply chain?

The main contributions of our research are as follows:

(1) We add to supply chain research by analyzing the portfolio contracts under two practically relevant assumptions: that the retailer is risk-averse and there is a non-trivial shortage cost. We derive the optimal ordering policy of the retailer and the optimal production policy of the supplier.
(2) We find that the retailer's optimal total ordering quantity is independent of the wholesale price, and does not have a monotonic relationship with the risk aversion and exercise price which does not occur when risk aversion and shortage costs are considered in isolation.
(3) We show that when the option order is incorporated into the firm order, this benefits both the retailer and the supplier, and improves the performance of the whole supply chain.

[1] There are multiple measures of risk aversion widely adopted in the literature such as mean-variance, value at risk (VaR) and CVaR [20], [21]. However, in comparison with mean-variance and VaR, CVaR is a relatively conservative criterion for decision makers. In addition, CVaR has been proven to be a coherent risk measure that is easily computed and commonly used in marketing [21], [22], supply chain management [23] and finance [24].

The rest of the article is structured as follows. Section 2 provides a brief review of relevant literature. In Section 3, the model formulation and assumptions about our problem are presented. In Section 4, the retailer's order policy and the supplier's production policy are studied, respectively. The effects of the introduction of the call option contract on the retailer and the supplier are analyzed in Section 5. Section 6 derives the condition under which the supply chain can be coordinated. Section 7 presents the results of the numerical examples to validate our findings. Finally, Section 8 concludes the paper with a summary of our results, managerial implications, and potential directions for future research.

II. LITERATURE REVIEW

Our work draws upon three major streams of supply chain literature: (1) the use of call options in supply chain contracts, (2) supply chain contract performance with risk averse players and (3) the consideration of shortage costs within supply chains. Below, we briefly review relevant literature that connects these areas and highlight our contributions.

A. CALL OPTION CONTRACTS

The use of call options in supply chains with risk neutral players has received considerable attention under various situations. Generally, this research can be classified into two categories. The first is to consider an independent contract, and the second is to consider the portfolio contracts consisting of the call option contract and other contracts. When call option contracts operate as an independent form of contracts in a supply chain (SC), they generate many benefits. By adopting a two-period model with correlated demand, Barnes-Schuster *et al.* [12] explored the role of the call option contract in a SC involving a single supplier and a single retailer and showed that the SC can be coordinated only if the exercise price is allowed to be piecewise linear. They also derived the corresponding sufficient conditions on the cost parameters for the linear prices to coordinate the SC. Wang and Liu [7] developed a retailer-led SC model to study risk sharing and SC coordination. They obtained the condition of SC coordination. Moreover, they showed that the option contract benefits to each party. Zhao *et al.* [8] developed an option contract model by taking a cooperative game approach. Compared with the wholesale price contract, they demonstrated that option contracts can coordinate the SC and improve its performance. Xu [25] characterized the supplier's the optimal production quantity and derived the manufacturer's optimal option order quantity. In addition, with the introduction of a call option contract, they demonstrated that both the supplier and the manufacturer can be better off. Luo and Chen [26] examined the effects of a call option contract on the SC members' optimal decisions with random yield in a spot market and derived the conditions under which the SC was coordinated and/or Pareto-improvement was achieved.

In addition to being used independently, call option contracts have been combined with other forms of contracts, referred to as portfolio contracts. Burnetas and Ritchken [27] investigated the efficiency of a SC involving one supplier and one retailer, where the retailer can purchase reorder options together with inventory purchases. Further, they studied the role of the option contract in a SC with a downward-sloping demand curve, and found that the impact of the option contract on the retailer's optimal order policy changed with the degree of demand fluctuation. Fu *et al.* [28] explored the value of portfolio procurement from various angles. When both random demand and spot price were considered, the optimal portfolio procurement policies were derived. Moreover, to obtain the optimal procurement solution for the general problem, they introduced a shortest-monotone path algorithm. Chen and Shen [29] obtained the channel agents' optimal ordering policy and production policy, in the presence of a service requirement, with and without the option contract. Through model comparison and analysis, they found that when the option contract was introduced, the SC efficiency improved. In a recent work, Wang and Chen [11] investigated the price-setting newsvendor's decision with a call option contract, they showed that when both single ordering and mixed ordering were available, the newsvendor's optimal ordering policy was mixed ordering. On top of this, they also showed that mixed ordering was more capable for hedging against supply price volatility risk. Wang and Chen [30] obtained the channel agents' optimal decisions with portfolio contracts and circulation loss. They showed that the optimal option pricing policy of the supplier is unrelated to the wholesale price and demand risk. Moreover, they found that with portfolio contracts, the SC was coordinated and Pareto-improvement was achieved. These works highlight the benefits and robustness of call option contracts but only consider their performance when supply chain members are risk neutral. Eriksson [31] investigated a dyadic supply chain in a multi-period setting. By combining the base stock model and the option mechanism, they presented an algorithm to address that key problem on how to coordinate the supply chain. Moreover, they illustrated the effects of the algorithm and showed how two decentralised companies can each maximise profits while reaching the optimal centralised system level by numerical experiments.

B. INCLUSION OF RISK AVERSE MEMBERS

Our research assumes that the retailer's risk aversion is characterized specifically by the CVaR criterion. This criterion is a measure that accounts for both upside rewards and downside penalties, making it particularly appropriate for describing a retailer's risk aversion. Several supply chain contracts have been studied while considering risk averse members that adopt the CVaR criterion with a large focus on supply chains containing a single risk neutral supplier and single risk averse retailer. Gan *et al.* [32], Yang *et al.* [20], Shang and Yang [33] and Xie *et al.* [34] investigate a supply chain with a single risk neutral supplier and a single risk averse retailer. Gan *et al.* [32] find that a buyback contract can coordinate the supply chain when it is contingent on the retailer's order quantity with an upper bound on returns while Yang *et al.* [20] and Xie *et al.* [34] demonstrate mild conditions for which buyback contracts, or revenue sharing contracts, are coordinating. Shang and Yang [33] focus on profit sharing contracts and demonstrate that when incorporating negotiating power and risk preferences, a Pareto-optimal contract can be designed. Based on their models, Vipin and Amit [35] find that loss aversion can significantly improve the utility function's performance in predicting the rational behavior. Under recourse option, they extend the analysis to a supply chain setting and establish coordinating contract between a loss averse retailer and a risk neutral supplier. Further, they also find that the contract parameters are independent on the loss aversion.

Other research has expanded upon this by considering multiple players at each level of the supplier chain. Yao *et al.* [36] and Hsieh and Lu [37] consider a supply chain that consists of a single supplier but multiple risk averse retailers. Both demonstrate that there are large differences when compared with a case of no retailer competition with the former characterizing an optimal returns policy and the latter showing that as retailers become more risk averse, the buyback price approaches the wholesale price. Expanding even further, Chen *et al.* [38] research coordinating contracts in a supply chain with multiple suppliers and multiple retailers. It is found that some of these contracts are not stable under competition but when the lowest-cost supplier handles all production and the least risk-averse retailer bears the total risk, the supply chain can be coordinated. These works demonstrate that contract performance is significantly impacted by the presence of risk averse players and supply chain coordination or Pareto-improvement is only achieved under restricted conditions.

The primary focus of our research is at the intersection of risk aversion and call option contracts which a small stream of literature has addressed. For the CVaR criterion, Wu *et al.* [39] derive optimal ordering policies for a risk averse manufacturer under call option contract and found that as the manufacturer's risk-aversion increases, the optimal ordering quantity decreases. Lee et al. [40] develop an efficient algorithm to compute a loss-averse newsvendor's optimal solution with uncertain demand and multiple options. Wang *et al.* [41] expand upon this by considering a supply chain with two risk averse retailers and demonstrate that the channel can be coordinated with a call option contract. Considering only the call option contract without any firm orders such as those under a wholesale price contract for a two-echelon supply chain with a risk-averse retailer and risk-neutral supplier, Juanjuan *et al.* [42] show that under some specific conditions, the supply chain can be coordinated. While these works begin to consider call option contract performance under risk aversion, they do not take into account the impact of a non-trivial shortage cost. In a setting with a risk averse retailer, shortage costs can have a substantial impact on behavior.

C. SHORTAGE COSTS WITH RISK AVERSE MEMBERS

A small number of papers exist that study shortage costs and risk aversion concurrently. Wang and Webster [13] study a loss-averse newsvendor problem with a non-trivial shortage cost and find that as the newsvendor's loss aversion increases, this leads to an increase in his optimal order quantity. Xu and Li [43] study a risk-averse newsvendor model, specifically adopting the CVaR criterion, with and without the shortage cost. The results show that when a substantial shortage cost is incorporated into the model, the complexity of the solution increases. Although both of these papers consider shortage costs, they do not consider them in a contractual environment where both a supplier and retailer are making decisions. Liu *et al.* [14] studies a newsvendor problem with random supply capacity. They show that under different conditions, when the shortage cost is considered, a loss-averse retailer may order less than, equal to or larger than the risk-neutral one. Further, when the shortage cost is less than a critical value, the loss-averse retailer's optimal order quantity is less than the risk-neutral retailer's.

To the best of our knowledge, only Chen *et al.* [10] and Yuan *et al.* [44] consider risk aversion and shortage costs in the context of contract design. Chen *et al.* [10] investigate a supply chain with a single risk neutral supplier and a single risk averse retailer who faces shortage costs. In this context, they study the performance of a call option contract as an independent contract and derive conditions for coordination. Different from our work, Chen *et al.* [10] do not consider the call option in combination with a wholesale price contract and do not adopt the CVaR criterion to characterize risk aversion. Yuan *et al.* [44] consider a supply chain consisting of a risk-neutral supplier and a risk-averse retailer, where the retailer orders call option from the supplier with an emergency order opportunity. When the emergency purchase price is low, the analytical model shows that the optimal order quantity of a risk-averse retailer is less than that of a risk-neutral retailer. When it is moderate or high, the risk-averse retailer may order more than, equal to, or less than a risk-neutral one. Computational studies show a risk-averse retailer may get higher profit than a risk-neutral one. Different from our work, Yuan *et al.* [44] do not consider the call option in combination with a wholesale price contract, and consider the retailer has an emergency order opportunity.

The extant literature shows that both retailer risk aversion and call option contracts have been widely studied. However, work that considers the intersection of these two streams is scarce. Furthermore, the consideration of shortage costs when retailer's are risk averse has received little attention. We contribute to these streams of literature by considering all three factors concurrently. In doing do so, we (1) provide new insights regarding the impact of shortage costs on optimal policies and supply chain performance, (2) highlight the importance of considering shortage costs when retailer's are risk averse and (3) provide recommendations for how to effectively manage inventory with call option contracts under new conditions.

III. MODEL FORMULATION AND ASSUMPTIONS

Consider a single-period two-party SC where a risk-neutral supplier who produces seasonal products sells via a risk-averse retailer to the end-users. Since the selling season is short enough and/or the production lead-time is long enough such that the supplier must produce all units prior to the start of the selling season. Therefore, the retailer has no chance to replenish inventory during the selling season and must make ordering decisions before the selling season.

In addition to placing initial firm orders (by a wholesale price contract), the retailer can purchase and exercise call options (by a call option contract) from the supplier to adjust his initial firm orders. The wholesale price contract is characterized by one parameter, i.e. the wholesale price, which is a per unit price the supplier charges the retailer for units the retailer commits to buying (i.e., firm orders). The call option contract is characterized by two parameters, namely, the option price and the exercise price. The option price is a per unit price the supplier charges the retailer to secure the right to purchase additional units (i.e., option orders). The exercise price is a per unit price the supplier charges the retailer for any additional units he actually purchases (i.e., options exercised). Therefore, the retailer fully commits to purchase firm orders but is not obligated to purchase option orders. However, the supplier is required to produce both firm and option orders, ensuring that they are available.

The notation used throughout this paper are listed in Table 1.

TABLE 1. Notation.

X	Random variable representing demand during the selling season, which is a continuous and differentiable, $X \geq 0$, $E(X) = \mu$ and $D(X) = \sigma^2$
$f(x)$	Probability density function of X
$F(x)$	Distribution function of X, which is a strictly increasing and invertible function, $F(0) = 0$ and $f(x) = F'(x)$
p	Unit retail price
c	Unit production cost
q_1	Firm order quantity of the retailer
q_2	Option order quantity of the retailer
q	Total order quantity of the retailer, $q = q_1 + q_2$
Q	Production quantity of the supplier
w	Unit wholesale price through a firm order
o	Unit option price
e	Unit exercise price
v	Unit salvage value after the selling season
h_r	Unit shortage cost of the retailer for market demand that cannot be filled

The sequence of events is illustrated in Figure 1 and unfolds over three distinct time periods marked by t_1 (start of the production season), t_2 (start of the selling season), and t_3 (end of the selling season). Before the production season (before t_1), the supplier offers the retailer a wholesale price contract (w), as well as a call option contract (o, e). Based on a preliminary demand forecast and a shortage cost

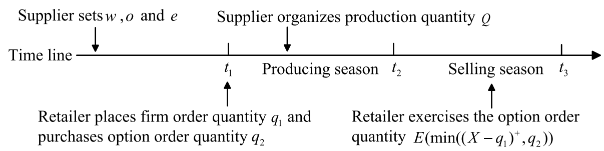

FIGURE 1. Sequence of events under the Portfolio Contracts.

penalty (h_r), at time t_1, the retailer considers the portfolio contracts and places orders with the supplier. Specifically, the retailer places a firm order for q_1 units and an option order for q_2 units, resulting in payments of wq_1 and oq_2 respectively, to the supplier. During the production season (between t_1 and t_2), the supplier will produce Q units resulting in a cost of cQ. At the start of the selling season (t_2) the retailer receives q_1. During the selling season (between t_2 and t_3), the retailer learns what realized demand is and determines whether or not to exercise the option contract. If $X < q_1$, the retailer will not exercise the options contract. At t_3, the retailer will salvage each of the remaining $q_1 - X$ units for v, resulting in revenue of $pX + v(q_1 - X)$. If $q_1 < X$, the retailer will exercise the call option, paying the supplier $e \min(X - q_1, q_2)$ and earning revenue of $p \min(X, q_1 + q_2)$. In addition, if $q_1 + q_2 < X$, the retailer will incur a per-unit shortage cost of h_r at time t_3 resulting in a cost of $h_r(X - q_1 - q_2)$.

To avoid trivial or impractical cases, we impose a series of assumptions regarding the costs in the SC. To ensure that the contracts are profitable for both parties, we assume $p > o + e > w > c$. Having $o + e > c$ and $w > c$ ensures profitability for the supplier while having $p > o + e$ and $p > w$ ensures profitability for the retailer. To ensure retailer participation, we further assume that $o + e > w > e$ and $h_r > w$. When the total cost of purchasing optional units exceeds the wholesale price, this ensures the retailer will place firm orders while having the wholesale price exceed the exercise price ensures the retailer will place option orders. By requiring the shortage cost to exceed the wholesale price, we create an incentive for the retailer to satisfy demand. Additionally, we assume that $c > o + v$ to avoid an arbitrage opportunity for the supplier. Finally, we assume that all information is known to both parties (i.e., is symmetric).

Since the retailer is risk-averse and wants to control the potential risk, we assume that the retailer takes CVaR as his performance measure. That is, the retailer's decision problem is to maximize CVaR. The definition of CVaR [20], [45], [46] is as follows

$$\mathrm{CVaR}_\eta(\pi_r) = \max_{\xi \in R} \left\{ \xi - \frac{1}{\eta} E[\xi - \pi_r]^+ \right\}.$$

Under this utility function, π_r is the retailer's random profit, ξ is a real number denoted as the target level of the profit, E is the expectation taken on the random demand X, and $\eta \in (0, 1]$ is a threshold quantile that reflects the retailer's degree of risk aversion. Note that if $\eta = 1$, then the value of CVaR is equal to the expected profit, and the retailer is risk-neutral.

If $0 < \eta < 1$, then the value of CVaR is less than the expected profit, and the retailer is risk-averse. Higher values of η imply a lower degree of risk-aversion.

IV. THE OPTIMAL ORDERING AND PRODUCTION POLICIES WITH PORTFOLIO CONTRACTS

The retailer is provided with the wholesale price contract (w) and the call option contract (o, e) at the beginning of the production season but prior to demand being realized. In consideration of the portfolio contracts, random demand (X) and the shortage cost (h_r), the retailer will determine the firm order quantity (q_1) and the option order quantity (q_2). Given the decision setting in Section III, the retailer's profit function is:

$$\pi_r(X, q_1, q_2) = p\min(q_1+q_2, X) + v[q_1-X]^+ - wq_1 - oq_2 - e\min(q_2, (X-q_1)^+) - h_r(X - q_1 - q_2)^+.$$

The first two terms above are the expected revenue from selling products and from salvage of overstock, respectively. The third, fourth and fifth terms capture the costs of purchasing the firm order and the option order, and of exercising the option order as required, respectively. The last term is the shortage penalty cost. Further, the retailer's corresponding CVaR measure is

$$\mathrm{CVaR}_\eta(\pi_r(X, q_1, q_2)) = \max_{\xi \in R}\left\{\xi - \frac{1}{\eta}E[\xi - \pi_r(X, q_1, q_2)]^+\right\}$$

Clearly, our goal is to find the optimal ordering decisions when the retailer is risk-averse under CVaR criterion, i.e.,

$$\max_{q_1>0, q_2>0} \mathrm{CVaR}_\eta(\pi_r(X, q_1, q_2)). \quad (1)$$

Let q_1^* and q_2^* be the optimal solutions to the decision problem presented in (1). The following theorem gives explicit expressions for q_1^* and q_2^*[2].

Theorem 1: With the portfolio contracts, $CVaR_\eta(\pi_r(X, q_1, q_2))$ is jointly concave with q_1 and q_2. The retailer's optimal firm order quantity is $q_1^* = F^{-1}(A)$ and optimal option order quantity is $q_2^* = \frac{1}{p+h_r-e}[(p-e)(F^{-1}(B) - F^{-1}(A)) + h_r(F^{-1}(C) - F^{-1}(A))]$, where $A = \frac{(e+o-w)\eta}{e-v}$, $B = \frac{(p+h_r-o-e)\eta}{p+h_r-e}$ and $C = 1 - \frac{o\eta}{p+h_r-e}$.

By characterizing the retailer's optimal ordering policy in Theorem 1, we can make two initial observations regarding q_1^* and q_2^*. First, since $e + o > w$ and $e > v$, we find that $q_1^* > 0$. Second, we find that $q_2^* > 0$ for any risk aversion level provided that $o \leq \frac{(w-v)(p+h_r-e)}{p+h_r-v}$. Together, these two results demonstrate that, regardless of the retailer's risk aversion level, he will always place a firm order and an option order as long as the option price, o, is low enough. For the remainder of this paper, we impose this condition on o to ensure that the retailer places both types of orders.

In characterizing the retailer's optimal firm and option quantities, we can derive the total order quantity for the retailer: $q^* = q_1^* + q_2^* = (p + h_r - e)^{-1}[(p - e)F^{-1}(B) + h_rF^{-1}(C)]$. Note that, if the retailer is risk-neutral (i.e., $\eta = 1$), then $q^* = F^{-1}(\frac{p+h_r-o-e}{p+h_r-e})$ and if shortage cost is ignored (i.e., $h_r = 0$), then $q^* = F^{-1}(\frac{(p-o-e)\eta}{p-e})$. Given that q^* is dependent on both η and h_r, the retailer's optimal ordering policy is more complicated than the case when the retailer is risk-neutral or shortage cost is not considered.

Next we investigate the impact of the retailer's level of risk aversion on his optimal ordering policy (q_1^*, q^*). Let $M(h_r, \eta) = (p-e)(p+h_r-o-e)f(F^{-1}(C)) - oh_rf(F^{-1}(B))$, then

Theorem 2: The risk-averse retailer's optimal order policy (q_1^*, q^*) has the following relationships with risk aversion coefficient:

(a) $\frac{dq_1^*}{d\eta} > 0$;
(b) if $M(h_r, \eta) > 0$, then $\frac{dq^*}{d\eta} > 0$, if $M(h_r, \eta) = 0$, then $\frac{dq^*}{d\eta} = 0$, otherwise, $\frac{dq^*}{d\eta} < 0$.

Theorem 2(a) shows that q_1^* is increasing in η. Since lower values of η indicate higher levels of retailer's risk-aversion, the optimal firm order quantity (q_1^*) decreases as the retailer becomes more risk-averse. Since $\eta = 1$ for a risk-neutral retailer, Theorem 2(a) also shows the risk-neutral q_1^* is strictly greater than the q_1^* for a risk-averse retailer. Furthermore, these results are independent of the shortage cost. Different from the case of the firm order quantity, Theorem 2(b) shows that q^*, and by extension, q_2^*, is not monotonic in η. The risk-averse retailer's optimal total order quantity can be less than, equal to or larger than that of the risk-neutral retailer. When $M(h_r, \eta) > 0(< 0)$, q^* is increasing (decreasing) in η, indicating that the total optimal order quantity decreases (increases) as the retailer's level of risk aversion increases. When $M(h_r, \eta) = 0$, q^* does not change as the retailer's level of risk aversion changes.

Therorem 2(b) also highlights the importance of considering the retailer's shortage cost when he is risk-averse. Recall that if $h_r = 0$, then $q^* = F^{-1}(\frac{(p-o-e)\eta}{p+-e})$ and thus, $\frac{dq^*}{d\eta} > 0$. This result is consistent with our intuition, showing that the risk-averse retailer's total optimal order quantity monotonically decreases as the retailer's level of risk aversion increases. However, our results show that, when the shortage cost is considered, under certain conditions, higher levels of risk aversion can lead the retailer to actually increase his total order quantity. A risk-averse retailer perceives a larger loss when profit is negative. Negative profit can be caused by a large overstock or a large understock. Without considering a cost of stocking out, the cost of understocking looms larger and the retailer will be incentivized only to lower his total order quantity. Conversely, when the cost of stocking out dominates that of overstocking, the potential negative profit from understocking becomes salient, causing the retailer to order more. Based on these results, we further investigate the relationship between the risk-averse retailer's optimal ordering policy and the shortage cost.

[2]Proofs of all theorems and corollaries are provided in the Appendix.

Theorem 3: q_1^* is constant in h_r, i.e, $\frac{dq_1^*}{dh_r} = 0$, while both q^* and q_2^* are strictly increasing in h_r, and $\frac{dq^*}{dh_r} = \frac{dq_2^*}{dh_r}$.

Theorem 3 states that as the shortage cost increases, the risk-averse retailer's optimal firm order quantity remains constant, while optimal total order quantity and optimal option order quantity strictly increase. Furthermore, if $\eta = 1$ (the retailer is risk-neutral), then the results of Theorem 3 still hold, showing that the retailer's risk aversion has no effect on the relationship between q^* and h_r. Therefore, by providing the risk-averse retailer the opportunity to purchase both firm and option orders, the portfolio contracts can address both the risk of overstocking and of understocking. A risk-averse retailer's fear of low or negative profits due to overstocking can be addressed by lowering the firm order quantity. The retailer can use option orders to manage the risk that realized demand may be higher than the firm order quantity since he is able to exercise those options after demand is realized.

The next corollary demonstrates how the retailer's optimal ordering policy changes as the wholesale price, the option price or the exercise price increases.

Corollary 1: The effects of price parameters of the portfolio contracts on the optimal order policy (q_1^*, q^*) are as follows:

(a) q_1^* is decreasing in w, and q^* is constant in w;
(b) q_1^* is increasing in o, while q^* is decreasing in o;
(c) q_1^* is increasing in e, while q^* is complicated in e. If $H(h_r, \eta) > 0$, then q^* is increasing in e, if $H(h_r, \eta) = 0$, then q^* is constant in e, otherwise, q^* is decreasing in e, where $H(h_r, \eta) = h_r[F^{-1}(C) - F^{-1}(B)] - \frac{(p-e)o\eta}{(p+h_r-e)f(F^{-1}(B))} - \frac{oh_r\eta}{(p+h_r-e)f(F^{-1}(C))}$.

Corollary 1(a) shows that when the wholesale price, w, increases, the retailer will decrease his optimal firm order quantity. However, as w increases, the total optimal order quantity remains constant, indicating that q_2^* increases. This implies that optimal total order quantity is unrelated to the wholesale price w. However, Corollary 1(b) shows that q^* does change as the option price, o, changes. Specfically, q_1^* is increasing in o while q^* is decreasing in o. Since $\frac{dq_2^*}{do} < -\frac{dq_1^*}{do} < 0$, the optimal firm order quantity is less sensitive to changes in o than the optimal option order quantity. These results mean that when the option becomes more expensive, the retailer reacts by increasing his firm order quantity. At the same time, the retailer decreases his option order quantity (q_2^*). The increase in the firm order quantity is less than the decrease in the option order quantity, resulting in a decrease in the total order quantity. Therefore, when all else remains constant, the supplier's mechanism to alter the retailer's total order quantity is the option price, o.

Corollary 1(c) shows that, similar to the relationship between risk aversion and the total optimal order quantity, the relationship between the exercise price and the total optimal order quantity is not monotonic. If the retailer is risk-neutral, then $H(h_r, 1) = -\frac{o}{f(F^{-1}(\frac{p+h_r-o-e}{p+h_r-e}))} < 0$ and if shortage cost is not considered, then $H(0, \eta) = -\frac{o\eta}{f\left(F^{-1}\left(\frac{(p-o-e)\eta}{p-e}\right)\right)} < 0$. Therefore, when only one factor is considered, q^* is monotonically decreasing in e. However, when both risk aversion and shortage cost are considered, the retailer's optimal total order quantity ($q*$) may be increasing or constant in e, both of which will never occur when the retailer is risk-neutral or shortage cost is not considered. That is because $q*$ is influenced not only by η and h_r, but is also influenced by demand uncertainty. When the retailer has the opportunity to purchase both firm and option orders, he can adjust both quantities simultaneously to account for the trade-off between demand uncertainty and shortage/overage costs.

Corollary 1 demonstrates the benefits of the portfolio contracts in allowing the retailer to account for demand uncertainty in both directions. Based on these results, we analyze the effect of demand uncertainty on the optimal order policy. For simplicity, we consider only the case when demand is normally distributed with mean μ and standard deviation σ.

Corollary 2: When market demand uncertainty changes, the change in the optimal order policy is as follows:

(a) if $2(e+o-w)\eta > e-v$, then q_1^* is increasing in σ, and $q_1^* > \mu$; if $2(e+o-w)\eta = e-v$, then q_1^* is constant in σ, and $q_1^* = \mu$; otherwise, q_1^* is decreasing in σ, and $q_1^* < \mu$
(b) q_2^* is strictly increasing in σ, and $q_2^* > \mu$.

Corollary 2(a) shows that the the relationship between the retailer's optimal firm order quantity and demand uncertainty is dependent on the relationship between the portfolio contract parameters and the level of risk aversion. The retailer's optimal firm order quantity can be either greater than μ and increasing in σ, equal to μ and constant in σ or less than μ, and decreasing in σ. Contrastingly, the retailer's optimal option quantity, q_2^*, is strictly greater than μ, increasing in σ, and independent of η. This means that, regardless of risk aversion, as variance in demand grows, the retailer relies more on the right to exercise options to satisfy demand once it is realized. Accounting for risk aversion and additional contract parameters, when the exercise price is large enough, the retailer increases the firm order quantity at the same time to avoid the necessity to exercise the option. When the exercise price is low enough, the retailer counters this action by decreasing the firm order quantity.

In what follows, we consider the supplier's optimal production policy. At the beginning of the selling season, although the retailer may exercise either part of or all of the call options due to seasonal product's intrinsic attributes, the supplier has to manufacture the exact quantity the retailer orders. In fact, the call option contract transfers random demand risk from the retailer to the supplier. Thus, the optimal production quantity of the supplier is equal to the total optimal quantity the retailer orders, i.e., $Q^* = q_1^* + q_2^*$. Given the decision setting presented in Section 3, the supplier's expected profit function is

$$\pi_s(Q^*) = wq_1^* + oq_2^* + eE[\min(q_2^*, (x - q_1^*)^+)] + vE(q_2^* - (x - q_1^*)^+)^+ - c(q_1^* + q_2^*).$$

The first term represents revenue that the supplier derives from the retailer's firm order. The second and third terms are the revenue from option sales and exercised options. Note that revenue from the firm order quantity and the option sales are known while the revenue from the exercised options is not known until demand is realized. Therefore, the introduction of the call option contract transfers some of the risk from demand uncertainty from the retailer to the supplier. The fourth term is the revenue from salvaging the excess option units not exercised by the retailer and the last term is the production cost. Then

$$\pi_s(Q^*) = wq_1^* + oq_2^* + e\left(q_2^* - \int_{q_1^*}^{q_1^*+q_2^*} F(x)dx\right) + v\left(\int_{q_1^*}^{q_1^*+q_2^*} F(x)dx\right) - c(q_1^* + q_2^*). \quad (2)$$

For the retailer, we analyzed how the parameters of the portfolio contracts and the shortage cost impacted the optimal ordering policy. Interestingly, we found that q^* was not impacted by w (Corollary 1(a)) and that while h_r had no impact on q_1^*, both q^* and q_2^* were increasing in h_r (Theorem 3). The following results analyze how these effects impact the supplier's optimal expected profit.

Theorem 4: Under the portfolio contracts, the optimal expected profit of the supplier is increasing in w, i.e., $\frac{d\pi_s(Q^*)}{dw} > 0$.

Recall that, according to Corollary 1(a), the retailer's optimal total order quantity is independent of the wholesale price. Theorem 4 shows that as the wholesale price increases, the optimal expected profit of the supplier increases. It is clear that the supplier can improve his expected profit by setting a relatively high wholesale price without influencing the retailer's total order quantity. Corollary 1(a) also shows that when the wholesale price increases, the retailer decreases his optimal firm order and increases his optimal option order. Since $w < o + e$, the retailer's total order cost is increasing in w. Then the retailer's optimal expected profit is decreasing in w. These results, in conjunction with Theorem 4, imply that the wholesale price w can be used to adjust profit distribution between the supplier and the retailer.

From Theorem 2, when the retailer is risk-averse and adopts the CVaR criterion, the presence of a shortage cost creates an intricate effect on the retailer's optimal order policy. By extension, the shortage cost will also impact the supplier's optimal expected profits.

Theorem 5: If $o+e-(e-v)F(q^*) > c$, *then* $\frac{d\pi_s(Q^*)}{dh_r} > 0$; *if* $o+e-(e-v)F(q^*) = c$, *then* $\frac{d\pi_s(Q^*)}{dh_r} = 0$; *if* $o+e-(e-v)F(q^*) < c$, *then* $\frac{d\pi_s(Q^*)}{dh_r} < 0$.

Theorem 5 shows that the relationship between the supplier's optimal expected profit and the shortage cost is determined by production cost, option price, exercise price, total order quantity and the demand distribution function. Also, note that this relationship is independent of w. Recall from Theorem 2 that the retailer's firm order quantity was not impacted by h_r. Since the firm order quantity is the only quantity associated with w, the wholesale price does not interact with h_r to impact the supplier's optimal expected profit. We find that when the expected revenue of the supplier from option sales and exercised options is larger than production cost (i.e., $o + e - (e - v)F(q^*) > c$), the supplier's optimal expected profit is increasing in h_r. When the expected revenue of the supplier from option sales and exercised options is equal to production cost (i.e., $o + e - (e - v)F(q^*) = c$), the supplier's optimal expected profit is constant in h_r. When the expected revenue of the supplier from option sales and exercised options is less than production cost (i.e., $o + e - (e - v)F(q^*) < c$), the supplier's optimal expected profit is decreasing in h_r.

The intuition for these results is easily reconciled with results regarding the retailer's optimal option quantity (q_2^*). Recall that as h_r increases, the retailer's option order quantity increases. When the revenue from options outweighs the supplier's production cost, it is beneficial to the supplier to have the retailer place more option orders. However, if the production cost outweighs the revenue from options, then this hurts the supplier. These results also highlight the need for the supplier to make efforts to decrease production cost.

V. THE EFFECT OF THE CALL OPTION CONTRACT

The above results demonstrate that the introduction of the call option contract is an effective tool for the retailer and the supplier when risk aversion and a shortage cost are present. In this section, we further investigate the effect of the introduction of the call option contract by comparing it to a case when only a wholesale price contract is offered. Consider a base model where the risk-averse retailer cannot purchase a call option. This means that there is no purchase or supply flexibility and the retailer's order quantity will be the supplier's production quantity. To keep this baseline setting distinct from the case of the introduction of the call option contract, we will refer to the retailer's order quantity as q_0. In this case, the retailer's expected profit function reduces to that of a wholesale price contract with shortage costs.

$$\pi_r(X, q_0) = p\min(q_0, X) + v[q_0 - X]^+ - wq_0 - h_r(X - q_0)^+.$$

As in Section 4, we assume that the retailer is risk averse and adopts the CVaR criterion leading to the following function.

$$\mathrm{CVaR}_\eta(\pi_r(X, q_0)) = \max_{\xi \in R}\left\{\xi - \frac{1}{\eta}E[\xi - \pi_r(X, q_0)]^+\right\}$$

Under this assumption, the retailer solves for the single order quantity, q_0, that solves the following decision problem.

$$max_{q_0 \geq 0}\mathrm{CVaR}_\eta(\pi_r(X, q_0)). \quad (3)$$

Let q_0^* be the optimal solution for the problem presented in (3). The following lemma characterizes the retailer's optimal order quantity under CVaR without the call option contract.

Lemma 1: Without the call option contract, $CVaR_\eta(\pi_r(X, q_0))$ is a concave function with q_0, and the optimal order quantity is given by $q_0^* = \frac{1}{p+h_r-v}[(p-v)F^{-1}(B') + h_rF^{-1}(C')]$, where $B' = \frac{(p+h_r-w)\eta}{p+h_r-v}$ and $C' = 1 - \frac{(w-v)\eta}{p+h_r-v}$.

Similar to Theorem 1, Lemma 1 derives that when the retailer places orders only by the wholesale price contract, the retailer's optimal ordering quantity exists and is unique. As seen in the characterization of q_0^* in Lemma 1, the optimal order quantity is dependent on w, whereas, in the case of the introduction of the call option contract, the optimal total order quantity q^* was independent of w. Note that the optimal order quantity q_0^* without the call option contract and q^* with the call option contract have similar structural form. The following theorem characterizes the specific relationship between these two order quantities.

Theorem 6: The retailer's total optimal order quantity q^* with the call option contract is larger than the optimal order quantity q_0^* without the call option contract, while the firm order quantity q_1^* with the call option contract is less than that without the call option contract, i.e., $q^* > q_0^* > q_1^*$.

This result indicates that under the portfolio contracts, the quantity that the retailer commits to purchasing is less than what they purchase when the call option contract is not offered. When the call option contract is offered, because the the retailer buys options, he has the flexibility to purchase a larger total amount than with the wholesale price contract. In other words, the portfolio contracts allow the retailer to better manage the risk of overstocking by placing a lower firm order. But, it also allows the retailer to better manage the risk of understocking and potentially achieve higher levels of revenue by reserving access to a larger number of total units that are only used if necessary.

Theorem 6 demonstrates the benefit of the call option contract with respect to flexibility for the retailer. The next result investigates whether or not the call option contract benefits the retailer by comparing the maximum CVaR derived from the portfolio contracts, $\text{CVaR}_\eta(\pi_r(X, q_1^*, q_2^*))$, and the wholesale price contract, $\text{CVaR}_\eta(\pi_r(X, q_0^*))$. By model comparison, the following theorem is obtained.

Theorem 7: The retailer's maximum CVaR with the call option contract is better off than that without the call option contract, i.e., $CVaR_\eta(\pi_r(X, q_1^*, q_2^*)) > CVaR_\eta(\pi_r(X, q_0^*))$.

Theorem 7 states that when the retailer is risk-averse and there is a shortage cost, with the call option contract, the retailer will always obtain greater CVaR than without. In combination with Theorem 6, this demonstrates that the introduction of the call option contract benefits the retailer by providing greater flexibility without sacrificing CVaR. In fact, it achieves even higher CVaR than a wholesale price contract.

We now consider whether or not the introduction of the call option contract is beneficial for the supplier. When the retailer obtains products only through the wholesale price contract, the supplier's optimal production quantity, Q_0^*, is equal to the retailer's optimal order quantity, q_0^*. The optimal expected profit of the supplier is $\pi_s(Q_0^*) = (w-c)q_0^*$. Given Theorem 6 and that $Q_0^* = q_0^*$, we can conclude that $Q^* > Q_0^*$. In other words, the supplier's optimal production quantity is always larger when the call option contract is offered. However, since part of this production quantity is due to options, the supplier may not sell the entire quantity to the retailer. Therefore, we now examine the effect of the call option contract on the optimal expected profit of the supplier. We can obtain the following theorem.

Theorem 8: The optimal expected profit of the supplier with the call option contract is larger than that without the call option contract, i.e., $\pi_s(Q^*) > \pi_s(Q_0^*)$.

This theorem clearly states that the supplier is always better off when he offers the portfolio contracts that include a call option contract rather than only a wholesale price contract. Based on Theorems 7 and 8, we can easily obtain that $\text{CVaR}_\eta(\pi_r(X, q_1^*, q_2^*)) + \pi_s(Q^*) > \text{CVaR}_\eta(\pi_r(X, q_0^*)) + \pi_s(Q_0^*)$ proving that the introduction of the call option contract benefits each player individually as well as the SC.

VI. SUPPLY CHAIN COORDINATION

In the previous sections, we have discussed how the introduction of the call option contract impacts optimal SC decisions and shown that portfolio contracts provide benefits to both parties in the SC when the retailer is risk averse and faces shortage costs. However, the baseline for comparison in the prior section was a wholesale price contract, a contract that does not coordinate the SC. Therefore, one issue left to address is whether the portfolio contracts merely provide increased profits over the wholesale price contract or if they can also coordinate the SC. As a benchmark, we assume that a single, risk-neutral decision maker controls the SC and decides the production quantity to maximize the expected profit of the SC [10]. We assume that the single decision maker's production quantity is q_c and the production cost, shortage cost, salvage value and retailer price are the same as in prior sections. Under these assumptions, the SC's expected profit is:

$$E[\pi(X, q_c)] = pE\min(q_c, X) + vE(q_c - X)^+ - h_rE(X - q_c)^+ - cq_c,$$

where the first two terms are the system's expected revenue and salvage revenue, the third term is the shortage cost, and the last term is the production cost. Then

$$E[\pi(X, q_c)] = (p+h_r-v)\left(\int_0^{q_c} xf(x)dx + \int_{q_c}^{\infty} q_cf(x)dx\right) + (v-c)q_c - h_r\mu.$$

The centralized SC's decision problem is $\max_{q_c\geq 0} E[\pi(X, q_c)]$. It is not difficult to obtain that $E[\pi(X, q_c)]$ is concave in q_c. Thus the optimal production quantity of the system is $q_c^* = F^{-1}\left(\frac{p+h_r-c}{p+h_r-v}\right)$. Note that since the supplier adopts the make-to-order supply policy and supplies product up to q^* under the portfolio contracts, to achieve the maximum expected profit of the system, it is sufficient for the supplier to provide a call option contract to push the retailer to pursue a total order quantity up to the production quantity of the system, i.e., $q^* = q_c^*$. This observation leads to the following result.

Theorem 9: When

$$(p + h_r - e)F^{-1}(D) = (p - e)F^{-1}(B) + h_r F^{-1}(C) \quad (4)$$

is satisfied, the SC can be coordinated by the portfolio contracts, where $D = \frac{p+h_r-c}{p+h_r-v}$.

Theorem 9 states that the channel coordination is determined by the relationship between the level of risk aversion (η), option price (o), exercise price (e), retail price (p), production cost (c), shortage cost (h_r) and demand uncertainty. Recall that from Theorem 1, the risk-averse retailer's optimal total order quantity was independent of w under the portfolio contracts. Similarly, we find that the condition on SC coordination is independent of w. Although we saw in Theorem 4 that w increases the supplier's expected profit and decreases the retailer's expected profit, it does not impact the total profit of the SC. Instead, w is merely a mechanism that controls the allocation of profit between the supplier and retailer when introducing the call option contract. In a non-coordinated system, SC expected profits are lower than in a coordinated system. Thus, when compared with a non-coordinating contract, the portfolio contracts can achieve Pareto optimality by regulating wholesale price. That is, the two parties' profits do not decrease and at least on party is strictly better off.

In addition to being independent of w, SC coordination is also independent of demand uncertainty under specific conditions. According to Theorem 9, if the retailer is risk-neutral (i.e., $\eta = 1$), we can obtain from Equation (4) that when $o = \frac{(p+h_r-e)(c-v)}{(p+h_r-v)}$, the SC can be coordinated. If the shortage cost is ignored ($h_r = 0$), then we can obtain from Equation (4) that when $o = [1 - \frac{p-c}{(p-v)\eta}](p-e)$, the SC can be coordinated. These results reveal that when the retailer is risk-neutral or shortage costs are absent, the portfolio contracts coordinate the SC, regardless of the demand distribution. However, when both risk aversion and shortage costs are present, Theorem 9 shows that the condition of channel coordination depends on the demand distribution. This dependency occurs because the optimal total order quantity (q^*) is not monotonic in η and its direction is determined by the relationship between η, h_r and demand uncertainty (i.e, $M(h_r, \eta)$).

VII. NUMERICAL EXPERIMENTS

In this section, we make use of numerical experiments to further validate the impact of the introduction of the call option contract on optimal decisions and illustrate the effects of risk aversion and shortage cost on the optimal ordering policy. Let $p = 30$, $c = 10$, $o = 5$ and $v = 2$. It is assumed that the random demand satisfies the normal distribution with mean ($\mu = 100$) and standard deviation (σ). Note that although the parameters are assigned specific values and normal demand distribution are used, the results of the theoretical analysis are independent on the specific values and distribution of demand. To evaluate the effect of risk aversion on the optimal total order quantity q^*, we vary the level of risk aversion, η, from 0.1 to 1 in steps of 0.1 with fixed values of $h_r = 25$, $w = 22$ and $e = 20$. In addition, we consider three different levels of demand variation: $\sigma = 75, \sigma = 50$ and $\sigma = 25$. The results of this numerical experiment are presented in Figure 2.

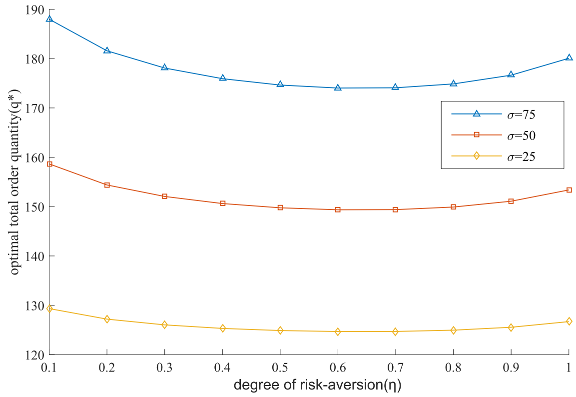

FIGURE 2. **The effect of risk aversion on the optimal order quantity.**

For each level of demand variation, the optimal total order quantity is first decreasing in η and decreases more slowly as η increases. At a certain point, q^* begins to increase in η and increases more rapidly as η continues to increase. In addition, Figure 2 shows that the risk averse retailer can order less than, equal to or larger than the corresponding risk neutral retailer ($\eta = 1$). These results are consistent with Theorem 2. Figure 2 also illustrates that the higher the level of demand variation, the larger the optimal total order quantity. Further, from these figures, we can observe that as σ increases, the U-shaped relationship between η and q^* becomes steeper. Therefore, as demand variation grows, the role of risk aversion becomes more prominent.

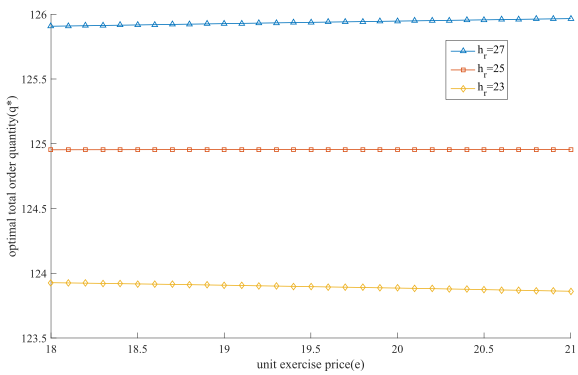

FIGURE 3. **The effect of exercise price on the optimal order quantity.**

In Figure 3, we illustrate the effect of the exercise price on the optimal total order quantity. In this experiment, we consider three shortage costs ($h_r = 23$, 25, and 27), and fix $w = 22$, $\sigma = 25$ and $\eta = 0.8$. In addition, we vary e from

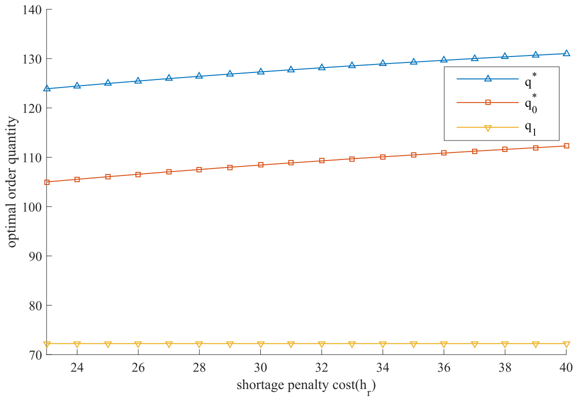

FIGURE 4. The effect of shortage penalty cost on the optimal order quantity.

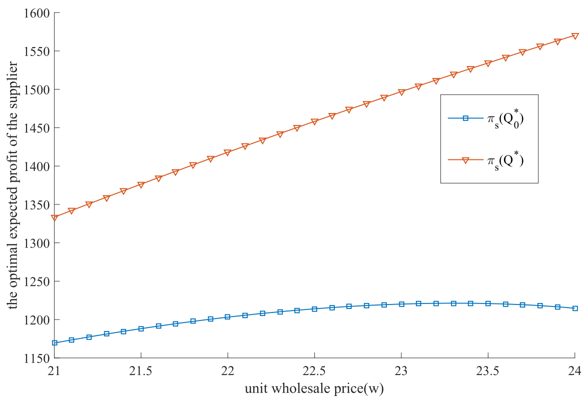

FIGURE 5. The effect of wholesale price on the optimal expected profit.

18 to 21 in steps of 0.1. As Figure 3 demonstrates, under different shortage costs, the exercise price can have varying effects on the optimal total order quantity, which is consistent with Corollary 1. We see that when $h_r = 23$, q^* decreases as e increases while when $h_r = 27$, q^* increases as e increases. When $h_r = 25$, the relationship between q^* and e is not monotonic.

Next, we analyze the effect of the shortage cost on the optimal ordering policy by comparing q^*, q_1^* and q_0^*. We vary h_r from 23 to 40, in steps of 1 and fix $w = 22$, $\sigma = 25$, $\eta = 0.8$ and $e = 20$. The results are illustrated by Figure 4. We can observe that the optimal firm order quantity q_1^* is constant with respect to h_r, while both q^* and q_0^* are strictly increasing in h_r. This result is in accordance with Theorem 3. Moreover, Figure 4 illustrates that the optimal order quantity without the call option contract, q_0^* is smaller than the total optimal order quantity with the call option contract, q^*, but is larger than optimal firm order quantity, q_1^*, which is consistent with Theorem 6.

To illustrate the impact of the introduction of the call option contract on the supplier's optimal profit, we fix $h_r = 25$, $\sigma = 25$, $\eta = 0.8$ and $e = 20$, and vary w from 21 to 24, in steps of 0.1. Figure 5 illustrates the supplier's optimal expected profit with respect to wholesale price, with and without the call option contract. As shown in Figure 5, the optimal expected profit of the supplier is increasing in w when the call option contract is offered while it is not monotonic in w without the call option contract. Moreover, the optimal expected profit of the supplier when the call option contract is introduced (i.e.,$\pi_s(Q^*)$) is larger than the optimal expected profit when it is not offered (i.e.,$\pi_s(Q_0^*)$). These results are consistent with Theorem 4 and Theorem 8. Figure 5 also illustrates that as wholesale price increases, $\pi_s(Q^*) - \pi_s(Q_0^*)$ becomes larger, indicating that the effects of risk aversion and demand uncertainty on the optimal expected profit of the supplier are reduced when the call option contract is introduced.

VIII. CONCLUSION

This paper investigated a supply chain involving a single risk-neutral supplier and a single risk-averse retailer, where the retailer adopts CVaR as his performance measure. Different from prior analytical research that considers this supply chain setup, we assume that the retailer faces a non-trivial shortage cost. The inclusion of this cost emphasizes that not only does the retailer face substantial risk in over-ordering but also in under-ordering. To hedge against the risk of not satisfying demand, we allow the retailer to purchase option orders, in addition to firm orders, from the supplier through a call option contract. The presence of both order types allows the retailer to effectively manage the risk of over-ordering (through the firm order) and under-ordering (through the option order). In this context, we explore the supplier's optimal production policy and the retailer's optimal order policy and compare results with that of a simple wholesale price contract.

Our analytical findings provide several insights. First, we find that when shortage cost is considered, the retailer's optimal total order quantity is not monotonic in the risk aversion coefficient, which is different from the case without shortage cost. Second, we find that the retailer's optimal total order quantity may be increasing in exercise price when both risk aversion and shortage cost are considered, which never happen, when the retailer is risk-neutral or shortage cost is not considered. Finally, compared to a case when the retailer only places a firm order (a wholesale price contract), we find that the introduction of the call option contract is Pareto-improving for both players and we are able to establish conditions under which the supply chain is coordinated. Although the conditions for coordination are complex, they provide important insight by demonstrating that both the level of retailer risk aversion and demand uncertainty play a role, which is not the case when the retailer is risk neutral or shortage cost is not considered.

These analytical findings generate many managerial implications. When designing contracts in practice, our results underscore the need to account for the retailer's risk aversion. Although it adds to the supplier's cost, it is beneficial to the supplier's overall profit to invest in flexible production policies. Not allowing the retailer to place a second order in the future may benefit the supplier in the short term, but providing this flexibility allows the supply chain to more efficiently match demand. In practice, retailers often face considerable costs for not satisfying demand. Our research calls attention to the importance of knowing what these shortage costs are. When these costs are large, being able to satisfy demand more often can be crucial for the long term success of a supply chain.

Given the results of our research, we suggest some potential directions for future research. First, our work assumed that both the retailer and supplier are aware of the retailer's level of risk aversion. To represent a more realistic scenario, one possible extension of this work is to consider that the information regarding the retailer's risk aversion is asymmetric. This would add to the complexity of the supplier's contract parameter decisions. Second, we considered only a two-party channel problem. This framework could be extended to include multiple suppliers, multiple retailers or both. Considering multiple players at each level of the supply chain introduces competition which could lead to useful results that mimic practice more closely. Third, we have considered uncertainty in demand, but not uncertainty in supply which can exist in practice. Given that demand uncertainty impacts the ability to coordinate the supply chain, it would be interesting to include supply uncertainty in the framework. Finally, in practice, retailers may interact with suppliers over multiple time periods but our model only considers a single time period. While this simplified scenario provides insights regarding the role of shortage costs and option orders, the consideration of multiple time periods could potentially increase our understanding of the benefits of allowing flexible ordering. The provision of this option could lead to better relationships between suppliers and retailers and satisfying demand more often might impact future shortage costs.

APPENDIX

Proof of Theorem 1: Let $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}E[\xi - \pi_r(X, q_1, q_2)]^+$, $\text{CVaR}_\eta(\pi_r(X, q_1, q_2) = \max_{\xi \in R} g(\xi, q_1, q_2)$. For any fixed q_1 and q_2, we can maximize $g(\xi, q_1, q_2)$ over $\xi \in R$. Obviously, we have

$$\begin{aligned}
g(\xi, q_1, q_2) &= \xi - \frac{1}{\eta}\int_0^{q_1}[\xi + (w-v)q_1 + oq_2 - x(p-v)]^+ f(x)dx \\
&\quad - \frac{1}{\eta}\int_{q_1}^{q_1+q_2}[\xi + (w-e)q_1 + oq_2 - x(p-e)]^+ f(x)dx \\
&\quad - \frac{1}{\eta}\int_{q_1+q_2}^{\infty}[\xi + h_r x - (p + h_r - w)q_1 \\
&\quad - (p + h_r - o - e)q_2]^+ f(x)dx.
\end{aligned}$$

If $\xi \le -(w-v)q_1 - oq_2$, then $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r}}^{\infty}[\xi + h_r x - (p + h_r - w)q_1 - (p + h_r - o - e)q_2]f(x)dx$, and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta}[1 - F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r})]$. Set $\xi^1(q_1, q_2) = (p + h_r - w)q_1 + (p + h_r - o - e)q_2 - h_r F^{-1}(1 - \eta)$. Obviously, if $\xi^1(q_1, q_2) \le -(w-v)q_1 - oq_2$, i.e, $(p + h_r - v)q_1 + (p + h_r - e)q_2 \le h_r F^{-1}(1 - \eta)$, then $\xi^1(q_1, q_2)$ is the solution of $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 0$ for fixed q_1 and q_2. Therefore, $g(\xi^1(q_1, q_2), q_1, q_2) = \xi^1(q_1, q_2) - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^1(q_1,q_2)}{h_r}}^{\infty}[\xi^1(q_1, q_2) + h_r x - (p + h_r - w)q_1 - (p + h_r - o - e)q_2]f(x)dx$. However, $\frac{\partial g(\xi^1(q_1,q_2),q_1,q_2)}{\partial q_1} = p + h_r - w > 0$ and $\frac{\partial g(\xi^1(q_1,q_2),q_1,q_2)}{\partial q_2} = p + h_r - o - e > 0$, which indicates that $\xi^1(q_1, q_2)$ is not the solution of $\max_{\xi \in R} g(\xi, q_1, q_2)$. Thus $(p + h_r - v)q_1 + (p + h_r - e)q_2 > h_r F^{-1}(1 - \eta)$.

If $-(w-v)q_1 - oq_2 < \xi \le (p-w)q_1 - oq_2$, then $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}\int_0^{\frac{\xi+(w-v)q_1+oq_2}{p-v}}[\xi + (w-v)q_1 + oq_2 - x(p-v)]f(x)dx - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r}}^{\infty}[\xi + h_r x - (p + h_r - w)q_1 - (p + h_r - o - e)q_2]f(x)dx$, and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta}[F(\frac{\xi+(w-v)q_1+oq_2}{p-v}) + 1 - F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r})]$. Note that $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi}|_{\xi=-(w-v)q_1-oq_2} = \frac{1}{\eta}F(\frac{(p+h_r-v)q_1+(p+h_r-e)q_2}{h_r}) + 1 - \frac{1}{\eta} > 0$. If $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi}|_{\xi=(p-w)q_1-oq_2} = 1 - \frac{1}{\eta}[F(q_1) + 1 - F(\frac{h_r q_1+(p+h_r-e)q_2}{h_r})] < 0$, then $\xi^2(q_1, q_2)$ satisfy $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 0$, and we have $F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^2(q_1,q_2)}{h_r}) - F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}) = 1 - \eta$. $\frac{\partial g(\xi^2(q_1,q_2),q_1,q_2)}{\partial q_1} = \frac{1}{\eta}[(v - w)F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}) + (1 - F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^2(q_1,q_2)}{h_r}))(p + h_r - w)) = \frac{1}{\eta}[(v - w)F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}) + (p + h_r - w)(\eta - F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v})) = \frac{1}{\eta}[(p + g - w)\eta - (p + g - v)F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v})).$ $\frac{\partial g(\xi^2(q_1,q_2),q_1,q_2)}{\partial q_2} = \frac{1}{\eta}[-oF(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}) + (1 - F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^2(q_1,q_2)}{h_r}))(p + h_r - o - e)) = \frac{1}{\eta}[-oF(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}) + (p + h_r - o - e)(\eta - F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v})) = \frac{1}{\eta}[(p + g - o - e)\eta - (p + g - e)F(\frac{\xi^2(q_1,q_2)+(w-v)q_1+oq_2}{p-v}))$. Clearly, there does not exist $\xi^2(q_1, q_2)$ which satisfies both $\frac{\partial g(\xi^2(q_1,q_2),q_1,q_2)}{\partial q_1} = 0$ and $\frac{\partial g(\xi^2(q_1,q_2),q_1,q_2)}{\partial q_2} = 0$. Thus $1 - \frac{1}{\eta}[F(q_1) + 1 - F(\frac{h_r q_1+(p+h_r-e)q_2}{h_r})] > 0$.

If $(p-w)q_1 + (p-o-e)q_2 < \xi$, then $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}\int_0^{q_1}[\xi + (w-v)q_1 + oq_2 - x(p-v)]f(x)dx - \frac{1}{\eta}\int_{q_1}^{q_1+q_2}[\xi + (w-e)q_1 + oq_2 - x(p-e)]f(x)dx - \frac{1}{\eta}\int_{q_1+q_2}^{\infty}[\xi + h_r x - (p + h_r - w)q_1 - (p + h_r - o - e)q_2]f(x)dx$, and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta} < 0$.

If $(p-w)q_1 - oq_2 < \xi \le (p-w)q_1 + (p-o-e)q_2$, then $g(\xi, q_1, q_2) = \xi - \frac{1}{\eta}\int_0^{q_1}[\xi + (w-v)q_1 + oq_2 - x(p-v)]f(x)dx - \frac{1}{\eta}\int_{q_1}^{\frac{\xi+(w-e)q_1+oq_2}{p-e}}[\xi + (w-e)q_1 + oq_2 - x(p-e)]f(x)dx - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r}}^{\infty}[\xi + h_r x - (p + h_r - w)q_1 - (p + h_r - o - e)q_2]f(x)dx$, and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi} = 1 - \frac{1}{\eta}[F(\frac{\xi+(w-e)q_1+oq_2}{p-e}) + 1 - F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi}{h_r})]$. Note that $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi}|_{\xi=(p-w)q_1-oq_2} = \frac{1}{\eta}F(\frac{h_r q_1+(p+h_r-e)q_2}{h_r}) - \frac{1}{\eta}[F(q_1) + 1 - \frac{1}{\eta} > 0$ and $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi}|_{\xi=(p-w)q_1-oq_2} =$

$1-\frac{1}{\eta}<0$. Therefore, there must exist $\xi^3(q_1,q_2)$ which satisfies $\frac{\partial g(\xi,q_1,q_2)}{\partial \xi}=0$, i.e, $\xi^3(q_1,q_2)=arg\max_{\xi\in R} g(\xi,q_1,q_2)$, then $CVaR_\eta(\pi_r(X,q_1,q_2)=g(\xi^3(q_1,q_2),q_1,q_2)$. To facilitate analysis, we further denote $A=F(q_1)$, $B=F(\frac{\xi^3(q_1,q_2)+(w-e)q_1+oq_2}{p-e})$ and $C=F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^3(q_1,q_2)}{h_r})$, then $C-B=1-\eta$. Taking the first partial derivative of $g(\xi^3(q_1,q_2),q_1,q_2)$ with respect to q_1 and q_2, we have $\frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_1}=\frac{1}{\eta}[(v-e)A+(v-w)B+(p+h_r-w)(1-C)]=\frac{-1}{\eta}[(e-v)A+(p+h_r-v)B+(p+h_r-w)\eta]$ and $\frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_2}=\frac{1}{\eta}[-oB+(p+h_r-o-e)(1-C)]=\frac{-1}{\eta}[(p+h_r-e)B-(p+h_r-o-e)\eta]$. Further, taking the first partial derivatives of $\frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_1}$ and $\frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_2}$ with respect to q_1 and q_2, we have $\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial {q_1}^2}=\frac{-1}{\eta}[(e-v)f(F^{-1}(A))+\frac{(p+h_r-v)(w-e)}{p-e}f(F^{-1}(B))]<0$, $\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial {q_2}^2}=\frac{-1}{\eta}[\frac{o(p+h_r-e)}{p-e}f(F^{-1}(B))]<0$, $\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_1\partial q_2}=\frac{-o(p+h_r-v)}{(p-e)\eta}f(F^{-1}(B))$ and $\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_2\partial q_1}=\frac{-(p+h_r-v)(w-e)}{(p-e)\eta}f(F^{-1}(B))$. With some algebra, we have $\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial {q_1}^2}\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial {q_2}^2}-\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_1\partial q_2}\frac{\partial^2 g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_2\partial q_1}=\frac{1}{\eta^2}[\frac{o(e-v)(p+h_r-e)}{p-e}f(F^{-1}(A))f(F^{-1}(B))]>0$, which indicates that $g(\xi^3(q_1,q_2),q_1,q_2)$ is strictly jointly concave in q_1 and q_2. According to the first order optimality condition, the optimal solution is given below:

$$\begin{cases}\frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_1}\\ \quad=\frac{-1}{\eta}[(e-v)A+(p+h_r-v)B]-p-h_r+w=0,\\ \frac{\partial g(\xi^3(q_1,q_2),q_1,q_2)}{\partial q_2}\\ \quad=\frac{-1}{\eta}(p+h_r-e)B+(p+h_r-o-e)=0.\end{cases}$$

Combined with $C-B=1-\eta$, we obtain $A=F(q_1)=\frac{(e+o-w)\eta}{e-v}$, $B=F(\frac{\xi^3(q_1,q_2)+(w-e)q_1+oq_2}{p-e})=\frac{(p+h_r-o-e)\eta}{(p+h_r-e)}$ and $C=F(\frac{(p+h_r-w)q_1+(p+h_r-o-e)q_2-\xi^3(q_1,q_2)}{h_r})=1-\frac{-o\eta}{(p+h_r-e)}$. Further, we can obtain optimal firm order quantity $q_1^*=F^{-1}(A)$ and optimal total order quantity $q^*=q_1^*+q_2^*=\frac{1}{p+h_r-e}[(p-e)F^{-1}(B)+h_rF^{-1}(C)]$. Since $q_2^*=q^*-q_1^*$, we can derive $q_2^*=\frac{1}{p+h_r-e}[(p-e)(F^{-1}(B)-F^{-1}(A))+h_r(F^{-1}(C)-F^{-1}(A))]$. □

Proof of Theorem 2: From Theorem 1, $q_1^*=F^{-1}(A)$ and $q^*=\frac{1}{p+h_r-e}[(p-e)F^{-1}(B)+h_rF^{-1}(C)]$. We can obtain $\frac{dq_1^*}{d\eta}=\frac{e+o-w}{(e-v)f(F^{-1}(A))}>0$, and

$$\begin{aligned}\frac{dq^*}{d\eta}&=\frac{p-e}{p+h_r-e}\frac{1}{f(F^{-1}(B))}\frac{p+h_r-o-e}{p+h_r-e}\\&\quad-\frac{h_r}{p+h_r-e}\frac{1}{f(F^{-1}(C))}\frac{o}{p+h_r-e}\\&=\left[\frac{(p-e)(p+h_r-o-e)}{f(F^{-1}(B))}-\frac{oh_r}{f(F^{-1}(C))}\right]\frac{1}{(p+h_r-e)^2}\\&=\frac{[(p-e)(p+h_r-o-e)f(F^{-1}(C))-oh_rf(F^{-1}(B))]}{f(F^{-1}(C))f(F^{-1}(B))(p+h_r-e)^2}\\&=\frac{M(h_r,\eta)}{f(F^{-1}(C))f(F^{-1}(B))(p+h_r-e)^2},\end{aligned}$$

where $M(h_r,\eta)=(p-e)(p+h_r-o-e)f(F^{-1}(C))-oh_rf(F^{-1}(B))$. Since $(p+h_r-e)^2f(F^{-1}(C))f(F^{-1}(B))$ is always positive, it is clear that if $M(h_r,\eta)>0$, then $\frac{dq^*}{d\eta}>0$. If $M(h_r,\eta)=0$, then $\frac{dq^*}{d\eta}=0$, otherwise, $\frac{dq^*}{d\eta}<0$. □

Proof of Theorem 3: From Theorem 1, we can obtain $q_1^*=F^{-1}(A)$ and $q^*=\frac{1}{p+h_r-e}[(p-e)F^{-1}(B)+h_rF^{-1}(C)]$. Note that q_1^* is independent of h_r, so $\frac{dq_1^*}{dh_r}=0$, and q_1^* is constant in h_r. The derivative of q^* with respect to h_r is $\frac{dq^*}{dh_r}=\frac{1}{(p+h_r-e)^2}[\frac{o\eta}{p+h_r-e}(\frac{p-e}{f(F^{-1}(B))}+\frac{h_r}{f(F^{-1}(C))})+(p-e)(F^{-1}(C)-F^{-1}(B))]$. Since $C-B=1-\eta>0$, then $F^{-1}(C)-F^{-1}(B)>0$. It is clear that $\frac{o\eta}{p+h_r-e}(\frac{p-e}{f(F^{-1}(B))}+\frac{h_r}{f(F^{-1}(C))})>0$ and $p>e$. It is easy to obtain that $\frac{dq^*}{dh_r}>0$. Since $\frac{dq_2^*}{dh_r}=\frac{dq^*}{dh_r}-\frac{dq_1^*}{dh_r}$, then $\frac{dq_2^*}{dh_r}=\frac{dq^*}{dh_r}>0$, both q^* and q_2^* are all strictly increasing in h_r. □

Proof of Corollary 1:

(a) By Theorem 1, we have $\frac{dq_1^*}{dw}=\frac{-\eta}{(e-v)f(F^{-1}(A))}<0$, which implies that q_1^* is decreasing in w, while q^* is independent of w, q^* is constant in w.

(b) By taking the derivative of q_1^* and q^* with respect to o, we have $\frac{dq_1^*}{do}=\frac{\eta}{(e-v)f(F^{-1}(A))}>0$, i.e, q_1^* is increasing in o, while $\frac{dq^*}{do}=\frac{-\eta}{(p+h_r-e)^2}[\frac{p-e}{f(F^{-1}(B))}+\frac{h_r}{f(F^{-1}(C))}]<0$, then q^* is decreasing in o.

(c) By taking derivative of q_1^* and q^* with respect to e, respectively, we have $\frac{dq_1^*}{de}=\frac{\eta(w-o-v)}{(e-v)^2f(F^{-1}(A))}$ and $\frac{dq^*}{de}=\frac{1}{(p+h_r-e)^2}[h_r(F^{-1}(c)-F^{-1}(B))-\frac{o\eta}{p+h_r-e}(\frac{p-e}{f(F^{-1}(B))}+\frac{h_r}{f(F^{-1}(C))})]$. Since $w>o+v$, $\frac{dq_1^*}{de}>0$, which means that q_1^* is decreasing in e. Since $H(h_r,\eta)=h_r[F^{-1}(C)-F^{-1}(B)]-\frac{(p-e)o\eta}{(p+h_r-e)f(F^{-1}(B))}-\frac{oh_r\eta}{(p+h_r-e)f(F^{-1}(C))}$, $\frac{dq^*}{de}=\frac{1}{(p+h_r-e)^2}H(h_r,\eta)$, $\frac{dq^*}{de}$ is related to $H(h_r,\eta)$'s sign. If $H(h_r,\eta)>0$, then $\frac{dq^*}{de}>0$, and q^* is increasing in e, if $H(h_r,\eta)=0$, then $\frac{dq^*}{de}=0$, and q^* is constant in e, otherwise, $\frac{dq^*}{de}<0$, and q^* is decreasing in e. □

Proof of Corollary 2: The demand is assumed to be normally distributed with mean $E(X)=\mu$ and the standard deviation $\sqrt[2]{D(X)}=\sigma$. Φ and ϕ denote the distribution and probability density functions of the standard normal distribution, respectively. Set $z^1=\Phi^{-1}(A)=\Phi^{-1}(\frac{(e+o-w)\eta}{e-v})$, $z^2=\Phi^{-1}(B)=\Phi^{-1}(\frac{(p+h_r-o-e)\eta}{p+h_r-e})$, $z^3=\Phi^{-1}(C)=\Phi^{-1}(1-\frac{o\eta}{p+h_r-e})$, $q_B^*=F^{-1}(B)$ and $q_C^*=F^{-1}(C)$, then we can obtain $q_1^*=\mu+\sigma z^1$, $q_B^*=\mu+\sigma z^2$ and $q_C^*=\mu+\sigma z^3$, where z^1, z^2 and z^3 are the optimal quantities.

(a) Since $q_1^*=\mu+\sigma z^1=\mu+\sigma\Phi^{-1}(\frac{(e+o-w)\eta}{e-v})$, it is obvious that if $2(e+o-w)\eta>e-v$, then q_1^* is increasing in σ, and $q_1^*>\mu$; if $2(e+o-w)\eta=e-v$, then q_1^* is constant in σ, and $q_1^*=\mu$; otherwise, q_1^* is decreasing in σ, and $q_1^*<\mu$.

(b) Since $q_B^* = F^{-1}(B) = \mu + \sigma z^2$ and $q_C^* = F^{-1}(C) = \mu + \sigma z^3$, combined with $q_2^* = q^* - q_1^*$, it is easy to obtain $q_2^* = \frac{(p-e)q_B^* + h_r q_C^*}{p+h_r-e}$. That is, $q_2^* = \mu + \sigma \frac{(p-e)(z^2-z^1)+h_r(z^3-z^1)}{p+h_r-e}$. Since $z^2 > z^1$ and $z^3 > z^1$, $\frac{(p-e)(z^2-z^1)+h_r(z^3-z^1)}{p+h_r-e} > 0$, which implies that q_2^* is strictly increasing in σ, and $q_2^* > \mu$. Thus the proof is complete. □

Proof of Theorem 4: According to Equation (2), with some algebra, we have $\pi_s(Q^*) = q^* + (w-o)q_1^* + (e-c)q^* - eq_1^* - (e-v)\int_{q_1^*}^{q^*} F(x)dx$. Further, $\pi_s(Q^*) = (o + e - c)q^* + (w - o - e)q_1^* + (e-v)\int_{q^*}^{q_1^*} F(x)dx$. By taking the derivative of $\pi_s(Q^*)$ with respect to w, we have $\frac{d\pi_s(Q^*)}{dw} = q_1^* + [(w-o-e)+(e-v)F(q_1^*)]\frac{dq_1^*}{dw} = q_1^* + (o+e-w)(\eta-1)\frac{dq_1^*}{dw}$. Since $w < o+e$, $\eta < 1$ and $\frac{dq_1^*}{dw} < 0$, then $\frac{d\pi_s(Q^*)}{dw} > 0$, i.e, the optimal expected profit of the supplier is increasing in w. □

Proof of Theorem 5: According to the proof of Theorem 4, $\pi_s(Q^*) = (o+e-c)q^* + (w-o-e)q_1^* + (v-e)\int_{q_1^*}^{q^*} F(x)dx$. By taking the derivative of $\pi_s(Q^*)$ with respect to h_r, we have $\frac{d\pi_s(Q^*)}{dh_r} = [o+e-(e-v)F(q)-c]\frac{dq^*}{dh_r}$. By Theorem 3, $\frac{dq^*}{dh_r} > 0$, so $\frac{d\pi_s(Q^*)}{dh_r}$'s sign is related to $o+e-(e-v)F(q)-c$. If $o+e-(e-v)F(q^*) > c$, then $\frac{d\pi_s(Q^*)}{dh_r} > 0$. If $o+e-(e-v)F(q^*) = c$, then $\frac{d\pi_s(Q^*)}{dh_r} = 0$. Otherwise, $\frac{d\pi_s(Q^*)}{dh_r} < 0$. □

Proof of Lemma 1: Let $g(\xi, q_0) = \xi - \frac{1}{\eta}E[\xi - \pi_r(X, q_0)]^+$, then $\mathrm{CVaR}_\eta(\pi_r(X, q_0)) = \max_{\xi\in R} g(\xi, q_0)$. For any fixed q_0, we first maximize $g(\xi, q_0)$ over $\xi \in R$. Obviously, we have

$$g(\xi, q_0) = \xi - \frac{1}{\eta}\int_0^{q_0}[\xi + (w-v)q_0 - x(p-v)]^+ f(x)dx - \frac{1}{\eta}\int_{q_0}^{\infty}[\xi + h_r x - (p+h_r-w)q_0]^+ f(x)dx.$$

If $\xi \le -(w-v)q_0$, then $g(\xi, q_0) = \xi - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_0-\xi}{h_r}}^{\infty}[\xi + h_r x - (p+h_r-w)q_0]f(x)dx$, and $\frac{\partial g(\xi,q_0)}{\partial\xi} = 1 - \frac{1}{\eta}[1 - F(\frac{(p+h_r-w)q_0-\xi}{h_r})]$. Set $\xi^0(q_0) = (p+h_r-w)q_0 - h_r F^{-1}(1-\eta)$. Obviously, if $\xi^1(q_0) \le -(w-v)q_0$, i.e, $(p+h_r-v)q_0 \le h_r F^{-1}(1-\eta)$, then $\xi^0(q_0)$ is the solution of equation $\frac{\partial g(\xi,q_0)}{\partial\xi} = 0$ for fixed q_0. Therefore, $g(\xi^0(q_0), q_0) = \xi^0(q_0) - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_0-\xi^0(q_0)}{h_r}}^{\infty}[\xi^0(q_0) + h_r x - (p+h_r-w)q_0]f(x)dx$. However, $\frac{\partial g(\xi^0(q_0),q_0)}{\partial q_0} = p + h_r - w > 0$, which indicates that $\xi^0(q_0)$ is not the solution of $\max_{\xi\in R} g(\xi, q_0)$. Thus $(p + h_r - v)q_0 > h_r F^{-1}(1-\eta)$.

If $(p-w)q_0 < \xi$, then $g(\xi, q_0) = \xi - \frac{1}{\eta}\int_0^{q_0}[\xi + (w-v)q_0 - x(p-v)]f(x)dx - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_0-\xi}{h_r}}^{\infty}[\xi + h_r x - (p+h_r-w)q_0]f(x)dx$, and $\frac{\partial g(\xi,q_0)}{\partial\xi} = 1 - \frac{1}{\eta} < 0$.

If $-(w-v)q_0 < \xi \le (p-w)q_0$, then $g(\xi, q_0) = \xi - \frac{1}{\eta}\int_0^{\frac{\xi+(w-v)q_0}{p-v}}[\xi + (w-v)q_0 - x(p-v)]f(x)dx - \frac{1}{\eta}\int_{\frac{(p+h_r-w)q_0-\xi}{h_r}}^{\infty}[\xi + h_r x - (p+h_r-w)q_0]f(x)dx$, and $\frac{\partial g(\xi,q_0)}{\partial\xi} = 1 - \frac{1}{\eta}[F(\frac{\xi+(w-v)q_0}{p-v}) + 1 - F(\frac{(p+h_r-w)q_0-\xi}{h_r})]$. It is noted that $\frac{\partial g(\xi,q_0)}{\partial\xi}|_{\xi=-(w-v)q_0} = 1 - \frac{1}{\eta}[1 - F(\frac{(p+h_r-w)q_0-\xi}{h_r})] > 0$ and $\frac{\partial g(\xi,q_0)}{\partial\xi}|_{\xi=(p-w)q_0} = 1 - \frac{1}{\eta} < 0$. Therefore, there must exist $\xi'(q_0)$ satisfying $\frac{\partial g(\xi,q_0)}{\partial\xi} = 0$, i.e, $\xi'(q_0) = arg\max_{\xi\in R} g(\xi, q_0)$. Then $\mathrm{CVaR}_\eta(\pi_r(x, q_0)) = g(\xi'(q_0), q_0)$. To facilitate analysis, denote $B' = F(\frac{\xi'(q_0)+(w-v)q_0}{p-v})$ and $C' = F(\frac{(p+h_r-w)q_0-\xi'(q_0)}{h_r})$, then $C' - B' = 1 - \eta$. Taking the first and second partial derivatives of $g(\xi'(q_0), q_0)$ with respect to q_0, we have $\frac{\partial g(\xi'(q_0),q_0)}{\partial q_0} = \frac{1}{\eta}[(v-w)B' + (p+h_r-w)(1-C)] = \frac{-1}{\eta}[(p+h_r-v)B' - (p+h_r-w)\eta]$ and $\frac{\partial^2 g(\xi'(q_0),q_0)}{\partial^2 q_0} = \frac{-1}{\eta}[\frac{(p+h_r-v)(w-v)}{p-v}f(F^{-1}B')] < 0$, which indicates that $g(\xi'(q_0), q_0)$ is strictly jointly concave in q_0. According to the first order optimality condition, the optimal solution is $\frac{\partial g(\xi'(q_0),q_0)}{\partial q_0} = \frac{-1}{\eta}[(p+h_r-v)B' - (p+h_r-w)\eta] = 0$. Combining $C' - B' = 1-\eta$, we obtain $B' = F(\frac{\xi'(q_0)+(w-v)q_0}{p-v}) = \frac{(p+h_r-w)\eta}{(p+h_r-v)}$ and $C' = F(\frac{(p+h_r-w)q_0-\xi'(q_0)}{h_r}) = 1 - \frac{(w-v)\eta}{(p+h_r-v)}$. Further, we can obtain optimal order quantity $q^* = \frac{1}{p+h_r-v}[(p-v)F^{-1}(B') + h_r F^{-1}(C')]$. □

Proof of Theorem 6: From Theorem 1 and Lemma 1, $B = \frac{(p+h_r-o-e)\eta}{p+h_r-e}$, $C = 1 - \frac{o\eta}{p+h_r-e}$, $B' = \frac{(p+h_r-w)\eta}{p+h_r-v}$ and $C' = 1 - \frac{(w-v)\eta}{p+h_r-v}$. Since $o < \frac{(w-v)(p+h_r-e)}{p+h_r-v}$, then $B = \frac{(p+h_r-o-e)\eta}{p+h_r-e} = \eta - \frac{o\eta}{p+h_r-e} > \eta - \frac{(w-v)\eta}{p+h_r-v} = \frac{(p+h_r-w)\eta}{p+h_r-v} = B'$ and $C = 1 - \frac{o\eta}{p+h_r-e} > 1 - \frac{(w-v)\eta}{p+h_r-v} = C'$. It follows that $F^{-1}(B) > F^{-1}(B')$ and $F^{-1}(C) > F^{-1}(C')$. Since $\frac{1}{p+h_r-e}[(p-e)F^{-1}(B) + h_r F^{-1}(C)] > \frac{1}{p+h_r-v}[(p-v)F^{-1}(B) + h_r F^{-1}(C)] = \frac{1}{p+h_r-e}[(p-e)F^{-1}(B') + h_r F^{-1}(C')]$, then $q^* > q_0^*$. Since $B' < C'$, then $F^{-1}(B') < F^{-1}(C')$, and $q_0^* = \frac{1}{p+h_r-v}[(p-v)F^{-1}(B') + h_r F^{-1}(C')] > F^{-1}(B')$. Since $A = \frac{(e+o-w)\eta}{e-v} < \frac{(e-w)\eta}{e-v} + \frac{\eta}{e-v}\frac{(w-v)(p+h_r-e)}{p+h_r-v} < \eta - \frac{(w-v)\eta}{p+h_r-v} = \frac{(p+h_r-w)\eta}{p+h_r-v} = B'$ then $F^{-1}(A) < F^{-1}(B')$. It follows that $q_0^* > F^{-1}(B') > F^{-1}(A) = q_1^*$, so $q^* > q_0^* > q_1^*$. □

Proof of Theorem 7: Since both q_1^* and q_2^* are the optimal solutions of $CVaR_\eta(\pi_r(X, q_1, q_2))$, we can easily obtain that $\mathrm{CVaR}_\eta(\pi_r(X, q_1^*, q_2^*)) > \mathrm{CVaR}_\eta(\pi_r(X, q_1, 0))$, where $\mathrm{CVaR}_\eta(\pi_r(X, q_1, 0))$ is the retailer's risk measure with the wholesale price-only contract. Obviously, the retailer with only a wholesale price contract is a special case of the retailer with the portfolio contract. Thus $\mathrm{CVaR}_\eta(\pi_r(X, q_1^*, q_2^*)) > \mathrm{CVaR}_\eta(\pi_r(X, q_0^*))$. □

Proof of Theorem 8: From the proof of Theorem 4, $\pi_s(Q^*) = (o+e-c)q^* + (w-o-e)q_1^* + (e-v)\int_{q^*}^{q_1^*} F(x)dx$. Since $\pi_s(Q_0^*) = (w-c)q_0^*$, then $\pi_s(Q^*) - \pi_s(Q_0^*) = (o + e - c)q^* + (w-o-e)q_1^* + (e-v)\int_{q^*}^{q_1^*} F(x)dx - (w-c)q_0^*$. Clearly, from Theorem 1 and Corollary 1(a), $q_2^* = q^* - q_1^*$ and $\frac{dq_2^*}{dw} > 0$. Assuming $w = w_0$, then $q_2^* = 0$, which means that if wholesale price w is a sufficiently small, then the retailer orders products only with the wholesale price contract. It follows that if $w > w_0$, then $q_2^* > 0$. Otherwise, $q_2^* = 0$, $q_1^* = q_0^*$, and $\pi_s(Q^*) = \pi_s(Q_0^*)$. By taking the derivative of $\pi_s(Q^*) - \pi_s(Q_0^*)$ with respect to w, we have $\frac{d\pi_s(Q^*)}{dw}|_{w=w_0} - \frac{d\pi_s(Q_0^*)}{dw}|_{w=w_0} = q_1^* + (o + e - w)(\eta - 1)\frac{dq_1^*}{dw} - q_0^* - (w - c)\frac{dq_0^*}{dw}$. Since $\frac{dq_1^*}{dw} < 0$ and $\frac{dq_0^*}{dw} < 0$, then $(\eta - 1)\frac{dq_1^*}{dw} > 0$ and $-(w - c)\frac{dq_0^*}{dw} > 0$, it follows that $\frac{d\pi_s(Q^*)}{dw}|_{w=w_0} - \frac{d\pi_s(Q_0^*)}{dw}|_{w=w_0} > 0$. Since $w > w_0$, we obtain that $\pi_s(Q^*) > \pi_s(Q_0^*)$. □

Proof of Theorem 9: Since $Q^* = q^*$ and the optimal production quantity of the SC is $q_c^* = F^{-1}\left(\frac{p+h_r-c}{p+h_r-v}\right)$, to coordinate the SC, it is sufficient for the supplier to offer wholesale price and call option portfolio contracts to motivate the retailer to pursue total order quantity q_c^*. From Theorem 1, the retailer's optimal total order quantity is $q^* = \frac{1}{p+h_r-e}[(p-e)F^{-1}(B) + h_r F^{-1}(C)]$, which implies that when $Q^* = q^* = q_c^*$, the SC can be coordinated by portfolio contracts. That is, $\frac{1}{p+h_r-e}[(p-e)F^{-1}(B) + h_r F^{-1}(C)] = F^{-1}\left(\frac{p+h_r-c}{p+h_r-v}\right)$, i.e, $(p + h_r - e)F^{-1}(D) = (p - e)F^{-1}(B) + h_r F^{-1}(C)$. □

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