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A Cydia Pomonella Integrated Management Predator-Prey Model With Smith Growth and Linear Feedback Control

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ABSTRACT Cydia Pomonella has great influence on the output and quality of apple, therefore, it is an interesting subject to find out the optimum control strategy of Cydia Pomonella. A Cydia Pomonella integrated management predator-prey model with Smith growth and linear feedback control is established in this paper, and the pulse parameter linearity is dependent on controlling level. Firstly, we prove the existence and uniqueness of order one periodic solution (OOPS) by using differential equation geometry theory and successor function method. Secondly, the stability of OOPS is proved by using the geometric method. Finally, with the discussion of different parameters, we verify the theoretical results by numerical simulations, and demonstrate the advantages and disadvantages of biological control strategy, chemical control strategy and integrated control strategy. Moreover, the optimization model with minimum cost is established by using OOPS and optimal control level of the model is obtained.

INDEX TERMS Integrated pest management, optimization, periodic solution, Smith growth.

I. INTRODUCTION

Apple is one of the four most famous fruits in the world. It can supply the body with necessary nutritious elements, besides, it has very rich medicinal value and a lot of therapeutic effects. However, the output and quality of apple are greatly influenced by Cydia Pomonella. Because Cydia Pomonella is a kind of omnivorous drill borer, and it has very strong adaptability, resistance, and reproductive performance, and it has been spread extensively in almost all of the apple-producing areas of six continents in the world. It is a kind of destructive pest of kernel fruit trees, and it has serious influence on the productivity and selling of the fruits at home and abroad. Consequently, in order to decrease the management cost and ensure the quality and quantity of apple, we need to establish a practical biologic mathematical model and do the research of the controlling strategy of Cydia Pomonella.

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In biological mathematics, Logistic growth model

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)$$

is a simple but important biological system and it is built up on the basis of the assumption that relative growth rate of population size $\frac{dx}{xdt}$ is linear function $1 - \frac{x}{K}$ of population size, where $x(t)$ denotes the densities of population at time t , r is the intrinsic rate of growth, and K is the constant carrying capacity. A lot of scholars have established many models with logistic growth including Beddington-DeAngelis type [1], Holling type [2]–[4], ratio-dependent type [5] and they also have obtained some dynamical properties. Some scholars believe that, to a certain extent, Logistic equations confirmed the logical pattern of population growth and the limited resources, however, individual reproduction and nutrition supply has not been taken into consideration. Logistic growth model is mainly applied to low biological population, such as bacteria, yeasts, and floating algae etc [6]. In 1963, Smith

studied a kind of algae called Daphnia in the lab. He found out that the relevant data of this population did not accord with linear rule [7]. Smith assumed that the relative growth rate of the population density at time t is proportional to the amount of remaining food at that time. i.e.

$$\frac{1}{x} \frac{dx}{dt} = r \left(1 - \frac{G(t)}{\nu} \right),$$

where G is the rate of food demand of a population with a population density of x at time t . ν is the demand rate for food when the population reaches the saturation state. Smith supposed that the food consumed by the population was mainly the food needed to sustain the organism itself: $c_1 x(t)$ and the food needed for the population to reproduce: $c_2 \frac{dx}{dt}$. That is, the food $F(x)$ consumed by the population can be expressed as

$$F(x) = c_1 x(t) + c_2 \frac{dx}{dt}$$

Then,

$$\frac{dx}{dt} = rx \left(1 - \frac{\nu - c_1 x}{\nu + rc_2 x} \right).$$

When $x(t) = K$, the population no longer grows, the food is only used for survival: $\nu = F(t) = c_1 K$. Set $D = \frac{rc_2}{c_1}$, we shall get

$$\frac{dx}{dt} = rx \left(1 - \frac{K - x}{K + Dx} \right).$$

Smith model used hyperbolic function $\frac{\nu - c_1 x}{\nu + rc_2 x}$ to replace linear function in Logistic model. It is a further improvement of Logistics model.

Because there are many interactions between the prey and the predator [9], [10], [46], so we assume that the predation is linearly dependent on the Cydia Pomonella and Trichogramma densities with a predation coefficient q , i.e. $qx(t)y(t)$, and assume that the per capita conversion rate from Cydia Pomonella to Trichogramma is saturated, i.e. $\frac{lx(t)y(t)}{x(t)+b}$. The Cydia Pomonella studied in this paper and the Daphnia are both arthropods, so we think that Cydia Pomonella is suitable for Smith growth rate [7]. In addition, we know that Trichogramma belongs to invertebrates and invertebrates can be described by Holling Type II functional response function [8], so we think that Trichogramma can be described by Holling Type II functional response function. Therefore, a predator-prey model with Smith growth is structured as follows:

$$\begin{cases} \dot{x}(t) = \frac{px(t) - rx^2(t)}{k + \xi x(t)} - qx(t)y(t), \\ \dot{y}(t) = \frac{lx(t)y(t)}{x(t) + b} - my(t). \end{cases} \quad (1)$$

where $x(t)$ represents the density of the Cydia Pomonella at time t , $y(t)$ represents the density of the Trichogrammatid at time t . And the term $\frac{lx(t)}{x(t)+b}$ is a Holling type II

functional response function. The parameters p, r, k, ξ, q, m, b , are all positive.

Some other scholars have done a lot of work in the control of pests such as biological control, chemical control, and integrated pest management. Chemical control strategy is to kill the pests directly by using chemicals [11], [12]. This kind of method has very quick and obvious effect and it is not very much limited by the regions and seasons, but the natural enemies and the useful organisms are easily killed by large quantity of pesticides, and the secondary insects will prevail. At the meantime, chemical control has a great impact on both human health and the environment. Biological control is a kind of pests control method by using the methods of natural enemies preying on pests and virus prevention and some other measures [13]–[18]. This method do no harm to the natural enemies, and is very friendly to the environment, but it alters the biological structure, so it is only put into use when the quantity of insects is not so large. Integrated pest management (IPM) combines the chemical control and biological control, and this method encourages to control the pests within economic threshold, not to exterminate the pests [10], [22]–[27].

However, with the diversity of human intervention, the control strategy of pests will lead to the fundamental change of species population, and this phenomenon exists in many dynamic systems. These phenomena can all be described by impulsive differential equations [28]. On the basis of this theory, a lot of researchers have established human intervention model with periodic impulse, for example, periodic spraying of pesticides combined with periodic release of infected pests [29]–[32], periodic release of natural enemies and infected pests [33]–[38], periodic release of natural enemies [19]–[21], [39]–[41], regular release of pests infected with the disease [42]–[46]. We assume that the control action in the above-mentioned models is carried out at the same pest level. Zhang and Chen [47], Zhao *et al.* [48], Wang *et al.* [49], [50] studied the control action which is carried out on different pest levels. We will consider the control strategy when the number of Cydia Pomonella is between two control levels.

The organization of this article is as follows. In Section 2, a Cydia Pomonella integrated management predator-prey model is structured, and qualitative analysis of the model is given. The existence, uniqueness and asymptotic stability of the OOPS of system (2) are proved in Section 3. In Section 4, numerical simulations of specific model are performed to complement our theoretical results. Furthermore, in order to reduce the total cost of Cydia Pomonella control, we formulate the optimization problem. The final conclusion is drawn in Section 5.

II. CYDIA POMONELLA MANAGEMENT MODEL AND QUALITATIVE ANALYSIS OF SYSTEM (1)

Based on the above introduction, the interaction between Cydia Pomonella and Trichogrammatid is taken as examples.

We establish the following pulse feedback control predator-prey model with Smith growth.

$$\begin{cases} \dot{x}(t) = \frac{px(t) - rx^2(t)}{k + \xi x(t)} - qx(t)y(t), \\ \dot{y}(t) = \frac{lx(t)y(t)}{x(t) + b} - my(t), \\ \Delta x(t) = -c(x)x(t), \\ \Delta y(t) = -b(x)y(t) + \delta(x), \end{cases} \begin{matrix} x < ET, \\ x = ET, \quad y \leq \bar{y}, \end{matrix} \quad (2)$$

where $ET \in [SHT, EIT]$ denotes the threshold of Cydia Pomonella, SHT is the biological control level (i.e. the threshold with slight damage to the apple), EIT is the chemical control level (i.e. Cydia Pomonella economic harm level). We adopt the integrated control strategy when $x = ET$. \bar{y} represents Trichogrammatid threshold at Cydia Pomonella level ET . $\delta(x)$ is defined as the number of artificial Trichogrammatid. The control parameters $\delta(x)$, $c(x)$, $b(x)$ are continuous functions defined on $[SHT, EIT]$, which satisfy $\delta(SHT) = \delta_{max}$, $\delta(EIT) = \delta_{min}$, $c(SHT) = 0$, $c(EIT) = c_{max}$, $b(SHT) = 0$, $b(EIT) = b_{max}$. In this paper, the functions $\delta(x)$, $b(x)$, $c(x)$ are linearly dependent on the Cydia Pomonella control level x_{ET} [52],

$$\begin{cases} \delta(x) = \delta_{max} - (\delta_{max} - \delta_{min}) \frac{x - SHT}{EIT - SHT}, \\ b(x) = b_{max} \frac{x - SHT}{EIT - SHT}, \\ c(x) = c_{max} \frac{x - SHT}{EIT - SHT}, \end{cases} \quad (3)$$

where $b_{max} < c_{max}$ and $c_{max} \leq 1 - \frac{SHT}{EIT}$ in practice.

From the biological point of view, we only consider this region: $\Omega = \{(x, y) | 0 < x \leq ET, 0 \leq y < H - \rho x\}$, where H is a sufficiently large constant satisfying $d\phi/dt|_{\phi=0} < 0$ where $\phi : y + \rho x + b = 0$.

Theorem 2.1: If $l > m + \frac{mbr}{p}$ holds, then system (1) possesses two boundary equilibriums: $E_0(0, 0)$ and $E_1(\frac{p}{r}, 0)$, and an internal equilibrium: $E_*(x^*, y^*)$, where $x^* = \frac{mb}{l-m}$, $y^* = \frac{pl-pm-rmb}{q(kl-km+\xi mb)}$. Furthermore, $E_0(0, 0)$ and $E_1(\frac{p}{r}, 0)$ are saddle points, and $E_*(x^*, y^*)$ is a node or focus which is locally asymptotically stable (see Fig. 1).

Proof: It is easy to prove that $E_0(0, 0)$ is always a saddle point. The Jacobian matrix at $E_1(\frac{p}{r}, 0)$ is

$$J(E_1) = \begin{pmatrix} -\frac{pr}{kr + p\xi} & -\frac{pq}{r} \\ 0 & \frac{pl}{p + br} - m \end{pmatrix}.$$

By calculations, we shall get

$$\text{Det}(J(E_1)) = -\frac{pr}{kr + p\xi} (\frac{pl}{p + br} - m) < 0,$$

then $E_1(\frac{p}{r}, 0)$ is saddle point.

$$J(E_*) = \begin{pmatrix} -\frac{\xi rx^{*2} - 2krx^* + pk}{(k + \xi x^*)^2} - qy^* & -qx^* \\ \frac{bly^*}{(x^* + b)^2} & \frac{lx^*}{x^* + b} - m \end{pmatrix}$$

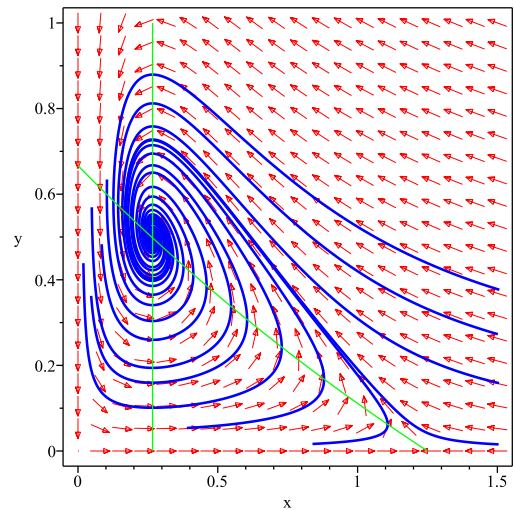


FIGURE 1. Phase diagram of system (1) with $p = 1, r = 0.8, k = 3, \xi = 0.6, q = 0.5, l = 0.5, b = 0.4, m = 0.2$.

$$= \begin{pmatrix} -\frac{krx^* + p\xi x^*}{(k + \xi x^*)^2} & -qx^* \\ \frac{bly^*}{(x^* + b)^2} & 0 \end{pmatrix},$$

by calculations, we have

$$\begin{aligned} \text{Det}(J(E_*)) &= qx^* \frac{bly^*}{(x^* + b)^2} > 0, \\ \text{Tr}(J(E_*)) &= -\frac{krx^* + p\xi x^*}{(k + \xi x^*)^2} < 0, \end{aligned}$$

then $E_*(x^*, y^*)$ is a locally asymptotically stable node or focus. ■

For system (2), set $M = \{(x, y) | x = ET, 0 \leq y \leq \bar{y}\}$ and set $N = \{(x, y) | x = (1 - c(ET))ET, 0 \leq y \leq c(ET) + \delta_{max}\}$ are denoted as the the impulse set and the phase set respectively. The isoclinic lines are $l_1 = \{(x, y) | x = \frac{mb}{l-m}, y \geq 0\}$ and $l_2 = \{(x, y) | y = \frac{p-rx}{q(k+\xi x)}, x \geq 0\}$. For any point $A(x_A, y_A)$, x_A and y_A are denoted as its horizontal coordinate and ordinate coordinate respectively. We define the first integral of system (2) as $G(x, y) = G_0$. It can be seen from the qualitative analysis of system (1) that implicit function $G(x, y) = G_0$ is divided into upper and lower sections by isoclinic line $l_2 : \frac{dx}{dt} = 0$. Upper and lower sections are respectively denoted as $y_{G^+}(x, A_0)$ and $y_{G^-}(x, A_0)$, where $A_0((1 - c(ET))ET, y_{A_0})$ is the initial point.

III. DYNAMICAL ANALYSIS OF SYSTEM (2)

A. EXISTENCE OF THE ORDER ONE PERIODIC SOLUTION OF SYSTEM (2)

1) CASE I: $SHT \leq ET \leq \text{MIN}\{EIT, X^*\}$

Theorem 3.1: If $l > m + \frac{mbr}{p}$ and $SHT \leq ET \leq \text{min}\{EIT, x^*\}$ hold, then system (2) admits a unique OOPS.

Proof: Assume that the isoclinic line l_2 intersects with the phase set N at the point $B(x_B, y_B)$, where $x_B = (1 - c(ET))ET$, $y_B = \frac{p-r(1-c(ET))ET}{q(k+\xi(1-c(ET))ET)} \triangleq \sigma$. The trajectory

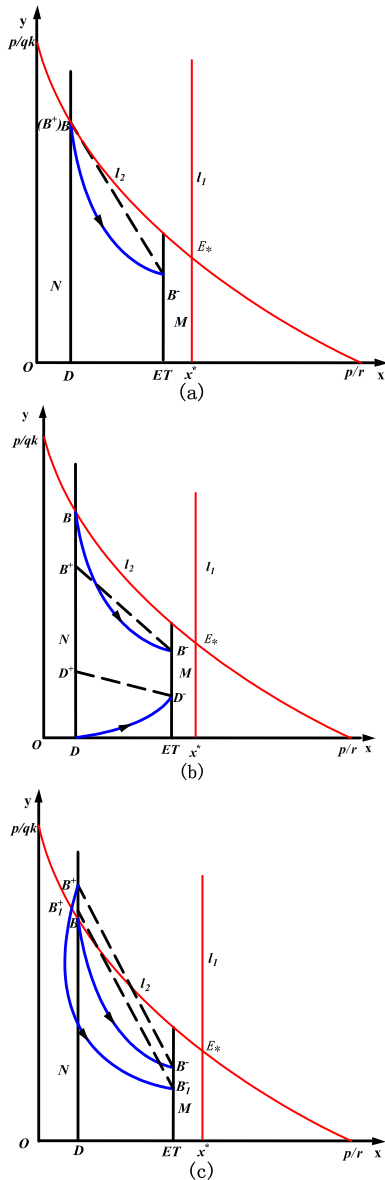


FIGURE 2. The existence of the OOPS of system (2) in case I. (a) $\sigma = (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$. (b) $\sigma > (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$. (c) $\sigma < (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$.

$y_{G^-}(x, B)$ with the initial point B intersects with the impulse set M at point B^- after time t , then point B^- jumps to the point $B^+(x_{B^+}, (1 - c(ET))ET)$ after the impulsive effects, where $x_{B^+} = x_B, y_{B^+} = (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$. According to the location of point B^+ , there have two cases:

(i): $\sigma \geq (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$.

If $\sigma = (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$, then B^+ coincides with B and the successor function of B is $f(B) = y_{B^+} - y_B = 0$, so system (2) admits an OOPS $\overline{BB^-B^+}$ (see Fig. 2(a)).

If $\sigma > (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$, then point B^+ is below point B , so the successor function of B is $f(B) = y_{B^+} - y_B < 0$. The intersection of phase set and X-axis is denoted as point D . The trajectory with the initial point D intersects the impulse set M at point $D^-(x_{D^-}, y_{D^-})$, where $x_{D^-} = ET, y_{D^-} = y_{G^-}(ET, D)$, then point D^- jumps

to the point $D^+(x_{D^+}, y_{D^+})$ after the impulse effects, where $y_{D^+} = (1 - b(ET))y_{G^-}(ET, D) + \delta(ET) > y_{D^-}$. Therefore, the successor function of point D is $f(D) = y_{D^+} - y_{D^-} > 0$. According to [53, Theorem 2.2], there is a point P_* located in \overline{DB} and meets $f(P_*) = 0$. Therefore, system (2) possesses an OOPS (see Fig. 2(b)).

(ii): $\sigma < (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$.

In this case, we shall get that point B^+ is above the point B , then the successor function of point B is $f(B) = y_{B^+} - y_B > 0$. The trajectory $G(x, y) = G(B^+)$ with the initial point B^+ intersects the impulse set M at point $B_1^-(x_{B_1^-}, y_{B_1^-})$, where $x_{B_1^-} = ET, y_{B_1^-} = y_{G^-}(ET, B^+)$. Then the trajectory jumps to the point $B_1^+(x_{B_1^+}, y_{B_1^+})$ after the impulse effects. In view of the fact that any two tracks of the system (2) do not intersect, we shall get $y_{G^-}(ET, B^+) < y_{G^-}(ET, B)$, that is, $y_{B_1^+} < y_{B^+}$. Therefore, the successor function of point B^+ is $f(B^+) = y_{B_1^+} - y_{B^+} < 0$. According to [53, Theorem 2.2], there is a point P_* that locates in $\overline{BB^+}$ and meets $f(P_*) = 0$. Therefore, system (2) admits an OOPS (see Fig. 2(c)).

Now, we discuss the uniqueness of OOPS of system (2). First, the uniqueness of OOPS of system (2) in the case of $\sigma \geq (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$ is proved. We assume that there exist two OOPSs $P_1P_1^-P_1^+$ and $P_2P_2^-P_2^+$ with points $P_1 \in \overline{DB}$ and $P_2 \in \overline{DB}$, that is to say,

$$(1 - b(ET))y_{G^-}(ET, P_1) + \delta(ET) = y_{P_1},$$

$$(1 - b(ET))y_{G^-}(ET, P_2) + \delta(ET) = y_{P_2},$$

where $y_{P_1} < y_{P_2}$.

Denote $d_{P_1P_2}(x) = y_{G^-}(x, P_1) - y_{G^-}(x, P_2), x \in [(1 - c(ET))ET, ET]$. Because of

$$\begin{aligned} & d'_{P_1P_2}(x) \\ &= y'_{G^-}(x, P_1) - y'_{G^-}(x, P_2) \\ &= \left(\frac{l}{x + b} - mx \right) \cdot \left[\left(\frac{p-rx}{k+\xi x} - qy_{P_1} \right)^{-1} y_{P_1} - \left(\frac{p-rx}{k+\xi x} - qy_{P_2} \right)^{-1} y_{P_2} \right], \end{aligned}$$

where $l(y) = \left(\frac{p-rx}{k+\xi x} - qy \right)^{-1} y$ with $l'(y) = \frac{p-rx}{k+\xi x} \left(\frac{p-rx}{k+\xi x} - qy \right)^{-2} > 0$. Thus $l(y_1) < l(y_2)$, i.e., $d'_{P_1P_2}(x) < 0$. According to the above analysis, we can get $d_{P_1P_2}((1 - c(ET))ET) > d_{P_1P_2}(ET)$. It is known from system (2),

$$\begin{aligned} \delta(ET) &= y_{P_1} - (1 - b(ET))y_{P_1^-} \\ &= y_{P_2} + d_{P_1P_2}((1 - c(ET))ET) \\ &\quad - (1 - b(ET))[y_{P_2^-} + d_{P_1P_2}(ET)] \\ &> y_{P_2} - (1 - b(ET))y_{P_2^-} = \delta(ET) \end{aligned}$$

which leads to a contradiction. Therefore, the OOPS is unique when $\sigma \geq (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$.

For $\sigma < (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$, there is a point $P_* \in \overline{BB^+}$, which makes the trajectory $P_*P_*^-P_*$ the OOPS,

that is to say, $y_{P^+} = y_{P^*} = (1 - b(ET))y_{P^*} + \delta(ET)$. For any point $P' \in \overline{BB^+}$, the successor function of point P' is

$$\begin{aligned} f(P') &= y_{P'^+} - y_{P'} \\ &= (1 - b(ET))y_{P'^-} + \delta(ET) - y_{P'} \\ &= (1 - b(ET))y_{P'^-} - y_{P'} + y_{P^*} - (1 - b(ET))y_{P^*} \\ &= y_{P^*} - y_{P'} + (1 - b(ET))(y_{P'^-} - y_{P^*}), \end{aligned}$$

that is,

$$f(P') = \begin{cases} -d_{P^*P'}((1 - c(ET))ET) \\ + (1 - b(ET))d_{P^*P'^-}(ET), & \text{if } y_{P'} > y_{P^*}, \\ d_{P^*P'}((1 - c(ET))ET) \\ - (1 - b(ET))d_{P^*P'^-}(ET), & \text{if } y_{P'} < y_{P^*}. \end{cases}$$

There is $f(P') < 0$ for any $P' \in \overline{BB^+}$ with $y_{P'} > y_{P^*}$ and $f(P') > 0$ for any $P' \in \overline{BB^+}$ with $y_{P'} < y_{P^*}$. Thus the OOPS is unique in the case of $\sigma < (1 - b(ET))y_{G^-}(ET, B) + \delta(ET)$. ■

2) CASE II: $(1 - C(ET))ET < X^* < ET < EIT$

Theorem 3.2: If $l > m + \frac{mbr}{p}$, $(1 - c(ET))ET < x^* < ET < EIT$ and $ET > \tau_0$ hold, then system (2) does not possess OOPS; if $l > m + \frac{mbr}{p}$, $(1 - c(ET))ET < x^* < ET < EIT$ and $ET \leq \tau_0$ hold, then system (2) possesses a unique OOPS.

Proof: Denote $\tau_0 = \max_{x^* < x \leq EIT} \{x|y_{G^-}(x, ((1 - c(ET))ET, 0)) < y_{G^-}(ET, D)\}$. According to the magnitudes between ET and τ_0 , two cases are discussed: (i): $ET > \tau_0$

The trajectory with the initial point D will approach the stationary point E_* after time t . For any points in the region Ω will go on approaching the point E_* . Therefore, there does not exist an OOPS in this case.

(ii): $ET \leq \tau_0$.

We denote that the isoclinic line l_2 intersects the impulse set M at the point C . The trajectory passing point C intersects the phase set $x = (1 - c(ET))ET$ at the point A_1 and B_1 , where point A_1 and B_1 are located above and below isoclinic line $\frac{dx}{dt} = 0$, respectively. The successor point of point A_1 is point A_1^+ . By applying a similar method of Theorem 3.1, we can prove the existence and uniqueness of OOPS (see Fig. 3). ■

B. STABILITY OF THE ORDER ONE PERIODIC SOLUTION

Next, we prove the stability of the OOPS by using the geometric method. Considering system (4),

$$\left. \begin{aligned} \dot{x} &= \frac{px(t) - rx^2(t)}{k + \zeta x(t)} - qx(t)y(t) \triangleq f(x, y), \\ \dot{y} &= \frac{lx(t)y(t)}{x(t) + b} - my \triangleq g(x, y), \\ \Delta x &= -c(x)x(t), \\ \Delta y &= -b(x)y(t) + \delta(x), \end{aligned} \right\} \begin{aligned} x < ET, \\ x = ET, y \leq \bar{y}, \end{aligned} \quad (4)$$

we denote that the trajectory of OOPS of system (4) is \widehat{PQ} , and the period is T . For any constant $\varepsilon > 0$, we choose a

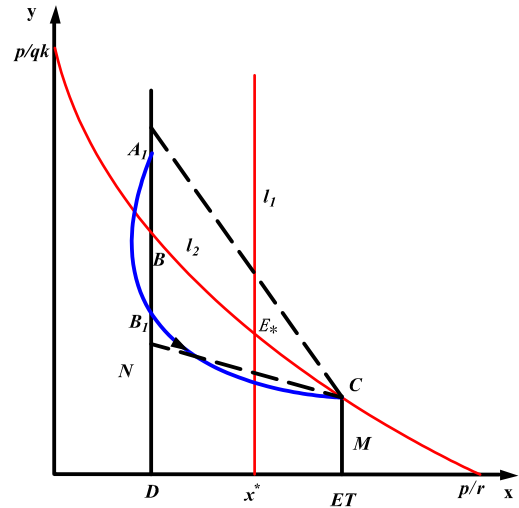


FIGURE 3. The existence of the OOPS of system (2) in case II.

point $P_1(x_{P_1}, y_{P_1})$, where $x_{P_1} = (1 - c(ET))ET$ and $y_{P_1} \in (y_P, y_P + \varepsilon)$. The trajectory $\varphi(t, P_1)$ with starting point P_1 intersects the impulse set at the point Q_1 , then point Q_1 jumps to the point P_1^+ after the impulse effects (see Fig. 4).

The normal lines at the point P and Q intersect the trajectory $\widehat{P_1Q_1}$ at the point P_0 and Q_0 , respectively. And these two normals are denoted as \vec{n}_1 and \vec{n}_2 , respectively. Let $|PP_0| = u$, $|QQ_0| = v$, according to [51, Lemma2.7], $v = v(u)$ is the successor function, we shall get $\frac{dv(u)}{du} = \exp\left(\int_0^T \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dt\right)$.

According to [54, Lemma 4.1], it follows that

$$\frac{v}{u} = \frac{\sqrt{f^2(P) + g^2(P)}}{\sqrt{f^2(Q) + g^2(Q)}} \exp\left(\int_0^T \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dt\right).$$

Let α and β be the angles between the normal line \vec{n}_1 and the y -axial, \vec{n}_2 and the y -axial. Because point P_1 is close enough to point P , we get

$$\begin{aligned} u &= |PP_0| \approx |PP_1| \cos \alpha, \\ v &= |QQ_0| \approx |QQ_1| \cos \beta. \end{aligned}$$

Because the tangent line $\vec{\tau}_1$ of the trajectory \widehat{PQ} at point P satisfy $\vec{\tau}_1 = (f(P), g(P))$, we get $\cos \alpha = \frac{f(P)}{\sqrt{f^2(P) + g^2(P)}}$.

Analogously, $\cos \beta = \frac{f(Q)}{\sqrt{f^2(Q) + g^2(Q)}}$. So we get

$$\begin{aligned} \frac{|QQ_1|}{|PP_1|} &= \frac{|QQ_0|}{\cos \beta} \cdot \frac{\cos \alpha}{|PP_0|} \\ &= \frac{v}{u} \cdot \frac{\cos \alpha}{\cos \beta} \\ &= \frac{\sqrt{f^2(P) + g^2(P)}}{\sqrt{f^2(Q) + g^2(Q)}} \cdot \frac{f(P)}{\sqrt{f^2(P) + g^2(P)}} \\ &= \frac{\sqrt{f^2(Q) + g^2(Q)}}{f(Q)} \cdot \exp\left(\int_0^T \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dt\right) \\ &= \frac{f(P)}{f(Q)} \int_0^T \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dt. \end{aligned}$$

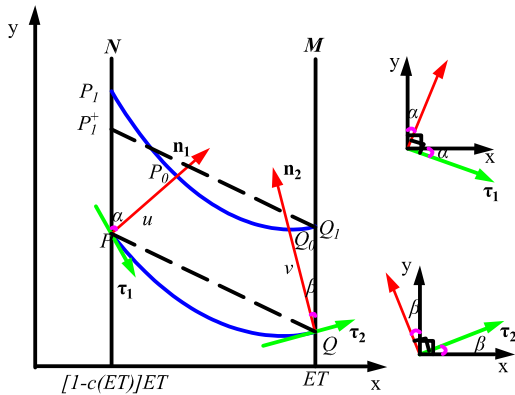


FIGURE 4. The illustration of OOPS of system (4).

Now, impulsive effects would be considered.

$$\begin{aligned}
 y_{P_1^+} &= y_{Q_1} - b(ET)y_{Q_1} + \delta(ET) \\
 y_P &= y_{Q^+} = y_Q - b(ET)y_Q + \delta(ET) \\
 |P_1^+ P| &= y_{P_1^+} - y_P \\
 &= y_{Q_1} - y_Q - b(ET)(y_{Q_1} - y_Q) \\
 &= [1 - b(ET)](y_{Q_1} - y_Q) \\
 &= [1 - b(ET)](|Q_1 Q|) \\
 \frac{|P_1^+ P|}{|PP_1|} &= (1 - b(ET)) \left(\frac{|Q_1 Q|}{|PP_1|} \right)
 \end{aligned}$$

If $\frac{|P_1^+ P|}{|PP_1|} < 1$, then we can get $|\varphi(t, P_1^+) - \varphi(t, P)| < \varepsilon, t > t_0$. So we can prove that the OOPS is asymptotically stable on the upper side of the region.

Similarly, the OOPS is asymptotically stable on the lower side of the region. And then we have the following theorem:

Theorem 3.3: Suppose that system (4) has a periodic trajectory \widehat{PQ} with point P and point Q locate in phase set and impulse set respectively. Let

$$\mu_0 = \frac{f(P)}{f(Q)} \exp \int_0^T \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} \right) dt$$

If $|\mu| = |(1 - b(ET))\mu_0| < 1$, then the OOPS of the system (2) is orbitally asymptotically stable.

IV. SIMULATIONS AND OPTIMIZATION

A. NUMERICAL SIMULATIONS

In this paper, we construct a pulse feedback control predator-prey model with Smith growth. We prove the existence, uniqueness and stability of the OOPS of system (2). We give a special example to verify the theoretical results in this section. For the specific model

$$\left\{ \begin{aligned} \dot{x} &= \frac{x - 0.8x^2}{3 + 0.6x} - 0.5xy, \\ \dot{y} &= \frac{0.5xy}{x + 0.4} - 0.2y, \end{aligned} \right\} x < ET, \quad (5)$$

$$\left\{ \begin{aligned} \Delta x &= -c(x)x, \\ \Delta y &= -b(x)y + \delta(x), \end{aligned} \right\} x = ET, y \leq \bar{y},$$

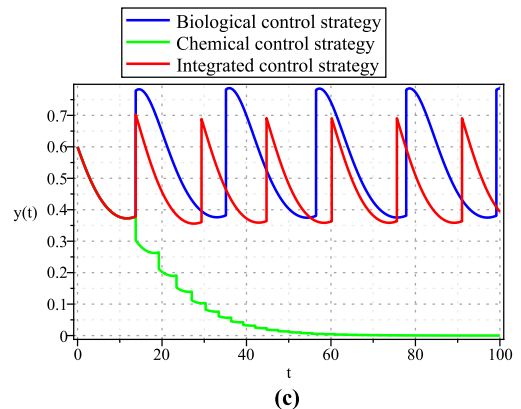
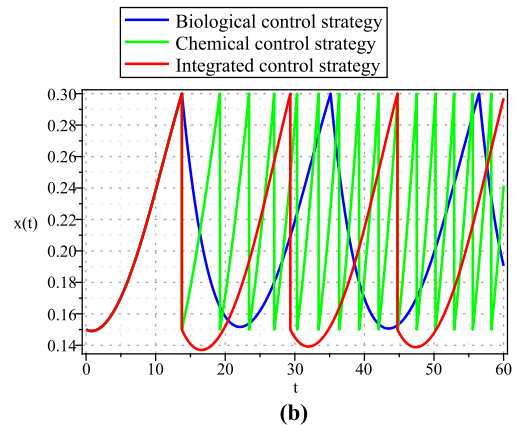
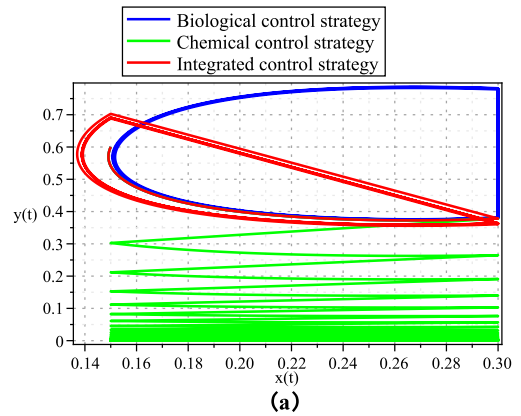


FIGURE 5. The phase diagram (a), time series of Cydia Pomonella (b) and Trichogrammatid (c) starting from $(x_0, y_0) = (0.15, 0.6)$. Control parameters: biological control strategy: $ET = 0.3, c = 0, b = 0$ and $\delta = 0.4$; chemical control strategy: $ET = 0.3, c = 0.5, b = 0.2$ and $\delta = 0$; integrated control strategy: $ET = 0.3, c = 0.5, b = 0.2$ and $\delta = 0.4$.

with a simple calculation, the internal equilibrium of the free system is $E_x(x^*, y^*) = (0.267, 0.498)$, which is orbitally asymptotically stable. let $SHT = 0.1, EIT = 0.4, \delta_{max} = 0.1, \delta_{min} = 10\% \delta_{max} = 0.1, c_{max} = 0.75, b_{max} = 0.3$. The simulations are carried out by changing one main parameter ET and fixing all other parameters. The control parameters δ, c, b are computed in equation (3).

Firstly, the biological control strategy, the chemical control strategy and the integrated control strategy are compared by numerical simulation and are illustrated in Fig.5. Fig.5 shows

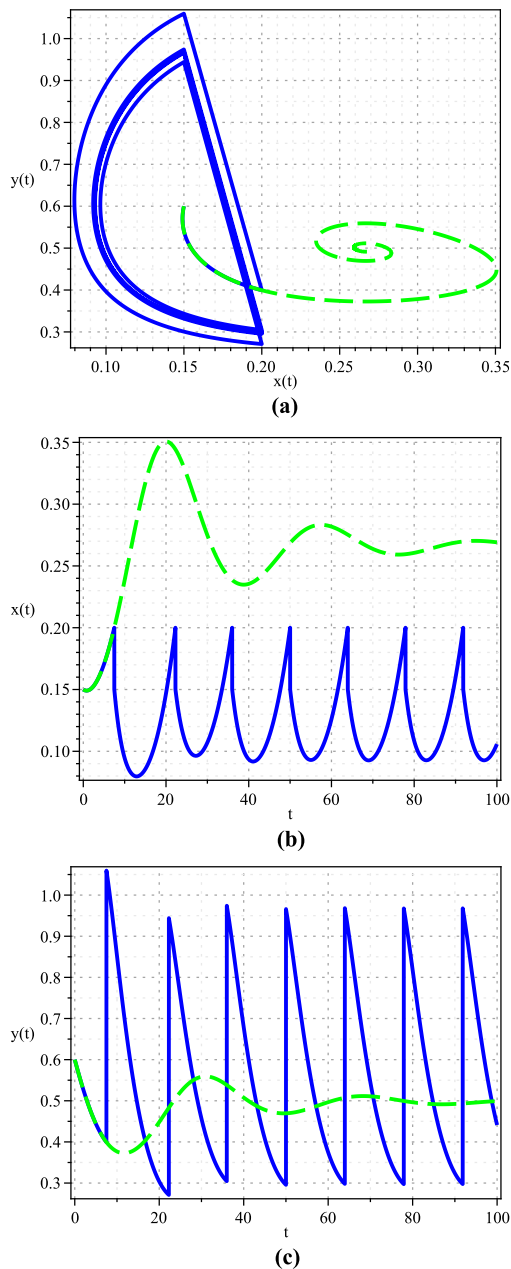


FIGURE 6. The phase diagram (a), time series of Cydia Pomonella (b) and Trichogrammatid (c) starting from $(x_0, y_0) = (0.15, 0.6)$. Control parameters: $ET = 0.2$, $c = 0.25$, $b = 0.1$ and $\delta = 0.7$. The solution of system (2) is presented in blue full line and the solution of free system (1) is represented in green dotted lines.

that the integrated control strategy is more effective than biological control strategy and chemical control strategy in Cydia Pomonella management.

For case I in Sect 3.1.1, the Cydia Pomonella control level is selected as $ET = 0.2$. The phase portrait of Cydia Pomonella density and Trichogrammatid density and time series starting from $(x_0, y_0) = (0.15, 0.6)$ are showed in Fig.6. It shows that the trajectory admits the OOPS. For case II in Sect 3.1.2, the Cydia Pomonella control level is selected as $ET = 0.35$. The phase

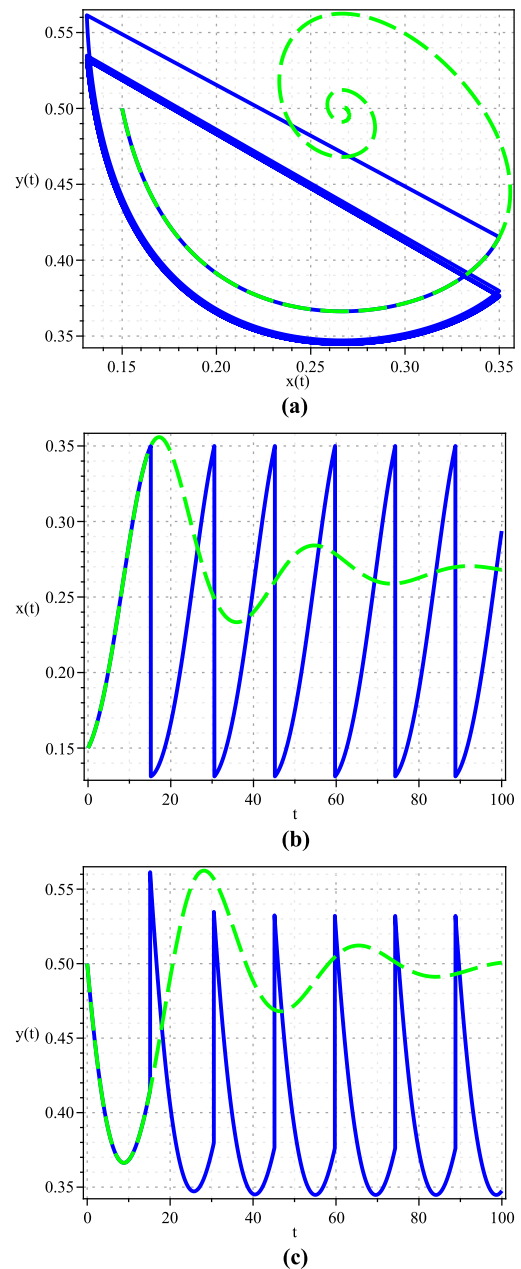


FIGURE 7. The phase diagram (a), time series of Cydia Pomonella (b) and Trichogrammatid (c) starting from $(x_0, y_0) = (0.15, 0.5)$. Control parameters: $ET = 0.35$, $c = 0.625$, $b = 0.25$ and $\delta = 0.25$. The solution of system (2) is presented in blue full line and the solution of free system (1) is represented in green dotted lines.

portrait of Cydia Pomonella density and Trichogrammatid density and time series starting from $(x_0, y_0) = (0.15, 0.5)$ are showed in Fig.7. It shows that the trajectory tends to be periodic (i.e., the OOPS exists). For a little higher Cydia Pomonella control level, such as, $ET = 0.4$, the phase portrait of Cydia Pomonella density and Trichogrammatid density and time series starting from $(x_0, y_0) = (0.15, 0.5)$ are shown in Fig.8 and it shows that the trajectory tends to the positive equilibrium $E_*(x^*, y^*) = (0.267, 0.498)$.

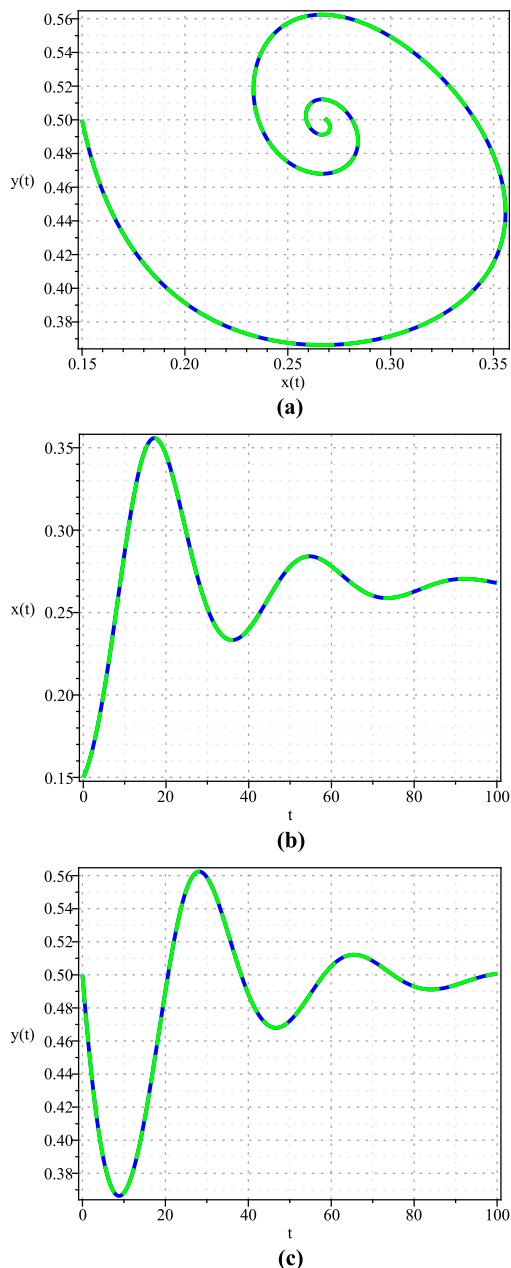


FIGURE 8. The phase diagram (a), time series of Cydia Pomonella (b) and Trichogrammatid (c) starting from $(x_0, y_0) = (0.15, 0.5)$. Control parameters: $ET = 0.4$, $c = 0.75$, $b = 0.3$ and $\delta = 0.1$. The solution of system (2) is presented in blue full line and the solution of free system (1) is represented in green dotted lines.

B. OPTIMAL CYDIA POMONELLA CONTROL LEVEL DETERMINATION

Our objective is to select a Cydia Pomonella level between SHT and EIT and minimize the cost of per unit period. The following optimization problem is considered in order to solve the this problem.

Let d_1 denote the unit cost of releases of the Trichogrammatid, d_2 be the unit cost of the chemical control (i.e. the price of pesticide and the price of the environment disruption). Our final objective is to make the cost per unit time lowest in the

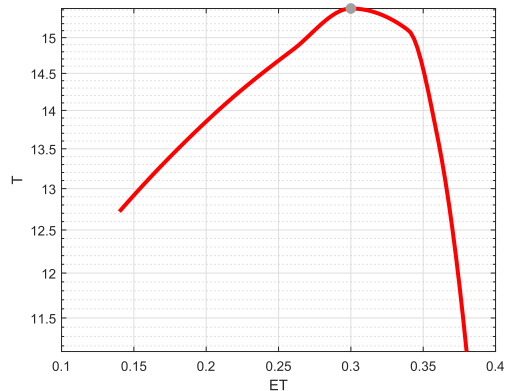


FIGURE 9. Impulse period T of the OOPS varies with the cydia pomonella control level ET .

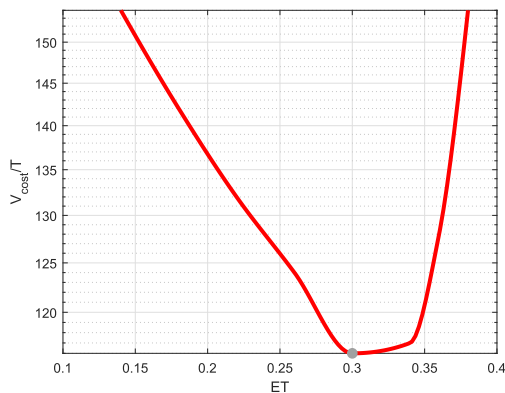


FIGURE 10. The cost per unit time $\frac{V_{cost}}{T}$ on the cydia pomonella control level ET .

process of Cydia Pomonella control. Let V_{cost} represent the total cost in single period, which is a function of the yield of releases of Trichogrammatid $\delta(ET)$ and chemical control strength $c(ET)$. Then $V_{cost} = d_1\delta(ET) + d_2c(ET)$. Thus, the optimization model is constructed as follows

$$\begin{aligned} &\min \frac{V_{cost}(ET)}{T(ET)} \\ &\text{s.t. } SHT \leq ET \leq EIT. \end{aligned}$$

The optimum economic threshold ET^* is obtained after solving the objective function problem, which results in the optimum release amount of the Trichogrammatid $\delta^* = \delta(ET^*)$, the optimal chemical control strength $c^* = c(ET^*)$ and the optimal control period of chemical control is $T^* = T(\delta^*, c^*)$. Certainly, it should be pointed out that the optimum Cydia Pomonella control level ET is dependent on the ratio of $\zeta = \frac{d_2}{d_1}$.

The impulse period T of the OOPS varies with the Cydia Pomonella control level ET , as is shown in Fig.9 (parameter values are presented in Table.1), and the cost per unit time $\frac{V_{cost}}{T}$ is shown in Fig.10. Assume $d_1 = d_2 = 2000$, we shall get $\zeta = 1$. From Fig.9 and Fig.10, the optimal Cydia Pomonella threshold is $ET^* = 0.3$, the optimal chemical control strength is $c^* = c(ET^*) = 0.5$, the optimal release amount of the Trichogrammatid is $\delta^* = \delta(ET^*) = 0.4$, and the optimum frequency of the chemical control is $T^* = 15.42$. According to Fig.11, we shall get that the

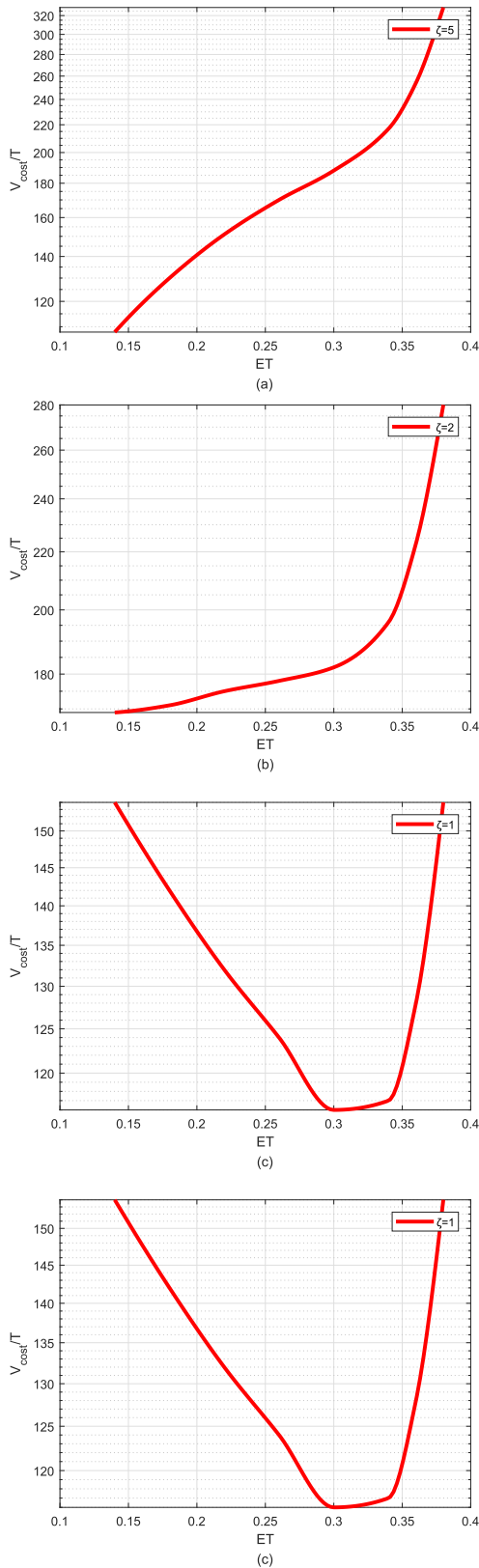


FIGURE 11. The change in the cost per unit time V_{cost} on the cydia pomonella control level ET for $\zeta = 5, 2, 1, 1/5$.

optimum Cydia Pomonella level ET^* is dependent on the economic ratio of $\zeta = \frac{d_2}{d_1}$.

TABLE 1. Value of parameters.

ET	c	b	δ	T	$\frac{V_{cost}}{T}$
0.14	0.1	0.04	0.88	12.72	154
0.18	0.2	0.88	0.76	13.49	142
0.22	0.3	0.12	0.64	14.2	132
0.26	0.4	0.16	0.52	14.83	124
0.3	0.5	0.2	0.4	15.42	117
0.34	0.6	0.24	0.28	15.10	116
0.36	0.65	0.26	0.22	13.64	128
0.38	0.7	0.28	0.16	11.137	154

V. CONCLUSION

The interaction between Cydia Pomonella and Trichogrammatid is taken as examples in this paper. We establish a pest integrated management predator-prey model with Smith growth and pulse feedback control. Cydia Pomonella integrated management strategy is adopted between the biological control level and the chemical control level. The purpose of this study is to obtain the optimal control level of the Cydia Pomonella and minimize the cost of Cydia Pomonella control.

Based on the above analysis, the OOPS always exists for any release amount of the Trichogrammatid and chemical control strength if the Cydia Pomonella control level ET satisfies $SHT \leq ET \leq \min\{EIT, x^*\}$. While for $(1 - c(ET))ET < x^* < ET < EIT$, the OOPS exists only when the Cydia Pomonella control level ET satisfies $ET \leq \tau_0$. In order to reduce the cost of Cydia Pomonella management, we set up optimization model and obtain a optimal control level. We verify that the integrated control strategy is more effective than chemical control and biological control by numerical simulation. Moreover, numerical simulations complement our theoretical results.

In this paper, system (2) is the case of biological control of [49] when $x = h_1$, system (2) is the case of comprehensive control of [49] when $x = h_2$. Therefore, our proposed control strategy is more general. Meanwhile, we formulate the optimization problem to minimize the total cost of Cydia Pomonella control.

In addition, the optimization problem proposed in this paper needs to be further improved. In future research, we will consider the specific functional relationship between cost and period, and cost and threshold to make our optimization more scientific and more realistic.

COMPETING INTERESTS

The authors declare that they have no competing interests.

AUTHOR'S CONTRIBUTIONS

All authors read and approved the final manuscript.

REFERENCES

[1] T. C. Gard, "Global analysis of the predator-prey system with Beddington-DeAngelis functional response," *J. Math. Anal. Appl.*, vol. 281, pp. 395-401, May 2002.

- [2] M. A. Idlango, J. J. Shepherd, and J. A. Gear, "Logistic growth with a slowly varying Holling type II harvesting term," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 49, pp. 81–92, Aug. 2017.
- [3] Y. Z. Pei, H. N. Wang, C. G. Li, and S. J. Gao, "An immune model with logistic growth and Holling type-II functional response," *Acta Math. Sinica*, vol. 54, no. 2, pp. 301–312, 2011.
- [4] H. Cheng, F. Wang, and T. Zhang, "Multi-state dependent impulsive control for Holling I predator-prey model," *Discrete Dyn. Nature Soc.*, vol. 2012, Apr. 2012, Art. no. 181752.
- [5] L. N. Guin and P. K. Mandal, "Spatial pattern in a diffusive predator-prey model with sigmoid ratio-dependent functional response," *Int. J. Biomath.*, vol. 7, no. 5, 2014, Art. no. 1450047.
- [6] W. Fengying and W. Ke, "Economic harvesting model with variable price and cost for population with smith growth and cui lawson growth," *J. Biomath.*, vol. 19, no. 3, pp. 328–336, 2004.
- [7] F. E. Smith, "Population dynamics in daphnia magna and a new model for population growth," *Ecology*, vol. 44, no. 4, pp. 651–663, 1963.
- [8] L. S. Chen, X. Y. Song, and Z. Y. Lu, *Mathematical Models and Methods in Ecology*. Chengdu, China: Sichuan Education Publishing House, 2003.
- [9] L. Nie, Z. Teng, L. Hu, and J. Peng, "Existence and stability of periodic solution of a predator-prey model with state-dependent impulsive effects," *Math. Comput. Simul.*, vol. 79, no. 7, pp. 2122–2134, 2009.
- [10] K. Sun, T. Zhang, and Y. Tian, "Dynamics analysis and control optimization of a pest management predator-prey model with an integrated control strategy," *Appl. Math. Comput.*, vol. 293, pp. 253–271, Jan. 2017.
- [11] B. Liu, Y. Zhang, and L. Chen, "The dynamical behaviors of a Lotka-Volterra predator-prey model concerning integrated pest management," *Nonlinear Anal., Real World Appl.*, vol. 6, pp. 227–243, Apr. 2005.
- [12] T. Zhang, X. Meng, and Y. Song, "The dynamics of a high-dimensional delayed pest management model with impulsive pesticide input and harvesting prey at different fixed moments," *Nonlinear Dyn.*, vol. 64, pp. 1–12, Apr. 2011.
- [13] S. J. Walde, "Immigration and the dynamics of a predator-prey interaction in biological control," *J. Animal Ecology*, vol. 63, no. 2, pp. 337–346, 1994.
- [14] O. Arino, A. El Abdllaoui, J. Mikram, and J. Chattopadhyay, "Infection in prey population may act as a biological control in ratio-dependent predator-prey models," *Nonlinearity*, vol. 17, no. 3, p. 1101, 2004.
- [15] A. J. Terry, "Biocontrol in an impulsive predator-prey model," *Math. Biosci.*, vol. 256, pp. 102–115, Oct. 2014.
- [16] F. Li, X. Meng, and X. Wang, "Analysis and numerical simulations of a stochastic SEIQR epidemic system with quarantine-adjusted incidence and imperfect vaccination," *Comput. Math. Methods Med.*, vol. 2018, Feb. 2018, Art. no. 7873902.
- [17] F. Liu and F. Wang, "Entropy-expansiveness of geodesic flows on closed manifolds without conjugate points," *Acta Math. Sinica*, vol. 32, no. 4, pp. 507–520, 2016.
- [18] Y. Wang, "Beyond regular semigroups," *Semigroup Forum*, vol. 92, pp. 418–448, Apr. 2016.
- [19] Y. Zhang, B. Liu, and L. Chen, "Extinction and permanence of a two-prey one-predator system with impulsive effect," *Math. Med. Biol.*, vol. 20, no. 4, pp. 309–325, 2003.
- [20] Y. Tian, K. Sun, and L. Chen, "Geometric approach to the stability analysis of the periodic solution in a semi-continuous dynamic system," *Int. J. Biomath.*, vol. 7, no. 2, 2014, Art. no. 1450018.
- [21] Y. Li, D. Xie, and J. Cui, "Complex dynamics of a predator-prey model with impulsive state feedback control," *Appl. Math. Comput.*, vol. 230, pp. 395–405, Mar. 2014.
- [22] R. Shi, X. Jiang, and L. Chen, "A predator-prey model with disease in the prey and two impulses for integrated pest management," *Appl. Math. Model.*, vol. 33, pp. 2248–2256, May 2009.
- [23] Y. Li, H. Cheng, J. Wang, and Y. Wang, "Dynamic analysis of unilateral diffusion Gompertz model with impulsive control strategy," *Adv. Difference Equ.*, vol. 2018, Jan. 2018, Art. no. 32.
- [24] Z. Liang, G. Pang, X. Zeng, and Y. Liang, "Qualitative analysis of a predator-prey system with mutual interference and impulsive state feedback control," *Nonlinear Dyn.*, vol. 87, no. 3, pp. 1495–1509, 2017.
- [25] H. Cheng and T. Zhang, "A new predator-prey model with a profitless delay of digestion and impulsive perturbation on the prey," *Appl. Math. Comput.*, vol. 217, no. 22, pp. 9198–9208, 2011.
- [26] Y. Zhang, F. Wang, J. Wang, and Y. Huang, "Adaptive finite time control of nonlinear quantized systems with actuator dead-zone," *IEEE Access*, to be published. doi: [10.1109/ACCESS.2019.2922748](https://doi.org/10.1109/ACCESS.2019.2922748).
- [27] Y. Li, H. Cheng, and Y. Wang, "A *Lycyon pictus* impulsive state feedback control model with Allee effect and continuous time delay," *Adv. Difference Equ.*, vol. 2018, p. 367, Dec. 2018.
- [28] L. S. Chen, "Pest control and geometric of semi continuous dynamical system," *J. Beihua Univ.*, vol. 12, no. 1, pp. 1–9, Oct. 2011.
- [29] Y. Tian, T. Zhang, and K. Sun, "Dynamics analysis of a pest management prey-predator model by means of interval state monitoring and control," *Nonlinear Anal., Hybrid Syst.*, vol. 23, pp. 122–141, Feb. 2017.
- [30] G. Liu, X. Wang, X. Meng, and S. Gao, "Extinction and persistence in mean of a novel delay impulsive stochastic infected predator-prey system with jumps," *Complexity*, vol. 2017, Jun. 2017, Art. no. 1950970.
- [31] K. Liu, T. Zhang, and L. Chen, "State-dependent pulse vaccination and therapeutic strategy in an SI epidemic model with nonlinear incidence rate," *Comput. Math. Methods Med.*, vol. 2019, Feb. 2019, Art. no. 3859815.
- [32] N. Gao, Y. Song, X. Wang, and J. Liu, "Dynamics of a stochastic SIS epidemic model with nonlinear incidence rates," *Adv. Difference Equ.*, vol. 1, p. 41, Dec. 2019.
- [33] H. Zhang, P. Georgescu, and L. Chen, "On the impulsive controllability and bifurcation of a predator-pest model of IPM," *Biosystems*, vol. 93, pp. 151–171, Sep. 2008.
- [34] Y. Pei, G. Zeng, and L. Chen, "Species extinction and permanence in a prey-predator model with two-type functional responses and impulsive biological control," *Nonlinear Dyn.*, vol. 52, pp. 71–81, Apr. 2008.
- [35] S. Zhang, X. Meng, and X. Wang, "Application of stochastic inequalities to global analysis of a nonlinear stochastic SIRS epidemic model with saturated treatment function," *Adv. Difference Equ.*, vol. 2018, no. 1, p. 50, Dec. 2018.
- [36] F. Liu, "Continuity and approximate differentiability of multisublinear fractional maximal functions," *Math. Inequalities Appl.*, vol. 21, no. 1, pp. 25–40, 2018.
- [37] F. Liu, Q. Xue, and K. Yabuta, "Rough maximal singular integral and maximal operators supported by subvarieties on Triebel-Lizorkin spaces," *Nonlinear Anal.*, vol. 171, pp. 41–72, Jun. 2018.
- [38] A. Miao, T. Zhang, J. Zhang, and C. Wang, "Dynamics of a stochastic SIR model with both horizontal and vertical transmission," *J. Appl. Anal. Comput.*, vol. 8, no. 4, pp. 1108–1121, 2018.
- [39] J. Wang, H. Cheng, H. Liu, and Y. Wang, "Periodic solution and control optimization of a prey-predator model with two types of harvesting," *Adv. Difference Equ.*, vol. 2018, Jan. 2018, Art. no. 41.
- [40] W. Lv and F. Wang, "Adaptive tracking control for a class of uncertain nonlinear systems with infinite number of actuator failures using neural networks," *Adv. Difference Equ.*, vol. 2017, Dec. 2017, Art. no. 374.
- [41] L. Mailleret and F. Gognard, "Global stability and optimisation of a general impulsive biological control model," *Math. Biosci.*, vol. 221, pp. 91–100, Oct. 2009.
- [42] L. Wang, L. Chen, and J. J. Nieto, "The dynamics of an epidemic model for pest control with impulsive effect," *Nonlinear Anal., Real World Appl.*, vol. 11, pp. 1374–1386, Jun. 2010.
- [43] T. Zhang, X. Meng, and T. Zhang, "Global analysis for a delayed SIV model with direct and environmental transmissions," *J. Appl. Anal. Comput.*, vol. 6, no. 2, pp. 479–491, 2016.
- [44] Z. Shi, Y. Li, and H. Cheng, "Dynamic analysis of a pest management smith model with impulsive state feedback control and continuous delay," *Mathematics*, vol. 7, no. 7, p. 591, 2019.
- [45] X. Lv, L. Wang, and X. Meng, "Global analysis of a new nonlinear stochastic differential competition system with impulsive effect," *Adv. Difference Equ.*, vol. 2017, Sep. 2017, Art. no. 296.
- [46] Z. Shi, J. Wang, Q. Li, and H. Cheng, "Control optimization and homoclinic bifurcation of a prey-predator model with ratio-dependent," *Adv. Difference Equ.*, vol. 2019, p. 2, Dec. 2019.
- [47] H. Zhang, L. Chen, and P. Georgescu, "Impulsive control strategies for pest management," *J. Biol. Syst.*, vol. 15, no. 2, pp. 235–260, 2007.
- [48] L. Zhao, L. Chen, and Q. Zhang, "The geometrical analysis of a predator-prey model with two state impulses," *Math. Biosci.*, vol. 238, no. 2, pp. 55–64, 2012.
- [49] J. Wang, H. Cheng, Y. Li, and X. Zhang, "The geometrical analysis of a predator-prey model with multi-state dependent impulses," *J. Appl. Anal. Comput.*, vol. 8, no. 2, pp. 427–442, 2018.
- [50] J. Wang, H. Cheng, X. Meng, and B. G. S. A. Pradeep, "Geometrical analysis and control optimization of a predator-prey model with multi state-dependent impulse," *Adv. Difference Equ.*, vol. 2017, Aug. 2017, Art. no. 252.

- [51] Z. Y. Zhang, *Geometrical Methods in the Theory of Ordinary Differential Equations*. Beijing, China: Peking Univ. Press, 1987, pp. 262–272.
- [52] K. Sun, T. Zhang, and Y. Tian, “Theoretical study and control optimization of an integrated pest management predator–prey model with power growth rate,” *Math. Biosci.*, vol. 279, pp. 13–26, Sep. 2016.
- [53] T. Zhang, X. Meng, R. Liu, and T. Zhang, “Periodic solution of a pest management Gompertz model with impulsive state feedback control,” *Nonlinear Dyn.*, vol. 78, no. 2, pp. 921–938, 2014.
- [54] H. Guo, L. Chen, and X. Song, “Dynamical properties of a kind of SIR model with constant vaccination rate and impulsive state feedback control,” *Int. J. Biomath.*, vol. 10, no. 7, 2017, Art. no. 1750093.



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