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# Adaptive Neural Constraint Output Control for a Class of Quantized Input Switched Nonlinear System

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**ABSTRACT** In this paper, an adaptive neural control issue is addressed for a class of switched unknown strict-feedback nonlinear system under constraint output, in which the input signal is quantized. The control goal is to design a quantized controller to ensure that the system's output signal follows a given reference signal, meanwhile, the system output signal meets the asymmetric constraint requirement. To this end, the radial basis function neural networks (RBFNNs) are employed to approximate the unknown nonlinear functions. Adaptive backstepping technique and barrier Lyapunov function method are utilized to design the tracking controller and analyze the closed-loop stability. The proposed control strategy is shown to deal with the presented problem well. Finally, two simulation examples are presented to illustrate the efficacy of the design scheme.

**INDEX TERMS** Adaptive neural control, asymmetric constraint, backstepping, quantized control, switched systems.

## I. INTRODUCTION

In recent years, the problems of quantized control have been paid a lot of attention. Plenty of practical systems are studied by considering quantized input [1]–[5]. There are two main reasons that quantization needs to be considered in practical systems. On the one hand, quantization is inevitable since the control input signals to plants will be transmitted into piece-wise constant. For instance, the standard amplifier, a stepping motor, these devices could be viewed as input quantizers. On the other hand, the quantization scheme requires a low communication rate. Due to the importance of the theoretical and practical applications, the quantized feedback method has been investigated a lot for linear or nonlinear systems in the literature [6]–[11]. These early results on the quantization control are limited to the systems with precise mathematical models. However, the real controlled plants are not always precisely known because of the modeling error or some uncertainties. The robust control technique is thus introduced to deal with the quantization control issue of the systems with uncertainties. By adaptive control approach and backstepping technique, some quantization control schemes are presented for strict-feedback nonlinear uncertain systems [12]. Besides, many scholars apply adaptive neural or fuzzy control approaches to address the control problems of unknown nonlinear systems, since RBFNNs or fuzzy logic systems (FLS) are useful tools to approximate the unknown nonlinear functions [13]–[19]. Particularly, the recent works in [20], [21] give out some new quantized adaptive neural/fuzzy control strategies for a class of nonlinear uncertain systems in strict-feedback form.

Though significant results have been obtained respectively, there are still further problems needed to be addressed. Also, all the aforementioned results on the quantized control are proposed without considering the cases of output or state constraints. Then in practice, the system output may be constrained during operation range. In this case, the method of Barrier Lyapunov Functions (BLF) is applied in control design and stability analysis in [22], [23]. And then this approach is extended to discuss asymmetric output constraints. In [24], a new adaptive neural control scheme is addressed for uncertain non-affine systems with output constraint. In [25], [26], the authors apply log-type BLFs to

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ensure the output signal tracking the given reference signal, meanwhile, the system output meets the constraint requirement. Recently, a novel design approach is presented in [27], which works well for whatever having constraint requirement or no constraint requirement.

Due to its important applications, the control issues of switched systems have been paid a lot of attention during the past decades [28], [29]. Since the stability analysis and control design of switched system are more complicated, some approaches have been developed, including common Lyapunov function(CLF) method [30], [31], average dwell time(ADT) method [32]–[35] and persistent dwell time(PDT) method [36], [37]. Compared with ADT method and PDT method, the advantage of CLF method is that it allows arbitrary switchings among subsystems. Multiple Lyapunov functions approaches, such as ADT method and PDT method, need the switching signal stay at a subsystem for a certain time, the CLF method relaxes this limitation [33]. In [33], the established control approach employs the CLF method to construct an adaptive switching control law. However, the output constraint issue hasn't been addressed in their work and it is important to consider the output constraint requirement in practice, for example, as the autonomous fleet move along the sea, every ship should be constrained so that it can prevent collision.

Although some scholars have discussed the problem of constraint output by adaptive neural/fuzzy control approach for usual unknown nonlinear systems in a strict feedback form and give some interesting results, the corresponding results cannot be directly extended to switched uncertain nonlinear systems with input quantization. Motivated by the above discussions, the presented paper mainly focuses on quantization control for a class of switched nonlinear uncertain systems with output constraint. The main contributions of the paper lie in that (1) Based on common Lyapunov method, a new backstepping design scheme is proposed. Different from the existing results, the proposed adaptive laws do not require their initial value must be nonnegative. Then in the existing results, to guarantee the stability of closed-loop systems these initial values of adaptive variables must be negative. (2) During controller design, we consider the cases of input quantization and output constraint. The proposed controller guarantees that the system's output follows the reference signal with the quantized input signal and meets the constraint. (3) The proposed control scheme ensures the achievement of the desired control issue under the arbitrary switching among the subsystems.

### A. SOME PRELIMINARIES

Considered the following switched dynamic system with input quantized signal

$$\begin{aligned} \dot{\eta}_i &= f_{i,\sigma(t)}(\bar{\eta}_i) + g_{i,\sigma(t)}(\bar{\eta}_i)\eta_{i+1} \\ \dot{\eta}_n &= f_{n,\sigma(t)}(\bar{\eta}_n) + Q_u(u_{\sigma(t)}) \\ y &= \eta_1, \end{aligned} \tag{1}$$

where $\bar{\eta}_i = [\eta_1, \eta_2, \ldots, \eta_i]^T (i = 1, \ldots, n)$ is the state vector of system. $f_{i,\sigma(t)}(\cdot) : R^i \to R$, $g_{j,\sigma(t)}(\cdot) : R^j \to R, (j = 1, 2, \ldots, n-1)$ are smooth unknown nonlinear functions and $f_{i,\sigma(t)}(0) = 0$, $g_{j,\sigma(t)}$ are referred to the unknown bounded functions. $\sigma(t)$ is a piecewise function which is the switched signal among different subsystems, $\sigma(t) : R_+ \to M = \{1, 2, \ldots, m\}$. For the sake of simplicity, $\sigma(t)$ is replaced by $k$ in the rest of this paper. It denotes that the $k$th subsystem is running when $\sigma(t) = k$. Let $\{t_0, t_1, t_2, \ldots\}$ denote the switching times, which means the switching occurs at time $t_i$. $u$ denotes the designed input controller for the switched system. $Q_u(\cdot)$ is the input quantizer.

Quantized input $Q_u$ is taken into consideration to construct an adaptive neural controller. According to [39], the quantizer has the following character

$$|Q_u(u) - u| \le \delta_u^* |u| + d_u^* \tag{2}$$

where $0 < \delta_u^* < 1$ and $d_u^*$ are quantizer parameters.

*Remark 1:* In real applications, different practical quantizers are used to design the adaptive controller, such as uniform,logarithmic, and hysteresis quantizers, and all these practical quantizers satisfy the condition (2), see [39]. In this paper, hysteresis quantizer is used to quantize the input, which is described as

$$Q_u(u) = \begin{cases} sgn(u)u_b^*, & \frac{u_b^*}{1+\delta_u^*} < |u| \le u_b^*, \dot{u} < 0, or \\ & u_b^* < |u| \le \frac{u_b^*}{1-\delta_u^*}, \dot{u} > 0 \\ sgn(u)u_b^*(1+\delta_u^*), & u_b^* < |u| \le \frac{u_b^*}{1-\delta_u^*}\dot{u} < 0, or \\ & \frac{u_b^*}{1-\delta_u^*} \le \frac{u_b^*(1+\delta_u^*)}{1-\delta_u^*}, \dot{u} > 0 \\ 0, & 0 \le |u| < \frac{d_u^*}{1+\delta_u^*}, \dot{u} < 0, or \\ & \frac{d_u^*}{1+\delta_u^*} \le u \le d_u^*, \dot{u} > 0, \\ Q_u(u(t^-)), & \dot{u} = 0 \end{cases}$$

where $u_b^* = \varpi_u^{1-i} d_u^*$ with $i \in N_+$ and $\varpi_u = \frac{1-\delta_u^*}{1+\delta_u^*}$. $Q_u(u) \in \{0, \pm u_b^*\}$, $d_u^*$ determines the size of the quantizer of the dead-zone.

*Assumption 1:* Assume the desired output signal $y_r(t)$ is continuous and has up to the $n$th order derivative. Let $\bar{y}_{ri} = [y_r, y_r^{(1)}, \cdots, y_r^{(i)}]^T$ (i = 1, 2, ..., n), which satisfies $||\bar{y}_{ri}|| \le y_r^*$, where $y_r^*$ is a positive constant, and $y_r^{(i)}$ denotes the $i$th order time derivative of $y_r(t)$.

*Assumption 2:* Assume that the signs of the unknown function $g_{i,k}$ are consistent and known, which satisfy the following inequality

$$\underline{g}_i^* < |g_{i,k}| < \bar{g}_i^*, \tag{3}$$

where $\underline{g}_i^*$ and $\bar{g}_i^*$ are positive constants. For simplicity, $g_{i,k} > 0$ is assumed to be positive in the following context.

*Lemma 1 [40]:* For any unknown continuous function $f(\eta): R^n \to R$, it can be described as $f(\eta) = \phi^T \Psi(\eta) + \Delta(\eta)$, $\phi^T \Psi(\eta)$ is the neural network, $\phi = [\phi_1, \phi_2, \ldots, \phi_l] \in R^l$ is the weight vector, $\Psi(\eta) = [\Psi_1(\eta), \Psi_2(\eta), \ldots, \Psi_l(\eta)]^T \in R^l$ is the radial basis function vector, $\Psi_i(\eta) = \exp[-\frac{1}{\rho}||\eta - \mu_i||^2]$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$, $\mu_{ii}$ and $\rho$ are the center and the width of the chosen Gaussian functions, respectively. If the node number $l$ is large enough, for any given positive constant $\Delta^*$, $|\Delta(\eta)| < \Delta^*$ will be satisfied.

*Lemma 2 [41]:* For any $\xi \in R$ and $\epsilon^* \geq 0$, $0 \leq |\xi| \leq \xi \tanh(\frac{\xi}{\epsilon^*}) + \zeta^* \epsilon^*$ is true and $\zeta^* = 0.2785$.

Suppose the constraint requirements act on the output signal $y$ and the given reference signal $y_r$, i,e., $-B_{Ly}(t) < y < B_{Hy}(t)$, $-B_{Ly}(t) < y_r < B_{Hy}(t)$, where $B_{Ly}(t)$, $B_{Hy}(t)$ are known smooth functions which are the constraint requirement on the output signal. Thus we have $-B_{Ly}(t) < y - y_r + y_r < B_{Hy}(t)$, define the output error $z_1 := y - y_r$, $B_L(t) = B_{Ly}(t) + y_r > 0$, $B_H(t) = B_{Hy}(t) - y_r > 0$, a further calculating shows that

$$-B_L(t) < z_1 < B_H(t), \tag{4}$$

where $B_L(t) > 0$ and $B_H(t) > 0$ are continuous functions which have up to $n$th order derivative.

## II. MAIN RESULTS

In this section, the control issue of the system (1) is investigated, and an adaptive neural quantized controller is designed to guarantee the output error convergence to a small neighbor of origin.

The considered asymmetric barrier function has the following form

$$V_b = \frac{1}{2}\Gamma^2, \quad \Gamma = \frac{B_L B_H z_1}{(B_H - z_1)(B_L + z_1)}$$
$$-B_L(0) < z_1(0) < B_H(0). \tag{5}$$

*Remark 2:* For the asymmetric barrier function $\Gamma$, $\Gamma \to +\infty$ if $z_1 \to B_H$, then the BLF $V_b \to +\infty$. Considering $z_1 \to -B_L$, $\Gamma \to -\infty$, then the BLF $V_b \to +\infty$. By making $z_1$ satisfy the constraint $-B_L < z_1 < B_H$, then $-B_{Ly} < y < B_{Hy}$ can be guaranteed.

From (5), the dynamic of $\Gamma$ is

$$\dot{\Gamma} = \bar{\Gamma} + \Omega \dot{z}_1, \tag{6}$$

where $\bar{\Gamma} = \Lambda_1 \dot{B}_H + \Lambda_2 \dot{B}_L$, $\Lambda_1 = \frac{\partial \Gamma}{\partial B_H} = \frac{B_L z_1^2}{(B_H - z_1)^2 (B_L + z_1)}$, $\Lambda_2 = \frac{\partial \Gamma}{\partial B_L} = \frac{B_H z_1^2}{(B_H - z_1)(B_L + z_1)^2}$, $\Omega = \frac{\partial \Gamma}{\partial z_1} = \frac{B_H B_L (z_1^2 + B_H B_L)}{(B_H - z_1)^2 (B_L + z_1)^2}$.

In the following, backstepping technique is applied to construct the controller and to obtain the main results.

**Step 1:** Considering $V_b = \frac{1}{2}\Gamma^2$ and $z_1 = y - y_r$, and then

$$\dot{V}_b = \Gamma \bar{\Gamma} + \Gamma \Omega (f_{1,k} + g_1 \eta_2 - \dot{y}_r). \tag{7}$$

According to Lemma 2, the unknown nonlinear function $f_{1,k}$ can be described as $f_{1,k} = \phi_{1,k}^T \Phi(\eta_1) + \Delta_{1,k}(\eta_1)$, and then the following inequality holds

$$\begin{aligned} \Gamma \Omega f_{1,k} &\leq |\Gamma \Omega||\phi_{1,k}^T \Phi(\eta_1) + \Delta_{1,k}(\eta_1)| \\ &\leq |\Gamma \Omega| \theta_1^* U_1, \end{aligned} \tag{8}$$

where $U_1 = \| \Phi(\eta_1) \| + 1$, $\theta_1^* = \max\{|\Delta_{1,k}|, \| \phi_{1,k} \|\}$.

*Remark 3:* By applying inequality (8), $\theta_1^*$ has nothing to do with switching signal $k$, that ensures common Lyapunov function method can be used to analyze the stability of the switched system. In the following procedure, an update law will be presented to estimate the unknown parameter $\theta_1^*$.

By Lemma 3, the above inequality implies that

$$|\Gamma \Omega| \theta_1^* U_1 \leq \theta_1^* U_1 \Gamma \Omega \tanh(\frac{U_1 \Gamma \Omega}{\epsilon_1^*}) + \theta_1^* \zeta^* \epsilon_1^* \tag{9}$$

Define Lynapunov function candidate as

$$V_1 = V_b + \frac{1}{2 r_1^*} \tilde{\theta}_1^2, \tag{10}$$

where $\tilde{\theta}_1 = \theta_1^* - \hat{\theta}_1$, $r_1^*$ is a positive constant. Differentiating (10) and by simple calculation,

$$\begin{aligned} \dot{V}_1 \leq\ & \Gamma \bar{\Gamma} + \Gamma \Omega (g_{1,k} z_2 + g_{1,k} \alpha_1 - \dot{y}_r) - \frac{1}{r_1^*} \dot{\hat{\theta}}_1 \tilde{\theta}_1 \\ & + \theta_1^* U_1 \Gamma \Omega \tanh(\frac{U_1 \Gamma \Omega}{\epsilon_1^*}) + \theta_1^* \zeta^* \epsilon_1^*. \end{aligned} \tag{11}$$

Then, the above inequality (11) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq\ & -k_1^* \Gamma^2 + \Gamma \breve{\alpha}_1 + \Gamma \Omega g_{1,k} z_2 + \Gamma \Omega g_{1,k} \alpha_1 + \theta_1^* \zeta^* \epsilon_1^* \\ & + \tilde{\theta}_1 U_1 \Gamma \Omega \tanh(\frac{U_1 \Gamma \Omega}{\epsilon_1^*}) - \frac{1}{r_1^*} \dot{\hat{\theta}}_1 \tilde{\theta}_1, \end{aligned} \tag{12}$$

where $k_1^*$ is a design positive constant and

$$\breve{\alpha}_1 = k_1^* \Gamma + \bar{\Gamma} - \Omega \dot{y}_r + \hat{\theta}_1 \Omega U_1 \tanh(\frac{\Gamma \Omega U_1}{\epsilon_1^*}). \tag{13}$$

Design the virtual control coefficient $\alpha_1$ as

$$\begin{aligned} \alpha_1 &= -\frac{1}{\bar{g}_1^*} \bar{\alpha}_1 \tanh(\frac{\Gamma \Omega \bar{\alpha}_1}{\epsilon_1^*}) \\ \bar{\alpha}_1 &= \frac{1}{\Omega} \breve{\alpha}_1. \end{aligned} \tag{14}$$

Designing the update law $\dot{\hat{\theta}}_1$ as

$$\dot{\hat{\theta}}_1 = r_1^* U_1 \Gamma \Omega \tanh(\frac{U_1 \Gamma \Omega}{\epsilon_1^*}) - \nu_1^* \hat{\theta}_1, \tag{15}$$

where $\nu_1^*$ is a design positive constant. For the term $\Gamma \Omega g_{1,k} \alpha_1$, one has

$$\begin{aligned} \Gamma \Omega g_{1,k} \alpha_1 &\leq -\Gamma \Omega \bar{\alpha}_1 \tanh(\frac{\Gamma \Omega \bar{\alpha}_1}{\epsilon_1^*}) \\ &\leq \epsilon_1^* \zeta^* - |\Gamma \Omega \bar{\alpha}_1| \\ &\leq \epsilon_1^* \zeta^* - \Gamma \Omega \bar{\alpha}_1. \end{aligned} \tag{16}$$

From (12)-(16), it is easy to obtain that

$$\dot{V}_1 \leq -k_1^* \Gamma^2 + \Gamma \Omega g_{1,k} z_2 - \frac{\nu_1^*}{r_1^*} \hat{\theta}_1 \tilde{\theta}_1 + C_1^*, \tag{17}$$

where $C_1^* = \theta_1^* \zeta^* \epsilon_1^* + \zeta^* \epsilon_1^*$.

*Remark 4:* In the above proof procedure, we have not required the initial value of $\hat{\theta}_1(0)$ being positive. However, such a requirement is necessary for current adaptive neural or fuzzy control design.

**Step i.** Define $z_i = \eta_i - \alpha_{i-1} (2 \le i \le n-1)$, $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$, and choose Lyapunov candidate function as

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2r_i^*} \tilde{\theta}_i^2. \tag{18}$$

Differentiating (18), one has

$$\dot{V}_i = \dot{V}_{i-1} - z_{i-1} g_{i-1,k} z_i + z_i \bar{f}_{i,k} + z_i g_{i,k} (z_{i+1} + \alpha_i) - \frac{1}{r_i^*} \dot{\hat{\theta}}_i \tilde{\theta}_i, \tag{19}$$

where

$$\bar{f}_{i,k}(Z_i) = f_{i,k} + g_{i-1,k} z_{i-1} - \dot{\alpha}_{i-1}, \tag{20}$$

where $Z_i = [\bar{\eta}_i^T, \bar{y}_{ri}^T, \hat{\bar{\theta}}_i^T]^T$, $\hat{\bar{\theta}}_i = [\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_i]^T$,

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \eta_j} (f_{j,k} + g_{j,k} \eta_{j+1}) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)}.$$

Note that, when $i = 2$, $\bar{f}_{2,k}(Z_i) = f_{2,k} + g_{2,k} \Gamma \Omega - \dot{\alpha}_1$. By Lemma 2, it follows that

$$\begin{aligned} z_i \bar{f}_{i,k} &\le | z_i | \theta_i^* (\| \Phi_i(Z_i) \| + 1) = | z_i | \theta_i^* U_i \\ &\le z_i \theta_i^* U_i \tanh(\frac{z_i U_i}{\epsilon_i^*}) + \theta_i^* \zeta^* \epsilon_i^*, \end{aligned} \tag{21}$$

where $\theta_i^* = \max\{|\Delta_{i,k}|, \| \phi_i \|\}$ and $U_i = \| \Phi_i(Z_i) \| + 1$. Design the virtual control coefficient $\alpha_i$ as

$$\alpha_i = -\frac{1}{\bar{g}_i^*} \bar{\alpha}_i \tanh(\frac{\bar{\alpha}_i z_i}{\epsilon_i^*}) \tag{22}$$

$$\bar{\alpha}_i = k_i^* z_i + \hat{\theta}_i U_i \tanh(\frac{z_i U_i}{\epsilon_i^*}), \tag{23}$$

where $k_i^*$ is a design positive constant, $\zeta^* = 0.2785$. Taking (21) and (22) into consideration, (19) can be expressed as

$$\dot{V}_i = \dot{V}_{i-1} - k_i^* z_i^2 - z_{i-1} g_{i-1,k} z_i + z_i g_{i,k} z_{i+1} + z_i \bar{\alpha}_i \\ z_i g_i \alpha_i - \frac{1}{r_i^*} \tilde{\theta}_i (\dot{\hat{\theta}}_i - r_i^* z_i U_i \tanh(\frac{z_i U_i}{\epsilon_i^*})), \tag{24}$$

where $r_i^*$ is a design positive constant. Similar to inequality (16), for the term $z_i g_{i,k} \alpha_i$, one has

$$z_i g_{i,k} \alpha_i \le \epsilon_i^* \zeta^* - z_i \bar{\alpha}_i. \tag{25}$$

Design the update law $\hat{\theta}_i$ as

$$\dot{\hat{\theta}}_i = r_i^* U_i \tanh(\frac{z_i U_i}{\epsilon_i^*}) - \frac{\nu_i^*}{r_i^*} \hat{\theta}_i, \tag{26}$$

where $\nu_i^*$ is a design constant. Therefore, by using (25) and the above (26), (24) can be rewritten as

$$\dot{V}_i \le -k_1^* \Gamma^2 - \sum_{j=2}^{i} k_j^* z_j^2 - z_{i-1} g_{i-1,k} z_i + z_i g_{i,k} z_{i+1} - \sum_{j=1}^{i} \frac{\nu_j^*}{r_j^*} \tilde{\theta}_j \hat{\theta}_j + C_i^*, \tag{27}$$

where $C_i^* = \sum_{j=1}^{i} (\theta_i^* \zeta^* \epsilon_j^* + \zeta^* \epsilon_j^*)$.

**Step n:** In this step, quantized controller will be designed for system (1) to track the given reference signal.

Define $z_n = \eta_n - \alpha_{n-1}$, $\tilde{\theta}_n = \theta_n^* - \hat{\theta}_n$, and choose Lyapunov candidate function as

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2r_n^*} \tilde{\theta}_n^2. \tag{28}$$

The time derivative of (28) is

$$\dot{V}_n = \dot{V}_{n-1} + z_n \bar{f}_{n,k} + z_n Q_u - \frac{1}{r_n^*} \dot{\hat{\theta}}_n \tilde{\theta}_n, \tag{29}$$

where $\bar{f}_{n,k}(Z_n) = f_{n,k} + g_{n-1,k} z_{n-1}$, $Z_n = [\bar{\eta}_n^T, \bar{y}_{rn}^T, \hat{\bar{\theta}}_n^T]^T$. Design the real control law $u$ as

$$u = -\bar{u} \tanh(\frac{z_n \bar{u}}{\epsilon_n^*}) \tag{30}$$

$$\bar{u} = -\frac{1}{1 - \delta_u^*} (\bar{\alpha}_n - d_u^* \tanh(\frac{d_u^* z_n}{\epsilon_n^*}))$$

$$\bar{\alpha}_n = -k_n^* z_n - \hat{\theta}_n U_n \tanh(\frac{z_n U_n}{\epsilon_n^*}),$$

where $k_n^*$ is a design positive constant. From the definition of the quantizer, multiply $|z_n|$ on both side of the equation (2) resulting in

$$|z_n||Q_u - u| \le \delta_u^* |u||z_n| + d_u^* |z_n|. \tag{31}$$

Then, (31) can be further extended as

$$\begin{aligned} z_n Q_u &\le z_n u + \delta_u^* |u||z_n| + d_u^* |z_n| \\ &\le -z_n \bar{u} \tanh(\frac{z_n \bar{u}}{\epsilon_n^*}) + \delta_u^* z_n \bar{u} \tanh(\frac{z_n \bar{u}}{\epsilon_n^*}) + d_u^* |z_n| \\ &\le -(1 - \delta_u^*) z_n \bar{u} \tanh(\frac{z_n \bar{u}}{\epsilon_n^*}) + d_u^* |z_n| \\ &\le -(1 - \delta_u^*) z_n \bar{u} + d_u^* |z_n| + (1 - \delta_u^*) \epsilon_n^*. \end{aligned} \tag{32}$$

Similar to inequality (21), one can get

$$\begin{aligned} z_n \bar{f}_n &\le | z_n | \theta_n^* (\| \Phi_n(Z_n) \| + 1) = | z_n | \theta_n^* U_n \\ &\le z_n \theta_n^* U_n \tanh(\frac{z_n U_n}{\epsilon_n^*}) + \theta_n^* \zeta^* \epsilon_n^* \end{aligned} \tag{33}$$

where $\theta_n^* = \max\{|\Delta_{n,k}|, \| \phi_n \|\}$ and $U_n = \| \Phi_n(Z_n) \| + 1$. Apparently, combining from (28) to (33) gives

$$\dot{V}_n \le \dot{V}_{n-1} - k_n z_n^n - z_{n-1} g_{n-1} z_n + \bar{C}_n^* - \frac{1}{r_n^*} \tilde{\theta}_n (\dot{\hat{\theta}}_n - r_n^* z_n U_n \tanh(\frac{z_n U_n}{\epsilon_n^*})), \tag{34}$$

where $\bar{C}_n^* = \theta_n^* \zeta^* \epsilon_n^* + (2 - \delta_u^*) \epsilon_n^*$. Design the update law as

$$\dot{\hat{\theta}}_n = r_n^* z_n U_n \tanh(\frac{z_n U_n}{\epsilon_n^*}) - \frac{\nu_n^*}{r_n^*} \hat{\theta}_n. \tag{35}$$

Then (34) becomes

$$\dot{V}_n \le -k_1^* \Gamma^2 - \sum_{j=2}^{n} k_j^* z_j^2 - \sum_{j=1}^{n} \frac{\nu_j^*}{r_j^*} \tilde{\theta}_j \hat{\theta}_j + C_n^*, \tag{36}$$

where $C_n^* = C_i^* + \bar{C}_n^*$. Note that,

$$\frac{\nu_j^*}{r_j^*}\tilde{\theta}_i\hat{\theta}_i = \frac{\nu_j^*}{r_j^*}\tilde{\theta}_i(\theta_i^* - \tilde{\theta}_i) \leq -\frac{1}{2}\frac{\nu_j^*}{r_j^*}\tilde{\theta}_i^2 + \frac{1}{2}\frac{\nu_j^*}{r_j^*}\theta_i^{*2}. \quad (37)$$

Then, (36) can be rewritten as

$$\dot{V}_n \leq -a^* V_n + C^*, \quad (38)$$

where $a^* = \min\{2k_i^*, \nu_j^*\}$, $C^* = C_n^* + \sum_{j=1}^{n} \frac{1}{2}\frac{\nu_j^*}{r_j}\theta_j^{*2}$, and (38) can be further expressed as

$$V_n(t) \leq V(0)e^{-a^* t} + \frac{C^*}{a^*} \quad (39)$$

From (39), the following Theorem is concluded.

*Theorem 1:* Considering system (1) under Assumptions 1-2, for any initial condition $-B_{Ly}(0) < y(0) < B_{Hy}(0)$, $-B_{Ly}(0) < y_r(0) < B_{Hy}(0)$, associated with the update law (26) and the virtual controllers $\alpha_i$ $(i = 1, 2, \ldots, n)$, the designed controller (30) guarantees the system output signal tracking the given reference efficacy, meanwhile, the tracking error meets the constraint tracking requirement.

*Remark 5:* From (39), it is easy to obtain that all signals of the closed-loop system are bounded. Meanwhile, as a result, $\lim_{t\to\infty} \Gamma_1^2 \leq 2\frac{C^*}{a^*}$. By choosing the appropriate design parameters, the tracking error $z_1$ will meet the constraint tracking requirement.

## III. SIMULATION EXAMPLE

In this section, two simulation examples with constraint output and input quantization are demonstrated to test the validity of our proposed strategy.

*Example 1:* A two-order switched nonlinear system with required constraint on output tracking is considered

$$\begin{aligned} \dot{\eta}_1 &= f_{1,k} + g_{1,k}\eta_2 \\ \dot{\eta}_2 &= f_{2,k} + Q_u \\ y &= \eta_1, \end{aligned} \quad (40)$$

$Q_u$ is the input quantizer, the parameters are given as $\delta_u^* = 0.2$, $d_u^* = 0.1$, $k \in \{1, 2\}$. $f_{1,1} = f_{1,2} = 0$, $f_{2,1} = \eta_1^2\cos(\eta_2)$, $f_{2,2} = \eta_1\eta_2^2$, $g_{1,1} = 1$, $g_{1,2} = 1.5 + \sin(\eta_1)$. Neural network system $\Phi_2(Z_2)$ is adopted to approximate the unknown nonlinear functions $f_{2,1}$ and $f_{2,2}$. $\rho = 2$ is chosen for the gaussian functions, the nodes of the centers $\mu_i$ are evenly distributed on the interval $\overbrace{[-1.5, 1.5] \times \ldots \times [-1.5, 1.5]}^{6}$. Choosing the initial state $\eta_1 = 0.3$, $\eta_2 = 0.1$, $\hat{\theta}_1 = \hat{\theta}_2 = 0$. The required constraint on the output signal $B_{Ly}(t)$ and $B_{Hy}(t)$ satisfy

$$\begin{aligned} B_{Hy}(t) &= 0.45 + 0.3e^{-0.6t} + \sin(t) \\ B_{Ly}(t) &= -0.2 + 0.3e^{-0.5t} - \sin(t). \end{aligned} \quad (41)$$

The given reference signal is $y_r = \sin(t) + 0.3$, then we have $B_L(t) = 0.1 + 0.3e^{-0.5t}$ and $B_H(t) = 0.15 + 0.3e^{-0.6t}$. The initial state satisfy $-B_{Ly}(0) < y(0) < B_{Hy}(0)$. The parameters are chosen as $k_1^* = 10$, $k_2^* = 15$, $\epsilon_1^* = \epsilon_1^* = 0.1$, $r_1^* = r_2^* = 1$, $\nu_1^* = \nu_2^* = 0.4$.

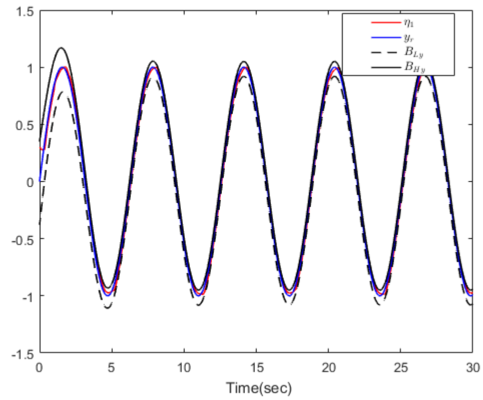


**FIGURE 1.** Trajectories of $y$, given signal $y_r$, the tracking error $z_1$ the Lowerbound $B_{Ly}$ and the upperbound $B_{Hy}$ for example 1.

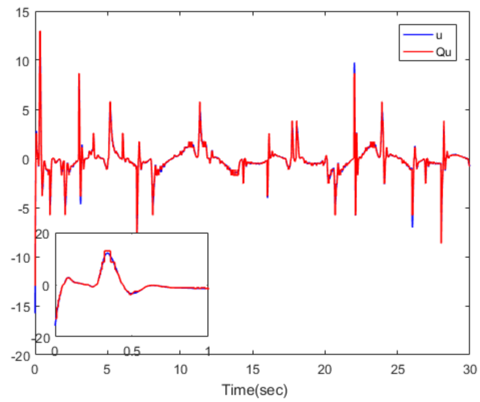


**FIGURE 2.** The trajectories of input $u$ and quantized input $Q_u$ for example 1.

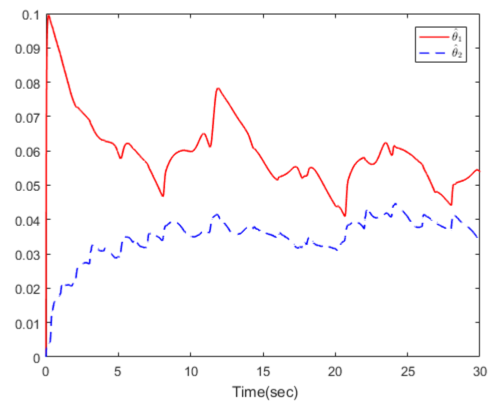


**FIGURE 3.** The adaptive parameter $\hat{\theta}_1$ and $\hat{\theta}_2$ for example 1.

Figs. 1-4 demonstrate the simulation results. Fig. 1 shows that the output of system (1) tracks the given reference well, also the output meets the constraint requirements. Fig.2 shows the trajectory of $\eta_2$. Fig.3 shows the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$. Fig.4 shows the input signal and the quantized signal. Fig.5 shows the switched signal.

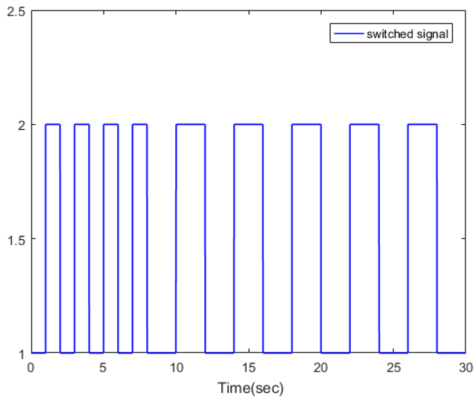


**FIGURE 4.** The switched signal for example 1.

*Example 2:* In this example, a single-link robot arm system is taken from [37] and [38] to further test the effectiveness of the proposed scheme. The single-link robot arm system is described by the following dynamic equation

$$\ddot{\eta}(t) = -\frac{m^i gl}{J^i}\sin(\eta(t)) - \frac{D^i}{J^i}\dot{\eta}(t) + \frac{1}{J^i}u(t), \tag{42}$$

where $\eta(t)$, $u(t)$ denote the angle of the arm and the control input, respectively. Since the system mass $m^i$ , inertia $J^i$ and damping $D^i$ form a set of discrete sequences $q^i = (m^i, J^i, D^i)$ are changing depending on the angle $\eta(t)$, so the robot arm can be viewed as switched system. In this example, we choose the same parameter as in [38], $q^1 = (1, 1, 2)$, $q^2 = (5, 5, 2)$, $q^3 = (10, 10, 2)$. $q^i$ denotes the three different subsystems which means $k \in \{1, 2, 3\}$. Here, we define $\eta_1(t) = \eta(t)$, $\eta_2(t) = \dot{\eta}(t)$. The single-link robot arm can be rewritten as

$$\begin{aligned}\dot{\eta}_1 &= \eta_2\\ \dot{\eta}_2 &= -\frac{m^i gl}{J^i}\sin(\eta_1) - \frac{D^i}{J^i}\eta_2 + Q_u\\ y &= \eta_1,\end{aligned} \tag{43}$$

the length of arm is $l = 0.5m$ and the gravitational constant $g = 9.8$. Different from the example in [37] and [38], we take the quantized input $Q_u$ and output constraint requirement into consideration. Choose the quantized parameter as $\delta_u^* = 0.1$, $d_u^* = 0.1$ and the required constraint output as

$$\begin{aligned}B_{Hy}(t) &= 0.1 + 0.3e^{-0.2t} + \sin(t)\\ B_{Ly}(t) &= 0.1 + 0.3e^{-0.2t} - \sin(t).\end{aligned} \tag{44}$$

The given reference signal is $y_r = \sin(t)$, and then one has $B_H(t) = 0.1 + 0.3e^{-0.2t}$, $B_L(t) = 0.1 + 0.3e^{-0.2t}$. The chosen gaussian functions with $\rho = 2$ is presented to approximate the nonlinear functions $-\frac{m^i gl}{J^i}\sin(\eta_1) - \frac{D^i}{J^i}\eta_2$, the nodes of the centers $\mu_i$ are evenly distributed on the interval $\overbrace{[-1.5, 1.5] \times \ldots \times [-1.5, 1.5]}^{6}$. Choosing the initial state $\eta_1 = 0.1$, $\eta_2 = 0.3$, $\hat{\theta}_1 = -0.1$, $\hat{\theta}_2 = 0$. The parameters are chosen as $k_1^* = 10$, $k_2^* = 15$, $\epsilon_1^* = \epsilon_2^* = 0.1$, $r_1^* = r_2^* = 1$, $\nu_1^* = \nu_2^* = 0.4$.

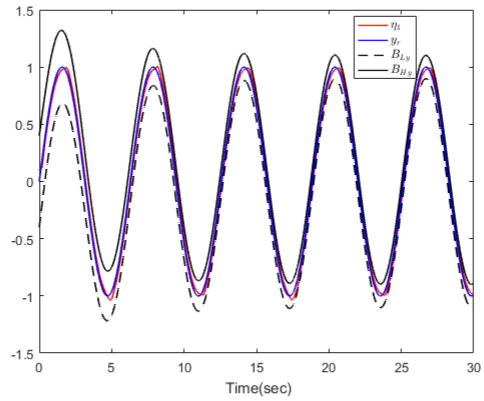


**FIGURE 5.** Trajectories of $y$, given signal $y_r$, the tracking error $z_1$ the lowerbound $B_{Ly}$ and the upperbound $B_{Hy}$ for example 2.

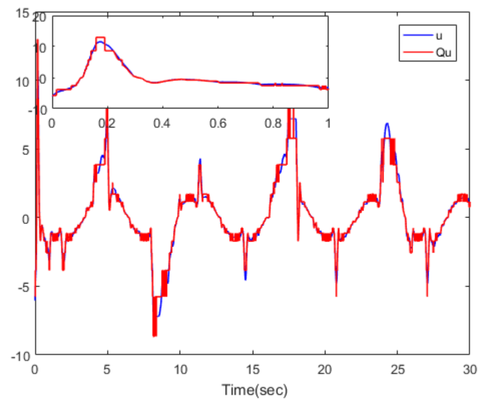


**FIGURE 6.** The trajectories of input $u$ and quantized input $Q_u$ for example 2.

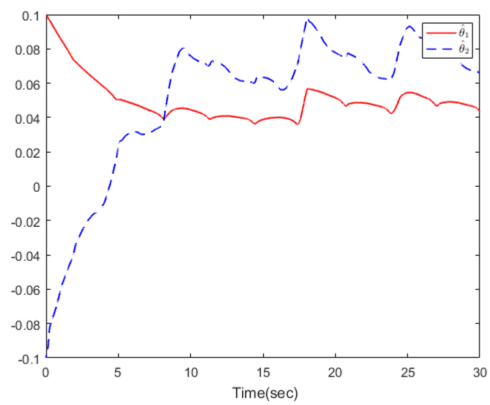


**FIGURE 7.** The adaptive parameter $\hat{\theta}_1$, $\hat{\theta}_2$ for example 2.

Figs. 5-8 demonstrate the simulation results. Fig. 5 shows that the output of the system (1) tracking the given reference well, also the output signal meets the constraint requirements. Fig.6 shows the adaptive parameters $\theta_1$ and $\theta_2$. Fig.7 shows the input signal and the quantized input signal. Fig. 8 shows the switched signal.

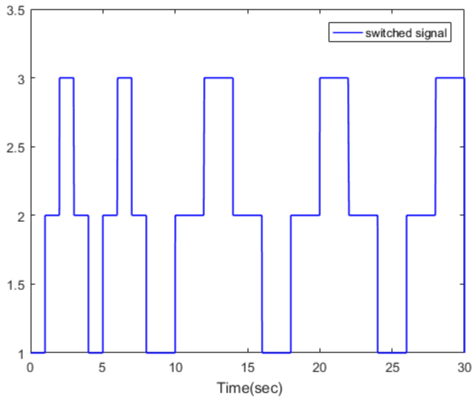


**FIGURE 8.** The switched signal for example 2.

## IV. CONCLUSION

The current research develops an adaptive neural constraint output control scheme for a class of switched unknown strict-feedback nonlinear systems. In this study, a quantized input signal has been taken into consideration, an adaptive neural quantized controller is constructed. The designed controller ensures the output signal tracks the given reference well, meanwhile, the constraint output requirement is satisfied. Under the action of the proposed quantized controller, it is shown that all the signals of the closed-loop system are bounded.

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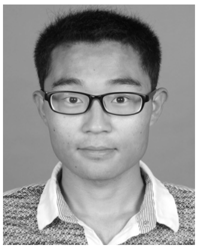

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