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Optimal Relay Deployment in Bidirectional AF Relaying Systems

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ABSTRACT Recently, the node deployment problem of a relay network has brought a hot discussion. This paper studies utility function minimum maximization problem of bidirectional amplify and forward relaying systems from the perspective of relay deployment. Since many optimization problems are considered separately, different algorithms are proposed for each optimization problem. So we consider a unified utility function instead of the separate optimization goals. Two different optimization problems are considered. One is to find the optimal relay deployment and optimize the utility function with fixed power allocation. The other is joint optimization of relay deployment and power allocation. Especially when power is fixed, the relay deployment is not a fixed location, but a piecewise function. In order to balance the two opposite transmission directions in bidirectional relaying networks, the relay deployment is related to channel gain and path loss. In addition, approximate and accurate expressions of average relay deployment are investigated when only statistical channel information is available. Simulation results validate that the reasonable relay deployment improves the system performance.

INDEX TERMS Bidirectional relaying, relay deployment, power allocation.

I. INTRODUCTION

Compared to traditional one-way relaying transmission, bidirectional relay can improve energy efficiency and spectral efficiency because it has one more transmission direction than one-way counterpart. When a certain transmission direction is turned off, a bidirectional transmission can be degraded to a corresponding one-way counterpart. Therefore, the one-way relaying network can be regarded as a special case of the bidirectional relaying one from the perspective of the transmission direction.

A. RELATED WORK

Current research on the resource allocation of bidirectional relay technology is the use of various mathematical skills to optimize one or more objective functions. Different mathematical skills determine the amount of resources available to each user in wireless networks. Specifically, secondary users' data rate was maximized in bidirectional cognitive relay networks [1], where secondary users helped primary users to enhance their rate through joint subcarrier and power allocation. Similar problem was investigated in [2], but in a more practical scenario, imperfect spectrum sensing. In addition, the feasibility, sensitivity, convergence and complexity were analyzed. The signal to noise ratio (SNR) was maximized under the achievable SNR region constraint [3], where a semi closed solution was found for relay beamforming and power allocation. The average sum rate of secondary users was regarded as a optimization goal in bidirectional multi-relay networks [4], where iterative convex searches was proposed under different power constraints. Orthogonal frequency division multiplexing based bidirectional simultaneous wireless information and power transfer relay networks was considered in [5], where subcarrier grouping, pairing and power allocation problem were proposed for sum rate maximization. Applying barrier based interior point method, a non convex geometric programming problem was developed in [6] for outage probability optimization in bidirectional cognitive

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relay networks. The successive convex approximation was presented in [7] by transforming non convex problem into a sequence of convex programs in non concurrent bidirectional networks.

Among a great lot of resource allocation problems, the relay deployment issue determines where the relay should be deposited for establishing a bidirectional communication channel between a pair of transceivers. In general, optimal relay deployment takes into account many factors, such as power, distance, energy and so on. Specifically, according to asymptotic outage probability in multiuser bidirectional relay networks, optimal relay deployment problem was considered in [8] under asymmetric and symmetric traffics. Furthermore, Ref. [9] extended the work of Ref. [8] to cover cochannel interference and channel estimation error. However, the calculation process and ideas were similar to those in [8]. Three-phase bidirectional networks were studied in [10], where relay deployment was investigated still based on outage probability minimization problem. Based on Taylor series expansion of outage probability, the relay deployment was obtained in [11], [12] for full duplex bidirectional relay networks. But the path loss exponent was specified as 4, which was very limited in practice. Similarly, Ref. [13] only assumed a path loss exponent of 4. Relay deployments under a specific path loss exponent were not universal. The optimal relay deployment was determined according to the throughput, also related to the outage probability [14]. Unfortunately, no closed form expression of relay deployment was provided, so the authors in [14] only turned to numerical search. In fact, this problem has not been solved. Similar shortcoming also exists in [15]. The optimal relay deployment was obtained by taking the first derivation of objective function to zero, the solution of the relevant algebraic equation had not been found though [15]. The objective function was replaced by an asymptotic symbol error probability (SEP) and ergodic sum rate in high SNR regime in [16]. A non ideal power amplifier model was integrated into energy efficiency in [17], where relay deployment was derived by energy efficiency optimization problem.

It is worthwhile to note that the idea of most previous works are very similar. The optimal relay deployment is built on an asymptotic outage probability [8]–[12], [14] or symbol error probability [15], [16] in a sufficiently high SNR region. The benefit of asymptotic formula instead of exact one is obvious. Generally speaking, the asymptotic curve is a straight line in a simulation figure, simpler than a complex accurate one. Therefore, the closed formula of relay deployment is easier found by virtue of linear characteristics of the asymptotic outage probability. However, the relay deployment based on the asymptotic formula may not be suitable for low or medium SNR level.

B. OUR CONTRIBUTION

In a scalable network environment, a good relay deployment is a critical task in saving energy, increasing efficiency and reducing overhead. This paper studies relay deployment in bidirectional amplify and forward (AF) relaying networks. Optimal relay deployment is achieved by maximizing the minimum utility function problem under distance and power constraints. Two types of problems are considered. One is to determine the optimal relay deployment while maximizing the minimum utility function at a fixed power allocation. The other is a joint relay deployment and power allocation algorithm while optimizing the object function. Specifically, in the relay deployment problem, it is relatively rare not to use asymptotic outage probability as an optimization goal. Then as an example, the average SEP is compared between optimal relay deployment and joint algorithm.

Via theoretical analysis, the following important conclusions are drawn. First, under a fixed power allocation, it is impossible always to balance the utility of the two links between two transceivers and relay. However, it is very easy to achieve a balanced state in joint relay deployment and power allocation problem. Second, under an asymmetric traffic condition, the system performance is limited by a smaller link traffic, whose maximum determines the optimal relay deployment. Third, as a double check, the optimal relay position of the joint optimization problem still satisfies the conditions of the relay deployment problem under fixed power allocation. Finally, if the instantaneous channel state information is not available, the closed form expression of average relay deployment serves as a useful tool instead, which has been scarcely considered in previous works.

II. SYSTEM MODEL

Consider a bidirectional relay network where two transceivers S_1 and S_2 establish a bidirectional communication with each other via an AF relay node R. Assume that d_0, d_1 and d_2 are the distances between S_1 and S_2, S_1 and R, S_2 and R, respectively. Let h_1 and h_2 be the channel coefficients between S_1 and R, S_2 and R. For simplicity, the relay is deployed on a straight line between the two transceivers. The SNRs received at S_1 and S_2 are respectively given by

$$\begin{aligned}\gamma_1 &= \frac{p_2 p_r \frac{ab}{d_1^n d_2^n}}{p_r \frac{a}{d_1^n} + p_1 \frac{a}{d_1^n} + p_2 \frac{b}{d_2^n} + 1} \\ &\approx \frac{p_2 p_r \frac{ab}{d_1^n d_2^n}}{p_r \frac{a}{d_1^n} + p_1 \frac{a}{d_1^n} + p_2 \frac{b}{d_2^n}} \end{aligned} \tag{1}$$

$$\begin{aligned}\gamma_2 &= \frac{p_1 p_r \frac{ab}{d_1^n d_2^n}}{p_r \frac{b}{d_2^n} + p_1 \frac{a}{d_1^n} + p_2 \frac{b}{d_2^n} + 1} \\ &\approx \frac{p_1 p_r \frac{ab}{d_1^n d_2^n}}{p_r \frac{b}{d_2^n} + p_1 \frac{a}{d_1^n} + p_2 \frac{b}{d_2^n}} \end{aligned} \tag{2}$$

where p_1, p_2 and p_r are the power consumptions of nodes S_1, S_2 and R, respectively, n is path loss exponent, and $a = |h_1|^2/\sigma^2$ and $b = |h_2|^2/\sigma^2$ with σ^2 as the noise variance. This approximations in (1)-(2) have been widely adopted without much loss [8], [15]. Firstly, we study the optimal relay deployment while maximizing the minimum utility function under a fixed power allocation. This problem is formulated as

$$\max_{\{d_1,d_2\}} \min\left[U\left(\gamma_1\right), U\left(\gamma_2\right)\right] \quad \text{s.t. } d_1+d_2=d_0 \tag{3}$$

where $U(\cdot)$ is a unity function. In general, the utility function is chosen to be a monotonically increasing function, such as (1) outage probability, $\max\left\{\min\left[-\Pr\left(\gamma_1<\gamma_{th}\right), -\Pr\left(\gamma_2<\gamma_{th}\right)\right]\right\}$, where γ_{th} is a preset threshold; (2) data rate, $\max\left\{\min\left[\log_2\left(1+\gamma_1\right), \log_2\left(1+\gamma_2\right)\right]\right\}$; (3) SEP, $\max\left\{\min\left[-uQ\left(\sqrt{v\gamma_1}\right), -uQ\left(\sqrt{v\gamma_2}\right)\right]\right\}$, where u and v are constants depend on modulation constellation, and $Q(\cdot)$ is Gaussian function; (4) SEP in the context of generalized noise environment, $\max\left\{\min\left[-uQ_\alpha\left(\sqrt{v\gamma_1}\right), -uQ_\alpha\left(\sqrt{v\gamma_2}\right)\right]\right\}$, where $Q_\alpha(\cdot)$ is a generalized Gaussian function [18] and α defines the type of generalized Gaussian noise and so on.

Secondly, we investigate the joint relay deployment and power allocation algorithm while maximizing the minimum utility function. This problem is given by

$$\begin{aligned}\max_{\{d_1,d_2,p_1,p_2,p_r\}} \ & \min\left[U\left(\gamma_1\right), U\left(\gamma_2\right)\right]\\ \text{s.t. } & d_1+d_2=d_0\\ & p_1+p_2+p_r=P_T\end{aligned} \tag{4}$$

where P_T is the total power budget.

III. OPTIMAL RELAY DEPLOYMENT UNDER FIXED POWER ALLOCATION

A. OPTIMIZATION ALGORITHM

Here we are looking for the optimal relay deployment when the power is fixed. The problem (3) can be converted to the following one

$$\max_{\{d_1,d_2\}} \min\left(\gamma_1, \gamma_2\right) \quad \text{s.t. } d_1+d_2=d_0 \tag{5}$$

Theorem 1: If the power allocation has been fixed in the bidirectional relay transmission, then the optimal relay deployment is

$$d_1^\star=\begin{cases} d_{1,1} & \text{if } p_1\geq p_2+p_r \text{ or if } p_2<p_1+p_r,\\ & p_1<p_2+p_r, \dfrac{a}{b}<\xi_1\\ d_{1,eq} & \text{if } p_2<p_1+p_r, p_1<p_2+p_r,\\ & \xi_1\leq\dfrac{a}{b}\leq\xi_2\\ d_{1,2} & \text{if } p_2\geq p_1+p_r \text{ or if } p_2<p_1+p_r,\\ & p_1<p_2+p_r, \xi_2<\dfrac{a}{b}\end{cases} \tag{6}$$

where

$$d_{1,1}=\frac{d_0}{1+\left(\frac{bp_2}{a\left(p_1+p_r\right)}\right)^{\frac{1}{n-1}}} \tag{7}$$

$$d_{1,eq}=\frac{d_0}{1+\left(\frac{bp_2\left(p_r+p_2-p_1\right)}{ap_1\left(p_1+p_r-p_2\right)}\right)^{\frac{1}{n}}} \tag{8}$$

$$d_{1,2}=\frac{d_0}{1+\left(\frac{b\left(p_2+p_r\right)}{ap_1}\right)^{\frac{1}{n-1}}} \tag{9}$$

$$\xi_1=\frac{p_2\left[p_1\left(p_1+p_r-p_2\right)\right]^{n-1}}{\left(p_1+p_r\right)^n\left(p_2+p_r-p_1\right)^{n-1}} \tag{10}$$

$$\xi_2=\frac{\left(p_2+p_r\right)^n\left(p_1+p_r-p_2\right)^{n-1}}{p_1\left[p_2\left(p_2+p_r-p_1\right)\right]^{n-1}} \tag{11}$$

Proof: See Appendix A ∎

From the above formulas, the explicit optimal relay deployment is related to the fixed power, channel coefficient and path loss exponent. Because the power and channel coefficients are non-negative, we can conclude that $d_1^\star$ is always in the interval $[0, d_0]$, in accordance with the actual physical meaning. Furthermore, relay deployment is not a simple formula, but the result of a classification discussion. In a symmetric system, the optimal relay deployment is achieved in a balanced state since $d_{1,eq}$ is obtained according to $\gamma_1=\gamma_2$. In an asymmetric system, the optimal relay deployment depends on the extreme point of the smaller link since $d_{1,1}$ is obtained according to $\gamma_1<\gamma_2$ while $d_{1,2}$ is obtained according to $\gamma_1>\gamma_2$.

B. PERFORMANCE ANALYSIS

Here, we prepare to quantitatively analyze relay deployment variable d_1. Assume that all links suffer from Nakagami-m fading. Then we can capture the statistical behavior of the relay deployment. First, the cumulative distribution function (CDF) of d_1 is presented in the following theorem.

Theorem 2: If the power allocation has been fixed in the bidirectional relay transmission, then CDF $F_{d_1}(z)$ of the optimal relay deployment d_1 is $F_{d_1}(z)=0$ for $z\leq 0$ and $F_{d_1}(z)=1$ for $z\geq d_0$. When $0<z<d_0$, the CDF is divided into three cases as follows.

(1) If $p_1\geq p_2+p_r$, the CDF of d_1 is given by

$$\begin{aligned}F_{d_1}(z)=F_{d_{1,1}}(z)&=\frac{\Gamma\left(m_1+m_2\right)}{\Gamma\left(m_2\right)\Gamma\left(m_1+1\right)}\\ &\times\left[\frac{\beta_1p_2}{\beta_2\left(p_1+p_r\right)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1}\\ &\times {}_2F_1\left[m_1, m_1+m_2; m_1+1;\right.\\ &\left.-\frac{\beta_1p_2}{\beta_2\left(p_1+p_r\right)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]\end{aligned} \tag{12}$$

where $F_X(\cdot)$ is the CDF of variable X, m_1 and m_2 are fading parameters of links $S_1\leftrightarrow R$ and $S_2\leftrightarrow R$. $\beta_1=m_1\sigma^2/E\left(|h_1|^2\right)$ and $\beta_2=m_2\sigma^2/E\left(|h_2|^2\right)$ are the corresponding shape parameters. $\Gamma(\cdot)$ is the gamma function [19, eq.(8.311)] and ${}_2F_1(\cdot)$ is the Gaussian hypergeometric function [19, eq.(9.100)].

(2) If $p_2\geq p_1+p_r$, the CDF of d_1 is given by

$$F_{d_1}(z)=F_{d_{1,2}}(z)=\frac{\Gamma\left(m_1+m_2\right)}{\Gamma\left(m_2\right)\Gamma\left(m_1+1\right)}$$

$$\times\left[\frac{\beta_1(p_2+p_r)}{\beta_2 p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1}\times{}_2F_1\left[m_1,m_1+m_2;m_1+1;-\frac{\beta_1(p_2+p_r)}{\beta_2 p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right] \tag{13}$$

(3) If $p_2 < p_1+p_r$ and $p_1 < p_2+p_r$, the CDF of d_1 is further divided in three segments. When $0 < z < \frac{d_0p_1(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)}$, the CDF is given by $F_{d_1}(z)=F_{d_{1,1}}(z)$; when $\frac{d_0p_1(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)} \le z \le \frac{d_0(p_2+p_r)(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)}$, the CDF is given by

$$F_{d_1}(z)=F_{d_{1,eq}}(z)=\frac{\Gamma(m_1+m_2)}{\Gamma(m_2)\Gamma(m_1+1)}\times\left[\frac{\beta_1p_2(p_2+p_r-p_1)}{\beta_2p_1(p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^n}\right]^{m_1}\times{}_2F_1\left[m_1,m_1+m_2;m_1+1;-\frac{\beta_1p_2(p_2+p_r-p_1)}{\beta_2p_1(p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^n}\right] \tag{14}$$

and when $\frac{d_0(p_2+p_r)(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)} < z$, the CDF is given by $F_{d_1}(z)=F_{d_{1,2}}(z)$.

Proof: See Appendix B. ■

Differentiating the CDF $F_{d_1}(z)$ with respect to z yields the corresponding the probability density function (PDF). The PDF $f_{d_1}(z)$ of the optimal relay deployment d_1 is $f_{d_1}(z)=0$ for $z\le 0$ or $z\ge d_0$. When $0<z<d_0$, the PDF is divided into three cases as follows.

(1) If $p_1 \ge p_2+p_r$, the PDF of d_1 is given by

$$f_{d_1}(z)=f_{d_{1,1}}(z)=\frac{\Gamma(m_1+m_2)d_0(n-1)}{\Gamma(m_2)\Gamma(m_1)z(d_0-z)}\times\frac{\left[\frac{\beta_1p_2}{\beta_2(p_1+p_r)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1}}{\left[1+\frac{\beta_1p_2}{\beta_2(p_1+p_r)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1+m_2}} \tag{15}$$

where $f_X(x)$ is probability density function of the variable X.

(2) If $p_2 \ge p_1+p_r$, the PDF of d_1 is given by

$$f_{d_1}(z)=f_{d_{1,2}}(z)=\frac{\Gamma(m_1+m_2)d_0(n-1)}{\Gamma(m_2)\Gamma(m_1)z(d_0-z)}\times\frac{\left[\frac{\beta_1(p_2+p_r)}{\beta_2p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1}}{\left[1+\frac{\beta_1(p_2+p_r)}{\beta_2p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1+m_2}} \tag{16}$$

(2) If $p_2 < p_1+p_r$ and $p_1 < p_2+p_r$, the PDF of d_1 is further divided in three segments. When $0<z<\frac{d_0p_1(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)}$, the PDF is given by $f_{d_1}(z)=f_{d_{1,1}}(z)$; when $\frac{d_0p_1(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)}\le z\le\frac{d_0(p_2+p_r)(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)}$, the PDF is given by

$$f_{d_1}(z)=f_{d_{1,eq}}(z)=\frac{\Gamma(m_1+m_2)d_0n}{\Gamma(m_2)\Gamma(m_1)z(d_0-z)}\times\frac{\left[\frac{\beta_1p_2(p_2+p_r-p_1)}{\beta_2p_1(p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^n}\right]^{m_1}}{\left[1+\frac{\beta_1p_2(p_2+p_r-p_1)}{\beta_2p_1(p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^n}\right]^{m_1+m_2}} \tag{17}$$

and when $\frac{d_0(p_2+p_r)(p_1+p_r-p_2)}{p_r(p_1+p_2+p_r)} < z$, the CDF is given by $f_{d_1}(z)=f_{d_{1,2}}(z)$.

Sometimes, it is very difficult to capture perfect instantaneous channel state information due to the rapid change of the channel. However, the statistical information of the channel is generally known in advance judging from previous experience. So the average relay deployment serves as an important guideline where the relay node should be installed. Finally, the expectation of relay deployment is also divided into three cases.

(1) If $p_1\ge p_2+p_r$, the expectation of relay deployment is given by

$$E(d_1)=\int_0^{d_0}f_{d_1}(z)\,dz=d_0-\int_0^{d_0}F_{d_1}(z)\,dz \tag{18}$$

where $E(\cdot)$ is expectation operator. Make a change of $d_0/z-1=u$, then $z=d_0/(u+1)$. The expectation of relay deployment is further given by

$$E(d_1)=d_0-\int_0^{+\infty}\frac{d_0}{\Gamma(m_1)\Gamma(m_2)}\times H_{2,2}^{2,1}\left[\frac{\beta_2(p_1+p_r)u^{n-1}}{\beta_1p_2}\middle|\begin{matrix}(1-m_1,1),(1,1)\\(0,1),(m_2,1)\end{matrix}\right]\times H_{1,1}^{1,1}\left[u\middle|\begin{matrix}(-1,1)\\(0,1)\end{matrix}\right]du \tag{19}$$

$$=d_0-\frac{d_0}{\Gamma(m_1)\Gamma(m_2)}H_{3,3}^{3,2}\left[\frac{\beta_2(p_1+p_r)}{\beta_1p_2}\middle|\begin{matrix}(0,n-1),(1-m_1,1),(1,1)\\(0,1),(m_2,1),(1,n-1)\end{matrix}\right] \tag{20}$$

where $H(\cdot)$ is Fox's H function [20, eq.(1.1.1)]. (19) is derived by expressing the Gauss hypergeometric function in terms of Fox's H function according to [20, eq.(2.9.15)]. (20) is derived according to [20, eq.(2.8.4)].

(2) If $p_2\ge p_1+p_r$, following a similar procedure in above step, the expectation of relay deployment is given by

$$E(d_1)=d_0-\frac{d_0}{\Gamma(m_1)\Gamma(m_2)}H_{3,3}^{3,2}\left[\frac{\beta_2p_1}{\beta_1(p_2+p_r)}\middle|\begin{matrix}(0,n-1),(1-m_1,1),(1,1)\\(0,1),(m_2,1),(1,n-1)\end{matrix}\right] \tag{21}$$

(2) If $p_2 < p_1+p_r$ and $p_1 < p_2+p_r$, unfortunately, it's very difficult and mathematically untraceable to find the closed average deployment in this case. So we provide an approximate average relay deployment. By observing the

structure of the CDF of relay deployment, the approximation can be expressed as

$$\begin{aligned} E\left(d_1\right) \approx d_0 &- \Pr\left(\frac{a}{b} < \xi_1\right) \int_0^{d_1} F_{d_{1,1}}(z)\, dz \\ &- \Pr\left(\xi_1 \le \frac{a}{b} \le \xi_2\right) \int_0^{d_1} F_{d_{1,eq}}(z)\, dz \\ &- \Pr\left(\xi_2 < \frac{a}{b}\right) \int_0^{d_1} F_{d_{1,2}}(z)\, dz \end{aligned} \tag{22}$$

where

$$\begin{aligned} \Pr\left(\frac{a}{b} < \xi_1\right) &= \int_0^{+\infty} \int_0^{\xi_1 y} f_a(x) f_b(y)\, dxdy \\ &= 1 - \left(\frac{\beta_2}{\beta_1 \xi_1}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\quad \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_1}\right) \end{aligned} \tag{23}$$

$$\begin{aligned} \Pr\left(\xi_2 < \frac{a}{b}\right) &= \int_0^{+\infty} \int_{\xi_2 y}^{+\infty} f_a(x) f_b(y)\, dxdy \\ &= \left(\frac{\beta_2}{\beta_1 \xi_2}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\quad \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_2}\right) \end{aligned} \tag{24}$$

and

$$\begin{aligned} \Pr\left(\xi_1 \le \frac{a}{b} \le \xi_2\right) &= \int_0^{+\infty} \int_{\xi_1 y}^{\xi_2 y} f_a(x) f_b(y)\, dxdy \\ &= \left(\frac{\beta_2}{\beta_1 \xi_1}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\quad \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_1}\right) \\ &\quad - \left(\frac{\beta_2}{\beta_1 \xi_2}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\quad \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_2}\right) \end{aligned} \tag{25}$$

where $f_a(x)$ and $f_b(y)$ are the PDFs of the Gamma distribution, which can be found in [8]. Note that we have used [19, eq.(6.455)] to derive (23)-(25). Following the same approach as used in the above case, the approximate average relay deployment is given by

$$\begin{aligned} E\left(d_1\right) \approx d_0 &- \left[1 - \left(\frac{\beta_2}{\beta_1 \xi_1}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \right. \\ &\left. \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_1}\right)\right] \\ &\times \frac{d_0}{\Gamma(m_1)\Gamma(m_2)} H_{3,3}^{3,2}\left[\frac{\beta_2(p_1+p_r)}{\beta_1 p_2} \middle| \begin{matrix} (0,n-1),(1-m_1,1),(1,1) \\ (0,1),(m_2,1),(1,n-1) \end{matrix}\right] \\ &- \left[\left(\frac{\beta_2}{\beta_1 \xi_1}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \right. \\ &\times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_1}\right) \\ &- \left(\frac{\beta_2}{\beta_1 \xi_2}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\left. \times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_2}\right)\right] \\ &\times \frac{d_0}{\Gamma(m_1)\Gamma(m_2)} H_{3,3}^{3,2}\left[\frac{\beta_2 p_1(p_1+p_r-p_2)}{\beta_1 p_2(p_2+p_r-p_1)} \middle| \begin{matrix} (0,n),(1-m_1,1),(1,1) \\ (0,1),(m_2,1),(1,n) \end{matrix}\right] \\ &- \left(\frac{\beta_2}{\beta_1 \xi_2}\right)^{m_2} \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \\ &\times {}_2F_1\left(m_2, m_1+m_2; m_2+1; -\frac{\beta_2}{\beta_1\xi_2}\right) \\ &\times \frac{d_0}{\Gamma(m_1)\Gamma(m_2)} H_{3,3}^{3,2}\left[\frac{\beta_2 p_1}{\beta_1(p_2+p_r)} \middle| \begin{matrix} (0,n-1),(1-m_1,1),(1,1) \\ (0,1),(m_2,1),(1,n-1) \end{matrix}\right] \end{aligned} \tag{26}$$

It is worth mentioning that different from integer parameter m and path loss n in most previous works [8], [9], [11]–[14], [16], there is not this restriction here. This formulas of CDF, PDF and average relay deployment are applicable to arbitrary parameter m and path loss n.

IV. JOINT POWER ALLOCATION AND RELAY DEPLOYMENT

A. OPTIMIZATION ALGORITHM

This section consider a joint power allocation and relay deployment problem, which is formulated by

$$\begin{aligned} \max_{\{d_1,d_2,p_1,p_2,p_r\}} \quad & \min(\gamma_1, \gamma_2) \\ \text{s.t.} \quad & d_1 + d_2 = d_0 \\ & p_1 + p_2 + p_r = P_T \end{aligned} \tag{27}$$

Substituting $p_r = P_T - p_1 - p_2$ and $d_2 = d_0 - d_1$ into γ_1 and γ_2, then setting $\gamma_1 = \gamma_2$, we get

$$\begin{aligned} \gamma_1 - \gamma_2 = & \frac{ab(p_1+p_2-P_T)\left[a(d_0-d_1)^n p_1(2p_2-P_T)\right.}{\left(a(d_0-d_1)^n p_1 - bd_1^n(p_1-P_T)\right)} \\ & \times \frac{\left.-bd_1^n p_2(2p_1-P_T)\right]}{-bd_1^n p_2 + a(d_0-d_1)^n(p_2-P_T)} = 0 \end{aligned} \tag{28}$$

From (28), p_2 can be expressed by

$$p_2 = \frac{a(d_0-d_1)^n p_1 P_T}{2a(d_0-d_1)^n p_1 - 2bd_1^n p_1 + bd_1^n P_T} \tag{29}$$

Inserting p_2 into γ_1 or γ_2 yields

$$\gamma_1 = \gamma_2 = \frac{abp_1(P_T-2p_1)}{2a(d_0-d_1)^n p_1 + bd_1^n(P_T-2p_1)} \tag{30}$$

Taking the partial derivatives of γ_1 with respect to p_1 and d_1 and setting the derivatives to be zero, we obtain

$$\frac{\partial \gamma_1}{\partial p_1} = -\frac{ab\left(4a\left(d_0-d_1\right)^n p_1^2 - bd_1^n\left(P_T-2p_1\right)^2\right)}{\left(2a\left(d_0-d_1\right)^n p_1 + bd_1^n\left(P_T-2p_1\right)\right)^2} = 0 \tag{31}$$

$$\frac{\partial \gamma_1}{\partial d_1} = \frac{abp_1\left(2p_1-P_T\right)}{\left(2a\left(d_0-d_1\right)^n p_1 + bd_1^n\left(P_T-2p_1\right)\right)^2} \times \left(-2a\left(d_0-d_1\right)^{n-1} np_1 + bd_1^{n-1} n\left(P_T-2p_1\right)\right) = 0 \tag{32}$$

From (31) and (32), the optimal relay deployment is given by

$$d_1^\star = \frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}} \tag{33}$$

$$d_2^\star = \frac{d_0}{1+\left(\frac{a}{b}\right)^{\frac{1}{n-2}}} \tag{34}$$

Here the relay deployment is related to channel coefficient and path loss exponent, apparently distinct from that in the case of fixed power allocation. Subsequently, the power allocation is given by

$$p_1^\star = \frac{P_T\sqrt{b}\left(\frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}}}{2\left[\sqrt{b}\left(\frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}} + \sqrt{a}\left(\frac{d_0}{1+\left(\frac{a}{b}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}}\right]} \tag{35}$$

$$p_2^\star = \frac{P_T\sqrt{a}\left(\frac{d_0}{1+\left(\frac{a}{b}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}}}{2\left[\sqrt{b}\left(\frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}} + \sqrt{a}\left(\frac{d_0}{1+\left(\frac{a}{b}\right)^{\frac{1}{n-2}}}\right)^{\frac{n}{2}}\right]} \tag{36}$$

$$p_r^\star = \frac{P_T}{2} \tag{37}$$

From the result of power allocation, the relay R always occupies half of the total power. As a double check, substituting the powers into (8), we get $d_{1,eq}$ is exactly equal to (33). Similarly, substituting the powers into $d_{1,2}$, then we get

$$d_{1,2} = \frac{d_0}{1+\left[2\left(\frac{b}{a}\right)^{\frac{n-1}{n-2}} + \frac{b}{a}\right]^{\frac{1}{n-1}}} \leq \frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}} = d_{1,eq} \tag{38}$$

Substituting the powers into $d_{1,1}$, then we get

$$d_{1,1} = \frac{d_0}{1+\left[\frac{1}{\frac{a}{b}+2\left(\frac{a}{b}\right)^{\frac{n-1}{n-2}}}\right]^{\frac{1}{n-1}}} \geq \frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}} = d_{1,eq} \tag{39}$$

In the joint optimization problem, the three possible extreme points still satisfy the relationship $d_{1,2} \leq d_{1,eq} \leq d_{1,1}$ because the maximum occurs at $\gamma_1 = \gamma_2$. Next we take have a look at two special cases of interest.

(1) When the channel fading of the two links is the same, i.e., $a = b$, the relay is deployed at the midpoint, i.e., $d_1^\star = d_2^\star = d_0/2$. The power is allocated according to $p_1^\star = p_t/4$, $p_2^\star = p_t/4$ and $p_r = p_t/2$. This result is obviously in line with the intuitive understanding.

(2) When the path loss exponent $n = 2$, the relay deployment is given by

$$d_1^\star = \begin{cases} 0 & b > a \\ d_1 \in [0, d_0] & b = a \\ d_0 & b < a \end{cases} \tag{40}$$

and

$$d_2^\star = \begin{cases} 0 & a > b \\ 1 - d_1 & a = b \\ d_0 & a < b \end{cases} \tag{41}$$

The corresponding power allocation is given by

$$p_1^\star = \begin{cases} 0 & b > a \\ \frac{d_1 p_t}{2d_0} & b = a \\ \frac{p_t}{2} & b < a \end{cases} \tag{42}$$

$$p_2^\star = \begin{cases} 0 & a > b \\ \frac{d_2 p_t}{2d_0} & a = b \\ \frac{p_t}{2} & a < b \end{cases} \tag{43}$$

and $p_r^\star = p_t/2$. The corresponding SNRs are given by

$$\gamma_1 = \gamma_2 = \frac{p_t \max\{a, b\}}{2d_0^2} \tag{44}$$

In reality, a relay cannot be deployed on two endpoints $d_1 = 0$ and $d_1 = d_0$ because it cannot overlap with the transceivers. (40) and (41) tell us that there is a simple relationship between relay deployment and channel coefficients in an ideal free space ($n = 2$). If the quality of link $S_1 \leftrightarrow R$ is better than link $S_2 \leftrightarrow R$, i.e., $a > b$, the relay should be deployed in the vicinity of transceiver S_2. Otherwise, it is in the vicinity of the transceiver S_1. If the quality of the two links is evenly divided, the relay is deployed at the midpoint.

B. PERFORMANCE ANALYSIS

This section prepares a quantitative analysis of relay deployment in the joint optimization problem. Recall that this bidirectional relaying network is in a Nakagami-m fading environment. Clearly, according to the physical meaning, $F_{d_1}(z) = 0$ for $z \leq 0$ and $F_{d_1}(z) = 1$ for $z \geq d_0$. Furthermore, when $n > 2$ and $0 < z < d_0$, the CDF of d_1 is given by

$$F_{d_1}(z) = \Pr\left(\frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}} \leq z\right)$$

$$
\begin{aligned}
&= \int_{0}^{+\infty} \int_{x\left(\frac{d_0}{z}-1\right)^{n-2}}^{+\infty} f_b(y) f_a(x)\, dy dx \\
&= \frac{\Gamma(m_1+m_2)}{\Gamma(m_2)\Gamma(m_1+1)} \left[\frac{\beta_1}{\beta_2}\left(\frac{d_0}{z}-1\right)^{2-n}\right]^{m_1} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; -\frac{\beta_1\left(\frac{d_0}{z}-1\right)^{2-n}}{\beta_2}\right] \quad (45)
\end{aligned}
$$

where [19, eq.(6.455)] is used to derive (45). The PDF of d_1 is given by

$$
\begin{aligned}
f_{d_1}(z) &= \frac{\partial F_{d_1}(z)}{\partial d_1} \\
&= \frac{\Gamma(m_1+m_2)\, d_0 (n-2)}{\Gamma(m_2)\Gamma(m_1)\, z (d_0 - z)} \\
&\quad \times \frac{\left[\frac{\beta_1}{\beta_2}\left(\frac{d_0}{z}-1\right)^{2-n}\right]^{m_1}}{\left[1+\frac{\beta_1}{\beta_2}\left(\frac{d_0}{z}-1\right)^{2-n}\right]^{m_1+m_2}} \quad (46)
\end{aligned}
$$

The expectation of relay deployment is given by

$$
\begin{aligned}
E(d_1) &= d_0 - \frac{d_0}{\Gamma(m_1)\Gamma(m_2)} \\
&\quad \times H_{3,3}^{3,2}\left[\frac{\beta_2}{\beta_1}\middle| \begin{matrix} (0, n-2), (1-m_1, 1), (1, 1) \\ (0, 1), (m_2, 1), (1, n-2) \end{matrix}\right] \quad (47)
\end{aligned}
$$

If we only know the statistics of the channel without the knowledge of instantaneous channel state information, the relay can be deployed according to the expected value in (47).

Similarly, when $0 < n < 2$ and $0 < z < d_0$, the CDF of d_1 is given by

$$
\begin{aligned}
F_{d_1}(z) &= \Pr\left(\frac{d_0}{1+\left(\frac{b}{a}\right)^{\frac{1}{n-2}}} \le z\right) \\
&= \int_{0}^{+\infty} \int_{0}^{x\left(\frac{d_0}{z}-1\right)^{n-2}} f_b(y) f_a(x)\, dy dx \\
&= \frac{\Gamma(m_1+m_2)}{\Gamma(m_1)\Gamma(m_2+1)} \left[\frac{\beta_2}{\beta_1}\left(\frac{d_0}{z}-1\right)^{n-2}\right]^{m_2} \\
&\quad \times {}_2F_1\left[m_2, m_1+m_2; m_2+1; -\frac{\beta_2\left(\frac{d_0}{z}-1\right)^{n-2}}{\beta_1}\right] \quad (48)
\end{aligned}
$$

The PDF of d_1 is given by

$$
\begin{aligned}
f_{d_1}(z) &= \frac{\partial F_{d_1}(z)}{\partial d_1} \\
&= \frac{\Gamma(m_1+m_2)\, d_0 (2-n)}{\Gamma(m_2)\Gamma(m_1)\, z (d_0 - z)} \\
&\quad \times \frac{\left[\frac{\beta_1}{\beta_2}\left(\frac{d_0}{z}-1\right)^{2-n}\right]^{m_1}}{\left[1+\frac{\beta_1}{\beta_2}\left(\frac{d_0}{z}-1\right)^{2-n}\right]^{m_1+m_2}} \quad (49)
\end{aligned}
$$

The expectation of relay deployment is given by

$$
\begin{aligned}
E(d_1) &= d_0 - \frac{d_0}{\Gamma(m_1)\Gamma(m_2)} \\
&\quad \times H_{3,3}^{3,2}\left[\frac{\beta_1}{\beta_2}\middle| \begin{matrix} (0, 2-n), (1-m_2, 1), (1, 1) \\ (0, 1), (m_1, 1), (1, 2-n) \end{matrix}\right] \quad (50)
\end{aligned}
$$

When $n = 2$, d_1 becomes a two-point distribution given by

$$
\begin{aligned}
\Pr(d_1 = 0) &= \frac{\Gamma(m_1+m_2)\left(\frac{\beta_1}{\beta_2}\right)^{m_1}}{\Gamma(m_2)\Gamma(m_1+1)} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; -\frac{\beta_1}{\beta_2}\right] \quad (51)
\end{aligned}
$$

and

$$
\begin{aligned}
\Pr(d_1 = d_0) &= 1 - \frac{\Gamma(m_1+m_2)\left(\frac{\beta_1}{\beta_2}\right)^{m_1}}{\Gamma(m_2)\Gamma(m_1+1)} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; -\frac{\beta_1}{\beta_2}\right] \quad (52)
\end{aligned}
$$

Then, the expectation of relay deployment is given by

$$
\begin{aligned}
E(d_1) &= d_0 - \frac{\Gamma(m_1+m_2)\left(\frac{\beta_1}{\beta_2}\right)^{m_1} d_0}{\Gamma(m_2)\Gamma(m_1+1)} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; -\frac{\beta_1}{\beta_2}\right] \quad (53)
\end{aligned}
$$

V. SIMULATION RESULTS

This section studies the performance of utility function and verifies theoretical analysis by simulation experiments. The SEP is chosen as an example of a utility function. Other system performances can be simulated similarly, such data rate, outage probability and so on.

The influence of average SNR on average SEP of 4-pulse amplitude modulation (4PAM) is shown Fig.1, where the average SNR is defined as P_T/σ^2 and the path loss exponent is 3.3. Four typical deployment approaches are compared. (1) Joint algorithm: This is our proposed joint algorithm of power allocation and relay deployment in section IV. (2) Optimal deployment: The relay is placed at a fixed power according to theorem 1 in section III. (3) Fixed deployment: The relay is installed at a certain location and is not movable. Since the power is fixed in cases (2) and (3), it is assumed that $p_1 = 0.1P_T$, $p_2 = 0.2P_T$ and $p_r = 0.7P_T$. (4) Equal power: The total power is evenly distributed to two users and the relay. As expected, the joint algorithm performs the best among all approaches. For example, about 6dB SNR saved is achieved at a target SEP of 0.01 compared with equal power method. And power allocation can bring about a SNR gain of about 2 dB from joint algorithm and optimal deployment (red curves). The importance of power allocation can also be seen by observing equal power ($d_1/d_0 = 0.5$)

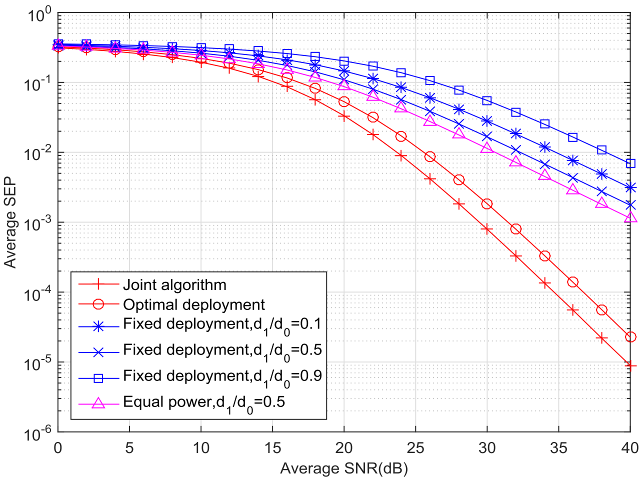

FIGURE 1. **Average SEP of 4PAM.**

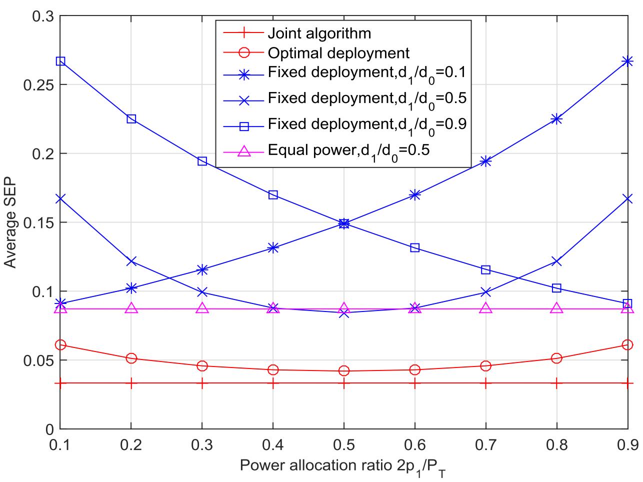

FIGURE 2. **Impact of power ratio on average SEP.**

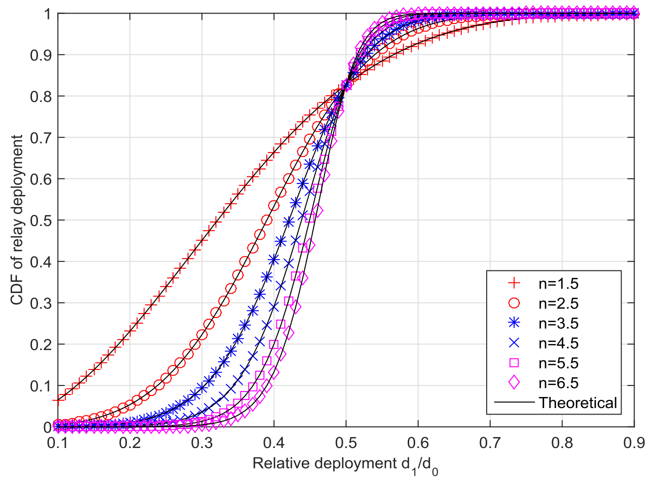

FIGURE 3. **The CDF of relay deployment under fixed power, average SNR = 10dB, $p_1 = 0.1P_T$, $p_2 = 0.2P_T$, $p_r = 0.7P_T$, $m_1 = 1.22$, $m_2 = 2.22$.**

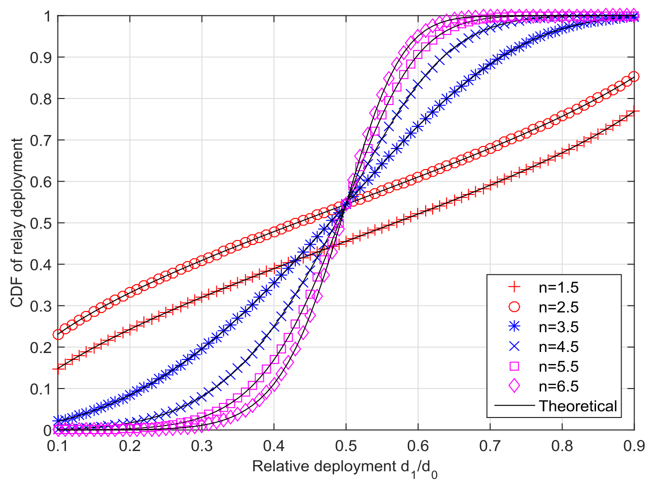

FIGURE 4. **The CDF of relay deployment of joint algorithm, average SNR = 10dB, $m_1 = 1.22$, $m_2 = 2.22$.**

and fixed deployment ($d_1/d_0 = 0.5$) curves. The same relay deployment, but different power allocations result in different SEP performances.

Next, we study the relationship between different power allocation ratios and SEP. Fig.2 shows the average SEP versus $2p_1/P_T$, where $p_2 = P_T/2 - p_1$ and $p_r = P_T/2$. Since the adaptive power allocation is applied in joint algorithm, its SEP remains constant. The similar principle is in equal power method. When $d_1/d_0 = 0.1$, the average SEP is an increasing function of $2p_1/P_T$, indicating that when R is close to S_1, more power should be allocated to S_2 because our target depends on the smaller SEP of these two links. Similarly, when $d_1/d_0 = 0.9$, the average SEP is an decreasing function of $2p_1/P_T$, indicating that when R is far away from S_1, more power should be allocated to S_1. When the relay is deployed at the midpoint, S_1 and S_2 should have equal power to balance the two links.

Figs.3-4 show the CDF of relay deployment. The simulated CDF curves are consistent with the theoretical results, confirming the correctness of the piecewise CDF. More specifically, in optimal deployment scenario, $d_1/d_0 = 0.5$ is a watershed. When $d_1/d_0 < 0.5$, the CDF under good channel conditions with small path loss is larger than that under bad channel conditions under heavy path loss. While $d_1/d_0 > 0.5$, it's the opposite.

Figs.5-6 illustrate the impact of path loss on average relay deployment in the presence of different channel quality. Note that the expressions of average relay deployment are approximate in optimal deployment, while accurate in joint algorithm. In the optimal deployment scenario, the approximation is very close to the simulation curve under heavy channel conditions, while the approximation is somewhat off the the simulation curve under good channel conditions. In joint algorithm scenario, the theoretical value agrees with simulation curves very well. The path loss exponent $n = 2$ is a watershed. There is a interesting phenomenon about the average relay deployment: the larger when $n < 2$, the smaller when $n > 2$. The reason is that the relay deployment is

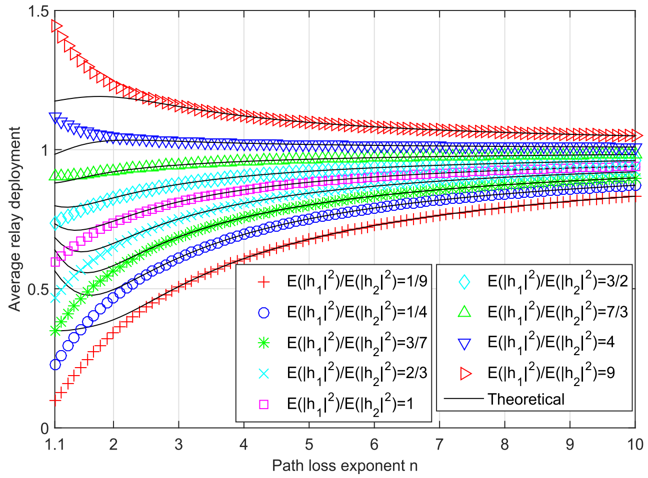

FIGURE 5. Average relay deployment under fixed power, average SNR = 10dB, $p_1 = 0.1P_T$, $p_2 = 0.2P_T$, $p_r = 0.7P_T$, $m_1 = 1.22$, $m_2 = 2.22$.

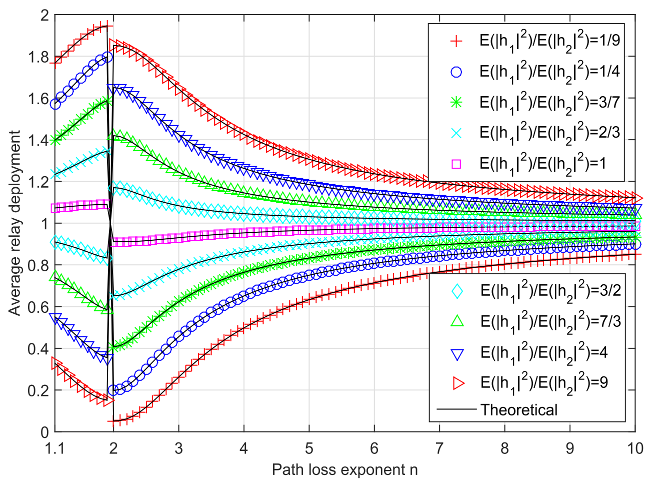

FIGURE 6. Average relay deployment of joint algorithm, average SNR = 10dB, $m_1 = 1.22$, $m_2 = 2.22$.

related to the exponent $1/(n-2)$. And when $n = 2$, the relay deployment variable obeys a two-point distribution.

VI. CONCLUSION

In this paper, we have investigated the relay deployment problem in bidirectional AF relay networks. Through the utility function optimization problem, we get the optimal relay deployment and joint optimization algorithm. Then, the CDF of relay deployment help us grasp the rule of relay mobility. Finally, the average relay deployment indicates the placement of relay installation in wireless fading channels. In future, the one-dimensional relay motion path will be extended to a two-dimensional one.

APPENDIX A
PROOF OF THEOREM 1

According to the characteristics of the max min problem, the problem (5) has three possible maximum points. First, when the two links $S_1 \leftrightarrow R$ and $S_2 \leftrightarrow R$ are balanced, there is a possible maximum point in problem (5). So when $\gamma_1 = \gamma_2$, we have

$$ad_1^{-n} - \frac{-bd_2^{-n}p_1p_2 + bd_2^{-n}p_2^2 + bd_2^{-n}p_2p_r}{p_1(p_1 - p_2 + p_r)} = 0 \tag{54}$$

Then, the relay deployment of balanced state is given by

$$d_{1,eq} = \frac{d_0}{1 + \left(\frac{bp_2(p_r+p_2-p_1)}{ap_1(p_1+p_r-p_2)}\right)^{\frac{1}{n}}} \tag{55}$$

$$d_{2,eq} = \frac{d_0}{1 + \left(\frac{ap_1(p_1+p_r-p_2)}{bp_2(p_r+p_2-p_1)}\right)^{\frac{1}{n}}} \tag{56}$$

Second, when the SNR of either link reaches a maximum, there is a possible maximum point in problem (5). So when γ_1 reaches its maximum, we have

$$\frac{\partial\gamma_1}{\partial d_1} = \frac{abnp_2p_r\left(a(d_0-d_1)^{n-1}(p_1+p_r) - bd_1^{n-1}p_2\right)}{\left(a(d_0-d_1)^n(p_1+p_r) + bd_1^np_2\right)^2} = 0 \tag{57}$$

Then the relay deployment of the link $S_1 \leftrightarrow R$ maximum is given by

$$d_{1,1} = \frac{d_0}{1 + \left(\frac{bp_2}{a(p_1+p_r)}\right)^{\frac{1}{n-1}}} \tag{58}$$

$$d_{2,1} = \frac{d_0}{1 + \left(\frac{a(p_1+p_r)}{bp_2}\right)^{\frac{1}{n-1}}} \tag{59}$$

Note that in reality, the path loss index is greater than 1, i.e., $n > 1$. From the partial derivative $\partial\gamma_1/\partial d_1$, we can infer that when $0 \le d_1 \le d_{1,1}$, the SNR γ_1 is monotonically increasing and when $d_1 \ge d_{1,1}$, the SNR γ_1 is monotonically decreasing, because of the relationship $\partial\gamma_1/\partial d_1 \ge 0$ when $0 \le d_1 \le d_{1,1}$ and $\partial\gamma_1/\partial d_1 \le 0$ when $d_1 \ge d_{1,1}$.

Similarly, when γ_2 reaches its maximum, we have

$$\frac{\partial\gamma_2}{\partial d_1} = \frac{abnp_1p_r\left(a(d_0-d_1)^{n-1}p_1 - bd_1^{n-1}(p_2+p_r)\right)}{\left(a(d_0-d_1)^np_1 + bd_1^n(p_2+p_r)\right)^2} = 0 \tag{60}$$

Then the relay deployment of the link $S_2 \leftrightarrow R$ maximum is given by

$$d_{1,2} = \frac{d_0}{1 + \left(\frac{b(p_2+p_r)}{ap_1}\right)^{\frac{1}{n-1}}} \tag{61}$$

$$d_{2,2} = \frac{d_0}{1 + \left(\frac{ap_1}{b(p_2+p_r)}\right)^{\frac{1}{n-1}}} \tag{62}$$

From the partial derivative $\partial\gamma_2/\partial d_1$, we can infer that when $0 \le d_1 \le d_{1,2}$, the SNR γ_2 is monotonically increasing and when $d_1 \ge d_{1,2}$, the SNR γ_2 is monotonically decreasing, because of the relationship $\partial\gamma_2/\partial d_1 \geq 0$ when $0 \leq d_1 \leq d_{1,2}$ and $\partial\gamma_2/\partial d_1 \leq 0$ when $d_1 \geq d_{1,2}$.

Next, optimal relay deployment will be discussed in categories. Before doing this, we first calculate the values of the SNRs at the two endpoints. When the relay is deployed at S_1, the SNRs are given by

$$\gamma_1|_{d_1=0} = \frac{bp_2p_r}{(p_1+p_r)\,d_0^n} \tag{63}$$

$$\gamma_2|_{d_1=0} = \frac{bp_r}{d_0^n} \tag{64}$$

When the relay is deployed at S_2, the SNRs are given by

$$\gamma_1|_{d_1=d_0} = \frac{ap_r}{d_0^n} \tag{65}$$

$$\gamma_2|_{d_1=d_0} = \frac{ap_1p_r}{(p_2+p_r)\,d_0^n} \tag{66}$$

From the above formulas, the relationship between γ_1 and γ_2 at two endpoints depends on the given power allocation. The optimal deployment is divided into five situations as follows.

(1) When the power of the transceiver S_2 is large enough to satisfy $p_2 \geq p_1 + p_r$, the SNR γ_1 is always no less than the SNR γ_2, i.e., $\gamma_1 \geq \gamma_2$. Therefore, the optimal relay deployment $d_1^\star$ depends on where the SNR γ_2 is maximized, i.e., $d_1^\star = d_{1,2}$. A illustrative diagram of this situation is shown in Fig.7.

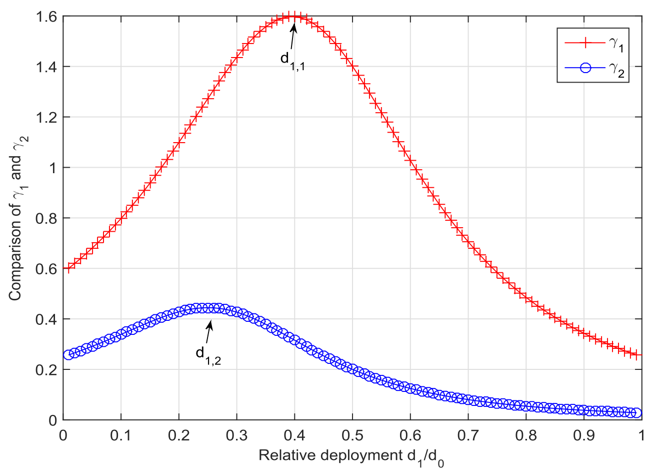

FIGURE 7. When $p_2 \geq p_1 + p_r$, the optimal relay deployment $d_1^\star = d_{1,2}$.

(2) When the power of the transceiver S_1 is large enough to satisfy $p_1 \geq p_2 + p_r$, the SNR γ_2 is always no less than the SNR γ_1, i.e., $\gamma_2 \geq \gamma_1$. Therefore, the optimal relay deployment $d_1^\star$ depends on where the SNR γ_1 is maximized, i.e., $d_1^\star = d_{1,1}$. A illustrative diagram of this situation is shown in Fig.8.

(3) When $p_2 < p_1 + p_r$ and $p_1 < p_2 + p_r$, we can infer that the two SNRs γ_1 and γ_2 must intersect in the interval $[0, d_1]$ due to the fact of $\gamma_1|_{d_1=0} < \gamma_2|_{d_1=0}$ and $\gamma_1|_{d_1=d_0} > \gamma_2|_{d_1=d_0}$. So the optimal relay deployment depends on the size relationship among the three possible maximum points. In reality, $n > 1$, so we have $d_{1,1} \geq d_{1,2}$. Furthermore, when three possible maximum points meet $d_{1,2} \leq d_{1,eq} \leq d_{1,1}$,

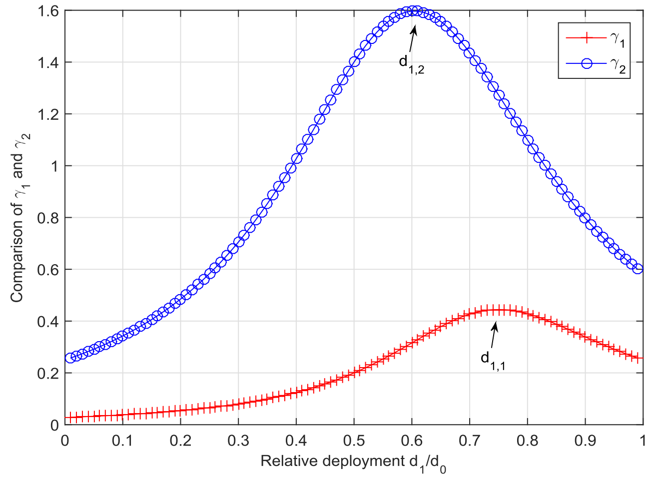

FIGURE 8. When $p_1 \geq p_2 + p_r$, the optimal relay deployment $d_1^\star = d_{1,1}$.

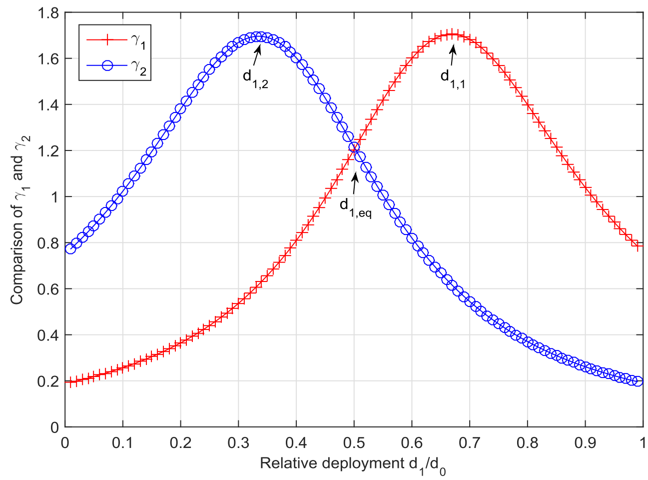

FIGURE 9. When $p_2 < p_1 + p_r$, $p_1 < p_2 + p_r$ and $\xi_1 \leq \frac{a}{b} \leq \xi_2$, the optimal relay deployment $d_1^\star = d_{1,eq}$.

equivalent to $\xi_1 \leq \frac{a}{b} \leq \xi_2$, the optimal relay deployment is given by $d_1^\star = d_{1,eq}$. A illustrative diagram of this situation is shown in Fig.9.

(4) Similarly, when $p_2 < p_1 + p_r$, $p_1 < p_2 + p_r$ and $d_{1,2} < d_{1,1} < d_{1,eq}$, the optimal relay deployment is given by $d_1^\star = d_{1,1}$. After some mathematical operations, the situation $d_{1,2} < d_{1,1} < d_{1,eq}$ is equivalent to $\frac{a}{b} < \xi_1$. A illustrative diagram of this situation is shown in Fig.10.

(5) Finally, when $p_2 < p_1 + p_r$, $p_1 < p_2 + p_r$ and $d_{1,eq} < d_{1,2} < d_{1,1}$, the optimal relay deployment is given by $d_1^\star = d_{1,2}$. After some mathematical operations, the situation $d_{1,2} < d_{1,1} < d_{1,eq}$ is equivalent to $\xi_2 < \frac{a}{b}$. A illustrative diagram of this situation is shown in Fig.11.

Summarizing the above five situations, the optimal relay deployment is expressed in theorem 1.

APPENDIX B
PROOF OF THEOREM 2

The CDF of relay deployment d_1 is divide into three cases according to the values of power allocation.

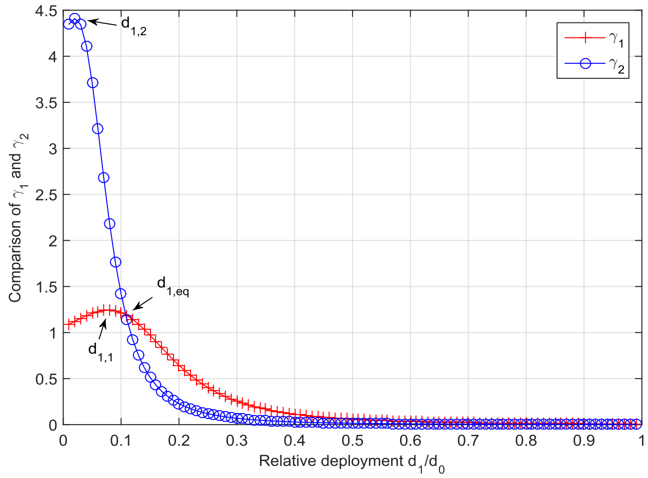

FIGURE 10. When $p_2 < p_1 + p_r, p_1 < p_2 + p_r$ and $\frac{a}{b} < \xi_1$, the optimal relay deployment $d_1^\star = d_{1,1}$.

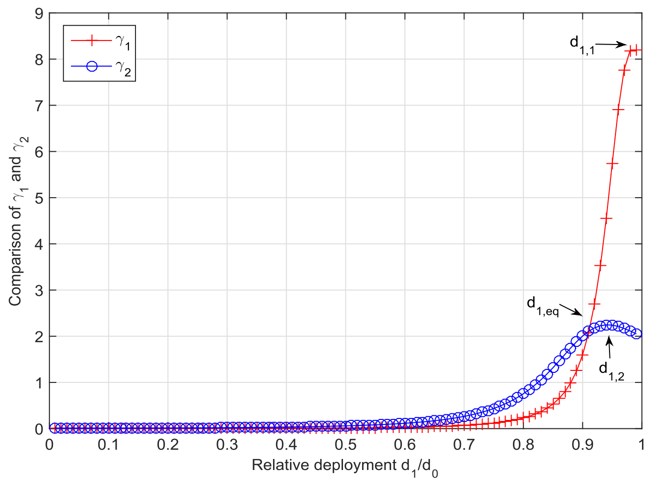

FIGURE 11. When $p_2 < p_1 + p_r, p_1 < p_2 + p_r$ and $\xi_2 < \frac{a}{b}$, the optimal relay deployment $d_1^\star = d_{1,2}$.

(1) If $p_1 \geq p_2 + p_r$, d_1 takes value of $d_{1,1}$, so the CDF of d_1 is given by

$$\begin{aligned}
F_{d_1}(z) &= F_{d_{1,1}}(z) = \Pr\left(d_{1,1} \leq z\right) \\
&= \int_0^{+\infty} \int_{\frac{(p_1+p_r)x\left(\frac{d_0}{z}-1\right)^{n-1}}{p_2}}^{+\infty} f_a(x) f_y(b)\, dydx \\
&= \frac{\Gamma(m_1+m_2)}{\Gamma(m_2)\Gamma(m_1+1)} \left[\frac{\beta_1 p_2}{\beta_2 (p_1+p_r)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; \right. \\
&\quad \left. -\frac{\beta_1 p_2}{\beta_2 (p_1+p_r)\left(\frac{d_0}{z}-1\right)^{n-1}}\right]
\end{aligned} \tag{67}$$

(2) If $p_2 \geq p_1 + p_r$, d_1 takes value of $d_{1,2}$, so the CDF of d_1 is given by

$$\begin{aligned}
F_{d_1}(z) &= F_{d_{1,2}}(z) = \Pr\left(d_{1,2} \leq z\right) \\
&= \int_0^{+\infty} \int_{\frac{p_1 x\left(\frac{d_0}{z}-1\right)^{n-1}}{p_2+p_r}}^{+\infty} f_a(x) f_y(b)\, dydx \\
&= \frac{\Gamma(m_1+m_2)}{\Gamma(m_2)\Gamma(m_1+1)} \left[\frac{\beta_1 (p_2+p_r)}{\beta_2 p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right]^{m_1} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; \right. \\
&\quad \left. -\frac{\beta_1 (p_2+p_r)}{\beta_2 p_1\left(\frac{d_0}{z}-1\right)^{n-1}}\right]
\end{aligned} \tag{68}$$

(3) If $p_2 < p_1 + p_r$, $p_1 < p_2 + p_r$, we need take a further discussion. When $\frac{a}{b} < \xi_1$, the optimal relay deployment d_1 takes $d_{1,1}$, whose range falls on $0 < d_{1,1} < \frac{d_0 p_1 (p_1+p_r-p_2)}{p_r (p_1+p_2+p_r)}$. Clearly, the CDF is $F_{d_1}(z) = F_{d_{1,1}}(z)$. When $\xi_1 \leq \frac{a}{b} \leq \xi_2$, the optimal relay deployment d_1 takes $d_{1,eq}$, whose range falls on $\frac{d_0 p_1 (p_1+p_r-p_2)}{p_r (p_1+p_2+p_r)} \leq d_{1,eq} \leq \frac{d_0 (p_2+p_r)(p_1+p_r-p_2)}{p_r (p_1+p_2+p_r)}$. Clearly, the CDF is given by

$$\begin{aligned}
F_{d_1}(z) &= F_{d_{1,eq}}(z) \\
&= \int_0^{+\infty} \int_{\frac{p_1 (p_1+p_r-p_2) x\left(\frac{d_0}{z}-1\right)^{n}}{p_2 (p_2+p_r-p_1)}}^{+\infty} f_a(x) f_y(b)\, dydx \\
&= \frac{\Gamma(m_1+m_2)}{\Gamma(m_2)\Gamma(m_1+1)} \left[\frac{\beta_1 p_2 (p_2+p_r-p_1)}{\beta_2 p_1 (p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^{n}}\right]^{m_1} \\
&\quad \times {}_2F_1\left[m_1, m_1+m_2; m_1+1; \right. \\
&\quad \left. -\frac{\beta_1 p_2 (p_2+p_r-p_1)}{\beta_2 p_1 (p_1+p_r-p_2)\left(\frac{d_0}{z}-1\right)^{n}}\right]
\end{aligned} \tag{69}$$

When $\xi_2 < \frac{a}{b}$, the optimal relay deployment d_1 takes $d_{1,2}$, whose range falls on $\frac{d_0 (p_2+p_r)(p_1+p_r-p_2)}{p_r (p_1+p_2+p_r)} < d_{1,2} < d_0$. The CDF is $F_{d_1}(z) = F_{d_{1,2}}(z)$.

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