IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received June 18, 2019, accepted July 28, 2019, date of publication August 21, 2019, date of current version September 23, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2936605

Stabilization of a Class of Robot Systems in Fractional-Order Hold Case via Sampling Zero Dynamic Stable Approach

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This work was supported in part by the National Natural Science Foundation of China under Grant 61771077 and Grant 61763004, in part by the Science and Technology Major Special Project of Guangxi under Grant GKAA17129002, in part by the Basic Research and Frontier Exploration in Chongqing Science and Technology Commission under Grant CSTC2018jcyjAX0815, and in part by the Key Research Program of Chongqing Science and Technology under Grant CSTC2017jcyjBX0025.

ABSTRACT Robots play a significant-and growing-role in many practical fields, for example, the corresponding application in industrial and medicine field. More importantly, for the control of a robot system, a good control performance is natural and necessary requirements. And since the control of system is usually realized by a digital computer, sampling is an inevitable process in actual engineering. However, it is a well known truth that unstable zero dynamics can greatly limit the control performance of the system. This paper investigates the stabilization of a class of robot system based on the corresponding sampled-data model, which generated from the discretization with fractional-order hold (FROH). Further, the sampled-data model of the two degree of freedom (DOF) robot system is obtained and the expression and stable conditions of sampling zero dynamics are also obtained. What is more, the control strategy of the corresponding robot system is acquired using the sampling zero dynamic stable approach. Finally, numerical example is provide to illustrate the effectiveness of the proposed approach in this paper.

INDEX TERMS Fractional-order hold, robot system, sampled-data model, stabilization, zero dynamic.

I. INTRODUCTION

It is well known that robots have great potential application and the fastest expansion in many practical engineering fields. Due to their abilities, robots can assist and even substitute humans to accomplish assignment [1]. For instance, in modern medicine, they play an important role in training the doctors, dentists, and nurses, and role in comforting and protecting patients. The first recorded robotic surgical procedure – a CT-guided brain biopsy – took place on 11 April 1985, at the Memorial Medical Center, Long Beach, CA,USA [2]. Since then, scholars have done many researches on medical robotics [3]–[5]. In medicine, robots are classified into five types based on the difference of actuation and applications, such as passive robots, active robots, semi-active robots, synergistic and intra corporal systems [4]. Due to the special environment and requirement of medical robots, a good control performance of the robots is very important which can increase the accurateness of operation and reduce human error. Any mistake and error made during surgery may lead to aggravate patients' misery or even loss their lives. Similarly, any mistake and error made during production may also lead to significant loss of economic and property. What is more, robot system is a highly complicated system with nonlinear dynamics characteristic. Thus, it is important to stable the robot systems accurately and rapidly. In addition, the research about robot control in other areas is also very important.

For the hand of robot to have human like motion, similar characteristics and methods of motion generation need to be properly considered. To solve this problem, some methods have been proposed, such as single objective optimization genetic algorithm [6] and multi objective evolutionary

The associate editor coordinating the review of this article and approving it for publication was Ning Sun.

algorithm [7]. Besides, considering the truth that control systems suffer from many input limitations in control process. A comprehensive control method including neural network and robust integral term in the control law can reduce final positioning errors due to input constraints [8], [9]. Moreover, because of the equations of motion of the robots are highly nonlinear, which will lead to make the control performance worsening. Therefore, the stabilization of the robot system is one of the hot research spots by a proper control strategy.

Since the control of plant is usually realized by a digital computer with the rapid development of microprocessor technology, the digital electronics have been become more and more important in the area of control engineering. In the area of nonlinear control, more and more scholars have paid attention to the sampled-data control [10]–[12]. However, a good approximation of sampled-data model for designing the controller based on the corresponding sampled-data model is necessary because the exact sampled-data model of nonlinear systems is usually difficult to obtained for controller designers. Further, the results in previous research were shown that the original exact system can be stabilized when the more accurate sampled-data model with stabile zero dynamics was obtained to design the corresponding control input [13]. Thus, finding a discrete time model to represent the original continuous time system in control process is a good choice, where the discrete time algorithm is typically used to control the original continuous time system [10]. Time discretization is an indispensable program for obtaining the discrete time model and based on the discrete time control law designed for a discrete time system is very attractive for dealing with the issue of sampling at some time instants [14]. what is more, because of the unstable zero dynamics exist in the obtained sampled-data model, the controller designed based on the sampled-data model can not stabilize the original continuous time system [15], [16]. In order to obtain the sampled-data model from the original continuous time system, the sampler and hold device provides a link between of them, such as the zero-order hold (ZOH), first-order hold (FOH) and fractional-order hold (FROH) are the generally signal reconstruction devices. Using FROH as control signal reconstruction, a more stable position of discrete zeros can be achieved compared to systems that use the more common ZOH or FOH device [17]. Therefore, this paper has introduced the FROH as the signal reconstruction device to discretization the robot system and analyzed the corresponding control performance.

Zero dynamic, an important notion, of the nonlinear sampled-data system obtained by sampling. In digital linear technology, the control performance of linear system was greatly limited due to the presence of unstable zeros in the corresponding sampled-data model [15]. And it is also a truth that unstable zero dynamics of the discrete time system greatly limit the control performance of the original nonlinear control system [18]–[22]. Zero dynamics of discrete time system are usually classified into two categories [23]. One of them is called intrinsic zero dynamic and it has counterpart in the continuous time system. Another one is called sampling zero dynamic which is generated during the sampling process. In recent years, many scholars have introduced zero dynamic approaches to design the controller. for instance, the zero dynamic stable approaches were used in the control of quad rotor [24], vertical take-off and landing [25], [26] and multimachine power system [27].

This paper is mainly focus on the stabilization of the robot system via sampling zero dynamic stable approach. Using FROH as the input signal reconstruction device to discretize the original continuous time robot system, the approximate sampled-data model is obtained. The main contributions of this paper are summarized to be: 1) The approximate sampled-data model of robot systems with FROH is proposed by using Taylor expansion method, and the order of the local truncation error between the resulting sampled-data model and exact sampled-data model is $r_i + 1$ with respect to sampling period T; 2) The expression of the zero dynamics about two degree of freedom (DOF) robot system is presented, and the stability condition of zero dynamic is also given; 3) Using the resulting approximate sampled-data model to design control strategy can obtain a good control performance.

The structure of this paper is organized as follows: the next section introduces some preliminaries of FROH and the system description of robot system; in Section III we state the approximate sampled-data model and the local truncation error of the robot system with FROH; zero dynamics of the corresponding sampled-data model is shown in Section IV; Section V provides the numerical simulation; finally, conclusions are presented in Section VI.

II. PRELIMINARIES AND SYSTEM DESCRIPTION

In this section, we begin by reviewing some well-known results and the robot system description to better understand the results of this paper. We are interested in the properties of discrete time model composed of a hold circuit,the continuous time system and a sampler and hold device in cascade, where the FROH [28]–[30] signal reconstruction method was considered to generate the input of the control system. i.e.,

$$u(t) = u(kT) + \beta \left[\frac{u(kT) - u((k-1)T)}{T} \right] (t - kT) \quad (1)$$

where $kT \leqslant t < (k+1)T, k = 0, 1, \cdots$, and β is a changeable real parameter of a FROH and T is sampling period. The signal reconstruction of a FROH in the case of $\beta = -0.5$ is shown in Figure 1 [28]–[30].

Remark 1: Obviously, the FROH reduces to ZOH for $\beta = 0$ while it becomes FOH for $\beta = 1$. Thus, it is universality to research the robot control system in FROH.

For convenience to understand the discrete time model, we express some results using δ-operator [31]. The operator is different from the Euler integration. In discrete time and complex variable domains, the shift operator was shown in the following and was used to obtain the corresponding results.

$$\delta = \frac{q-1}{T} \Leftrightarrow \gamma = \frac{z-1}{T} \quad (2)$$

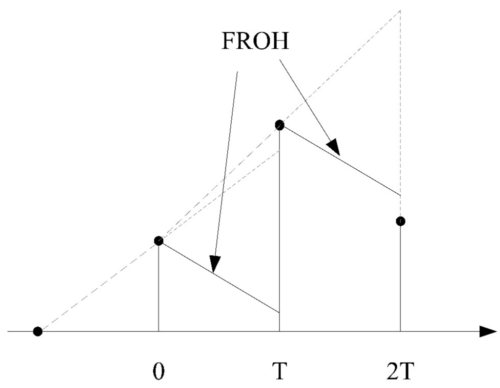

FIGURE 1. Signal reconstruction of a fractional-order hold with $\beta = -0.5$.

Actually, the use of the δ-operator with the transformation $q = \delta T + 1$ or $\delta = (q-1)/T$ is simply a way of reparameterisation any discrete time model. This reparameterisation has the advantage of highlighting the bridge between discrete time and continuous time domains and achieving improved numerical properties.

The dynamic model of the robot system can be expressed as a second order nonlinear differential equation [32]:

$$M(\rho)\ddot{\rho} + C(\rho, \dot{\rho})\dot{\rho} + G(\rho) + F(\dot{\rho}) = u \tag{3}$$

where $\rho \in R^N$ and $\dot{\rho} \in R^N$ are vector of the joint variable denotes the motion of the robotic arm and the velocity vector, respectively. N represents the degree of freedom (DOF) of the robotic arm. $M(\rho) \in R^{N\times N}$ is the symmetric positive definite inertial matrix, $C(\rho, \dot{\rho}) \in R^{N\times N}$ is the Coriolis and centrifugal matrix, $G(\rho) \in R^N$ is the gravitational force vector, $F(\dot{\rho}) \in R^N$ is the friction vector. $u \in R^N$ represents the control vector of joint torque/force exerting on the arm or the control input.

Further, the following multivariable nonlinear system is considered [33],

$$\begin{cases} \dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_1 = h_1(x) \\ \cdots \\ y_m = h_m(x) \end{cases} \tag{4}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^m$ are the state, input and output vectors, respectively. The vector fields $f(x)$ and $g_1(x), \cdots, g_m(x)$ and the output function $h_1(x), \cdots, h_m(x)$ will be assumed to be analytic on an open subset M. We also assume that x_e is an equilibrium vector for the original nonlinear system (4).

Denoting by $L_\tau(x)\lambda(x)$ the Lie derivative of the function λ along the vector field τ, we recall the following definition about the relative degree of the multivariable system.

Definition 1: The multivariable nonlinear system (4) has relative degree $r_1, \cdots, r_m$ about each output at the equilibrium if

(i) $L_{g_j} L_f^k h_i(x) = 0$ for all $1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant m$ and $k < r_i - 1$.

(ii) The matrix $A(x_e) \in R^{m\times m}$ is nonsingular.

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(x) & \cdots & L_{g_m} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & \cdots & L_{g_m} L_f^{r_2-1} h_2(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m(x) & \cdots & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix}$$

III. THE SAMPLED-DATA MODEL OF THE ROBOT SYSTEMS WITH FROH

Note that the ZOH and FOH are a special case of FROH. Thus, the research about the stabilization of a class of robot system in FROH is more valuable. In addition, the output of FROH has the following relations

$$\dot{u}_j(t) = \beta \left[\frac{u_j(kT) - u_j((k-1)T)}{T} \right]$$

and

$$\ddot{u}_j(t) = 0, \quad j = 1, \cdots, m$$

In order to show the derivation process more clearly, we have selected the DOF of the robot systems is two in this paper [34]. Robot systems with other DOF can be expanded using the similar method. Thus, the joint vector of the robot system is $\rho = \begin{bmatrix} \rho_1 & \rho_2 \end{bmatrix}$. And we let the robot system with

$$M(\rho) = \begin{bmatrix} m_1 + m_2\cos(\rho_2) & m_3\sin(\rho_2) \\ m_4\sin(\rho_2) & m_5 \end{bmatrix}$$

$$C(\rho, \dot{\rho}) = \begin{bmatrix} c_1\sin(\rho_2)\dot{\rho}_2 & c_2\cos(\rho_2)\dot{\rho}_2 \\ c_3\cos(\rho_1)\dot{\rho}_2 & 0 \end{bmatrix}$$

$$G(\rho) = \begin{bmatrix} g_1 g\cos(\rho_1 + \rho_2) \\ g_2 g\cos(\rho_1 + \rho_2) \end{bmatrix}$$

$$F(\dot{\rho}) = \begin{bmatrix} f_1\dot{\rho}_1 + f_2\sin(\dot{\rho}_1) \\ f_3\dot{\rho}_2 + f_4\sin(\dot{\rho}_2) \end{bmatrix}$$

those coefficients $m_l, l = 1, \cdots, 5$, $c_e, e = 1, 2, 3$, g_1, g_2 and $f_w, w = 1, 2, 3, 4$ are present the design parameters, and $g = 9.8m/s^2$ represents the gravitational acceleration.

Therefore, the state space of the robot system can be expressed as follows

$$\begin{cases} \dot{x}_1^1 = x_2^1 \\ \dot{x}_2^1 = b_1 + a_{11}u_1 + a_{12}u_2 \\ \dot{x}_1^2 = x_2^2 \\ \dot{x}_2^2 = b_2 + a_{21}u_1 + a_{22}u_2 \\ y_1 = x_1^1 \\ y_1 = x_1^2 \end{cases} \tag{5}$$

where $x_1^i = \rho_i$ and $x_2^i = \dot{\rho}_i$, $i = 1, 2$.

$$b_1 = \frac{-\left(\begin{array}{l} \left(m_5 c_1 \sin(x_1^2) x_2^2 - m_3 c_3 \sin(x_1^2)\cos(x_1^1) x_2^2\right) x_2^1 \\ + m_5 c_2 \cos(x_1^2) x_2^2 x_2^2 + g_1 g\cos(x_1^1 + x_1^2) + f_1 x_2^1 \\ + f_2 \sin(x_2^1) \end{array} \right)}{d}$$

$$b_2 = \frac{-\left(\begin{pmatrix}-m_4c_1\sin^2\left(x_1^2\right)x_2^2+m_1c_3\cos\left(x_1^1\right)x_2^2\\+m_2\cos\left(x_1^1\right)\cos\left(x_1^2\right)x_2^2\end{pmatrix}x_2^1\\-\ m_4c_2\sin\left(x_1^2\right)\cos\left(x_1^2\right)x_2^2x_2^2+g_2g\cos\left(x_1^1+x_1^2\right)\\+f_3x_2^2+f_4\sin\left(x_2^2\right)\right)}{d}$$

$a_{11}=m_5/d, a_{12}=-m_3\sin\left(x_1^2\right)/d, a_{21}=-m_4\sin\left(x_1^2\right)/d$, $a_{22}=\left(m_1+m_2\cos\left(x_1^2\right)\right)/d$ and d represents the determinant of $M(\rho)$. Obviously, the relative degree of the system is $r_1=r_2=2$. Then, for sufficiently small sampling period, the different derivatives of the outputs can be obtained

$$\begin{cases}\dot{y}_i=x_2^i\\ \ddot{y}_i=b_i+a_{i1}u_1+a_{i2}u_2\\ \dddot{y}_i=\dot{b}_i+\dot{a}_{i1}u_1+\dot{a}_{i2}u_2+a_{i1}\dot{u}_1+a_{i2}\dot{u}_2\\ \quad\approx\beta\sum_{j=1}^{2}a_{ij}\dfrac{u_j(kT)-u_j((K-1)T)}{T}\end{cases}\tag{6}$$

where $i=1,2$ and the final approximation result of $\dddot{y}_i$ is acquired from the fact that the fourth and fifth terms with β is dominant for sufficiently small T.

Applying the Taylor expansion formula for a sufficiently small sampling period T and using (6) acquired

$$\begin{aligned}x_{l+1,k+1}^i&=y_{i,k+1}^{(l)}\\&\approx y_{i,k}^{(l)}+Ty_{i,k}^{(l+1)}+\cdots+\frac{T^{r_i-l}}{(r_i-l)!}y_{i,k}^{(r_i)}\\&\quad+\frac{T^{r_i-l+1}}{(r_i-l+1)!}y_{i,k}^{(r_i+1)}\\&\approx x_{l+1,k}^i+Tx_{l+2,k}^i\\&\quad+\cdots+\frac{T^{r_i-l}}{(r_i-l)!}\left(b_{i,k}+\sum_{j=1}^{m}a_{i,j}^ku_{j,k}\right)\\&\quad+\frac{T^{r_i-l+1}}{(r_i-l+1)!}\beta\sum_{j=1}^{m}a_{i,j}^k\frac{u_{j,k}-u_{j,k-1}}{T}\\&\qquad i=1,\cdots,N.\quad l=0,\cdots,r_i-1.\end{aligned}\tag{7}$$

where, the subscripts $k-1$, k and $k+1$ represent the time instant $(k-1)T$, kT and $(k+1)T$, respectively.

Remark 2: From the higher-order Taylor expansion (7), this result can be expanded to N DOF of robot system. This paper selects two DOF as a didactic tool to simplify the exposition of results.

Hence, the sampled-data model for (5) with FROH is obtained as follows

$$\begin{cases}X_{k+1}=\Phi_\beta X_k+\sum_{i=1}^{2}\Gamma_{i,\beta}u_{i,k}+h_k\\ y_{1,k}=x_{1,k}^1\\ y_{1,k}=x_{1,k}^2\end{cases}\tag{8}$$

where

$$X_k=\begin{bmatrix}x_{1,k}^1 & x_{2,k}^1 & x_{1,k}^2 & x_{2,k}^2 & u_{1,k-1} & u_{2,k-1}\end{bmatrix}^T$$

$$\Phi_\beta=\begin{bmatrix}1 & T & 0 & 0 & -\frac{T^2\beta}{3!}a_{1,1}^k & -\frac{T^2\beta}{3!}a_{1,2}^k\\ 0 & 1 & 0 & 0 & -\frac{T\beta}{2!}a_{1,1}^k & -\frac{T\beta}{2!}a_{1,2}^k\\ 0 & 0 & 1 & T & -\frac{T^2\beta}{3!}a_{2,1}^k & -\frac{T^2\beta}{3!}a_{2,2}^k\\ 0 & 0 & 0 & 1 & -\frac{T\beta}{2!}a_{2,1}^k & -\frac{T\beta}{2!}a_{2,2}^k\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Gamma_{i,\beta}=\begin{bmatrix}\frac{T^2}{2!}a_{1,i}^k+\frac{T^2\beta}{3!}a_{1,i}^k\\ Ta_{1,i}^k+\frac{T\beta}{2!}a_{1,i}^k\\ \frac{T^2}{2!}a_{2,i}^k+\frac{T^2\beta}{3!}a_{2,i}^k\\ Ta_{2,i}^k+\frac{T\beta}{2!}a_{2,i}^k\\ 0\\ 0\end{bmatrix}$$

$$h_k=\begin{bmatrix}\frac{T^2}{2!}b_{1,k} & Tb_{1,k} & \frac{T^2}{2!}b_{2,k} & Tb_{2,k} & 0 & 0\end{bmatrix}^T$$

What is more, the local truncation error between the true system outputs and the ith output of the resulting sampled-data model is shown in following. It is assumed that the state of the sampled-data model is the same as the true system state at $t=kT$. Then, we compare the next sampling time $t=(k+1)T$ of the true system outputs $y_i((k+1)T)$ and the each first state $x_{1,k+1}^i$ of the resulting sampled-data model.

First, on the basis of the options in [23], the true system output $y_i((k+1)T)$ can be described as

$$\begin{aligned}&y_i((k+1)T)\\&\quad=x_{1,k}^i+Tx_{2,k}^i+\cdots+\frac{T^{r_i+1}}{(r_i+1)!}\\&\qquad\times\left[\dot{b}_{i,k}+\sum_{j=1}^{m}\dot{a}_{i,j}^ku_{j,k}+\sum_{j=1}^{m}a_{i,j}^k\dot{u}_{j,k}\right]_{t=\xi_1^i}\end{aligned}\tag{9}$$

with $kT<\xi_1^i<(k+1)T$.

The truncation errors of the output between of two systems are expressed as following.

$$\begin{aligned}e_{k+1}^i&=\left|y_i((k+1)T)-x_{1,k}^i\right|\\&=\frac{T^{r_i+1}}{(r_i+1)!}\left|\begin{matrix}\left[\dot{b}_{i,k}+\sum_{j=1}^{m}\dot{a}_{i,j}^ku_{j,k}+\sum_{j=1}^{m}a_{i,j}^k\dot{u}_{j,k}\right]_{t=\xi_1^i}\\-\left[\sum_{j=1}^{m}a_{i,j}^k\dot{u}_{j,k}\right]_{t=kT}\end{matrix}\right|\\&\leqslant\frac{T^{r_i+1}}{(r_i+1)!}L\left\|x\left(\xi_1^i\right)-x(kT)\right\|\end{aligned}\tag{10}$$

where L is the Lipschitz constant, which is determined by the following equation.

$$\begin{aligned}\left\| x\left(\xi_1^i\right) - x\left(kT\right)\right\| &\leqslant C \times \frac{e^{L\left|\xi_1^i - kT\right|} - 1}{L} \\ &< C \times \frac{e^{LT} - 1}{L} = O\left(T\right)\end{aligned} \tag{11}$$

Therefore, the local truncation error of the output between the true system and sampled-data with FROH is order T^{r_i+1}, which implies that the accuracy is the same order of as the Yuz and Goodwin's model [23]. Thus, the local error of the robot system with two DOF using the method in this paper is order three about small sampling period.

IV. ZERO DYNAMICS OF SAMPLED-DATA MODEL WITH FROH

In this section, we will show the results of zero dynamics of the sampled-data model in the previous section.

Based on the definition of the zero dynamic, we let the outputs of the robot systems are equal to zero. Thus, setting $x_{1,k+1}^1 = x_{1,k}^1 = 0$ and $x_{1,k+1}^2 = x_{1,k}^2 = 0$, then (8) leads to

$$\begin{bmatrix} x_{2,k+1}^1 \\ x_{2,k+1}^2 \\ u_{1,k} \\ u_{2,k} \\ 0 \\ 0 \end{bmatrix} = P\left(\beta\right) \begin{bmatrix} x_{2,k}^1 \\ x_{2,k}^2 \\ u_{1,k-1} \\ u_{2,k-1} \\ u_{1,k} \\ u_{2,k} \end{bmatrix} + \psi \tag{12}$$

where

$$P\left(\beta\right) = \begin{bmatrix} p_{21} & p_{22} & p_{23} \\ 0 & 0 & I_2 \\ p_{11} & p_{12} & p_{13} \end{bmatrix}$$

and I_2 represents a second order identity matrix.

$$p_{11} = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}, \quad p_{12} = \begin{bmatrix} -\frac{T^2\beta}{3!}a_{1,1}^k & -\frac{T^2\beta}{3!}a_{1,2}^k \\ -\frac{T^2\beta}{3!}a_{2,1}^k & -\frac{T^2\beta}{3!}a_{2,2}^k \end{bmatrix},$$

$$p_{13} = \begin{bmatrix} \left(\frac{T^2}{2!} + \frac{T^2\beta}{3!}\right)a_{1,1}^k & \left(\frac{T^2}{2!} + \frac{T^2\beta}{3!}\right)a_{1,2}^k \\ \left(\frac{T^2}{2!} + \frac{T^2\beta}{3!}\right)a_{2,1}^k & \left(\frac{T^2}{2!} + \frac{T^2\beta}{3!}\right)a_{2,2}^k \end{bmatrix},$$

$$p_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad p_{22} = \begin{bmatrix} -\frac{T\beta}{2!}a_{1,1}^k & -\frac{T\beta}{2!}a_{1,2}^k \\ -\frac{T\beta}{2!}a_{2,1}^k & -\frac{T\beta}{2!}a_{2,2}^k \end{bmatrix},$$

$$p_{23} = \begin{bmatrix} \left(T + \frac{T\beta}{2!}\right)a_{1,1}^k & \left(T + \frac{T\beta}{2!}\right)a_{1,2}^k \\ \left(T + \frac{T\beta}{2!}\right)a_{2,1}^k & \left(T + \frac{T\beta}{2!}\right)a_{2,2}^k \end{bmatrix},$$

$$\psi = \begin{bmatrix} Tb_{1,k} & Tb_{2,k} & 0 & 0 & \frac{T^2}{2!}b_{1,k} & \frac{T^2}{2!}b_{2,k} \end{bmatrix}^T.$$

Moreover, noting that the order of the each variable of ψ are higher order with respect to T than the corresponding one of p_{11} and p_{21}, applying z-transform to (12), we will obtain the sampling zero dynamics

$$\phi_\beta\left(z\right)\upsilon = 0 \tag{13}$$

where

$$\phi_\beta\left(z\right) = \begin{bmatrix} -p_{11} & -p_{12} & -p_{13} \\ zI - p_{21} & -p_{22} & -p_{23} \\ 0 & zI & -I \end{bmatrix}, \quad \upsilon = \begin{bmatrix} Z\left(\zeta_k\right) \\ Z\left(u_k\right) \end{bmatrix}$$

and $\zeta_k = \begin{bmatrix} x_{2,k}^1 & x_{2,k}^2 & u_{1,k-1} & u_{2,k-1} \end{bmatrix}^T$, $u_k = \begin{bmatrix} u_{1,k} & u_{2,k} \end{bmatrix}^T$. where the $Z\left[\cdot\right]$ represents the z-transform.

Consequently, the sampling zero dynamics could come from $\left|\phi_\beta\left(z\right)\right| = 0$, where the determinant $\left|\phi_\beta\left(z\right)\right|$ can be obtained by the following calculation process.

$$\begin{aligned}\left|\phi_\beta\left(z\right)\right| &= \begin{vmatrix} -p_{11} & -p_{12} & -p_{13} \\ zI - p_{21} & -p_{22} & -p_{23} \\ 0 & zI & -I \end{vmatrix} \\ &= \begin{vmatrix} -p_{11} & -p_{12} - zp_{13} & -p_{13} \\ zI - p_{21} & -p_{22} - zp_{23} & -p_{23} \\ 0 & 0 & -I \end{vmatrix} \\ &= \begin{vmatrix} -p_{11} & -p_{12} - zp_{13} \\ zI - p_{21} & -p_{22} - zp_{23} \end{vmatrix}\end{aligned} \tag{14}$$

Submitting the block matrix into (14), and using Schur complement obtains

$$\begin{aligned}\left|\phi_\beta\left(z\right)\right| &= \left| \left(-p_{22} - zp_{23}\right) - \left(zI - p_{21}\right)\left(-p_{11}\right)^{-1}\left(-p_{12} - zp_{13}\right) \right| \\ &= \frac{T\left(\left(3+\beta\right)z^2 + \left(3+\beta\right)z - 2\beta\right)}{6} \begin{vmatrix} a_{1,1}^k & a_{1,2}^k \\ a_{2,1}^k & a_{2,2}^k \end{vmatrix}\end{aligned} \tag{15}$$

Theorem 1: When a class of two DOF robot system as shown in this paper. Then, for sufficiently small sampling periods, the sampling zero dynamics of the corresponding sampled-data mode (8) with the FROH to reconstruct the control signal are determined by

$$\left(3+\beta\right)z^2 + \left(3+\beta\right)z - 2\beta = 0 \tag{16}$$

Thus, the zero dynamics of the sampled-data model with FROH are stable if all the zero dynamics are stable and $-1 < \beta < 0$.

Proof: The proof process of Theorem1 is not a difficult work. From the above deduced process and the result of (14), the result of (16) can be obtained. And then, according to the result(16), using bilinear transformation and Jury stability test [35] to obtain the stability condition $-1 < \beta < 0$. ■

Remark 3: When the FROH signal reconstruction device is used, the stability of sampling zero dynamics of the corresponding sampled-data multivariable models is only determined by the parameter β. And the conclusion of the paper about the zero dynamics stability condition is similar to the results of the linear multivariable case [30].

V. NUMERICAL EXAMPLE

This section presents an interesting class of robot system as the example to exemplify the ideas in this paper. And based on the sampled-data model to design the stabilization controller of the robot system when the sampling zero dynamics of the sampled-data model is stable. The following simulation results show that the outputs of the control system converge to the origin in FROH case.

Consider that the robot system with relative degree two (5). Further, the sampled-data model (8) of the robot system can be equaled to

$$\begin{cases} x^1_{1,k+1} = x^1_{1,k} + Tx^1_{2,k} + \frac{T^2}{2!}\left(b_{1,k} + a^k_{1,1}u_{1,k} + a^k_{1,2}u_{2,k}\right) \\ \qquad + \frac{T^3}{3!}\begin{pmatrix} a^k_{1,1}\frac{\beta}{T}\left(u_{1,k} - u_{1,k-1}\right) \\ + a^k_{1,2}\frac{\beta}{T}\left(u_{2,k} - u_{2,k-1}\right) \end{pmatrix} \\ x^1_{2,k+1} = x^1_{2,k} + T\left(b_{1,k} + a^k_{1,1}u_{1,k} + a^k_{1,2}u_{2,k}\right) \\ \qquad + \frac{T^2}{2!}\begin{pmatrix} a^k_{1,1}\frac{\beta}{T}\left(u_{1,k} - u_{1,k-1}\right) \\ + a^k_{1,2}\frac{\beta}{T}\left(u_{2,k} - u_{2,k-1}\right) \end{pmatrix} \\ x^2_{1,k+1} = x^2_{1,k} + Tx^2_{2,k} + \frac{T^2}{2!}\left(b_{2,k} + a^k_{2,1}u_{1,k} + a^k_{2,2}u_{2,k}\right) \\ \qquad + \frac{T^3}{3!}\begin{pmatrix} a^k_{2,1}\frac{\beta}{T}\left(u_{1,k} - u_{1,k-1}\right) \\ + a^k_{2,2}\frac{\beta}{T}\left(u_{2,k} - u_{2,k-1}\right) \end{pmatrix} \\ x^2_{2,k+1} = x^2_{2,k} + T\left(b_{2,k} + a^k_{2,1}u_{1,k} + a^k_{2,2}u_{2,k}\right) \\ \qquad + \frac{T^2}{2!}\begin{pmatrix} a^k_{2,1}\frac{\beta}{T}\left(u_{1,k} - u_{1,k-1}\right) \\ + a^k_{2,2}\frac{\beta}{T}\left(u_{2,k} - u_{2,k-1}\right) \end{pmatrix} \\ y_{1,k} = x^1_{1,k} \\ y_{2,k} = x^2_{1,k} \end{cases} \tag{17}$$

and the sampling zero dynamics of (17) are lead to

$$(3+\beta)z^2 + (3+\beta)z - 2\beta = 0 \tag{18}$$

Obviously, the sampling zero dynamics are stable if and only if $-1 < \beta < 0$.

What is more, the model following control was consider to converge the output of the robot sampled-data model in this paper to the origin. Thus, a discrete model following controller is designed using the model (17) as

$$u_{1,k} = \frac{3}{3+\beta}\cdot\frac{a^k_{2,2}}{a^k_{2,2}a^k_{1,1} - a^k_{1,2}}\begin{pmatrix} -b_{1,k} - \frac{a^k_{1,2}}{a^k_{2,2}}\Xi^1_{1,k} \\ + \Xi^1_{2,k} + \Xi^1_{3,k} \end{pmatrix}$$

$$u_{2,k} = \frac{3}{3+\beta}\cdot\frac{a^k_{1,1}}{a^k_{2,2}a^k_{1,1} - a^k_{2,1}}\begin{pmatrix} -b_{2,k} - \frac{a^k_{2,1}}{a^k_{1,1}}\Xi^2_{1,k} \\ + \Xi^2_{2,k} + \Xi^2_{3,k} \end{pmatrix}$$

where $\Xi^1_{1,k} = -b_{2,k} + \Xi^2_{2,k} + \Xi^2_{3,k}$, $\Xi^1_{2,k} = \frac{\beta}{3}a^k_{1,1}u_{1,k-1} + \frac{\beta}{3}a^k_{1,2}u_{2,k-1}$, $\Xi^1_{3,k} = \frac{2}{T^2}\left(-Tx^1_{2,k} + (\alpha_1 - 1)x^1_{1,k}\right)$,

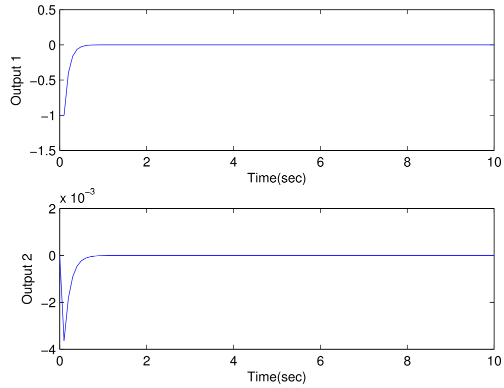

FIGURE 2. **Outputs of the robot system by the stabilization controller in the case of FROH for the first parameter group.**

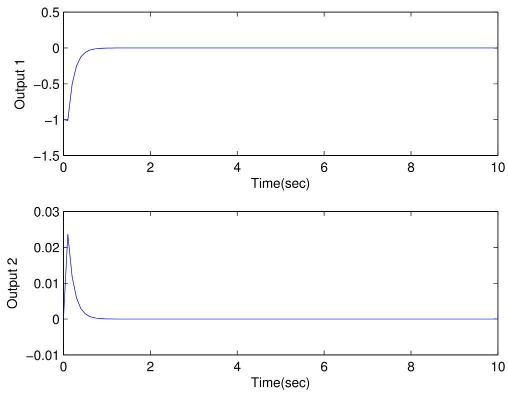

FIGURE 3. **Outputs of the robot system by the stabilization controller in the case of FROH for the second parameter group.**

$\Xi^2_{1,k} = -b_{1,k} + \Xi^1_{2,k} + \Xi^1_{3,k}$, $\Xi^2_{2,k} = \frac{\beta}{3}a^k_{2,1}u_{1,k-1} + \frac{\beta}{3}a^k_{2,2}u_{2,k-1}$ and $\Xi^2_{3,k} = \frac{2}{T^2}\left(-Tx^2_{2,k} + (\alpha_2 - 1)x^2_{1,k}\right)$. The coefficient parameters of the robot system (5) was selected as $m_1 = m_5 = 0.1$, $m_2 = m_3 = m_4 = 0.01$, $g_1 = g_2 = 0.01$, $c_1 = -0.005$, $c_2 = c_3 = 0.005$, $f_1 = f_3 = 0.1$ and $f_2 = f_4 = 0.05$. Next, we have selected two groups parameters to show the results in this paper is effective. In first group, let $T = 0.01$, $\beta = -0.45$, $\alpha_1 = 0.4$ and $\alpha_2 = 0.5$, and the second group parameters are $T = 0.03$, $\beta = -0.7$ and $\alpha_1 = \alpha_2 = 0.5$, consequently, the simulation results are shown in Figure 2 and Figure 3, respectively.

What is more, it can be seen from the simulation diagrams that the convergence of the outputs to origin with FROH is achieved. Those diagrams have also shown the control strategy of this paper based on the sampling zero dynamic stable approach is effective.

VI. CONCLUSION

This paper has used the sampling zero stable controller design approach to stabilize the robot system, where the sampled-data model of the robot system is obtained using FROH. We have derived several results on the sampling zero dynamic of sampled-data model for the continuous time robot system with FROH as the input signal reconstruction device. The expression of the zero dynamics about two DOF robot system is presented, and the stability condition of zero dynamics is also obtained. The stability condition of the zero dynamics of corresponding sampled-data system is similar to the results of the linear multivariable case [30]. And its effectiveness control performance is shown through simulation results. In future, we will incorporate disturbance into robot system and other input signal reconstruction device as our research content.

ACKNOWLEDGEMENT
The authors are very grateful to anonymous reviewers and editors for their helpful comments and suggestions, which have greatly improved the quality of this paper.

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