

Received August 8, 2019, accepted August 24, 2019, date of publication September 2, 2019, date of current version October 17, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2938814

# Robust Learning Control for Tank Gun Control Servo Systems Under Alignment Condition

GUANGMING ZHU<sup>1</sup>, XIUSHAN WU<sup>1</sup>, QIUZHEN YAN<sup>1</sup>, AND JIANPING CAI<sup>1</sup>

Zhejiang University of Water Resource and Electric Power, Hangzhou 310018, China

Corresponding author: Xiushan Wu (wuxiushan@cjlu.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant NSFC: 61573322, in part by the Scientific Research Project of the Water Conservancy Department of Zhejiang Province under Grant RC1858, and in part by the University Visiting Scholars Developing Project of the Zhejiang Province under Grant FX2017078.

**ABSTRACT** This paper proposes an adaptive learning control scheme to solve high-precision velocity tracking problem for tank gun control servo systems. Lyapunov approach is used to design the learning controller, with alignment condition used to cope with initial problem of iterative learning control. Robust control technique and adaptive learning control technique are synthesized to handle nonlinear uncertainties and external disturbances. The unknown parameters are estimated according to the full saturation difference learning strategy. As the iteration number increases, the system state can accurately track the reference signal over the whole time interval, and all signal are guaranteed to be bounded.

**INDEX TERMS** Tank gun control servo systems, iterative learning control, adaptive control.

## I. INTRODUCTION

As a kind of useful weapons in battle fields, tanks can both improve the efficiency of artillery firepower and strengthen the surviving ability. In actual fighting situations, military tanks usually need to track the moving targets in complex and harsh environments where there exists friction, various complex uncertainties and outside disturbances. For the reason that accuracy, stability and speed of response are essential to mission accomplishment, the gun control servo systems of tank should be well designed to have high tracking precision and good dynamic quality, but also strong robustness and survivability. Therefore, it is a meaningful job for us to investigate the control design for tank gun control servo systems.

The motion control of tank gun barrels has been an ongoing topic in the past three decades, and a lot of control schemes have been proposed for achieving better control performance. In [1], PID control strategy is applied to design control system to obtain the firing precise control for the tanks in motion. In [2], a variable structure control scheme is proposed to solve the position tracking for tank guns with large uncertainties. Refs. [3] and [4] studied the optimal control algorithm for tanks. In [5], sliding mode control based on optimization was reported to cope with the motion control of tank guns. In [6], Feng *et al.* investigated the adaptive fuzzy control method for tank systems. In [7], a tank gun elevation control system was

developed by using direct adaptive control method. In [8], adaptive robust control method for tank systems was discussed. Xia *et al.* proposed an active disturbance rejection control scheme to solve the problem of position tracking for a tank gun control system with inertia uncertainty and external disturbance, using the extended state observer to estimate the inertia uncertainty and external disturbance [9]. Hu *et al.* investigated the disturbance-observer base control [10] and adaptive neural network control [11] for tank gun control system, respectively. The above-mentioned works have presented meaningful results of tank gun control servo systems. However, in these above-mentioned existing works, it is still very hard to achieve high tracking precision in the complicated application environment for the great difficulties in accurate system modeling and the defects of control technologies themselves. Therefore, the precision control of tank systems is still a topic to be further studied.

On the other hand, iterative learning control (ILC) is effective in dealing with those repetitive control tasks over a finite time interval [12]–[21]. This control technique takes advantage of system error to updated control input cycle by cycle. As iteration number increases, the system output or state can follow its reference signal over the full interval. Up to now, ILC has been widely applied in many high-precision control cases, such as robotic manipulators, power electronic circuits, hard disk drives, and chemical plants [22]–[27]. In the past two decades, adaptive ILC has aroused great scholarly interest in ILC area. French *et al.* designed a differential

The associate editor coordinating the review of this article and approving it for publication was Okyay Kaynak.

learning law to estimate the unknown constant for the nonlinear systems over a fixed time interval [28]. Xu *et al.* proposed a difference learning control law to compensate unknown time-varying but iteration-independent vector [12]. Yin *et al.* investigated the trajectory-tracking problem for nonlinear system with unknown time-iteration-varying parameters [29]. Khanesar *et al.* proposed a sliding mode control theory-based learning algorithm which benefits from elliptic type-2 fuzzy membership functions [30]. In addition, the ILC research for nonparametric systems [31] and nonlinear parametric systems [32] have received close attentions in recent years.

In most existing ILC schemes, the initial system error is required to be zero at each iteration [33]–[35]. However, perfect system resetting for each iteration is not implementable in actual industries. Otherwise, a slight initial error may lead to divergence of the tracking error. Therefore, relaxing or removing the zero-error resetting condition has practical significance. Adaptive ILC without zero-error resetting condition has been explored in some works, and the presented solutions include time-varying boundary layer technique [36], error-tracking method [37]–[39], initial rectifying action [40]–[42] and so on. Besides, letting the final state of the previous iteration become the initial state of the current iteration, named as alignment condition, is effective at mitigating zero-error initial resetting condition for adaptive ILC where the reference trajectory is spatially closed, meaning that the starting point of the reference trajectory is also the end point in each iteration [43].

Up to now, there have been many ILC results on the position/velocity control of motors [45], [46]. In [47], an ILC scheme was proposed to reduce periodic torque pulsations in permanent magnet synchronous motors. In [48], the motion control of permanent magnet synchronous motors was considered, and ILC technique was used to eliminate the influence of force ripple for a position servo system. In [49], Precup *et al.* proposed a 2-DOF proportional-integral-fuzzy control scheme for a class of servo systems, and the extended symmetrical optimum method accompanied by an iterative feedback tuning algorithm was adopted to control law design. However, the ILC results on the trajectory-tracking problem for gun control servo systems of tank is few. How to develop ILC algorithms for gun control servo systems of tank is still an open issue.

In this paper, referring to the ILC algorithms design for PMLSMs and PMSMs, we want to solve the trajectory-tracking problem for gun control servo systems of tank by using ILC approaches. A robust learning control scheme is designed to obtain high-precision tracking performance regardless of nonzero initial errors. Compared with existing results, the main contributions of this work lie in the following:

(1) Difference from those existed results, in this work, robust adaptive iterative learning control technique is introduced to develop the control algorithm for tank gun control servo systems, which is helpful to get better tracking performance for the corresponding systems.

(2) In the process of ILC design for tank gun servo systems, alignment condition is used to remove/relax the zero initial error condition, which should be observed in most traditional ILC algorithms.

(3) The controller is designed by using Lyapunov approach, synthesizing learning control and robust control methods, which owns better capacity of handling the complex uncertainties. The parametric uncertainties and the bound of perturbations are estimated according to difference learning strategy, respectively.

The remainder of this paper is organized as follows. The problem formulation is given in Section 2. The design of iterative learning controller is given in Section 3. Section 4 presents the convergence analysis of the closed loop system. To demonstrate the effectiveness of the proposed ILC scheme, an illustrated example is shown in Section 5, followed by Section 6 which concludes the work.

## II. PROBLEM FORMULATION

### A. STRUCTURE OF TANK GUN CONTROL SYSTEMS

In a all-electric tank gun control system, the adjustments of turret and gun in both horizontal direction and vertical direction are accomplished by motor drive. Compared with the traditional electro-hydraulic/full-hydraulic gun control system, the full-electric one has some advantages including simple structure, excellent performance and high efficiency. Hence, it has been adopted widely in recent years. The structure diagram of vertical servo system of all-electrical tank gun is given in Fig 1. We can see that the controlled device mainly includes AC motor, speed reducer and barrel.

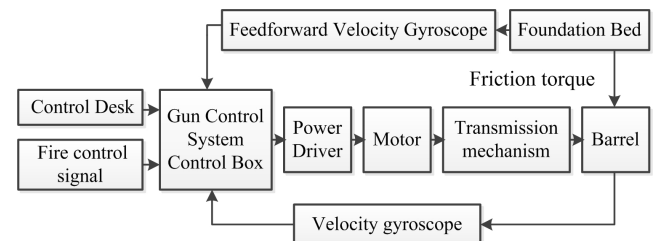


FIGURE 1. Structure diagram of vertical servo system of all-electrical tank gun.

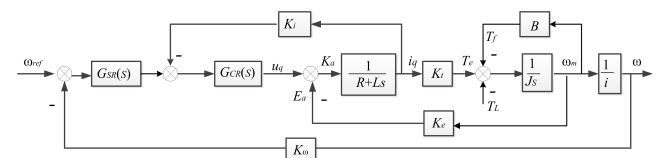


FIGURE 2. Block diagram of tank gun control systems.

### B. MODEL OF TANK GUN CONTROL SYSTEMS

The block diagram of tank gun control systems is presented in Fig. 2, where  $\omega_{ref}$  and  $\omega$  represent the desired angular velocity and the real angular velocity of the cannon, respectively.  $G_{SR}(s)$  is velocity regulator, and  $G_{CR}(s)$  is current regulator.  $u_q$  is the output voltage of the current loop.  $R$  and  $L$

represent the resistance and inductance of the motor armature circuit, respectively.  $K_a$  is the amplifier gain.  $E_a$  is the armature back electromotive force of motor.  $K_i$  is the current feedback coefficient of  $q$  axis.  $K_t$  is the motor torque factor.  $K_e$  denotes the electric torque coefficient.  $T_e$ ,  $T_L$  and  $T_f$  are the motor torque, load torque disturbance and friction torque disturbance, respectively.  $K_\omega$  is the angular velocity feedback coefficient of cannon.  $J$  is the total moment of inertia to the rotor.  $B$  is the viscous friction coefficient.  $i$  is the moderating ratio.  $s$  denotes the Laplace operator.

We can get the model of gun control servo systems of tank as

$$\dot{i}_q = -\frac{R}{L}i_q - \frac{K_e i}{L}\omega + \frac{K_a}{L}u_q, \quad (1)$$

$$\dot{\omega} = \frac{K_t}{J_i}i_q - \frac{1}{J_i}T_{Ls}, \quad (2)$$

where  $T_{Ls} = T_L + T_f$ . Combing (1) with (2), we obtain

$$\ddot{\omega} = -\frac{R}{L}\dot{\omega} - \frac{K_t K_e}{LJ}\omega + \frac{K_a K_t}{LJ_i}u_q - (\frac{R}{LJ_i}T_{Ls} + \frac{1}{J_i}\dot{T}_{Ls}) \quad (3)$$

Define  $x_1 = \omega$ ,  $x_2 = \dot{\omega}$ . Then, from (3), the dynamics of tank gun control systems at the  $k$ th iteration can be written as

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = -\frac{R}{L}x_{2,k} - \frac{K_t K_e}{LJ}x_{1,k} + \frac{K_a K_t}{LJ_i}u_{q,k} \\ \quad + \Delta f(\mathbf{x}_k, t), \end{cases} \quad (4)$$

where  $\mathbf{x}_k = (x_{1,k}, x_{2,k})$ ,  $\Delta f(\mathbf{x}_k, t) = -(\frac{R}{LJ_i}T_{Ls} + \frac{1}{J_i}\dot{T}_{Ls})$ , and  $k = 0, 1, 2, \dots$ , represents the number of iteration cycle.

For the given reference signal  $\mathbf{x}_d(t) = (x_d, \dot{x}_d)^T$ , which satisfies  $\mathbf{x}_d(t) \in C^1[0, T]$  and  $\mathbf{x}_d(T) = \mathbf{x}_d(0)$ , the control objective is to design proper adaptive learning control law so as to make the system state  $\mathbf{x}_k$  accurately track its reference signal  $\mathbf{x}_d$  over  $[0, T]$ , under the condition  $\mathbf{x}_k(0) = \mathbf{x}_{k-1}(T)$ . For the sake of brevity, the arguments in this paper are sometimes omitted when no confusion is likely to arise.

*Assumption 1:*

$$\Delta f(\mathbf{x}_k, t) = f_1(\mathbf{x}_k) + f_2(\mathbf{x}_k, t).$$

where  $f_1(\mathbf{x}_k)$  meets Liphitz continuous condition, i.e.,  $|f_1(\mathbf{x}_k) - f_1(\mathbf{x}_d)| \leq l\|\mathbf{x}_k - \mathbf{x}_d\|$  with  $l$  an unknown positive constant;  $f_2(\mathbf{x}_k, t)$  represents the noncontinuous but bounded perturbations. There exists an unknown smooth continuous function  $f_{2m}(t)$ ,  $f_2(\mathbf{x}_k, t) \leq f_{2m}(t)$ .

### III. CONTROL SYSTEM DESIGN

Let us define  $\mathbf{e}_k(t) = [e_{1,k}, \dots, e_{n,k}]^T = \mathbf{x}_k(t) - \mathbf{x}_d(t)$  and

$$\begin{aligned} s_k &= ce_{1,k} + e_{2,k}, \\ s_{\phi k} &= s_k - \phi \text{sat}(\frac{s_k}{\phi}) \end{aligned} \quad (5)$$

In this paper,  $\text{sat}(\cdot)$  represents saturation operator, defined as follows. For  $\hat{a} \in \mathbf{R}$ ,

$$\text{sat}_{\bar{a}}(\hat{a}) = \begin{cases} \hat{a}, & |a| < \bar{a} \\ \bar{a}\text{sign}(\hat{a}), & \text{else,} \end{cases}$$

in which,  $\bar{a}$  is the proper upper limit. While  $\bar{a} = 1$ , we denote  $\text{sat}_{\bar{a}}(\hat{a})$  briefly by  $\text{sat}(\hat{a})$ . For a vector  $\hat{\mathbf{a}} = (\hat{a}_1, \hat{a}_2, \dots, \hat{a}_m) \in \mathbf{R}^m$ ,  $\text{sat}_{\bar{a}}(\hat{\mathbf{a}}) \triangleq (\text{sat}_{\bar{a}}(\hat{a}_1), \text{sat}_{\bar{a}}(\hat{a}_2), \dots, \text{sat}_{\bar{a}}(\hat{a}_m))^T$ .

From (1), we can obtain

$$\begin{cases} \dot{e}_{i,k} = e_{i+1,k}, \\ \dot{e}_{2,k} = -\frac{R}{L}x_2 - \frac{K_t K_e}{LJ}x_1 + \frac{K_a K_t}{LJ_i}u_{q,k} + f_1(\mathbf{x}_k) \\ \quad + f_2(t) - \ddot{x}_d \end{cases}$$

and

$$\dot{s}_k = ce_{2,k} - \frac{R}{L}x_2 - \frac{K_t K_p}{LJ}x_1 + \frac{K_a K_t}{LJ_i}u_{q,k} + f_1(\mathbf{x}_k) + f_2(\mathbf{x}_k, t) - \ddot{x}_d. \quad (6)$$

Then, we choose a candidate control Lyapunov function at the  $k$ th iteration as

$$V_k = \frac{1}{2}s_{\phi k}^2 \quad (7)$$

Denoting  $\eta = \frac{K_a K_t}{LJ_i}$  and taking the time derivative to  $V_k$  yield

$$\begin{aligned} \dot{V}_k &= s_{\phi k} [ce_{2,k} - \frac{R}{L}x_2 - \frac{K_t K_e}{LJ}x_1 + \frac{K_a K_t}{LJ_i}u_{q,k} \\ &\quad + f_1(\mathbf{x}_k) + f_2(\mathbf{x}_k, t) - \ddot{x}_d] \\ &= s_{\phi k} \eta \left[ \frac{1}{\eta}(ce_{2,k} - \frac{R}{L}x_2 - \frac{K_t K_e}{LJ}x_1 + f_1(\mathbf{x}_k) \right. \\ &\quad \left. + f_2(\mathbf{x}_k, t) - \ddot{x}_d) + u_{q,k} \right]. \end{aligned} \quad (8)$$

While  $|s_k| > \phi$ , we have

$$\begin{aligned} \frac{1}{\eta}s_{\phi k}f_1(\mathbf{x}) &= \frac{1}{\eta}s_{\phi k}(f_1(\mathbf{x}) - f_1(\mathbf{x}_d)) + \frac{1}{\eta}s_{\phi k}f_1(\mathbf{x}_d) \\ &\leq \frac{l}{\eta}s_{\phi k}\|\mathbf{e}_k\| \text{sat}(\frac{s_k}{\phi}) + \frac{1}{\eta}s_{\phi k}f_1(\mathbf{x}_d) \end{aligned} \quad (9)$$

and

$$\frac{1}{\eta}s_{\phi k}f_2(\mathbf{x}_k, t) \leq \frac{1}{\eta}|s_{\phi k}|f_{2m}(t) = \frac{1}{\eta}s_{\phi k}f_{2m} \text{sat}(\frac{s_k}{\phi}). \quad (10)$$

Substituting (9) and (10) into (8) yields

$$\dot{V}_k = s_{\phi k} \eta (\boldsymbol{\theta}^T \boldsymbol{\varphi}_k + \boldsymbol{\vartheta}^T \boldsymbol{\psi}_k + u_{q,k}) \quad (11)$$

with

$$\begin{aligned} \boldsymbol{\theta} &\triangleq (\frac{1}{\eta}, -\frac{R}{\eta L}, -\frac{K_t K_e}{\eta L J}, \frac{f_1(\mathbf{x}_d)}{\eta})^T, \\ \boldsymbol{\varphi}_k &\triangleq (ce_{2,k} - \ddot{x}_d, x_{2,k}, x_{1,k}, 1)^T, \\ \boldsymbol{\vartheta} &\triangleq (\frac{1}{\eta}, \frac{c}{\eta}f_{2m})^T, \\ \boldsymbol{\psi}_k &\triangleq (\|\mathbf{e}_k\| \text{sat}(\frac{s_k}{\phi}), \text{sat}(\frac{s_k}{\phi}))^T. \end{aligned}$$

On the basis of (11), we propose the following learning control law for system (1) as

$$u_{q,k} = -\gamma_1 s_{\phi k} - \boldsymbol{\theta}_k^T \boldsymbol{\varphi}_k - \boldsymbol{\vartheta}_k^T \boldsymbol{\psi}_k, \quad (12)$$

in which,

$$\boldsymbol{\theta}_k = \text{sat}_{\bar{\theta}}(\boldsymbol{\theta}_{k-1}) + \gamma_2 s_{\phi k} \boldsymbol{\varphi}_k, \quad \boldsymbol{\theta}_{-1} = 0, \quad (13)$$

$$\boldsymbol{\vartheta}_k = \text{sat}_{\bar{\vartheta}}(\boldsymbol{\vartheta}_{k-1}) + \gamma_2 s_{\phi k} \boldsymbol{\psi}_k, \quad \boldsymbol{\vartheta}_{-1} = 0. \quad (14)$$

#### IV. CONVERGENCE ANALYSIS

In this section, we will analyze the stability and error convergence of closed-loop system. Here we present the main result in the following.

*Theorem 1:* For the closed loop dynamic system (4) with Assumption 1, control law (12) and learning laws (13)-(14), all system variables are guaranteed to be bounded at each iteration. Moreover, as the iteration number  $k$  increases,

$$|s_k(t)| \leq \phi, \quad \forall t \in [0, T] \quad (15)$$

and

$$|e_{1,k}(t)| \leq \frac{\phi}{c}, \quad \forall t \in [0, T] \quad (16)$$

can be obtained.

*Proof:*

*Part I Difference of  $L_k(t)$ :*

Let us calculate the difference of the Lyapunov functional  $L_k(t)$ , which is defined as

$$L_k = V_k + \frac{\eta}{2\gamma_2} \int_0^t \tilde{\theta}_k^T \tilde{\theta}_k d\tau + \frac{\eta}{2\gamma_3} \int_0^t \tilde{\vartheta}_k^T \tilde{\vartheta}_k d\tau, \quad (17)$$

with  $\tilde{\theta}_k = \theta - \theta_k$ ,  $\tilde{\vartheta}_k = \vartheta - \vartheta_k$ . For (11) and (12), we deduce that

$$V_k = V_k(0) - \gamma_1 \eta \int_0^t s_{\phi k}^2 d\tau + \eta \int_0^t s_{\phi k} (\tilde{\theta}_k^T \varphi_k + \tilde{\vartheta}_k^T \psi_k) d\tau.$$

Therefore, while  $k > 0$ , the difference of  $L_k(t)$  between two adjacent iterations is taken as

$$\begin{aligned} L_k - L_{k-1} &= V_k(0) - \gamma_1 \eta \int_0^t s_{\phi k}^2 d\tau + \eta \int_0^t s_{\phi k} (\tilde{\theta}_k^T \varphi_k + \tilde{\vartheta}_k^T \psi_k) d\tau \\ &\quad - V_{k-1} + \frac{\eta}{2\gamma_2} \int_0^t (\tilde{\theta}_k^T \tilde{\theta}_k - \tilde{\theta}_{k-1}^T \tilde{\theta}_{k-1}) d\tau \\ &\quad + \frac{\eta}{2\gamma_3} \int_0^t (\tilde{\vartheta}_k^T \tilde{\vartheta}_k - \tilde{\vartheta}_{k-1}^T \tilde{\vartheta}_{k-1}) d\tau \end{aligned} \quad (18)$$

From (13) and (14), we have

$$\begin{aligned} &\frac{1}{2\gamma_2} (\tilde{\theta}_k^T \tilde{\theta}_k - \tilde{\theta}_{k-1}^T \tilde{\theta}_{k-1}) + s_{\phi,k} \tilde{\theta}_k^T \varphi_k \\ &\leq \frac{1}{2\gamma_2} [(\theta - \theta_k)^T (\theta - \theta_k) - (\theta - \text{sat}_{\bar{\theta}}(\theta_{k-1}))^T \\ &\quad (\theta - \text{sat}_{\bar{\theta}}(\theta_{k-1}))] + s_{\phi,k} \tilde{\eta}_k^T \varphi_k \\ &\leq \frac{1}{2\gamma_2} (2\theta - \theta_k - \text{sat}_{\bar{\theta}}(\theta_{k-1}))^T (\text{sat}_{\bar{\theta}}(\theta_{k-1}) - \theta_k) \\ &\quad + s_{\phi,k} \tilde{\theta}_k^T \varphi_k \\ &\leq \frac{1}{\gamma_2} (\theta - \theta_k)^T (\text{sat}_{\bar{\theta}}(\theta_{k-1}) - \theta_k) + s_{\phi,k} \tilde{\theta}_k^T \varphi_k \\ &= 0 \end{aligned} \quad (19)$$

and

$$\begin{aligned} &\frac{1}{2\gamma_3} (\tilde{\vartheta}_k^T \tilde{\vartheta}_k - \tilde{\vartheta}_{k-1}^T \tilde{\vartheta}_{k-1}) + s_{\phi,k} \tilde{\vartheta}_k^T \psi_k \\ &\leq \frac{1}{2\gamma_3} [(\vartheta - \vartheta_k)^T (\vartheta - \vartheta_k) - (\vartheta - \text{sat}_{\bar{\vartheta}}(\vartheta_{k-1}))^T \\ &\quad (\vartheta - \text{sat}_{\bar{\vartheta}}(\vartheta_{k-1}))] + s_{\phi,k} \tilde{\eta}_k^T \psi_k \end{aligned}$$

$$\begin{aligned} &\leq \frac{1}{2\gamma_3} (2\vartheta - \vartheta_k - \text{sat}_{\bar{\vartheta}}(\vartheta_{k-1}))^T (\text{sat}_{\bar{\vartheta}}(\vartheta_{k-1}) - \vartheta_k) \\ &\quad + s_{\phi,k} \tilde{\vartheta}_k^T \psi_k \\ &\leq \frac{1}{\gamma_3} (\vartheta - \vartheta_k)^T (\text{sat}_{\bar{\vartheta}}(\vartheta_{k-1}) - \vartheta_k) + s_{\phi,k} \tilde{\vartheta}_k^T \psi_k \\ &= 0, \end{aligned} \quad (20)$$

respectively. Substituting (19) and (20) into (18), we have

$$L_k - L_{k-1} = V_k(0) - \gamma_1 \eta \int_0^t s_{\phi k}^2 d\tau - V_{k-1} \quad (21)$$

Under the alignment condition, both  $\mathbf{x}_{k-1}(T) = \mathbf{x}_k(0)$  and  $\mathbf{x}_d(T) = \mathbf{x}_d(0)$  hold. Hence,  $\mathbf{e}_{k+1}(0) = \mathbf{e}_k(T)$  holds for  $k = 0, 1, 2, 3, \dots$ . On the basis of this conclusion, now (21) becomes

$$L_k(T) - L_{k-1}(T) \leq -\gamma_1 \eta \int_0^T s_{\phi k}^2 d\tau, \quad (22)$$

which further implies

$$L_k(T) \leq L_0(T) - \gamma_1 \eta \sum_{j=1}^k \int_0^T s_{\phi j}^2 d\tau. \quad (23)$$

*Part II Finiteness of  $L_0(t)$ :*

By direct calculation, the time derivatives of

$$L_0 = V_0 + \frac{\eta}{2\gamma_2} \int_0^t \tilde{\theta}_0^T \tilde{\theta}_0 d\tau + \frac{\eta}{2\gamma_3} \int_0^t \tilde{\vartheta}_0^T \tilde{\vartheta}_0 d\tau, \quad (24)$$

may be obtained as

$$\begin{aligned} \dot{L}_0 &= -\gamma_1 g s_{\phi 0}^2 + \eta s_{\phi 0} (\tilde{\theta}_0^T \varphi_0 + \tilde{\vartheta}_0^T \psi_0) + \frac{\eta}{2\gamma_2} \tilde{\theta}_0^T \dot{\tilde{\theta}}_0 \\ &\quad + \frac{\eta}{2\gamma_3} \tilde{\vartheta}_0^T \dot{\tilde{\vartheta}}_0 \end{aligned} \quad (25)$$

From (13), we can see that  $\hat{\theta}_0 = \gamma_2 s_{\phi 0} \varphi_0$ . Thus,

$$\tilde{\theta}_0 = \theta - \gamma_2 s_{\phi 0} \varphi_0 \quad (26)$$

holds. Similarly, from (14), we obtain

$$\tilde{\vartheta}_0 = \vartheta - \gamma_3 s_{\phi 0} \psi_0. \quad (27)$$

Substituting (26) and (27) and into (25) shows

$$\begin{aligned} \dot{L}_0 &= -\gamma_1 \eta s_{\phi 0}^2 + \eta s_{\phi 0} \theta^T \varphi_0 - \gamma_2 \eta s_{\phi 0}^2 \varphi_0^T \varphi_0 \\ &\quad + \frac{1}{2\gamma_2} \eta \theta^T \theta + \frac{\gamma_2}{2} \eta s_{\phi 0}^2 \varphi_0^T \varphi_0 - \eta \theta s_{\phi 0} \varphi_0 \\ &\quad + \eta s_{\phi 0} \vartheta^T \psi_0 - \gamma_3 \eta s_{\phi 0}^2 \psi_0^T \psi_0 + \frac{1}{2\gamma_3} \eta \vartheta^T \vartheta \\ &\quad + \frac{\gamma_3}{2} \eta s_{\phi 0}^2 \psi_0^T \psi_0 - \eta \vartheta s_{\phi 0} \psi_0 \\ &\leq -\gamma_1 \eta s_{\phi 0}^2 + \frac{1}{2\gamma_2} \eta \theta^T \theta - \frac{\gamma_2}{2} \eta s_{\phi 0}^2 \varphi_0^T \varphi_0 \\ &\quad + \frac{1}{2\gamma_3} \eta \vartheta^T \vartheta - \frac{\gamma_3}{2} \eta s_{\phi 0}^2 \psi_0^T \psi_0 \\ &\leq \frac{1}{2\gamma_2} \eta \theta^T \theta + \frac{1}{2\gamma_3} \eta \vartheta^T \vartheta. \end{aligned} \quad (28)$$

Since  $0 \leq L_0(0) < +\infty$  and  $|\frac{1}{2\gamma_2}\eta\theta^T\theta + \frac{1}{2\gamma_3}\eta\vartheta^T\vartheta| < +\infty$ , from (28), we can see that  $L_0(t)$  is bounded for all  $t \in [0, T]$ , which implies

$$L_0(T) < +\infty. \tag{29}$$

*Part III Convergence of Tracking Errors:*

Combining (23) with (29) gives

$$\lim_{k \rightarrow +\infty} \int_0^T s_{\phi k}^2 d\tau = 0 \tag{30}$$

From (21), we can obtain

$$L_k(t) \leq L_{k-1}(T) - \gamma_1 \eta \int_0^T s_{\phi k}^2 d\tau \tag{31}$$

Therefore, by the definition of  $L_k$ , we can draw a conclusion that  $s_{\phi k}$  is bounded, which furthermore leads to the boundedness of  $e_k, \dot{e}_k$  and other signals. Then, by the definition of  $s_{\phi k}$ , we can see

$$|\dot{s}_{\phi k}| < +\infty, \tag{32}$$

which means  $s_{\phi k}$  is equicontinuous. On the basis of (32) and (30), we have

$$\lim_{k \rightarrow \infty} s_{\phi k}(t) = 0, \quad t \in [0, T], \tag{33}$$

which implies that

$$\lim_{k \rightarrow \infty} |s_k(t)| \leq \phi, \quad \forall t \in [0, T]. \tag{34}$$

From (34), we thus get that

$$|e_{1,k}(t)| \leq \frac{\phi}{c}, \quad \forall t \in [0, T], \tag{35}$$

holds for the enough large  $k$  [44]. Therefore, we can see that the predetermined control precision can be obtained by choosing an appropriate small positive number  $\phi$ .

By the property of saturation function, from (13) and (14), we can see that  $\hat{\theta}_k$  and  $\hat{\vartheta}$  are both bounded. Further, Since  $s_{\phi k}, \mathbf{x}_k, \mathbf{e}_k, \hat{\theta}_k$  and  $\hat{\vartheta}$  are all bounded, we can obtain the boundedness of  $u_k$  from (12). ■

To guarantee the boundedness of the estimation in learning laws, the partial saturation strategy is chosen in this work. One may also utilize the full saturation learning strategy to design the adaptive learning laws, which can also guarantee the parameter estimation bounded during the difference learning.

**V. NUMERICAL SIMULATION**

Consider the tank gun control system as follows [5]:

$$\begin{cases} \dot{x}_{1,k} = x_{2,k}, \\ \dot{x}_{2,k} = -\frac{R}{L}x_{2,k} - \frac{K_t K_e}{LJ}x_{1,k} + \frac{K_a K_t}{LJi}u_{q,k} \\ \quad + \Delta f(\mathbf{x}_k, t), \end{cases} \tag{36}$$

where  $R = 0.4\Omega, J = 5239\text{kg} \cdot \text{m}^2, i = 1039, L = 2.907 \times 10^{-3}\text{H}, K_t = 0.195\text{N} \cdot \text{m/A}, K_e = 0.197 \text{V}/(\text{rad} \cdot \text{s}^{-1}),$

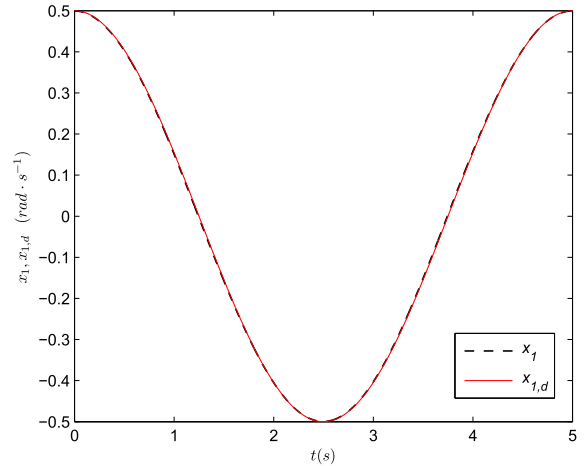


FIGURE 3.  $x_1$  and its reference signal  $x_{1,d}$  (ILC).

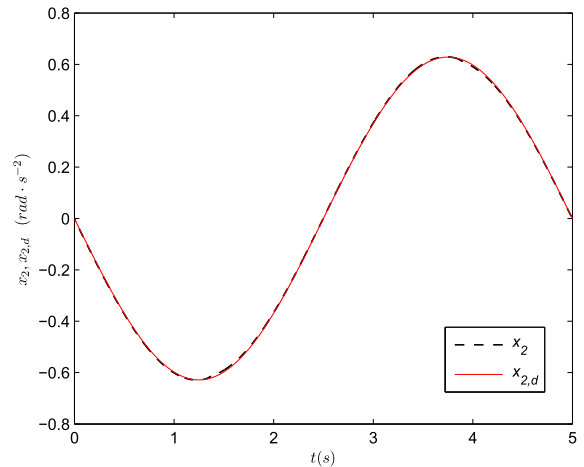


FIGURE 4.  $x_2$  and its reference signal  $x_{2,d}$  (ILC).

$B = 1.43 \times 10^{-4} \text{N} \cdot \text{m}, K_a = 2, \Delta f(\mathbf{x}_k, t) = 13.2 + 0.1 x_{1,k} + 0.2 x_{2,k} + 0.2\text{sign}(x_{2,k}) + 0.2\text{rand}(k) \sin(0.5t), \mathbf{x}_k(0) = [0.7, 0]^T, x_d = 0.5 \cos(0.5\pi t), T = 5.$  Here,  $\text{rand}(\cdot)$  denotes a random number between 0 and 1. We can easily verify that  $\Delta f(\mathbf{x}, t)$  satisfies Assumptions 1. The control objective is to make  $x_{1,k}$  accurately track its reference  $x_d$ . The robust adaptive ILC law (12) and adaptive learning laws (13)-(14) are used to do this simulation, and the control parameters are chosen as  $\gamma_1 = 5, \gamma_2 = 5, \gamma_3 = 0.05, \hat{\theta} = 50, \hat{\vartheta} = 20.$  After 70 cycles, the simulation results are shown in Figs. 3–8. Figs. 3-4 show the state profiles over  $[0, T]$  at the 70th iteration, with the corresponding state tracking error profiles presented in Figs. 5-6. According to Figs. 3–6, we conclude that  $x_1(t)$  can precisely track  $x_d(t)$  over  $[0, T]$  as the iteration number increases. The control input signal at the 70th iteration is shown in Fig. 7. Fig. 8 gives the convergence history of  $s_{\phi k}$ , where  $J_k \triangleq \max_{t \in [0, T]} |s_{\phi k}(t)|.$

For comparison, PID control algorithm for the tank gun control system (36) is simulated, with  $u_k(t) = k_p(x_d(t) - x_{1,k}(t)) + k_i[\sum_{j=0}^{k-1} \int_0^T (x_d(\tau) - x_{1,k}(\tau))d\tau + \int_0^t (x_d(\tau) - x_{1,k}(\tau))(\tau)d\tau] + k_d \frac{d(x_d(t) - x_{1,k}(t))}{dt}.$  The proportion

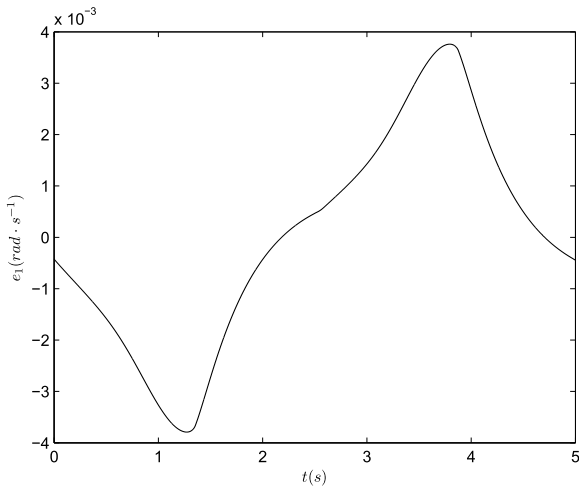


FIGURE 5. The error  $e_1$  (ILC).

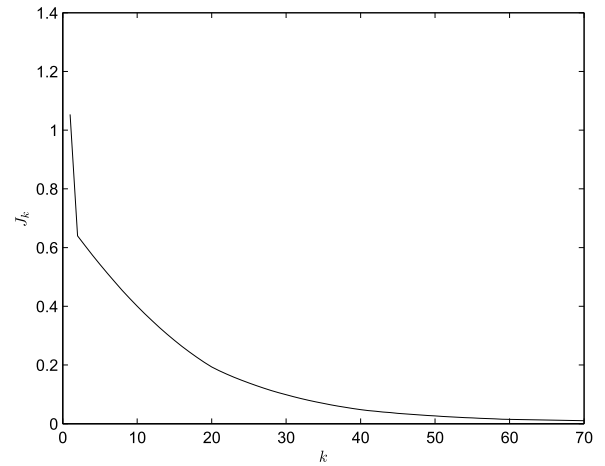


FIGURE 8. History of  $s_{\phi k}$  convergence (ILC).

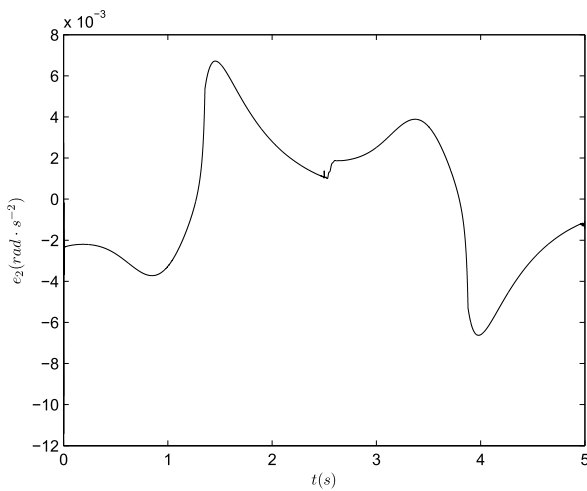


FIGURE 6. The error  $e_2$  (ILC).

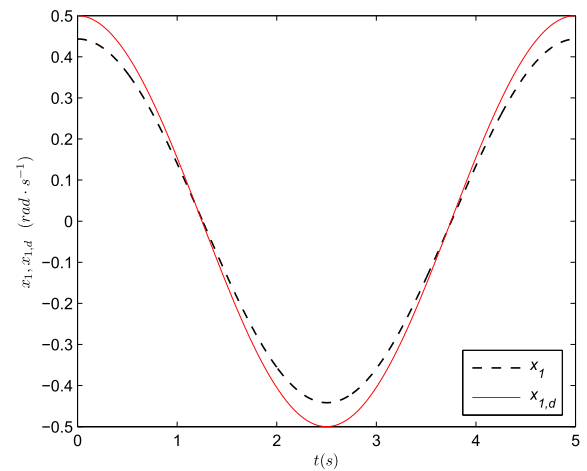


FIGURE 9.  $x_1$  and its reference signal  $x_{1,d}$  (PID).

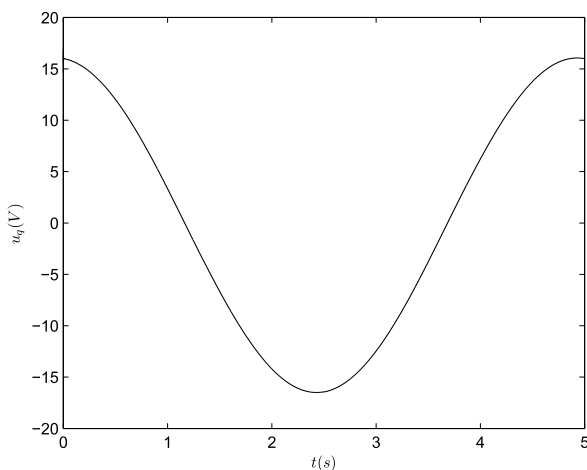


FIGURE 7. Control input (ILC).

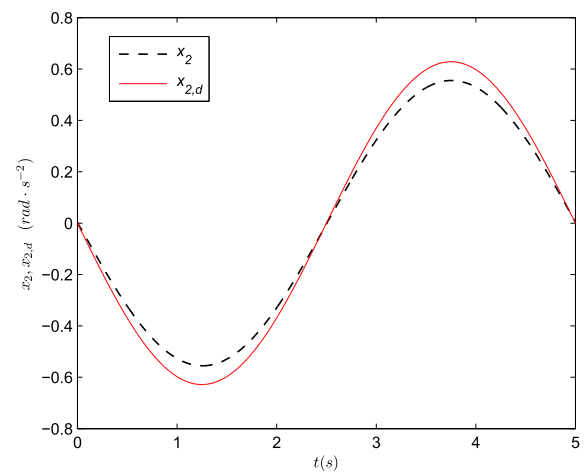


FIGURE 10.  $x_2$  and its reference signal  $x_{2,d}$  (PID).

parameter  $k_p$ , integral parameter  $k_i$  and differential coefficient  $k_d$  chosen to be 20, 10 and 5, respectively. The initial state, control objective and cycle number are the same as the

ones in the simulation of ILC algorithm. Simulation results are presented in Figs. 9–12. Figs. 9–10 show the system state profiles of PID control, while the corresponding error results are presented in Figs. 11–12. Comparing Figs. 9-10

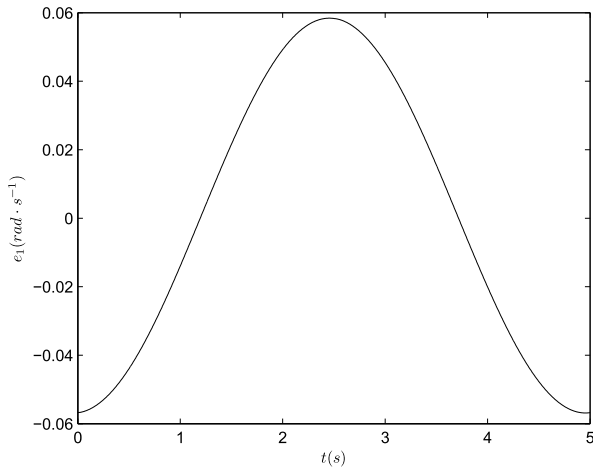


FIGURE 11. The error  $e_1$  (PID).

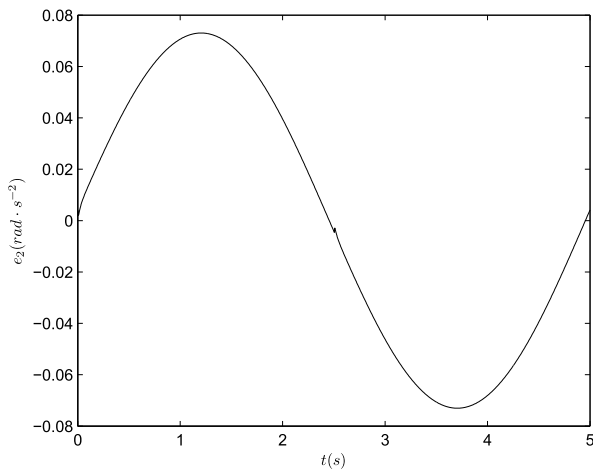


FIGURE 12. The error  $e_2$  (PID).

with Figs. 3-4, and comparing Figs. 11-12 with Figs. 5-6, we can see our proposed ILC scheme has higher control precision than PID control algorithm. The above simulation results verify the effectiveness of theoretical analysis in this paper.

## VI. CONCLUSION

This paper studies the velocity tracking problem for tank gun control servo systems. A robust adaptive learning control scheme is proposed to undertake the high-precision velocity tracking task for tank gun control servo systems. The alignment condition is used to remove zero initial error condition of iterative learning control, while robust control technique and learning control technique are combinedly applied to handle nonlinear uncertainties and external disturbances. As the iteration number increases, the system output can perfectly track its reference signal over the full time interval, and all signal are guaranteed to be bounded. Simulation results show the effectiveness of our proposed robust adaptive ILC scheme.

## REFERENCES

- [1] T. Jin, H. S. Yan, and D. X. Li, "PID control for tank firing in motion," *Ind. Control Comput.*, vol. 7, pp. 18–19, 2016.
- [2] R. Dana and E. Kreindler, "Variable structure control of a tank gun," in *Proc. 1st IEEE Conf. Control Appl.*, Sep. 1992, pp. 928–933.
- [3] W. Grega, "Time-optimal control of n-tank system," in *Proc. IEEE Int. Conf. Control Appl.*, vol. 1, Sep. 1998, pp. 522–526.
- [4] S. Shao-Jian, C. Gang, L. Bi-Lian, and L. Xiao-Feng, "Real-time optimal control for three-tank level system via improved ADDHP method," in *Proc. IEEE Int. Conf. Control Automat.*, Dec. 2009, pp. 564–568.
- [5] C. C. Sun, C. Jie, and L. Dou, "Variable structure control for sliding mode of tank stabilizer based on optimization," *Acta Armamentaria*, vol. 1, pp. 15–18, 2001.
- [6] L. Feng, X. J. Ma, Z. F. Yan, and H. Li, "Method of adaptive fuzzy sliding mode control of gun control system of tank," *Electr. Mach. Control*, vol. 11, no. 1, pp. 65–69, 2007.
- [7] N. Y. Li, K. C. Li, and Y. L. Liu, "Investigation of direct adaptive controller for tank gun elevation control system," *J. Syst. Simul.*, vol. 23, no. 4, pp. 762–765, 2011.
- [8] L.-J. Shen and J. P. Cai, "Adaptive robust control of gun control servo system of tank," *Math. Pract. Theory*, vol. 42, no. 7, pp. 170–175, 2012.
- [9] Y. Xia, L. Dai, M. Fu, C. Li, and C. Wang, "Application of active disturbance rejection control in tank gun control system," *J. Franklin Inst.*, vol. 351, no. 4, pp. 2299–2314, 2014.
- [10] J. H. Hu, Y. L. Hou, and Q. Gao, "Sliding-mode control for tank gun controlling system based on disturbance observer," *Electron. Opt. Control*, vol. 25, no. 2, pp. 98–101, 2018.
- [11] J. H. Hu, Y. L. Hou, and Q. Gao, "Method of neural network adaptive sliding mode control of gun control system of tank," *Fire Control Command Control*, vol. 43, no. 6, pp. 118–121, 2018.
- [12] J.-X. Xu and Y. Tan, "A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties," *IEEE Trans. Autom. Control*, vol. 47, no. 11, pp. 1940–1945, Nov. 2002.
- [13] T. Hu, K. H. Low, L. Shen, and X. Xu, "Effective phase tracking for bioinspired undulations of robotic fish models: A learning control approach," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 1, pp. 191–200, Feb. 2014.
- [14] L. Zhang and S. Liu, "Basis function based adaptive iterative learning control for non-minimum phase systems," *Acta Automat. Sinica*, vol. 40, no. 12, pp. 2716–2725, Dec. 2014.
- [15] X. Y. Li, "Iterative extended state observer and its application in iterative learning control," *Control Decis.*, vol. 30, no. 3, pp. 473–478, 2015.
- [16] Q.-Z. Yan and M. Sun, "Suboptimal learning control for nonlinear systems with both parametric and nonparametric uncertainties," *Acta Automatica Sinica*, vol. 41, no. 9, pp. 1660–1668, 2015.
- [17] R. Chi, D. Wang, Z. Hou, and S. Jin, "Data-driven optimal terminal iterative learning control," *J. Process Control*, vol. 22, no. 10, pp. 2026–2037, Dec. 2012.
- [18] Y. Wang, E. Dassau, and F. J. Doyle, III, "Closed-loop control of artificial pancreatic  $\beta$ -cell in type 1 diabetes mellitus using model predictive iterative learning control," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 2, pp. 211–219, Feb. 2010.
- [19] D. Shen and Y. Wang, "Survey on stochastic iterative learning control," *J. Process Control*, vol. 24, no. 12, pp. 64–77, Dec. 2014.
- [20] J. Wang, Y. Wang, W. Wang, L. Cao, and Q. Jin, "Adaptive iterative learning control based on unfalsified strategy applied in batch process," *J. Central South Univ. (Sci. Technol.)*, vol. 46, no. 4, pp. 1318–1325, 2015.
- [21] X.-H. Bu, Z.-S. Hou, and F. S. Yu, "Iterative learning control for a class of linear discrete-time switched systems," *Control Theory Appl.*, vol. 29, no. 8, pp. 1051–1056, 2012.
- [22] D.-Y. Meng, Y.-M. Jia, J.-P. Du, and F.-S. Yu, "Stability analysis of continuous-time iterative learning control systems with multiple state delays," *Acta Automatica Sinica*, vol. 36, no. 5, pp. 696–703, 2010.
- [23] Y. Li, Y. Chen, and H.-S. Ahn, "Convergence analysis of fractional-order iterative learning control," *Control Theory Appl.*, vol. 29, no. 8, pp. 1031–1037, 2012.
- [24] H. F. Tao, X. Q. Dong, and H. Z. Yang, "Optimal algorithm and application for point to point iterative learning control via updating reference trajectory," *Control Theory Appl.*, vol. 33, no. 9, pp. 1207–1213, 2016.
- [25] X. Dai, S. Tian, Y. Peng, and W. Luo, "Closed-loop P-type iterative learning control of uncertain linear distributed parameter systems," *IEEE/CAA J. Automatica Sinica*, vol. 1, no. 3, pp. 267–273, Jul. 2014.

- [26] C.-S. Teo, K.-K. Tan, and S.-Y. Lim, "Dynamic geometric compensation for gantry stage using iterative learning control," *IEEE Trans. Instrum. Meas.*, vol. 57, no. 2, pp. 413–419, Feb. 2008.
- [27] D. Huang and J.-X. Xu, "Steady-state iterative learning control for a class of nonlinear PDE processes," *J. Process Control*, vol. 21, no. 8, pp. 1155–1163, Sep. 2011.
- [28] M. French and E. Rogers, "Non-linear iterative learning by an adaptive Lyapunov technique," *Int. J. Control*, vol. 73, no. 10, pp. 840–850, 2000.
- [29] C. Yin, J.-X. Xu, and Z. Hou, "A high-order internal model based iterative learning control scheme for nonlinear systems with time-iteration-varying parameters," *IEEE Trans. Autom. Control*, vol. 55, no. 11, pp. 2665–2670, Nov. 2010.
- [30] M. A. Khanesar, E. Kayacan, M. Reyhanoglu, and O. Kaynak, "Feedback error learning control of magnetic satellites using type-2 fuzzy neural networks with elliptic membership functions," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 858–868, Apr. 2015.
- [31] X. Jin and J.-X. Xu, "Iterative learning control for output-constrained systems with both parametric and nonparametric uncertainties," *Automatica*, vol. 49, no. 8, pp. 2508–2516, 2013.
- [32] H. Ji, Z. Hou, L. Fan, and F. L. Lewis, "Adaptive iterative learning reliable control for a class of non-linearly parameterised systems with unknown state delays and input saturation," *IET Control Theory Appl.*, vol. 10, no. 17, pp. 2160–2174, Nov. 2016.
- [33] J.-X. Xu and R. Yan, "On initial conditions in iterative learning control," *IEEE Trans. Autom. Control*, vol. 50, no. 9, pp. 1349–1354, Sep. 2005.
- [34] X. E. Ruan and J. Zhao, "Pulse compensated iterative learning control to nonlinear systems with initial state uncertainty," *Control Theory Appl.*, vol. 29, no. 8, pp. 993–1000, 2012.
- [35] Q. Z. Yan and M. X. Sun, "Iterative learning control for nonlinear uncertain systems with arbitrary initial state," *Acta Autom. Sinica*, vol. 42, no. 4, pp. 545–555, Apr. 2016.
- [36] C.-J. Chien, C.-T. Hsu, and C.-Y. Yao, "Fuzzy system-based adaptive iterative learning control for nonlinear plants with initial state errors," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 5, pp. 724–732, Oct. 2004.
- [37] M.-X. Sun and Q.-Z. Yan, "Error tracking of iterative learning control systems," *Acta Autom. Sinica*, vol. 39, no. 3, pp. 251–262, Mar. 2013.
- [38] Q. Yan and M. Sun, "Error trajectory tracking by robust learning control for nonlinear systems," *Control Theory Appl.*, vol. 30, no. 1, pp. 23–30, Jan. 2013.
- [39] Q. Z. Yan, M. X. Sun, and H. Li, "Consensus-error-tracking learning control for nonparametric uncertain multi-agent systems," *Control Theory Appl.*, vol. 33, no. 6, pp. 793–799, 2016.
- [40] X.-D. Li, M.-M. Lv, and J. K. L. Ho, "Adaptive ILC algorithms of nonlinear continuous systems with non-parametric uncertainties for non-repetitive trajectory tracking," *Int. J. Syst. Sci.*, vol. 47, no. 10, pp. 2279–2289, 2016.
- [41] Q. Z. Yan, M. X. Sun, and J. P. Cai, "Filtering-error rectified iterative learning control for systems with input dead-zone," *Control Theory Appl.*, vol. 34, no. 1, pp. 77–84, 2017.
- [42] Q. Z. Yan, M. X. Sun, and J. P. Cai, "Reference-signal rectifying method of iterative learning control," *Acta Autom. Sinica*, vol. 43, no. 8, pp. 1470–1477, Aug. 2017.
- [43] Y. Yu, J. Wan, and H. Bi, "Suboptimal learning control for nonlinearly parametric time-delay systems under alignment condition," *IEEE Access*, vol. 6, pp. 2922–2929, 2018.
- [44] J.-J. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1991.
- [45] K. L. Moore, "Iterative learning control: An expository overview," in *Applied and Computational Control, Signals, and Circuits*. Boston, MA, USA: Birkhäuser, 1999, pp. 151–214.
- [46] Y. Chen, K. L. Moore, J. Yu, and T. Zhang, "Iterative learning control and repetitive control in hard disk drive industry—A tutorial," in *Proc. 45th IEEE Conf. Decis. Control*, Dec. 2006, pp. 2338–2351.
- [47] W. Qian, S. K. Panda, and J. X. Xu, "Torque ripple minimization in PM synchronous motors using iterative learning control," *IEEE Trans. Power Electron.*, vol. 19, no. 2, pp. 272–279, Mar. 2004.
- [48] W. Zhang, N. Nan, Y. Yang, W. Zhong, and Y. Chen, "Force ripple compensation in a PMLSM position servo system using periodic adaptive learning control," *ISA Trans.*, to be published.
- [49] R.-E. Precup, S. Preitl, I. J. Rudas, M. L. Tomescu, and J. K. Tar, "Design and experiments for a class of fuzzy controlled servo systems," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 1, pp. 22–35, Feb. 2008.
- [50] R. Marino and P. Tomei, "An iterative learning control for a class of partially feedback linearizable systems," *IEEE Trans. Autom. Control*, vol. 54, no. 8, pp. 1991–1996, Aug. 2009.



**GUANGMING ZHU** received the B.S. degree in computer science and technology and the M.S. degree in computer science from Hangzhou Dianzi University, Hangzhou, in 2011 and 2016, respectively. He is currently an Instructor of experiment with the Zhejiang University of Water Resources and Electric Power. His main research interests include computer numerical control and adaptive learning control.

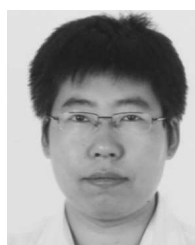


**XIUSHAN WU** received the Ph.D. degree in circuit and system from Southeast University, Nanjing, China, in 2009. Since 2009, he has been an Associate Professor with the College of Mechanical and Electrical Engineering, China Jiliang University. He is currently with the School of Electrical Engineering, Zhejiang University of Water Resources and Electric Power. His research interests include dynamic measurement and control, sensing technology, and RF-IC design.



Member of the Chinese Association of Automation.

**QIUZHEN YAN** received the M.S. degree in computer science and the Ph.D. degree in control science and engineering from the Zhejiang University of Technology, Hangzhou, China, in 2005 and 2015, respectively. Since 2005, he has been with the College of Information Engineering, Zhejiang University of Water Resources and Electric Power, where he is currently a Lecturer. His current research interests include iterative learning control and repetitive control. He is a Senior



**JIANPING CAI** was born in 1975. He received the Ph.D. degree from Zhejiang University, in 2014. He is currently an Associate Professor with the Zhejiang University of Water Resources and Electric Power. His main research interests include nonlinear systems and adaptive control.