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Equilibrium Analysis of Electricity Markets With Microgrids Based on Distributed Algorithm

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ABSTRACT Microgrid is an effective way to accommodate distributed renewable energy, and there is a need for microgrids to participate in electricity market competition to ensure its sustainable development. For this purpose, a market trading framework is presented where microgrids sell electricity by submitting bids in the distribution electricity market (DEM) while generators compete by submitting bids in the day-ahead wholesale market (DAWM). The retailers are considered to submit bids in the two markets to buy electricity to meet the demand of customers and an arbitrageur is introduced to buy and sell electricity between the DEM and the DAWM. Based on the market framework, a joint equilibrium model for the DEM and the DAWM is proposed. Moreover, the equilibrium problem is converted into a convex optimization problem, and the existence and uniqueness of Nash equilibrium for the DAWM and the DEM is theoretically demonstrated. Due to information asymmetry in practice, a distributed algorithm is applied to find equilibrium outcomes. Finally, numerical examples are presented to verify the effectiveness of the proposed model and algorithm.

INDEX TERMS Microgrid, distribution electricity market, day-ahead wholesale market, equilibrium model, distributed algorithm.

I. INTRODUCTION

Microgrids are small-scale power systems that can distribute energy in small geographic areas flexibly and reliably. They have become effective supplement to conventional centralized power grid because they offer the potential of lower cost, increased efficiency, reliability and security [1], [2]. A microgrid can be viewed as a prosumer because it is generally comprised of flexible loads, storage units, micro generation units and power generation from renewable energy sources [3]. Allowing microgrids to participate in the electricity market competition can help them make profit with their flexibility, thus contributing to sustainable development of microgrids. Direct participation of microgrids in the day-ahead wholesale market (DAWM) is not applicable for the following reasons: 1) technical limitations on voltage levels, generation, and demand capacity; 2) lack of technical infrastructure to provide microgrids with access to open access same-time information system and open access nondiscriminatory

transmission services; and 3) large scale of the day-ahead and real-time scheduling problem and corresponding computational burden of the independent system operator (ISO) due to increase in the number of microgrids with diverse capacities and coverage areas [4]–[6]. As microgrids are deployed in low or medium voltage distribution networks, they can participate in the distribution electricity market (DEM) [6]. In addition, if there is a price gap between the DAWM and the DEM, an arbitrageur can make profits by buying and selling electricity from the lower-price market to the higher-price market. The presence of the arbitrageur also leads to an interaction between the DAWM and the DEM. Therefore, it is necessary to examine the strategic behaviors of microgrids in the DEM and their impacts on the DAWM, which is helpful to the design of electricity market that involves microgrids.

Until now, a considerable amount of research has been conducted regarding the participation of microgrids in electricity trading market. Reference [7] proposes a distributed convex optimization framework for bilateral energy trading between islanded microgrids, where all microgrids agree to cooperate with one another in order to minimize the global

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operation cost. Reference [8] analyzes the pricing games among interconnected microgrids in a bilateral market, where the microgrids with deficit power buy electricity from the microgrids with excess power who quotes the lowest prices. Reference [9] proposes a bilateral transaction mechanism for energy trading among microgrids in a competitive market based on a multileader-multifollower Stackelberg game. The microgrids as sellers lead the game and the microgrids as buyers follow the sellers' actions by submitting a unit price bid to the sellers. In reference [10], a two-stage stochastic game approach for the day-ahead and real-time energy trading strategy is proposed for risk-averse microgrids and aggregator at distribution network level. Reference [11] presents a multiagent-based energy market for multi-microgrid systems, where the agents submit their bids in the day-ahead market aiming at maximizing the social welfare. These studies mainly deal with the trading and bidding mechanism of microgrids in the DEM, but the impacts of microgrids' participation in the DEM on the DAWM have not been investigated.

Recently, some research work has been published regarding the participation of microgrids in the wholesale market. Reference [12] proposes a hierarchical market framework in which small-scale microgrids exchange electricity with the real-time balancing market via a microgrid aggregator. However, the impact of the microgrids on the prices in the real-time balancing market was not addressed. Reference [6] proposes a hierarchical structure for the electricity market based on dynamic game, where the generation companies bid in the DAWM and the microgrids bid in a distribution market. Load aggregators trade electricity between the two markets. However, the retailers are not considered in the bidding competition in either market. In addition, the solution for the equilibrium models proposed in [6] needs to collect a comprehensive set of information, which is infeasible in real electricity markets due to the information asymmetry.

Recently, distributed algorithms have attracted increasing attention [13]. Reference [7] presents an iterative distributed algorithm to reach the minimum cost without collecting local cost functions and local consumptions of microgrids. A distributed algorithm is proposed to solve optimal power flow problem for microgrids in [14], which ensures scalability of the microgrid size and preserves data privacy and integrity. References [15] and [16] propose to transform the equilibrium problem into an equivalent optimization problem. The existence and uniqueness of the Nash equilibrium is studied and distributed algorithms are proposed to determine the equilibrium. Using distributed algorithms to solve equilibrium problems can protect the privacy of market participants, which also perfectly meets the practical needs of electricity markets with information asymmetry. In this regard, distributed algorithms possess valuable potentials to solve the equilibrium problems in real electricity markets.

Given the background above, the main contributions of this paper can be listed as follows:

1) A market trading framework is presented in which microgrids sell electricity by submitting bids in the form of

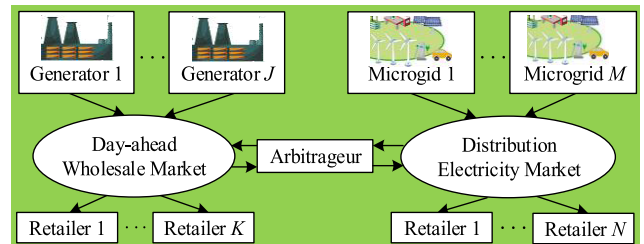


FIGURE 1. Market trading framework with microgrids.

supply function in the DEM while conventional generators bid in the form of supply function in the DAWM. Meanwhile, to reflect real electricity markets, retailers are considered to bid in the form of demand function in the two markets. In addition, an arbitrageur is allowed to participate in the DEM and the DAWM by buying electricity from the lower-price market and selling to the higher-price market. Based on this market framework, a joint equilibrium model of the DAWM and the DEM is proposed and the interactions between the two markets are studied.

2) The joint equilibrium model is solved by converting the equilibrium problems into convex optimization problems. The existence and uniqueness of the Nash equilibrium is theoretically demonstrated. Considering the information asymmetry in practical application, a distributed algorithm is proposed to find the equilibrium outcomes.

The rest of the paper is organized as follows: Section II presents the market framework. The joint equilibrium model is characterized in Section III and its solution method is proposed in section IV. Section V provides the numerical results. Finally, the paper is concluded in Section VI.

II. MARKET FRAMEWORK

Fig.1 shows the market trading framework with microgrids. In this framework, there are J generators and K retailers bidding in the DAWM, M microgrids and N retailers bidding in the DEM. Here, the retailers represent large customers or load serving entities buying electricity on behalf of customers. In order to address the interaction of the two markets, an arbitrageur is introduced as an intermediate agent that participates in both markets. If the price in the DAWM is higher than that in the DEM, the arbitrageur buys electricity from the DEM and sells the same volume to the DAWM. If the price in the DEM is higher than that in the DAWM, the arbitrageur purchases electricity from the DAWM and sells the same volume to the DEM. Following reference [15] and [16], a utility company is introduced to take the role of the ISO. The main responsibility of the utility company is to collect the bidding information from all market participants and update the prices of the two markets based on the bids and market clearing rule, then announce the prices to the market participants.

III. PROBLEM FORMULATION

A. BASIC ASSUMPTIONS

Assume that there are J strategic generators in the DAWM at a given time slot. Generator j 's ($j = 1, 2, \dots, J$) quadratic

generation cost function is $C_{1j}(Q_{1j}) = a_{1j}Q_{1j} + b_{1j}Q_{1j}^2$, where Q_{1j} represents the quantity of power generated by generator j , a_{1j} and b_{1j} are cost coefficients which are nonnegative. Since the supply function equilibrium model offers a more realistic view of electricity markets, generator j is assumed to submit a bid in a linear supply function (LSF) form $Q_{1j} = \beta_{1j} + \delta_{1j}P_1$ [17], [18]. P_1 represents the price in the DAWM. The LSF's slope δ_{1j} is the bidding variable of generator j . Generator j will bid more aggressively with increase of δ_{1j} . The LSF's intercept β_{1j} is assumed to be the intercept in the inverse function of the generator's marginal cost function $C'_{1j}(Q_{1j}) = a_{1j} + 2b_{1j}Q_{1j}$, i.e. $\beta_{1j} = -a_{1j}/2b_{1j}$.

Assume that there are K strategic retailers in the DAWM at the given time slot. Retailer k 's ($k = 1, 2, \dots, K$) quadratic utility function is $U_{2k}(Q_{2k}) = a_{2k}Q_{2k} - b_{2k}Q_{2k}^2$, where Q_{2k} represents the demand of retailer k , a_{2k} and b_{2k} are coefficients which are nonnegative. Retailer k bids in the DAWM in a demand function form $Q_{2k} = \beta_{2k} - \delta_{2k}P_1$. The slope δ_{2k} is the bidding variable of retailer k . Retailer k will bid more aggressively with decrease of δ_{2k} . The intercept β_{2k} is assumed to be the intercept in the inverse function of the retailer's marginal utility function $U'_{2k}(Q_{2k}) = a_{2k} - 2b_{2k}Q_{2k}$, i.e. $\beta_{2k} = a_{2k}/2b_{2k}$.

Assume that there are M strategic microgrids in the DEM at the given time slot. Microgrid m 's ($m = 1, 2, \dots, M$) quadratic generation cost function is $C_{3m}(Q_{3m}) = a_{3m}Q_{3m} + b_{3m}Q_{3m}^2$ [6], where Q_{3m} represents the quantity of power generated by microgrid m , a_{3m} and b_{3m} are cost coefficients which are nonnegative. Microgrid m bids in the DEM in a LSF form $Q_{3m} = \beta_{3m} + \delta_{3m}P_2$. P_2 represents the price in the DEM. Similar to the generators, the slope δ_{3m} is the bidding variable of microgrid m . Microgrid m will bid more aggressively with increase of δ_{3m} , and the intercept $\beta_{3m} = -a_{3m}/2b_{3m}$ is constant.

Assume that there are N strategic retailers in the DEM at the given time slot. Retailer n 's ($n = 1, 2, \dots, N$) quadratic utility function is $U_{4n}(Q_{4n}) = a_{4n}Q_{4n} - b_{4n}Q_{4n}^2$, where Q_{4n} represents the demand of retailer n , a_{4n} and b_{4n} are coefficients which are nonnegative. Retailer n bids in the DEM in a demand function form $Q_{4n} = \beta_{4n} - \delta_{4n}P_2$. The slope δ_{4n} is the bidding variable of retailer n . Retailer n will bid more aggressively with decrease of δ_{4n} , and the intercept $\beta_{4n} = a_{4n}/2b_{4n}$ is constant.

The arbitrageur is considered as an intermediate agent playing two different roles at the given time slot. If $P_1 > P_2$, the arbitrageur will bid in the DAWM with a LSF of $Q_1^{\text{out}} = \delta_{11}P_1$. The slope δ_{11} is the bidding variable of the arbitrageur in the DAWM. Q_1^{out} represents the quantity of electricity bought from the DEM by the arbitrageur. If $P_1 < P_2$, the arbitrageur will bid in the DEM with a LSF of $Q_1^{\text{in}} = \delta_{12}P_2$. The slope δ_{12} is the bidding variable of the arbitrageur in the DEM. Q_1^{in} represents the quantity of electricity bought from the DAWM by the arbitrageur.

B. JOINT EQUILIBRIUM MODEL

The joint equilibrium model is formed by combining equilibrium problems for strategic bidding games in the DAWM and the DEM. The equilibrium problem of the DAWM game is formed by combining the optimization problems for J generators and K retailers' bidding in the DAWM. The equilibrium problem of the DEM game is formed by combining the optimization problems for M microgrids and N retailers' bidding in the DEM. In addition, if the price in the DAWM is higher than that in the DEM, the equilibrium problem of the DAWM game should include the optimization problem for the arbitrageur's bidding. If the price in the DAWM is lower than that in the DEM, the equilibrium problem of the DEM game should include the optimization problem for the arbitrageur's bidding.

1) OPTIMIZATION PROBLEMS FOR GENERATORS' BIDDING IN THE DAWM

According to the assumptions, the optimization model for generator j 's ($j = 1, 2, \dots, J$) bidding in the DAWM can be formulated as follows:

$$\max_{\delta_{1j}} R_{1j} = P_1 \cdot Q_{1j} - C_{1j}(Q_{1j}) \quad (1)$$

$$s.t. Q_{1j} = \delta_{1j}P_1 + \beta_{1j} \quad (2)$$

$$\sum_{j=1}^J Q_{1j} + Q_1^{\text{out}} = \sum_{k=1}^K Q_{2k} + Q_1^{\text{in}}, \quad (3)$$

where R_{1j} is the profit of generator j . The demand is balanced by the total power output in the DAWM as shown in (3).

2) OPTIMIZATION PROBLEMS FOR RETAILERS' BIDDING IN THE DAWM

Retailer k 's ($k = 1, 2, \dots, K$) optimization problem in the DAWM is described as follows:

$$\max_{\delta_{2k}} R_{2k} = -P_1 \cdot Q_{2k} + U_{2k}(Q_{2k}) \quad (4)$$

$$s.t. Q_{2k} = -\delta_{2k}P_1 + \beta_{2k} \quad (5)$$

$$\sum_{j=1}^J Q_{1j} + Q_1^{\text{out}} = \sum_{k=1}^K Q_{2k} + Q_1^{\text{in}}, \quad (6)$$

where R_{2k} is the profit of retailer k in the DAWM.

3) OPTIMIZATION PROBLEMS FOR MICROGRIDS' BIDDING IN THE DEM

The optimization problem for microgrid m 's ($m = 1, 2, \dots, M$) bidding in the DEM can be expressed as the following quadratic program:

$$\max_{\delta_{3m}} R_{3m} = P_2 \cdot Q_{3m} - C_{3m}(Q_{3m}) \quad (7)$$

$$s.t. Q_{3m} = \delta_{3m}P_2 + \beta_{3m} \quad (8)$$

$$\sum_{m=1}^M Q_{3m} + Q_1^{\text{in}} = \sum_{n=1}^N Q_{4n} + Q_1^{\text{out}}, \quad (9)$$

where R_{3m} is the profit of microgrid m . The demand is balanced by the total power output in the DEM as shown in (9).

4) OPTIMIZATION PROBLEMS FOR RETAILERS' BIDDING IN THE DEM

The optimization problem for retailer k 's ($k = 1, 2, \dots, K$) bidding in the DEM can be formulated as follows:

$$\max_{\delta_{4n}} R_{4n} = -P_2 \cdot Q_{4n} + U_{4n}(Q_{4n}) \quad (10)$$

$$s.t. Q_{4n} = -\delta_{4n}P_2 + \beta_{4n} \quad (11)$$

$$\sum_{m=1}^M Q_{3m} + Q_1^{in} = \sum_{n=1}^N Q_{4n} + Q_1^{out}, \quad (12)$$

where R_{4n} is the profit of retailer n in the DEM.

5) OPTIMIZATION PROBLEM FOR THE ARBITRAGEUR'S BIDDING

If $P_1 > P_2$, the arbitrageur will participate in the competition of the DAWM. The price in the DEM is the cost of the electricity selling to the DAWM by the arbitrageur. Therefore, the arbitrageur's optimization problem to determine the best value of δ_{11} is as follows:

$$\max_{\delta_{11}} R_{11} = (P_1 - P_2) Q_1^{out} \quad (13)$$

$$s.t. Q_1^{out} = \delta_{11}P_1 \quad (14)$$

$$\sum_{j=1}^J Q_{1j} + Q_1^{out} = \sum_{k=1}^K Q_{2k} + Q_1^{in}, \quad (15)$$

where R_{11} represents the profit of the arbitrageur when the price in the DAWM is higher than that in the DEM.

If $P_1 < P_2$, the arbitrageur will participate in the competition of the DEM. The arbitrageur's optimization problem to chooses the best value of δ_{12} is as follows:

$$\max_{\delta_{12}} R_{12} = (P_2 - P_1) Q_1^{in} \quad (16)$$

$$s.t. Q_1^{in} = \delta_{12}P_2 \quad (17)$$

$$\sum_{m=1}^M Q_{3m} + Q_1^{in} = \sum_{n=1}^N Q_{4n} + Q_1^{out}, \quad (18)$$

where R_{12} represents the profit of the arbitrageur when the price in the DEM is higher than that in the DAWM.

6) EQUILIBRIUM PROBLEMS IN THE DAWM AND THE DEM

When $P_1 > P_2$, the equilibrium problem of the DAWM game is formed by combining J optimization problems of generators expressed by (1) ~ (3), K optimization problems of retailers in the DAWM expressed by (4) ~ (6) and the optimization problem of the arbitrageur expressed by (13) ~ (15). The equilibrium problem of the DEM game is formed by combining M optimization problems of microgrids expressed by (7) ~ (9), N optimization problems of retailers in the DEM expressed by (10) ~ (12).

When $P_1 < P_2$, the equilibrium problem of the DEM game is formed by combining M optimization problems of microgrids expressed by (7) ~ (9), N optimization problems of retailers in the DEM expressed by (10) ~ (12) and the optimization problem of the arbitrageur expressed

by(16) ~ (18). The equilibrium problem of the DAWM game is formed by combining J optimization problems of generators expressed by (1) ~ (3), K optimization problems of retailers in the DAWM expressed by (4) ~ (6).

It should be noted that the electricity transferred by the arbitrageur will explicitly affect the equilibrium outcomes of the two markets. Thus, considering the interaction between the DAWM and the DEM caused by the arbitrageur, the equilibrium of the two markets needs to be found by solving a joint equilibrium model, which is formed by combining the equilibrium problems of the two markets.

IV. SOLUTION METHOD

The joint equilibrium model is solved by converting the equilibrium problems into convex optimization problems and the existence and uniqueness of the Nash equilibrium is theoretically demonstrated. Then, considering the information asymmetry in practical application, a distributed algorithm is further proposed to find the equilibrium outcomes. In this section, we only discuss the solution process of the equilibrium model when $P_1 > P_2$. When $P_1 < P_2$ the solution process is similar and can be seen in the Appendix.

A. NASH EQUILIBRIUM FOR THE DAWM

When $P_1 > P_2$, the arbitrageur will bid in the DAWM and $Q_1^{in}=0$. The Nash equilibrium is a set of strategies for which no player has an incentive to change unilaterally. Let the tuple $\{\delta_{1j}^*, \delta_{2k}^*, \delta_{11}^*\}_{j=1,2,\dots,J,k=1,2,\dots,K}$ denotes the Nash equilibrium in the DAWM, P_1^* be the equilibrium price in the DAWM which is determined by the bids of the arbitrageur, generators and retailers in the DAWM. In the following, we will demonstrate the existence and uniqueness of the Nash equilibrium

Lemma 1: If $\{\delta_{1j}^*, \delta_{2k}^*, \delta_{11}^*\}_{j=1,2,\dots,J,k=1,2,\dots,K}$ is a Nash equilibrium of the DAWM, then $Q_{1j}^* < (A + \beta_{1j})/2$ for any generator $j(j = 1, 2, \dots, J)$, $Q_{2k}^* < \beta_{2k}$ for any retailer $k(k = 1, 2, \dots, K)$ and $Q_1^{out*} < A/2$ for the arbitrageur, where $A = -\sum_{j=1}^J \beta_{1j} + \sum_{k=1}^K \beta_{2k}$.

Proof: Since $Q_1^{in} = 0$, by substituting (2), (5), (14) into (3), we obtain $\sum_{j=1}^J (\delta_{1j}P_1 + \beta_{1j}) + \delta_{11}P_1 =$

$$\sum_{k=1}^K (-\delta_{2k}P_1 + \beta_{2k}), \text{ that is}$$

$$P_1 = \left(-\sum_{j=1}^J \beta_{1j} + \sum_{k=1}^K \beta_{2k} \right) / \left(\sum_{j=1}^J \delta_{1j} + \sum_{k=1}^K \delta_{2k} + \delta_{11} \right). \quad (19)$$

For generator $j(j = 1, 2, \dots, J)$, δ_{-1j} is introduced to denote $\sum_{i=1,i \neq j}^J \delta_{1i} + \sum_{k=1}^K \delta_{2k} + \delta_{11}$ for mathematic expression convenience, i.e. $\delta_{-1j} = \sum_{i=1,i \neq j}^J \delta_{1i} + \sum_{k=1}^K \delta_{2k} + \delta_{11}$. Thus (19) can be written as

$$P_1 = \frac{A}{\delta_{1j} + \delta_{-1j}}. \quad (20)$$

Substituting (20) into (2), we have

$$Q_{1j} = \frac{\delta_{1j}A}{\delta_{1j} + \delta_{-1j}} + \beta_{1j}. \quad (21)$$

From (1), (20) and (21), we get

$$\begin{aligned} \frac{\partial R_{1j}}{\partial \delta_{1j}} &= \frac{A^2}{(\delta_{1j} + \delta_{-1j})^2} \\ &\times \left[-\frac{\beta_{1j}}{A} + \frac{\delta_{-1j} - \delta_{1j}}{\delta_{-1j} + \delta_{1j}} \right. \\ &\quad \left. - \frac{\delta_{-1j}}{A} C'_{1j} \left(\frac{\delta_{1j}A}{\delta_{1j} + \delta_{-1j}} + \beta_{1j} \right) \right]. \end{aligned} \quad (22)$$

Since $\frac{\delta_{-1j} - \delta_{1j}}{\delta_{-1j} + \delta_{1j}} \leq 1$, if $-\frac{\beta_{1j}}{A} - \frac{\delta_{-1j}}{A} C'_{1j} \left(\frac{\delta_{1j}A}{\delta_{1j} + \delta_{-1j}} + \beta_{1j} \right) < -1$, then $\partial R_{1j} / \partial \delta_{1j} < 0$ for all δ_{1j} . Hence R_{1j} is strictly decreasing in δ_{1j} . Because $Q_{1j} = \delta_{1j}P_1 + \beta_{1j} \geq 0$, i.e. $\delta_{1j} \geq -\beta_{1j}/P_1$, so $\delta_{1j}^* = -\beta_{1j}/P_1$ maximizes generator j 's payoff R_{1j} for the given δ_{-1j} . As a result, $Q_{1j}^* = 0$, generator j does not participate in the competition of the DAWM.

If $-\frac{\beta_{1j}}{A} - \frac{\delta_{-1j}}{A} C'_{1j} \left(\frac{\delta_{1j}A}{\delta_{1j} + \delta_{-1j}} + \beta_{1j} \right) \geq -1$, then the optimal solution needs to satisfy:

$$-\frac{\beta_{1j}}{A} + \frac{\delta_{-1j}^* - \delta_{1j}^*}{\delta_{-1j}^* + \delta_{1j}^*} - \frac{\delta_{-1j}^*}{A} C'_{1j} \left(\frac{\delta_{1j}^*A}{\delta_{1j}^* + \delta_{-1j}^*} + \beta_{1j} \right) = 0. \quad (23)$$

Note that $\frac{\delta_{-1j}^*}{A} C'_{1j} \left(\frac{\delta_{1j}^*A}{\delta_{1j}^* + \delta_{-1j}^*} + \beta_{1j} \right) > 0$, so we get $-\frac{\beta_{1j}}{A} + \frac{\delta_{-1j}^* - \delta_{1j}^*}{\delta_{-1j}^* + \delta_{1j}^*} > 0$. Hence

$$-\beta_{1j} + P_1 (\delta_{-1j}^* - \delta_{1j}^*) > 0. \quad (24)$$

Since $P_1^* = \frac{A}{\delta_{-1j}^* + \delta_{1j}^*}$ and $Q_{1j}^* = \delta_{1j}^*P_1^* + \beta_{1j}$, we have

$$Q_{1j}^* < (A + \beta_{1j})/2. \quad (25)$$

For each retailer $k(k = 1, 2, \dots, K)$ in the DAWM, δ_{-2k} is introduced to denote $\sum_{j=1}^J \delta_{1j} + \sum_{i=1, i \neq k}^K \delta_{2i} + \delta_{11}$ for mathematic expression convenience, i.e. $\delta_{-2k} = \sum_{j=1}^J \delta_{1j} + \sum_{i=1, i \neq k}^K \delta_{2i} + \delta_{11}$. Similar to above mathematical deduction for generators, we can derive

$$\begin{aligned} \frac{\partial R_{2k}}{\partial \delta_{2k}} &= \frac{A^2}{(\delta_{2k} + \delta_{-2k})^2} \\ &\times \left[\frac{\beta_{2k}}{A} + \frac{\delta_{-2k} - \delta_{2k}}{\delta_{-2k} + \delta_{2k}} - \frac{\delta_{-2k}}{A} U'_{2k} \right. \\ &\quad \left. \times \left(-\frac{\delta_{2k}A}{\delta_{2k} + \delta_{-2k}} + \beta_{2k} \right) \right]. \end{aligned} \quad (26)$$

Since $\frac{\delta_{-2k} - \delta_{2k}}{\delta_{-2k} + \delta_{2k}} \geq -1$, if $\frac{\beta_{2k}}{A} - \frac{\delta_{-2k}}{A} U'_{2k} \left(-\frac{\delta_{2k}A}{\delta_{2k} + \delta_{-2k}} + \beta_{2k} \right) > 1$, then $\partial R_{2k} / \partial \delta_{2k} > 0$ for all δ_{2k} . Thus R_{2k} is strictly decreasing in δ_{2k} . Because $Q_{2k} = -\delta_{2k}P_1 + \beta_{2k} \geq 0$, i.e. $\delta_{2k} \geq \beta_{2k}/P_1$, so $\delta_{2k}^* = \beta_{2k}/P_1$ maximizes retailer k 's payoff R_{2k} for the

given δ_{-2k} . Thus $Q_{2k}^* = 0$, retailer k does not participate in the competition of the DAWM.

If $\frac{\beta_{2k}}{A} - \frac{\delta_{-2k}}{A} U'_{2k} \left(-\frac{\delta_{2k}A}{\delta_{2k} + \delta_{-2k}} + \beta_{2k} \right) \leq 1$, then the optimal solution needs to satisfy:

$$\frac{\beta_{2k}}{A} + \frac{\delta_{-2k} - \delta_{2k}^*}{\delta_{-2k}^* + \delta_{2k}^*} - \frac{\delta_{-2k}^*}{A} U'_{2k} \left(-\frac{\delta_{2k}^*A}{\delta_{2k}^* + \delta_{-2k}^*} + \beta_{2k} \right) = 0. \quad (27)$$

Note that $\frac{\beta_{2k}}{A} + \frac{\delta_{-2k}^* - \delta_{2k}^*}{\delta_{-2k}^* + \delta_{2k}^*} > 0$, so we have $\frac{\delta_{-2k}^*}{A} U'_{2k} \left(-\frac{\delta_{2k}^*A}{\delta_{2k}^* + \delta_{-2k}^*} + \beta_{2k} \right) > 0$. Hence $Q_{2k}^* < \beta_{2k}$. (28)

For the arbitrageur, δ_{-11} is introduced to denote $\sum_{j=1}^J \delta_{1j} + \sum_{k=1}^K \delta_{2k}$ for mathematic expression convenience, i.e. $\delta_{-11} = \sum_{j=1}^J \delta_{1j} + \sum_{k=1}^K \delta_{2k}$. Similar to above mathematical deduction for generators, we can get:

$$\frac{\partial R_{11}}{\partial \delta_{11}} = \frac{A^2}{(\delta_{-11} + \delta_{11})^2} \left(\frac{\delta_{-11} - \delta_{11}}{\delta_{-11} + \delta_{11}} - \frac{\delta_{-11}}{A} P_2 \right). \quad (29)$$

Since $\frac{\delta_{-11} - \delta_{11}}{\delta_{-11} + \delta_{11}} \leq 1$, if $\frac{\delta_{-11}}{A} P_2 > 1$, then $\partial R_{11} / \partial \delta_{11}$ for δ_{11} . This means R_{11} is strictly decreasing in δ_{11} and $\delta_{11}^* = 0$ maximizes the arbitrageur's payoff R_{11} for the given δ_{-11} . Thus $Q_{11}^* = 0$, the arbitrageur does not participate in the competition of the DAWM.

If $\frac{\delta_{-11}}{A} P_2 \leq 1$, then the optimal solution needs to satisfy:

$$\frac{A^2}{(\delta_{-11}^* + \delta_{11}^*)^2} \left(\frac{\delta_{-11}^* - \delta_{11}^*}{\delta_{-11}^* + \delta_{11}^*} - \frac{\delta_{-11}^*}{A} P_2 \right) = 0. \quad (30)$$

Notice that $\frac{\delta_{-11}^*}{A} P_2 > 0$, so $\frac{\delta_{-11}^* - \delta_{11}^*}{\delta_{-11}^* + \delta_{11}^*} > 0$, which requires $\delta_{11}^* < \delta_{-11}^*$. Hence

$$Q_1^{\text{out}*} = \frac{A\delta_{11}^*}{\delta_{-11}^* + \delta_{11}^*} < A/2. \quad (31)$$

Theorem 1: The equilibrium problem of the DAWM, expressed by (1)-(6) and (13)-(15) has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\begin{aligned} \min_{\substack{0 \leq Q_{1j} < \frac{A+\beta_{1j}}{2}, \\ 0 \leq Q_{2k} < \beta_{2k}, 0 \leq Q_1^{\text{out}} < \frac{A}{2}}} & \sum_{j=1}^J D_{1j}(Q_{1j}) \\ & - \sum_{k=1}^K D_{2k}(Q_{2k}) + d_1(Q_1^{\text{out}}) \end{aligned} \quad (32)$$

$$\text{s.t.} \quad \sum_{j=1}^J Q_{1j} + Q_1^{\text{out}} = \sum_{k=1}^K Q_{2k} \quad (33)$$

with

$$D'_{1j}(Q_{1j}) = \left(1 + \frac{Q_{1j}}{A - 2Q_{1j} + \beta_{1j}} \right) C'_{1j}(Q_{1j}) \quad (34)$$

$$D'_{2k}(Q_{2k}) = \left(1 - \frac{Q_{2k}}{A + 2Q_{2k} - \beta_{2k}} \right) U'_{2k}(Q_{2k}) \quad (35)$$

$$d'_1(Q_1^{\text{out}}) = \left(1 + \frac{Q_1^{\text{out}}}{A - 2Q_1^{\text{out}}} \right) P_2, \quad (36)$$

where, $D'_{1j}(Q_{1j})$ is the first derivative of $D_{1j}(Q_{1j})$ with respect to Q_{1j} , $D'_{2k}(Q_{2k})$ is the first derivative of $D_{2k}(Q_{2k})$ with respect to Q_{2k} , $d'_1(Q_1^{out})$ is the first derivative of $d_1(Q_1^{out})$ with respect to Q_1^{out} , $C'_{1j}(Q_{1j})$ is the first derivative of $C_{1j}(Q_{1j})$ with respect to Q_{1j} and $U'_{2k}(Q_{2k})$ is the first derivative of $U_{2k}(Q_{2k})$ with respect to Q_{2k} .

Proof: First, note that if $0 \leq Q_{1j} < (A + \beta_{1j})/2$, $0 \leq Q_{2k} < \beta_{2k}$ and $0 \leq Q_1^{out} < A/2$, then $D'_{1j}(Q_{1j}) > 0$, $D'_{2k}(Q_{2k}) > 0$ and $d'_1(Q_1^{out}) > 0$. Thus the optimization problem (32) and (33) is a strictly convex problem and has a unique optimal solution. The unique solution can be determined as follows.

For each generator $j(j = 1, 2, \dots, J)$,

$$\left[\left(1 + \frac{Q_{1j}^*}{A - 2Q_{1j}^* + \beta_{1j}} \right) C'_{1j}(Q_{1j}^*) - \omega_1 \right] (Q_{1j} - Q_{1j}^*) \geq 0 \quad \forall 0 \leq Q_{1j} < (A + \beta_{1j})/2. \quad (37)$$

For each retailer $k(k = 1, 2, \dots, K)$,

$$\left[- \left(1 - \frac{Q_{2k}^*}{A + 2Q_{2k}^* - \beta_{2k}} \right) U'_{2k}(Q_{2k}^*) + \omega_1 \right] (Q_{2k} - Q_{2k}^*) \geq 0 \quad \forall 0 \leq Q_{2k} < \beta_{2k}. \quad (38)$$

For the arbitrageur,

$$\left[\left(1 + \frac{Q_1^{out*}}{A - 2Q_1^{out*}} \right) P_2 - \omega_1 \right] (Q_1^{out} - Q_1^{out*}) \geq 0 \quad \forall 0 \leq Q_1^{out} < A/2, \quad (39)$$

where ω_1 is the Lagrange multiplier of the convex optimization problem.

The optimality condition of the DAWM equilibrium problem determined by (1) ~ (6) and (13) ~ (15) is as follows.

For each generator $j(j = 1, 2, \dots, J)$,

$$\left[-\frac{\beta_{1j}}{A} + \frac{\delta_{-1j}^* - \delta_{1j}^*}{\delta_{-1j}^* + \delta_{1j}^*} - \frac{\delta_{-1j}^*}{A} C'_{1j} \left(\frac{\delta_{1j}^* A}{\delta_{1j}^* + \delta_{-1j}^*} + \beta_{1j} \right) \right] \times (\delta_{-1j} - \delta_{1j}^*) \leq 0 \quad \forall \delta_{1j} \geq 0. \quad (40)$$

Since $P_1^* = A / (\delta_{-1j}^* + \delta_{1j}^*)$ and $Q_{1j}^* = \delta_{1j}^* P_1^* + \beta_{1j}$, we can write (40) as

$$\left[P_1^* - \left(1 + \frac{Q_{1j}^*}{A - 2Q_{1j}^* + \beta_{1j}} \right) C'_{1j}(Q_{1j}^*) \right] (Q_{1j} - Q_{1j}^*) \leq 0 \quad \forall 0 \leq Q_{1j} < (A + \beta_{1j})/2. \quad (41)$$

For each retailer $k(k = 1, 2, \dots, K)$,

$$\left[\frac{\beta_{2k}}{A} + \frac{\delta_{-2k}^* - \delta_{2k}^*}{\delta_{-2k}^* + \delta_{2k}^*} - \frac{\delta_{-2k}^*}{A} U'_{2k} \left(-\frac{\delta_{2k}^* A}{\delta_{2k}^* + \delta_{-2k}^*} + \beta_{2k} \right) \right] (\delta_{2k} - \delta_{2k}^*) \geq 0, \quad \forall \delta_{2k} \geq 0. \quad (42)$$

Since $Q_{2k}^* = -\delta_{2k}^* P_1^* + \beta_{2k}$, we can achieve

$$\left[P_1^* - \left(1 - \frac{Q_{2k}^*}{A + 2Q_{2k}^* - \beta_{2k}} \right) U'_{2k}(Q_{2k}^*) \right] (Q_{2k} - Q_{2k}^*) \geq 0 \quad \forall 0 \leq Q_{2k} < \beta_{2k}. \quad (43)$$

For the arbitrageur,

$$\left(\frac{\delta_{-11}^* - \delta_{11}^*}{\delta_{-11}^* + \delta_{11}^*} - \frac{\delta_{-11}^*}{A} P_2 \right) (\delta_{11} - \delta_{11}^*) \leq 0 \quad \forall \delta_{11} \geq 0. \quad (44)$$

Note that (44) can be written compactly as

$$\left[P_1^* - \left(1 + \frac{Q_1^{out*}}{A - 2Q_1^{out*}} \right) P_2 \right] (Q_1^{out} - Q_1^{out*}) \geq 0 \quad \forall 0 \leq Q_1^{out} < A/2. \quad (45)$$

The condition (41), (43) and (45) is equivalent to (37), (38) and (39) respectively. Thus, the Nash equilibrium of the DAWM satisfies the optimality conditions (37), (38) and (39) and solves the optimization problem (32) and (33). The existence and uniqueness of the Nash equilibrium is a result of the existence and uniqueness of the optimal solutions of (32) and (33).

B. NASH EQUILIBRIUM FOR THE DEM

When $P_1 > P_2$, the microgrids and retailers in the DEM will compete by submitting bids taking consideration of the arbitrageur's bidding in the DAWM. Let $\{\delta_{3m}^*, \delta_{4n}^*\}_{m=1,2,\dots,M,n=1,2,\dots,N}$ be a Nash equilibrium in the DEM, P_2^* be the equilibrium price in the DEM which is determined by the bids of the microgrids and retailers in the DEM.

Lemma 2: If $\{\delta_{3m}^*, \delta_{4n}^*\}_{m=1,2,\dots,M,n=1,2,\dots,N}$ is a Nash equilibrium of the DEM, then $Q_{3m}^* < (B + \beta_{3m})/2$ for any microgrid m ($m = 1, 2, \dots, M$) and $Q_{4n}^* < \beta_{4n}$ for any retailer n ($n = 1, 2, \dots, N$), where $B = -\sum_{m=1}^M \beta_{3m} + \sum_{n=1}^N \beta_{4n} + Q_1^{out}$.

Proof: For each microgrid m ($m = 1, 2, \dots, M$), δ_{-3m} is introduced to denote $\sum_{i=1, i \neq m}^M \delta_{3i} + \sum_{n=1}^N \delta_{4n}$ for mathematic expression convenience, i.e. $\delta_{-3m} = \sum_{i=1, i \neq m}^M \delta_{3i} + \sum_{n=1}^N \delta_{4n}$. Similar to the proof of Lemma 1, (8) and (9) can be written compactly as

$$\begin{cases} P_2 = \frac{B}{\delta_{3m} + \delta_{-3m}} \\ Q_{3m} = \frac{\delta_{3m} B}{\delta_{3m} + \delta_{-3m}} + \beta_{3m}. \end{cases} \quad (46)$$

From (7), we can derive

$$\begin{aligned} \frac{\partial R_{3m}}{\partial \delta_{3m}} &= \frac{B^2}{(\delta_{3m} + \delta_{-3m})^2} \\ &= \times \left[-\frac{\alpha_{3m}}{B} + \frac{\delta_{-3m} - \delta_{3m}}{\delta_{-3m} + \delta_{3m}} - \frac{\delta_{-3m}}{B} C'_{2m} \right. \\ &\quad \left. \times \left(\frac{\delta_{3m} B}{\delta_{3m} + \delta_{-3m}} + \beta_{3m} \right) \right]. \end{aligned} \quad (47)$$

Since $\frac{\delta_{-3m}-\delta_{3m}}{\delta_{-3m}+\delta_{3m}} \leq 1$, if $-\frac{\beta_{3m}}{B} - \frac{\delta_{-3m}}{B} C'_{3m} \left(\frac{\delta_{3m}B}{\delta_{3m}+\delta_{-3m}} + \beta_{3m} \right) < -1$, then $\partial R_{3m}/\partial \delta_{3m} < 0$ for all δ_{3m} . Hence R_{3m} is strictly decreasing in δ_{3m} . Because $Q_{3m} = \delta_{3m}P_2 + \beta_{3m} \geq 0$, i.e. $\delta_{3m} \geq -\beta_{3m}/P_2$, so $\delta_{3m}^* = -\beta_{3m}/P_2$ maximizes microgrid m 's payoff R_{3m} for the given δ_{-3m} . As a result, $Q_{3m}^* = 0$, microgrid m does not participate in the competition of the DEM.

If $-\frac{\beta_{3m}}{B} - \frac{\delta_{-3m}}{B} C'_{3m} \left(\frac{\delta_{3m}B}{\delta_{3m}+\delta_{-3m}} + \beta_{3m} \right) \geq -1$, then the optimal solution needs to satisfy:

$$-\frac{\beta_{3m}}{B} + \frac{\delta_{-3m} - \delta_{3m}^*}{\delta_{-3m}^* + \delta_{3m}^*} - \frac{\delta_{-3m}^*}{B} C'_{3m} \left(\frac{\delta_{3m}^* B}{\delta_{3m}^* + \delta_{-3m}^*} + \beta_{3m} \right) = 0. \quad (48)$$

Note that $\frac{\delta_{-3m}^*}{B} C'_{3m} \left(\frac{\delta_{3m}^* B}{\delta_{3m}^* + \delta_{-3m}^*} + \beta_{3m} \right) > 0$, so $-\frac{\beta_{3m}}{B} + \frac{\delta_{-3m}^* - \delta_{3m}^*}{\delta_{-3m}^* + \delta_{3m}^*} > 0$. Hence

$$-\beta_{3m} + P_2^* (\delta_{-3m}^* - \delta_{3m}^*) > 0. \quad (49)$$

Similar to the proof of Lemma 1, we get

$$Q_{3m}^* < (B + \beta_{3m})/2. \quad (50)$$

For each retailer $n(n = 1, 2, \dots, N)$ in the DEM, the proof process is similar to that of retailer k in the DAWM. Thus, we get

$$Q_{4n}^* < \beta_{4n}. \quad (51)$$

Theorem 2: The equilibrium problem of the DEM, constituted by (7)-(12) has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\min_{0 \leq Q_{3m} < \frac{B+\beta_{3m}}{2}, 0 \leq Q_{4n} < \beta_{4n}} \sum_{m=1}^M D_{3m}(Q_{3m}) - \sum_{n=1}^N D_{4n}(Q_{4n}) \quad (52)$$

$$s.t. \sum_{m=1}^M Q_{3m} = \sum_{n=1}^N Q_{4n} + Q_l^{out} \quad (53)$$

with

$$D'_{3m}(Q_{3m}) = \left(1 + \frac{Q_{3m}}{B - 2Q_{3m} + \beta_{3m}} \right) C'_{3m}(Q_{3m}) \quad (54)$$

$$D'_{4n}(Q_{4n}) = \left(1 - \frac{Q_{4n}}{B + 2Q_{4n} - \beta_{4n}} \right) U'_{4n}(Q_{4n}), \quad (55)$$

where, $D'_{3m}(Q_{3m})$ is the first derivative of $D_{3m}(Q_{3m})$ with respect to Q_{3m} , $D'_{4n}(Q_{4n})$ is the first derivative of $D_{4n}(Q_{4n})$ with respect to Q_{4n} , $C'_{3m}(Q_{3m})$ is the first derivative of $C_{3m}(Q_{3m})$ with respect to Q_{3m} and $U'_{4n}(Q_{4n})$ is the first derivative of $U_{4n}(Q_{4n})$ with respect to Q_{4n} .

Proof: First, note that if $0 \leq Q_{3m} < (B + \beta_{3m})/2$ and $0 \leq Q_{4n} < \beta_{4n}$, then we have $D'_{3m}(Q_{3m}) > 0$ and $-D'_{4n}(Q_{4n}) > 0$. Thus, the optimization problem (52) and (53) is a strictly convex problem and has a unique optimal solution. The unique solution is determined as follows.

For each microgrid $m(m = 1, 2, \dots, M)$,

$$\left[\left(1 + \frac{Q_{3m}^*}{B - 2Q_{3m}^* + \beta_{3m}} \right) C'_{3m}(Q_{3m}^*) - \omega_2 \right] (Q_{3m} - Q_{3m}^*) \geq 0 \quad \forall 0 \leq Q_{3m} < (B + \beta_{3m})/2. \quad (56)$$

For each retailer $n(n = 1, 2, \dots, N)$,

$$\left[- \left(1 - \frac{Q_{4n}^*}{B + 2Q_{4n}^* - \beta_{4n}} \right) U'_{4n}(Q_{4n}^*) + \omega_2 \right] (Q_{4n} - Q_{4n}^*) \geq 0 \quad \forall 0 \leq Q_{4n} < \beta_{4n}, \quad (57)$$

where ω_2 is the Lagrange multiplier of the convex optimization problem.

The optimality condition of the DEM equilibrium problem determined by (7) ~ (12) is as follows.

For each microgrid $m(m = 1, 2, \dots, M)$,

$$\left[-\frac{\beta_{3m}}{B} + \frac{\delta_{-3m}^* - \delta_{3m}^*}{\delta_{-3m}^* + \delta_{3m}^*} - \frac{\delta_{-3m}^*}{B} C'_{3m} \left(\frac{\delta_{3m}^* B}{\delta_{3m}^* + \delta_{-3m}^*} + \beta_{3m} \right) \right] \cdot (\delta_{3m} - \delta_{3m}^*) \leq 0 \quad \forall \delta_{3m} \geq 0. \quad (58)$$

Since $P_2^* = B/(\delta_{3m}^* + \delta_{-3m}^*)$ and $Q_{3m}^* = \delta_{3m}P_2^* + \beta_{3m}$, we obtain

$$\left[P_2^* - \left(1 + \frac{Q_{3m}^*}{B - 2Q_{3m}^* + \beta_{3m}} \right) C'_{3m}(Q_{3m}^*) \right] (Q_{3m} - Q_{3m}^*) \leq 0 \quad \forall 0 \leq Q_{3m} < (B + \beta_{3m})/2. \quad (59)$$

For each retailer $n(n = 1, 2, \dots, N)$, similar to the proof of Theorem 1, we obtain

$$\left[P_2^* - \left(1 - \frac{Q_{4n}^*}{B + 2Q_{4n}^* - \beta_{4n}} \right) U'_{4n}(Q_{4n}^*) \right] (Q_{4n} - Q_{4n}^*) \geq 0 \quad \forall 0 \leq Q_{4n} < \beta_{4n}. \quad (60)$$

The condition (59) and (60) is equivalent to (56) and (57) respectively. Similar to the DAWM, the Nash equilibrium of the DEM solves the optimization problem (52) and (53). The existence and uniqueness of the Nash equilibrium is a result of the existence and uniqueness of the optimal solutions of (52) and (53).

C. DISTRIBUTED ALGORITHM

One way to find the equilibrium is to directly solve the convex optimization problems (32)-(33) and (52)-(53), requiring the knowledge of cost (or utility) functions for all market participants. However, due to the information asymmetry, it may be impractical to collect the information of all market participants who are usually unwilling or unable to report their true information, motivating the needs of a distributed algorithm where the utility company sets the market prices while the market participants submit bids based on the prices, which requires only light communication and computation and can be easily scaled to large systems.

The convex optimization problems (32)-(33) and (52)-(53) can be easily solved in a distributed way by the dual gradient algorithm in [19]. Flow chart for solving the joint equilibrium model is shown in Fig.2. In the flow chart, e is a pre-specified threshold to control solution accuracy. Initially, the utility company picks initial prices $P_1(1)$ and $P_2(1)$ for the DAWM

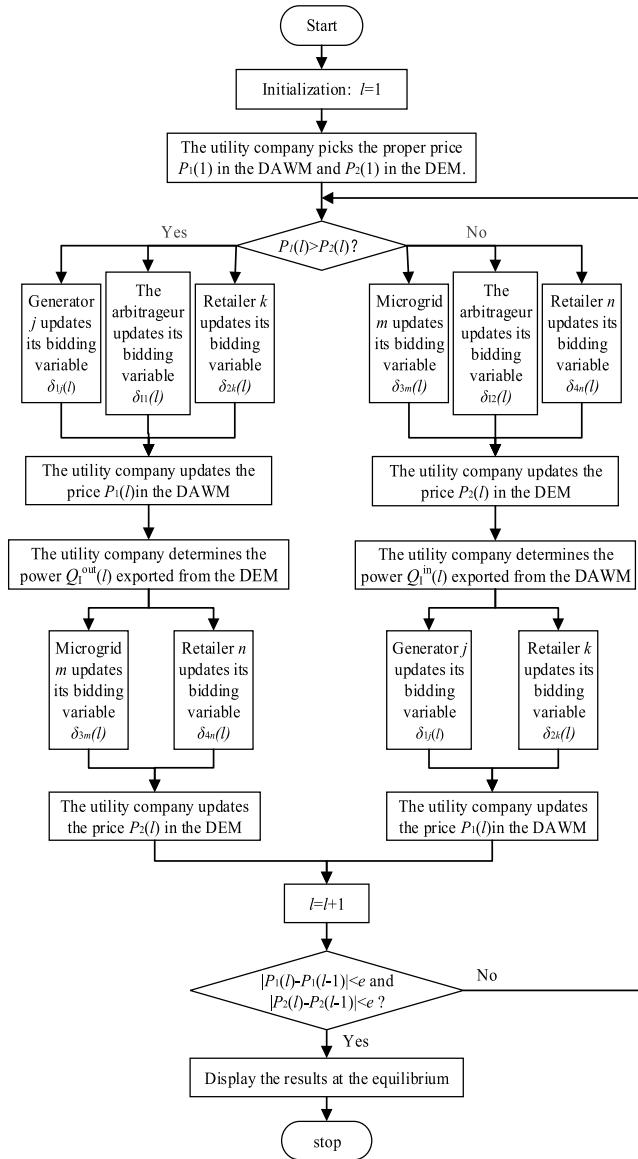


FIGURE 2. Flow chart for solving the joint equilibrium model.

and the DEM respectively and then announces the prices to each microgrid, generator, retailer and the arbitrageur over the communication network. At each iteration, when the price in the DAWM is higher than that in the DEM, the arbitrageur will bid in the DAWM along with the generators and retailers. The utility company determines the DAWM price and the quantity of electricity bought from the DEM by the arbitrageur based on the clearing results of the DAWM, and then the microgrids and retailers bid in the DEM to form the DEM price. When the price in the DEM is higher than that in the DAWM, the arbitrageur will bid in the DEM along with the microgrids and retailers, and then the generators and retailers bid in the DAWM.

At l th iteration, if $P_1(l) > P_2(l)$, the distributed algorithms for the DAWM and the DEM are as follows. The solution process for the case $P_1(l) < P_2(l)$ is given in the Appendix.

1) DISTRIBUTED ALGORITHM FOR THE DAWM

Generator j updates its bidding variable according to

$$\delta_{1j}(l) = \left[\frac{(D'_{1j})^{-1}(P_1(l)) - \beta_{1j}}{P_1(l)} \right]^+, \quad j = 1, 2, \dots, J. \quad (61)$$

The arbitrageur updates its bidding variable according to

$$\delta_{11}(l) = \left[\frac{(d_1)^{-1}(P_1(l))}{P_1(l)} \right]^+, \quad \delta_{12}(l) = 0. \quad (62)$$

Retailer k in the DAWM updates its bidding variable as follows

$$\delta_{2k}(l) = \left[-\frac{(D'_{2k})^{-1}(P_1(l)) - \beta_{2k}}{P_1(l)} \right]^+, \quad k = 1, 2, \dots, K. \quad (63)$$

Upon gathering the bids $\delta_{1j}(l)$ from generators, $\delta_{2k}(l)$ from retailers and $\delta_{11}(l)$ from the arbitrageur, the utility company updates the price in the DAWM as follows

$$P_1(l+1) = \left\{ P_1(l) - \gamma_1 \left[\sum_{j=1}^J (\delta_{1j}(l) P_1(l) + \beta_{1j}) + \delta_{11}(l) P_1(l) - \sum_{k=1}^K (-\delta_{2k}(l) P_1(l) + \beta_{2k}) \right] \right\}^+, \quad (64)$$

where parameter γ_1 denotes the step size, $[\cdot]^+$ is the projection onto the feasible set and $(\cdot)^{-1}$ is the inverse function.

2) DISTRIBUTED ALGORITHM FOR THE DEM

The utility company determines the cleared quantity of the arbitrageur based on the bidding information. Then each microgrid updates its bidding variable according to

$$\delta_{3m}(l) = \left[\frac{(D'_{3m})^{-1}(P_2(l)) - \beta_{3m}}{P_2(l)} \right]^+, \quad m = 1, 2, \dots, M. \quad (65)$$

Retailer n in the DEM updates its bidding variable according to

$$\delta_{4n}(l) = \left[-\frac{(D'_{4n})^{-1}(P_2(l)) - \beta_{4n}}{P_2(l)} \right]^+, \quad n = 1, 2, \dots, N. \quad (66)$$

Upon gathering the bids from the microgrids and retailers in the DEM, the utility company updates the price in the DEM as follows

$$P_2(l+1) = \left\{ P_2(l) - \gamma_2 \left[\sum_{m=1}^M (\delta_{3m}(l) P_2(l) + \beta_{3m}) - \sum_{n=1}^N (-\delta_{4n}(l) P_2(l) + \beta_{4n}) - \delta_{11}(l) P_1(l) \right] \right\}^+, \quad (67)$$

where parameter γ_2 denotes the step size.

Once the price in the DEM changes, the profit of the arbitrageur will also change and it will affect the equilibrium outcomes of the DAWM. Let $l = l + 1$, each generator, retailer and the arbitrageur will update its bidding variable again. Repeat the process until the price in the DEM will not change, which means the bidding variables of all microgrids, generators, retailers and the arbitrageur will no longer change and the iterative processes converge to the Nash equilibrium of the joint equilibrium model.

The above distributed algorithm is equivalent to the dual gradient method for the proposed optimization problems. Besides, it is demonstrated in [19] that the optimal solution to a linearly constrained convex separable optimization problem can be found using the dual gradient method. Hence, the above distributed algorithm will converge to the optimal solutions to the optimization problems (32)-(33) and (52)-(53) when sufficiently small step sizes γ_1 and γ_2 are used. Since the Nash equilibrium for both the DAWM and the DEM is unique, the iterative processes will converge to the Nash equilibrium of the joint equilibrium model. The selection of step size and the relation between step size and convergence speed can be referred to [20] and [21]. In addition, it does not need the profile of the other market participants since (61)-(63) and (65)-(66) only depend on its own bidding variable and the information announced by the utility company. This can guarantee the privacy of the market participants, which also perfectly meets the practical situation of information asymmetry in practical electricity markets.

It is noted that in the proposed model, we assume that the DAWM price P_1 and the DEM price P_2 are known to the arbitrageur in advance. This assumption is valid for the following reasons: 1) there is a multi-round bidding process in many actual electricity markets, which is simulated by the iterative distributed algorithm in this paper. 2) at the beginning of the first-round bidding, the arbitrageur and other participants can make a rough prediction of the prices in two markets according to the historical information, then make the first-round bids. The utility company collects the bidding information from all market participants and updates the prices of the two markets based on the bids and market clearing rule, then announces the prices to the market participants. 3) starting from the second-round bidding, the arbitrageur and other participants can know the prices in advance according to the announced prices from the utility company.

V. NUMERICAL EXAMPLES

In order to evaluate the effectiveness of the proposed approach, a DEM with two microgrids(i.e., MG1, MG2) and two retailers(H1, H2), a DAWM with two generators (i.e., G1, G2) and two retailers(i.e., C1, C2), and one arbitrageur (i.e., I) are assumed. As shown in Table 1, the following two cases are considered: *case1* for relatively high generation cost of microgrids; *case2* for relatively low generation cost of microgrids. The step size is set as 0.01.

TABLE 1. Market participants' parameters.

Market Participants		a (\$/MWh)		b (\$/(MW ² h))	
		Case 1	Case 2	Case 1	Case 2
Microgrids	MG1	15.00	3.000	0.150	0.030
	MG2	20.00	4.000	0.200	0.040
Retailers in the DEM	H1		35.00		0.160
	H2		40.00		0.180
Generators	G1		5.000		0.050
	G2		6.000		0.060
Retailers in the DAWM	C1		40.00		0.100
	C2		50.00		0.120

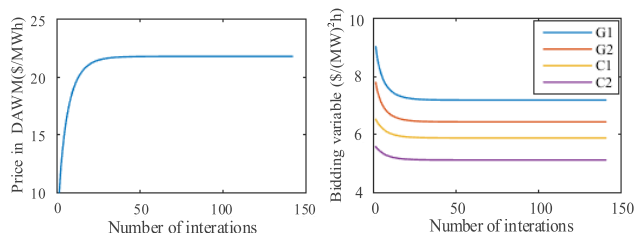


FIGURE 3. Evolution of price and bidding variables of generators and retailers in the DAWM.

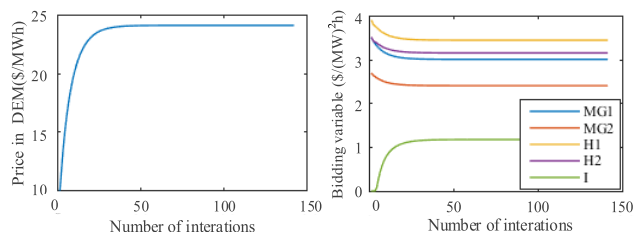


FIGURE 4. Evolution of price and bidding variables of microgrids, retailers and arbitrageur in the DEM.

A. CONVERGENCE OF DISTRIBUTED ALGORITHM

In this section, we consider *case 1* to demonstrate the effectiveness of the distributed algorithm. The step size is set as 0.01 and it takes about 0.1218s to obtain the solution. Fig.3 shows the evolution of the price and bidding variables of generators and retailers in the DAWM. Fig. 4 shows the evolution of the price and bidding variables of microgrids, retailers and the arbitrageur(I) in the DEM. It can be noted that when using the distributed algorithm to solve the equilibrium model proposed in this paper, the iterative process converges quickly, which illustrate effectiveness of the algorithm.

B. EQUILIBRIUM OUTCOMES ANALYSIS

In this section, we analyze the results for *case 1* and *case 2*, and examine the impacts of introducing the arbitrageur. The equilibrium outcomes for *case 1* and *case 2* with/without the arbitrageur(I) are listed in Table 2.

It can be found that in *case1* with the arbitrageur, the arbitrageur purchases electricity from the DAWM and sells to the DEM. Compared to the case without the arbitrageur, the price in the DEM drops while the price in the DAWM increases, thus the price difference between the two markets decreases. Therefore, the profits of microgrids decrease while the profits of retailers in the DEM increase; the profits of

TABLE 2. Equilibrium results.

		DEM			DAWM	
case		MG1	H1	I	G1	C1
Bidding variable ((MW ² ·h)/\$)	1 w/o I	2.921	3.364	/	7.283	6.009
	with I	3.023	3.464	1.182	7.190	5.878
Price (\$/MWh)	1 w/o I	10.36	3.846	/	7.283	6.009
	with I	9.767	3.652	3.394	7.767	6.167
Price (\$/MWh)	1 w/o I		27.31			20.31
	with I		24.17			21.81
Profit (\$/h)	1 w/o I		11.99			20.31
	with I		14.98			17.21
Profit (\$/h)	1 w/o I	233.4	85.64	/	1020	927.2
	with I	131.8	172.4	67.57	1225	790.6
Social welfare (\$/h)	1 w/o I	502.2	815.2	/	1020	927.2
	with I	875.2	616.4	130.5	671.8	1258
Social welfare (\$/h)	1 w/o I			5092		
	with I			5283		
Social welfare (\$/h)	1 w/o I			7224		
	with I			7703		

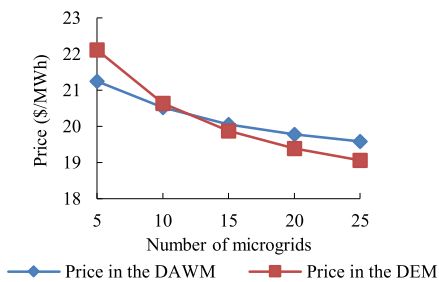


FIGURE 5. Electricity prices vs. number of microgrids.

generators increase while the profits of retailers in the DAWM decrease. Moreover, with the arbitrageur, the social welfare is improved.

For case2 with the arbitrageur, the arbitrageur purchases electricity from the DEM and sells it to the DAWM. Compared to the case without the arbitrageur, the market power abuse of generators in the DAWM is mitigated. The price in the DEM increases while the price in the DAWM drops. As a result, the profits of microgrids increase while the profits of retailers in the DEM decrease. The profits of generators decrease while the profits of retailers in the DAWM increase. In addition, the social welfare increases for the case with the arbitrageur.

Thus, under the market framework proposed in this paper, arbitrage between the two markets should be encouraged in order to increase the social welfare. In addition, the reduction in the generation cost of microgrids can help mitigate market power abuse of generators in the DAWM.

C. IMPACTS OF NUMBER OF MICROGRIDS ON EQUILIBRIUM OUTCOMES

In this section, we examine the impacts of the number of microgrids in the DEM. Assume that microgrids' a_{3m} and b_{3m} are isometrically drawn from 15 to 20 and 0.15 to 0.20, respectively. Fig.5 plots the prices in the DAWM and the DEM with different number of microgrids. Fig. 6 plots the

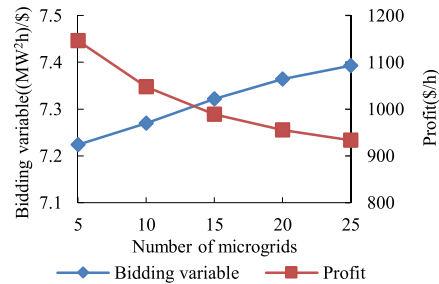


FIGURE 6. Bidding variable and profit of generator G1 vs. number of microgrids.

bidding variable and profit of generator G1 in the DAWM with different number of microgrids.

It can be found that the increase in the number of microgrids leads to a reduction in the DEM price and is followed by a price drop in the DAWM. More importantly, with an increase in the number of microgrids, generators increase their bidding variables, which means that they bid more aggressively in the DAWM. As a result, the market power abuse of generators in the DAWM is mitigated and their profits are lowered.

VI. CONCLUSION

This paper addresses the need for microgrids to participate in electricity market competition. For this purpose, a market trading framework is presented where microgrids sell electricity by submitting bids in the distribution electricity market (DEM) while conventional generators compete by submitting bids in the day-ahead wholesale market (DAWM). The arbitrageur is allowed to buy and sell electricity between the two markets. Based on this framework, a joint equilibrium model for strategic bidding games in the DAWM and the DEM is proposed. Considering the information asymmetry in practical application, the equilibrium model is solved by converting equilibrium problems into convex optimization problems. The existence and uniqueness of the Nash equilibrium is theoretically proved and a distributed algorithm is further proposed to find the equilibrium outcomes. Finally, the numerical examples are presented to verify the reasonableness and effectiveness of the proposed model and algorithm. It is shown that under the market framework proposed in this paper, arbitrage between the two markets should be encouraged in order to increase the social welfare. In addition, the reduction in the generation cost of microgrids and the increase in the number of microgrids can help mitigate market power abuse of generators in the DAWM.

APPENDIX

If $P_1 < P_2$, the arbitrageur bids in the DEM and $Q_i^{out} = 0$.

Lemma 3: It can be proved that if $\{\delta_{1j}^*, \delta_{2k}^*\}_{j=1,2,\dots,J,k=1,2,\dots,K}$ is a Nash equilibrium of the DAWM, then $Q_{1j}^* < (A + \beta_{1j})/2$ for any generator $j(j = 1, 2, \dots, J)$ and $Q_{2k}^* < \beta_{2k}$ for any retailer $k(k = 1, 2, \dots, K)$, where $A = -\sum_{j=1}^J \beta_{1j} + \sum_{k=1}^K \beta_{2k}$.

Lemma 4: If $\{\delta_{3m}^*, \delta_{4n}^*, \delta_{12}^*\}_{m=1,2,\dots,M, n=1,2,\dots,N}$ is a Nash equilibrium of the DEM, then $Q_{3m}^* < (B + \beta_{3m})/2$ for any microgrid $m(m = 1, 2, \dots, M)$, $Q_{4n}^* < \beta_{4n}$ for any retailer $n(n = 1, 2, \dots, N)$ and $Q_1^{\text{in}*} < B/2$ for the arbitrageur, where $B = -\sum_{m=1}^M \beta_{3m} + \sum_{n=1}^N \beta_{4n}$.

Theorem 3: The equilibrium model of the DAWM, constituted by (1)-(6) has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\min_{0 \leq Q_{1j} < \frac{A+\beta_{1j}}{2}, 0 \leq Q_{2k} < \beta_{2k}} \sum_{j=1}^J D_{1j}(Q_{1j}) - \sum_{k=1}^K D_{2k}(Q_{2k}) \quad (68)$$

$$\text{s.t.} \quad \sum_{j=1}^J Q_{1j} = \sum_{k=1}^K Q_{2k} + Q_1^{\text{in}}. \quad (69)$$

Theorem 4: The equilibrium model of the DEM, expressed by (7)-(12) and (16)-(18) has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

$$\min_{\substack{0 \leq Q_{3m} < \frac{B+\beta_{3m}}{2}, \\ 0 \leq Q_{4n} < \beta_{4n}, 0 \leq Q_1^{\text{in}} < \frac{B}{2}}} \sum_{m=1}^M D_{3m}(Q_{3m}) - \sum_{n=1}^N D_{4n}(Q_{4n}) + d_2(Q_1^{\text{in}}) \quad (70)$$

$$\text{s.t.} \quad \sum_{m=1}^M Q_{3m} + Q_1^{\text{in}} = \sum_{n=1}^N Q_{4n} \quad (71)$$

with

$$d_2'(Q_1^{\text{in}}) = \left(1 + \frac{Q_1^{\text{in}}}{B - 2Q_1^{\text{in}}}\right) P_1, \quad (72)$$

where, $d_2'(Q_1^{\text{in}})$ is the first derivative of $d_2(Q_1^{\text{in}})$ with respect to Q_1^{in} ,

If $P_1 < P_2$, the distributed algorithms for the DAWM and the DEM are as follows.

A. DISTRIBUTED ALGORITHM FOR THE DEM

Microgrid m updates its bidding variable according to

$$\delta_{3m}(l) = \left[\frac{(D'_{3m})^{-1}(P_2(l)) - \beta_{3m}}{P_2(l)} \right]^+, \quad m = 1, \dots, M. \quad (73)$$

The arbitrageur updates its bidding variable according to

$$\delta_{12}(l) = \left[\frac{(d_2')^{-1}(P_2(l))}{P_2(l)} \right]^+, \quad \delta_{11}(l) = 0. \quad (74)$$

Retailer n in the DEM updates its bidding variable according to

$$\delta_{4n}(l) = \left[-\frac{(D'_{4n})^{-1}(P_2(l)) - \beta_{4n}}{P_2(l)} \right]^+, \quad n = 1, \dots, N. \quad (75)$$

Upon gathering the bids $\delta_{3m}(l)$ from microgrids, $\delta_{4n}(l)$ from retailers and $\delta_{12}(l)$ from the arbitrageur, the utility company updates the price in the DEM as follows

$$P_2(l+1) = \left\{ P_2(l) - \gamma_2 \left[\sum_{m=1}^M (\delta_{3m}(l)P_2(l) + \beta_{3m}) + \delta_{12}(l)P_2(l) - \sum_{n=1}^N (-\delta_{4n}(l)P_2(l) + \beta_{4n}) \right] \right\}^+. \quad (76)$$

B. DISTRIBUTED ALGORITHM FOR THE DAWM

The utility company determines the cleared quantity of the arbitrageur based on the bidding information. Then each generator updates its bidding variable according to

$$\delta_{1j}(l) = \left[\frac{(D'_{1j})^{-1}(P_1(l)) - \beta_{1j}}{P_1(l)} \right]^+, \quad j = 1, \dots, J. \quad (77)$$

Retailer k in the DAWM updates its bidding variable according to

$$\delta_{2k}(l) = \left[-\frac{(D'_{2k})^{-1}(P_1(l)) - \beta_{2k}}{P_1(l)} \right]^+, \quad k = 1, 2, \dots, K. \quad (78)$$

Upon gathering the bids from the generators and retailers in the DAWM, the utility company updates the price in the DAWM as follows

$$P_1(l+1) = \left\{ P_1(l) - \gamma_1 \left[\sum_{j=1}^J (\delta_{1j}(l)P_1(l) + \beta_{1j}) - \sum_{k=1}^K (-\delta_{2k}(l)P_1(l) + \beta_{2k}) - \delta_{12}(l)P_2(l) \right] \right\}^+. \quad (79)$$

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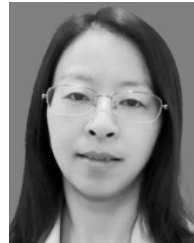
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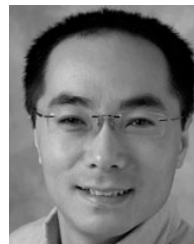
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