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# A Generalized One-Bit Control System Using a $\Delta\Sigma$ -Quantizer

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**ABSTRACT** This paper presents the use of a delta-sigma quantizer for generalized one-bit control processing. An equivalent control strategy based on sliding-mode control is employed to derive the necessary condition for the convergence of the proposed one-bit control system in both the continuous-time and discrete-time domains. Under the convergence condition, the binary signals generated by delta-sigma quantizers in the one-bit control system effectively replace their counterpart signals in conventional control systems. This enables a significant reduction in the number of multipliers and overall hardware cost for computing the control laws in one-bit control systems. Our result is applied to design a multiplier-less one-bit generalized proportional and integral controller for the position control of an experimental prototype of a DC motor. An implementation of the one-bit control system is carried out using an FPGA platform to demonstrate the behavior of one-bit generalized proportional and integral controller and compare the results with the standard in terms of implementation efficiency. The results of the simulation and experiment show that the one-bit generalized proportional and integral controller effectively controls the system and achieves the desired specifications. At the same time, the proposed one-bit control system consumes significantly fewer hardware resources than the standard control system.

**INDEX TERMS** One-bit control processing, delta-sigma quantizer, quantized control systems, equivalent control, sliding mode, generalized proportional and integral.

## I. INTRODUCTION

Different techniques of pulse code modulation (PCM) have found wide applications in fields such as communication systems, signal processing, power electronics, and control systems. In particular, the PCM-based delta-sigma quantizer ( $\Delta\Sigma$ -Q) is eminently suitable for applications where some signals are converted to switch signals represented by a sequence of binary values 0's and +1's such as ON/OFF power electronics systems and actuators [1]–[6] and networked control systems (NCSs) with a binary symmetric channel (i.e., transmitting only one digit 0 or 1 per sample time) [7]–[12].  $\Delta\Sigma$ -Qs have been used in different

state-space realizations of the so-called one-bit processing systems [13]–[16].

For practical considerations, the state-space realizations of transfer functions are often implemented in finite-word-length (FWL) integrated circuits such as field programmable gate arrays (FPGAs), digital signal processors (DSP), microprocessors and microcontrollers. These digital devices offer numerous advantages in terms of cost, performance and maintenance. However, some inherent challenges can arise from the use of such digital platforms due to the discretization effect and quantization error caused by FWL platforms [17]–[19]. Furthermore, inevitable latency is inherently introduced into discrete-time systems due to the nature of these digital systems. Another severe issue lies with the ill-conditioning and sensitivity caused by the high sampling

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systems using fixed-point arithmetic [15], [20]–[23]. These issues not only degrade the performance but, even worse, may lead to an instability of the entire system.

The above issues motivated researchers to insert  $\Delta\Sigma$ -Qs in linear systems with state-space representations such as linear controllers and digital filters. Such systems are referred to as one-bit control or one-bit signal processing systems [15], [21], [24]. The direct processing of one-bit signals, generated by  $\Delta\Sigma$ -Qs, has remarkable advantages for digital platforms with limited resources from a computational perspective [3], [15], [16]. The implementation of one-bit control laws alleviates the drawbacks associated with the latency, implementation and maintenance where the hardware and cost of products are constrained. Furthermore, large numbers of wires due to the actuators and sensors interface with controllers through multi-bit digital signals could be significantly reduced by adopting one-bit control techniques [7], [16], [25].

Although the aforementioned works have shown great results in the literature, it is worth noting that much more work is needed in terms of both theoretical and practical advancements. This paper, motivated by [15], presents different uses of  $\Delta\Sigma$ -Qs in both continuous-time (CT) and discrete-time (DT) control systems. In particular, a generalized one-bit control system full of trivial parameters<sup>1</sup> is established. According to the definition by Filippov [26], the proposed one-bit control system fits well with differential inclusion due to its discontinuous right-hand side (RHS). Thus, we adopt equivalent-control-based sliding-mode control (ECB-SMC) in our stability analysis of one-bit control systems. We show that the binary signals in a one-bit control system are equivalent to their counterpart signals in the conventional control system. The results of the theoretical analysis are validated through experiments by designing a multiplier-less one-bit generalized proportional and integral (GPI) controller considering an experimental prototype of a DC motor [28], [29]. The implementation of the one-bit control system is carried out using an FPGA platform. The results of the simulation and experiment demonstrate that the one-bit control system effectively controls the system and achieves the desired specifications while consuming fewer hardware resources than the standard control system.

In this paper, the notation  $I_m$  denotes the identity matrix with dimensions of  $\mathbb{R}^{m \times m}$ . The state variables  $x_i$  and  $\hat{x}_i$  ( $i = 1, 2, \dots, n$ ) denote the states of the nominal control system and the state of the one-bit control system, respectively.  $1_m = [1 \dots 1]^T \in \mathbb{Z}^m$  and  $0_m = [0 \dots 0]^T \in \mathbb{Z}^m$  denote vectors of ones and zeros, respectively. We use the standard *Euclidean* norm and the maximum norm of a vector  $x \in \mathbb{R}^n$  defined by  $\|x\|$  and  $\|x\|_\infty$ , respectively. Let  $\|A\|$  denote the matrix norm of  $A$  induced by a vector norm  $\|\cdot\|$  in  $\mathbb{R}^n$ , i.e.,  $\|A\| = \sup_{\|x\|=1} \|Ax\|$ . In our analysis, we use the following well-known mathematical facts:  $s^T \text{sgn}(s) \mathcal{D} \|s\|_1$ , and  $\|s\|_\infty \leq \|s\| \leq \|s\|_1 \leq \sqrt{m} \|s\|$  for all  $s \in \mathbb{R}^m$ .

<sup>1</sup>By *trivial parameters*, we mean those that are 0,  $\pm 1$ .

The rest of the paper is organized as follows. The description and stability analysis of the one-bit CT control system embedded with the  $\Delta\Sigma$ -Q are given in Section II. Section III discusses the formulated one-bit DT control system embedded with  $\Delta\Sigma$ -Q. Guidelines for designing the one-bit DT GPI for position control of a DC motor are briefly discussed in Section IV. The simulations and experimental results for the DC motor control system with the one-bit GPI are presented in Section V, and the concluding remarks are given in Section VI.

## II. ONE-BIT CT CONTROL SYSTEM

In this section, a precise formulation and stability analysis for the problem of one-bit CT control system are derived.

### A. NOMINAL CT CONTROL SYSTEM DESCRIPTION

Consider the single-input single-output linear time-invariant system given by

$$\dot{x}_t^p = A_p x_t^p + B_p u_t \tag{1a}$$

$$y_t^p = C_p x_t^p \tag{1b}$$

where  $x_t^p \in \mathbb{R}^{n_p}$  is the state-variable vector,  $u_t \in \mathbb{R}$  denotes the control signal to be designed and  $y_t^p \in \mathbb{R}$  denotes the output of the system. The state-space equations for the controller considered here are given by

$$\dot{x}_t^c = A_c x_t^c + B_{cy} y_t^p + B_{ce} e_t \tag{2a}$$

$$u_t = C_c x_t^c + d_{cy} y_t^p + d_{ce} e_t \tag{2b}$$

where  $x_t^c \in \mathbb{R}^{n_c}$  is the state-variable vector of the controller and  $e_t$  denotes the output tracking error

$$e_t = r - y_t^p$$

in which  $r$  is a constant reference signal. By combining (1) and (2), the state-space equations for the closed-loop control system become

$$\dot{x}_t = Ax_t + Be_t \tag{3a}$$

$$y_t = Cx_t \tag{3b}$$

where

$$x_t = \begin{bmatrix} x_t^p \\ x_t^c \end{bmatrix}, \quad A = \begin{bmatrix} A_p + d_{cy} B_p C_p & B_p C_c \\ B_{cy} C_p & A_c \end{bmatrix},$$

$$B = \begin{bmatrix} d_{ce} B_p \\ B_{ce} \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0_{n_c}^T \end{bmatrix}$$

$A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^n$  and  $C \in \mathbb{R}^{1 \times n}$  for all  $n = n_p + n_c$ .

*Assumption 1:* It is important that the controller (2) is designed such that  $A$  is a Hurwitz matrix and  $\lim_{t \rightarrow 0} e_t = 0$ .

For real-time applications of a power converter, one-bit control and an NCS with one-bit communication channels, some signals inside (3) are converted to binary signals. These signals can be either control signals in power converters, controller states in one-bit control processing or transmitted signals in the NCS. Depending on the applications, these signals that are to be converted are defined here as:

$$w_t := Ex_t + F e_t \tag{4}$$

where  $w_t \in \mathbb{R}^m$ ,  $E \in \mathbb{R}^{m \times n}$  and  $F \in \mathbb{R}^m$ . Note that  $E$  and  $F$  depend on several factors, including the intended real-time applications and types of signals to be converted.

Thus, combining (3) with (4) yields the following augmented system

$$\dot{\chi}_t = \mathcal{A}\chi_t + \mathcal{B}e_t \quad (5a)$$

$$\mathcal{Y}_t = \mathcal{C}\chi_t \quad (5b)$$

where  $\chi_t^T = [x_t^T w_t^T] \in \mathbb{R}^N$  with  $N = n + m$ ,  $\mathcal{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ ,  $\mathcal{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$ , and  $\mathcal{C} = [C \ 0_m^T]$  in which  $A_{11} \in \mathbb{R}^{n \times n}$ ,  $A_{12} \in \mathbb{R}^{n \times m}$ ,  $A_{21} \in \mathbb{R}^{m \times n}$ ,  $A_{22} \in \mathbb{R}^{m \times m}$ ,  $B_1 \in \mathbb{R}^n$  and  $B_2 \in \mathbb{R}^m$ . It should be stressed here that (3) is only a new form of (5); that is, both systems (3) and (5) represent the same control system with the only difference being the insertion of dependent vectors.

In the following example, we show how the values of the matrices  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\mathcal{C}$ ,  $E$  and  $F$  depend on the nature of the control problem to be solved. This in turn changes the various matrices in the augmented system.

*Example:* Consider the problem of an NCS where the communication channel with one-bit information is located between the sensors and the controller. Thus, the feedback signal  $y_t$  is converted to a binary signal to be transferred through the communication channel. For simplicity, let us consider an output-feedback control problem with  $r = 0$  in this example.

In this example, we want to represent  $w_t = y_t$ . Therefore,  $E = C$  and  $F = 0$  in (4). We consider *ideal* conversion where, as a result, the controller (2) can be rewritten as:

$$\dot{x}_t^c = A_c x_t^c + (B_{cy} - B_{ce}) w_t \quad (6a)$$

$$u_t = C_c x_t^c + (d_{cy} - d_{ce}) w_t \quad (6b)$$

The various matrices in the augmented system (5) are defined as follows. From (1a), and (6), it follows that

$$\dot{\chi}_t = A_{11}\chi_t + A_{12}w_t \quad (7)$$

where  $A_{11} = \begin{bmatrix} A_p & B_p C_c \\ 0_{n_c \times n_p} & A_c \end{bmatrix}$ , and  $A_{12} = \begin{bmatrix} (d_{cy} - d_{ce}) B_p \\ B_{cy} - B_{ce} \end{bmatrix}$ . Furthermore, using (5a) and (7), it can be shown that

$$\begin{aligned} \dot{w}_t &= C\dot{\chi}_t \\ &= A_{21}\chi_t + A_{22}w_t \end{aligned} \quad (8)$$

where  $A_{21} = CA_{11} = [C_p A_p \ C_p B_p C_c]$ , and  $A_{22} = (d_{cy} - d_{ce}) C_p B_p$ . Additionally,  $\mathcal{B} = \mathbf{0}_N$  and  $\mathcal{C} = [C_p \ 0_{n_c}^T \ 0_m^T]$ .

In this subsection, we only consider ideal conversion from conventional signals to binary signals with *zero* quantization error. Next, we introduce one-bit CT control where  $\Delta\Sigma$ -Q is embedded to convert  $w_t$  defined in (4) to binary signals.

### B. ONE-BIT CT CONTROL SYSTEMS EMBEDDED WITH THE $\Delta\Sigma$ -Q

In this section, we apply the  $\Delta\Sigma$ -Q to convert

$$\hat{w}_t = E\hat{x}_t + F\hat{e}_t \quad (9)$$

to binary signals.<sup>2</sup> The dynamic function of the  $\Delta\Sigma$ -Q, which operates here as a map  $\hat{w}_t \in \mathbb{R}^m \rightarrow \delta_t \in \{-1, 1\}, \dots, \{-1, 1\}^T \in \mathbb{Z}^m$ , is given by

$$\dot{s}_t = \hat{w}_t - \phi \delta_t \quad (10)$$

where  $s_t$  is the state-variable of the  $\Delta\Sigma$ -Q,  $\hat{w}_t$  is the input of the  $\Delta\Sigma$ -Q,  $\delta_t \mathcal{D} \text{sgn}(s_t) = [\text{sgn}(s_1), \dots, \text{sgn}(s_m)]^T$  is the binary output of the  $\Delta\Sigma$ -Q and  $\phi$  is the quantizer gain. The quantizer gain ( $\phi$ ) can be judiciously chosen to be greater than the upper bound of (4), i.e.,  $\|\hat{w}_t\|_\infty \leq \phi$ , (see [30], [31]).

The one-bit CT control systems are obtained by replacing  $\hat{w}_t$  with  $\delta_t$  using (10) such that

$$\dot{\hat{\chi}}_t = \bar{\mathcal{A}} \begin{bmatrix} \hat{x}_t \\ \delta_t \end{bmatrix} + \mathcal{B}\hat{e}_t \quad (11a)$$

$$\hat{\mathcal{Y}}_t = \mathcal{C}\hat{\chi}_t \quad (11b)$$

where  $\hat{\chi}_t^T = [\hat{x}_t^T \ \hat{w}_t^T]$  and  $\bar{\mathcal{A}} = \mathcal{A} \times \text{diag}\{I_n, \phi I_m\}$ .

*Theorem 1:* For the CT control system (3) with assumption 1, one-bit CT control systems (11), embedded with the oversampling  $\Delta\Sigma$ -Q described by (10), converges to the nominal control system (5) using ECB-SMC if  $\phi$  is designed such that

$$\Omega_t = \{w_t \in \mathbb{R}^m \mid \|w_t\|_\infty \leq \Psi_t < \phi\} \quad (12)$$

in which  $\Psi_t = \sqrt{m} (\|E\| \|x_t\| + \|F\| |e_t|)$ .

*Proof:* Consider the dynamic function of the  $\Delta\Sigma$ -Q given by (10), and let us define the Lyapunov function as  $V_t = \frac{1}{2} s_t^T s_t$ . The derivative of  $V_t$  with respect to time is expressed by  $\dot{V}_t = s_t^T \dot{s}_t$ . It follows from (10) that

$$\dot{V}_t = s_t^T \hat{w}_t - \phi s_t^T \delta_t \quad (13a)$$

$$= s_t^T \hat{w}_t - \phi \|s_t\|_1 \quad (13b)$$

$$\leq -\|s_t\| (\phi - \|\hat{w}_t\|_\infty) \quad (13c)$$

since  $\|s_t\|_2 \leq \|s_t\|_1$ . It is also obvious that (13c) is negative, i.e.,  $V_t$  decreases to ensure sliding mode, if  $\|\hat{w}_t\|_\infty < \phi$ . As a result, ECB-SMC can be taken into account to characterize one-bit CT control systems. For the equivalent control, the  $\Delta\Sigma$ -Q operates at an arbitrary high frequency, the so-called oversampling  $\Delta\Sigma$ -Q, which gives  $s_t = 0_m$ . This in turn implies that

$$\hat{w}_{t(eq)} = \phi \delta_{t(eq)} \quad (14)$$

in (10), where  $\hat{w}_{t(eq)}$  and  $\delta_{t(eq)}$  are the equivalent signals for  $\hat{w}_t$  and  $\delta_t$ , respectively. Both  $\hat{w}_{t(eq)}$  and  $\delta_{t(eq)}$  are obtained by filtering  $\hat{w}_t$  and  $\delta_t$  through a low-pass filter.<sup>3</sup>

<sup>2</sup>Here,  $\hat{\cdot}$  indicates quantized signals. Thus, we change the notation from  $w_t$  and  $x_t$  and the nominal signals in Section II to their associated quantized signals  $\hat{w}_t$  and  $\hat{x}_t$ .

<sup>3</sup>The equivalent signal (i.e., average) of the discontinuous function  $\delta_t$ , denoted as  $\delta_{t(eq)} \in [-1_m, 1_m]$ , can be obtained by passing  $\delta_t$  through a first-order low-pass filter such that  $\delta_{t(eq)} := \delta_t - \tau \dot{\delta}_{t(eq)}$ , where  $\tau$  is the time constant [32].

From a geometric point of view and during ideal sliding motion, ECB-SMC implies that the high-switching-frequency binary signals  $\phi\delta_t$  in one-bit CT control systems (11) can be replaced by their equivalent signals in (14) [30]–[32]. As a result, this gives

$$\dot{\chi}_t^* = \mathcal{A}\chi_t^* + \mathcal{B}e_t^* \quad (15)$$

where  $\chi_t^* = \begin{bmatrix} \mathbf{x}_t^* \\ \hat{\mathbf{w}}_{t(eq)} \end{bmatrix}$  and  $e_t^*$  denote the states and tracking error of the system under the equivalent control, respectively. It follows from (15) that  $\mathbf{w}_t \equiv \hat{\mathbf{w}}_{t(eq)} \equiv \hat{\mathbf{w}}_t$ . Hence, the quantizer gain  $\phi$  can be suitably designed using the upper bounds of  $\mathbf{w}_t$  defined in (4) in the original control system, such that

$$\Omega_t = \{\mathbf{w}_t \in \mathbb{R}^m \mid \|\mathbf{w}_t\|_\infty \leq \Psi_t < \phi\} \quad (16)$$

where  $\Psi_t = \sqrt{m}(\|E\| \|\mathbf{x}_t\| + \|F\| |e_t|)$ .

It is straightforward to see that if  $\hat{\chi}_t$  tends to  $\chi_t$ , then  $\hat{\mathcal{Y}}_t$  tends to  $\mathcal{Y}_t$  accordingly. Thus, we can conclude that the one-bit CT control system (11) converges to its nominal control system (5) with ECB-SMC if (16) holds. ■

*Remark 1:* In our analysis, we consider the *ideal* sliding mode, where sufficiently high-frequency switching devices should be implemented. However, this requirement is not necessarily valid for some power electronic converters with a low switching frequency, digital platforms with bandwidth constraints, and networks with a bandwidth-limited channel.

In what follows, we extend our analysis of one-bit CT control systems to the more realistic case of one-bit DT control systems where the switching frequency is finite.

### III. ONE-BIT DT CONTROL SYSTEM

#### A. NOMINAL DT-CONTROL SYSTEM DESCRIPTION

The DT transformation of (3), is described as:

$$\rho \mathbf{x}_k = A_d \mathbf{x}_k + B_d e_k \quad (17a)$$

$$y_k = C_d \mathbf{x}_k \quad (17b)$$

by defining  $\rho \triangleq zI_n - I_n$  and for all  $t \in [kh, (k+1)h]$ , where  $h > 0$  denotes the sampling period,  $\mathbf{x}_k$ ,  $u_k$  and  $y_k$  represent  $\mathbf{x}(kh)$ ,  $u(kh)$  and  $y(kh)$ , respectively. Euler's discretization method gives  $A_d = hA$ ,  $B_d = hB$  and  $C_d = C$ . Let  $\hat{A}_d = I_n + A_d$ , which implies that  $h$  should be chosen in the range  $0 \leq h \leq h_M$  such that  $\text{rank}\{\hat{A}_d\} = n$  and  $\varrho(\hat{A}_d) \leq 1$ ;  $\varrho(\hat{A}_d)$  is the spectral radius of  $\hat{A}_d$  in which  $\varrho(\hat{A}_d) = \max\{|\lambda_i(\hat{A}_d)|\}$  for  $(i = 1, \dots, n)$ . Note that the maximum sampling period  $h_M$  results in a marginally stable system where  $\varrho(\hat{A}_d) = 1$ .

The DT equivalent of (4) is defined as:

$$\mathbf{w}_k := E\mathbf{x}_k + F e_k. \quad (18)$$

The DT representation of the augmented nominal system (5) has the form

$$\rho \chi_k = \mathcal{A}_d \chi_k + \mathcal{B}_d e_k \quad (19a)$$

$$\mathcal{Y}_k = \mathcal{C}_d \chi_k \quad (19b)$$

where  $\chi_k^T = [\mathbf{x}_k^T \ \mathbf{w}_k^T]$ ,  $\mathcal{A}_d = h\mathcal{A}$ ,  $\mathcal{B}_d = h\mathcal{B}$ , and  $\mathcal{C}_d = \mathcal{C}$  for all  $N = n + m$ .

Note that both (17) and (19) are equivalent; i.e. they convey the same information. Next, we consider the Euler's discretization of the CT control system embedded with the  $\Delta\Sigma$ -Q.

#### B. ONE-BIT DT CONTROL SYSTEM EMBEDDED WITH THE $\Delta\Sigma$ -Q

The dynamic function in (10) is discretized as follows:

$$\rho s_k = \hat{\mathbf{w}}_k - \phi \delta_k \quad (20)$$

where  $\hat{\mathbf{w}}_k \in \mathbb{R}^m$  and  $\delta_k \in \mathbb{Z}^m$  are the input and output of the  $\Delta\Sigma$ -Q, respectively, and  $\phi$  denotes the quantizer gain.

Replacing the intended signals  $\hat{\mathbf{w}}_k$  by their associated binary signals  $\delta_k$  using (20) gives

$$\rho \hat{\chi}_k = \bar{\mathcal{A}}_d \begin{bmatrix} \hat{\mathbf{x}}_k \\ \delta_k \end{bmatrix} + \mathcal{B}_d \hat{e}_k \quad (21a)$$

$$\hat{\mathcal{Y}}_k = \mathcal{C}_d \hat{\chi}_k \quad (21b)$$

where  $\hat{\chi}_k^T = [\hat{\mathbf{x}}_k^T \ \hat{\mathbf{w}}_k^T]$ ,  $\bar{\mathcal{A}}_d = h\bar{\mathcal{A}} = h\mathcal{A} \times \text{diag}\{I_n, \phi I_m\}$  and  $\delta_k = \text{sgn}(s_k) = [\text{sgn}(s_1), \dots, \text{sgn}(s_m)]^T$ .

*Lemma 1:* Consider (20) with  $s_0 = \mathbf{0}_m$ . If the quantizer gains  $\phi$  is selected as

$$\|\hat{\mathbf{w}}_k\|_\infty < \phi, \quad (22)$$

then the following results are valid.

- 1) The dynamic function (20) is stable, and  $s_k$  is confined by  $\|s_k\| \leq \epsilon$  where  $\|\hat{\mathbf{w}}_k\|_\infty + \phi < 2\phi \leq \epsilon$ .
- 2) It can be verified that

$$\phi \delta_{(eq)_N} = \hat{\mathbf{w}}_{(eq)_N} \quad (23)$$

where  $\Xi_{(eq)_N} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=0}^{N-1} \Xi_i$ , denotes the  $N$ -moving average for all  $\Xi \in \{\hat{\mathbf{w}}, \delta\}$ .

*Proof:* Consider the scalar system of (20)

$$s_{j_{k+1}} = s_{j_k} + \hat{w}_{j_k} - \phi \delta_{j_k} \quad (24)$$

where  $\hat{w}_j \in \mathbb{R}$  and  $\delta_j = 1$  if  $s_j \geq 0$  and  $-1$  if  $s_j < 0$  for  $j = 1, \dots, m$ .

In what follows, the dynamical behavior and stability of (24) are studied in both possible cases:  $s_{j_k} \geq 0$ , and  $s_{j_k} < 0$ . For the case of  $s_{j_k} \geq 0$ , it can be shown that  $s_{j_{k+1}} = s_{j_k} + \hat{w}_{j_k} - \phi \leq s_{j_k}$  if  $|\hat{w}_{j_k}| < \phi$ . It can also be verified that  $s_{j_k} > s_{j_{k+1}} > -|\hat{w}_{j_k}| - \phi > -2\phi \geq -\epsilon$ . Similarly,  $s_{j_k} < 0$  implies that  $s_{j_{k+1}} > s_{j_k}$  and  $\epsilon > s_{j_{k+1}} > s_{j_k}$  if  $|\hat{w}_{j_k}| < \phi$  holds.

From the above analysis, it clear that  $s_{j_k}$  will be attracted to  $s_{j_k} \in [-\epsilon, \epsilon]$  if  $|\hat{w}_{j_k}| < \phi$  holds. This completes the proof of the first part.

Next, assume that (20) is stable. The addition of  $N$  iterations for (20) with  $s_0 = \mathbf{0}_m$  yields

$$s_N = \sum_{i=0}^{N-1} \hat{\mathbf{w}}_i - \phi \sum_{i=0}^{N-1} \delta_i \quad (25)$$

Given that  $\|s_k\| \leq \epsilon$ , then  $\lim_{N \rightarrow \infty} \frac{s_N}{N} = \mathbf{0}_m$ , which yields ECB-SMC (23). ■

*Definition 1:* Given that  $\|\chi_k\| < \infty$ , the one-bit DT control system is said to be stable if there exists  $\xi_k = \chi_k - \hat{\chi}_k$  such that  $\|\xi_k\| < \infty$ .

In the following theorem, the necessary conditions for the stability and convergence of the one-bit DT control system embedded with the  $\Delta\Sigma$ -Q described by (20), are investigated.

*Theorem 2:* If the system (17) is asymptotically stable, i.e.,  $\varrho(A_d) < 1$ , then there exist sampling periods  $h_m$  such that  $0 \leq h \leq h_m < h_M$ , where the one-bit DT control system (11), embedded with the oversampling  $\Delta\Sigma$ -Q described by (20) converges to the nominal control system (5) given that  $\phi$  satisfies

$$\Omega_k = \{w_k \in \mathbb{R}^m \mid \|w_k\|_\infty \leq \Psi_k < \phi\} \quad (26)$$

where  $\Psi_k = \sqrt{m}(\|E\| \|x_k\| + \|F\| \|e_k\|) + \mathcal{O}(h)$  with  $\lim_{h \rightarrow 0} \mathcal{O}(h) = 0$ .

*Proof:* Assume that the  $\Delta\Sigma$ -Q described by (20) is initially designed to be a stable, i.e., Lemma (1) holds. The quantization error generated by the  $\Delta\Sigma$ -Q (20) is defined as the difference between the input and the output

$$e_k = \hat{w}_k - \phi \delta_k. \quad (27)$$

Substituting (27) into one-bit DT control system (21) gives

$$\rho \hat{\chi}_k = \mathcal{A}_d \hat{\chi}_k + \mathcal{B}_d e_k + \xi_k \quad (28)$$

with

$$\xi_k = -\bar{\mathcal{A}}_d \begin{bmatrix} 0_n \\ \phi^{-1} e_k \end{bmatrix} = -h \mathcal{A} \begin{bmatrix} 0_n \\ e_k \end{bmatrix} \quad (29)$$

where  $\xi_k$  denotes the chattering behavior as per Definition 1. Notice that both one-bit DT control system (21), or equivalently (28), and the nominal system (19) should behave identically if  $\xi_k = \mathbf{0}_N$ . Hence, from (28) and (29), one can see that there exist sampling periods  $h_m$  such that  $0 < h \leq h_m \leq h_M$ , where the one-bit DT control system (21) converges to the nominal system (19) since  $\lim_{h \rightarrow 0} \xi_k = \mathbf{0}_N$ . It

is, also, straightforward to see that if  $\hat{\chi}_k$  tends to  $\chi_k$ , then  $\hat{\mathcal{Y}}_k$  tends to  $\mathcal{Y}_k$  accordingly.

Consider ECB-SMC as per Lemma 1, it can be assumed that  $\hat{w}_{k(eq)} \equiv \hat{w}_k \equiv w_k$ . Thus,  $\phi$  can be suitably designed using the upper bounds of  $w_k$  defined in (18) to satisfy the stability condition (26). ■

*Remark 2:* It follows from (27) that  $\|e_k\|_\infty < 2\phi$  if  $\|\hat{w}_k\|_\infty < \phi$ . It is then easy to verify that

$$\|\xi_k\|_\infty < 2\phi h \|\mathcal{A}\| \quad (30)$$

From (30), the chattering  $\xi_k$  can be reduced with a minimum quantizer gain  $\phi$ , short sampling period  $h$ , and small negative eigenvalues of  $\mathcal{A}$ , which result in slow dynamics such that the control system can effectively act as a low-pass filter. Roughly speaking, the optimal quantizer gain  $\phi$  can be selected, appropriately to be as close as possible to  $\|w_k\|_\infty$ . Thus, special care must be taken to judiciously select the

optimal quantizer gain  $\phi$ , which ensures both the stability and performance (i.e., less chattering) of the one-bit DT control system.

#### IV. ONE-BIT GPI CONTROLLER

This section illustrates the effectiveness of the proposed approach of designing a one-bit control system by implementing a GPI controller using an experimental prototype of a DC motor.

##### A. DESIGN OF THE GPI CONTROLLER FOR A DC MOTOR

The DC motor presented in this paper is modeled as a first-order system. The transfer function, with control input  $u_t$  and output angular speed  $\psi_t^r$ , is given by

$$\frac{\psi_t^r}{u_t} = \frac{\ell}{\tau_s s + 1} \quad \left( \text{with units of } \frac{\text{rad/s}}{\text{Volt}} \right)$$

where  $\ell$  and  $\tau_s$  denote the DC gain and rise time, respectively. The position control  $\theta_t$  of the DC motor is described by a second-order linear system of the form

$$\mathcal{P}_t : \begin{cases} \dot{x}_t^{p1} &= a x_t^{p1} + b u_t \\ \dot{x}_t^{p2} &= x_t^{p1} \\ y_t^p &= x_t^{p2} \end{cases} \quad (31)$$

where  $a = -1/\tau_s$ ,  $b = \ell/\tau_s$ ,  $x_t^{p1} = \omega_t^r$  and  $x_t^{p2} = \theta_t$ .

The GPI controller is designed so that the closed-loop characteristic polynomial is:

$$1p(s) = s^4 + k_3 s^3 + k_2 s^2 + k_1 s + k_0 = (s^2 + 2\zeta_r \omega_n s + \omega_n^2)^2$$

in which  $k_0 = \omega_n^4$ ,  $k_1 = 4\zeta_r \omega_n^3$ ,  $k_2 = 4\zeta_r^2 \omega_n^2 + 2\omega_n^2$  and  $k_3 = 4\zeta_r \omega_n$ , where  $\omega_n$  and  $\zeta_r$  are design parameters that denote the natural frequency and damping ratio of the transient response, respectively. The GPI controller is designed as  $u_t = u_t^{cy} + u_t^{ce}$ , where  $u_t^{cy}$  and  $u_t^{ce}$  are given by

$$C_t^{cy} : \begin{cases} \dot{x}_t^{c1y} &= -\bar{b} u_t^{cy} \\ u_t^{cy} &= x_t^{c1y} + \bar{a} y_t^p \end{cases} \quad (32a)$$

$$C_t^{ce} : \begin{cases} \dot{x}_t^{c1e} &= \frac{k_0}{b} e_t \\ \dot{x}_t^{c2e} &= x_t^{c1e} - \bar{b} u_t^{ce} + \frac{k_1}{b} e_t \\ u_t^{ce} &= x_t^{c2e} + \frac{1}{b} e_t \end{cases} \quad (32b)$$

where  $\bar{a} = (1 - a^2 - k_2 - ak_3)/b$ , and  $\bar{b} = k_3 + a$ .

From (31)-(32), the closed-loop control system can take the form of (3), such that

$$\begin{aligned} x_t^p &= [x_t^{p1} \ x_t^{p2}]^T, \quad x_t^c = [x_t^{c1y} \ x_t^{c1e} \ x_t^{c2e}]^T, \\ A_p &= \begin{bmatrix} a & 0 \\ 1 & 0 \end{bmatrix}, \quad B_p = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_p &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \quad A_c = \begin{bmatrix} -\bar{b} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -\bar{b} \end{bmatrix}, \end{aligned}$$

$$B_{cy} = \begin{bmatrix} -\bar{b}\bar{a} \\ -k_0/b \\ -(k_1 - \bar{b})/b \end{bmatrix}, B_{ce} = \begin{bmatrix} 0 \\ k_0/b \\ (k_1 - \bar{b})/b \end{bmatrix},$$

$$C_c = [1 \quad 0 \quad 1], \quad d_{cy} = \bar{a} - 1/b \text{ and } d_{cr} = 1/b.$$

Using Euler’s discretization, the discrete-time model for the the position control of the DC motor is given by

$$\mathcal{P}_k : \begin{cases} \rho x_k^{p1} = hax_k^{p1} + hbuk \\ \rho x_k^{p2} = hx_k^{p1} \\ y_k^p = x_k^{p2} \end{cases} \quad (33)$$

Similarly, the GPI controller is designed as  $u_k = u_k^{cy} + u_k^{ce}$  in which  $u_k^{cy}$  and  $u_k^{ce}$  are given by

$$C_k^y : \begin{cases} \rho x_k^{c1y} = -h\bar{b}u_k^{cy} \\ u_k^{cy} = x_k^{c1y} + \bar{a}y_k^p \end{cases} \quad (34a)$$

$$C_k^e : \begin{cases} \rho x_k^{c1e} = h\frac{k_0}{b}e_k \\ \rho x_k^{c2e} = hx_k^{c1e} - h\bar{b}u_k^{ce} + h\frac{k_1}{b}e_k \\ u_k^{ce} = x_k^{c2e} + \frac{1}{b}e_k \end{cases} \quad (34b)$$

where  $e_k$  denotes the output tracking error  $e_k = r - y_k^p$ .

**B. DC MOTOR CONTROL SYSTEM WITH THE ONE-BIT GPI CONTROLLER**

To meet the practical constraints of the ON/OFF power converter and further reduce the computational burden, the controller is implemented using one-bit techniques. Thus, we design (4) such that  $w_t = [u_t \ u_t^{cy} \ u_t^{ce} \ e_t]^T$ ,

$$E = \begin{bmatrix} 0 & \left(\bar{a} - \frac{1}{b}\right) & 1 & 0 & 1 \\ 0 & \bar{a} & 1 & 0 & 0 \\ 0 & -\frac{1}{b} & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}, \text{ and } F = \begin{bmatrix} 1 \\ \frac{1}{b} \\ 0 \\ \frac{1}{b} \\ 1 \end{bmatrix}.$$

Given that  $\hat{e}_t = r - \hat{y}_t^p$  and  $\hat{u}_t = \hat{u}_t^{ce} + \hat{u}_t^{cy}$ , the CT  $\Delta\Sigma$ -Q in (10) is designed to convert its input signals  $\hat{w}_t$  to binary signals  $\delta_t$ , where

$$\hat{w}_t = \begin{bmatrix} \hat{u}_t \\ \hat{u}_t^{cy} \\ \hat{u}_t^{ce} \\ \hat{e}_t \end{bmatrix} \xrightarrow{\Delta\Sigma-Q} \delta_t = \begin{bmatrix} \delta_t^u \\ \delta_t^{cy} \\ \delta_t^{ce} \\ \delta_t^e \end{bmatrix}. \quad (35)$$

The one-bit CT control subsystem counterparts of (31)-(32b) are then expressed as

$$\hat{\mathcal{P}}_t : \begin{cases} \hat{x}_t^{p1} = a\hat{x}_t^{p1} + b\phi \delta_t^u \\ \hat{x}_t^{p2} = \hat{x}_t^{p1} \\ \hat{y}_t^p = \hat{x}_t^{p2} \end{cases} \quad (36)$$

$$\hat{C}_t^y : \begin{cases} \hat{x}_t^{c1y} = -\bar{b}\phi \delta_t^{cy} \\ \hat{u}_t^{cy} = \hat{x}_t^{c1y} + \bar{a}\hat{y}_t^p \end{cases} \quad (37)$$

$$\hat{C}_t^e : \begin{cases} \hat{x}_t^{c1e} = \frac{k_0}{b}\phi \delta_t^e \\ \hat{x}_t^{c2e} = \hat{x}_t^{c1e} + \frac{k_1}{b}\phi \delta_t^e - \bar{b}\phi \delta_t^{ce} \\ \hat{u}_t^{ce} = \hat{x}_t^{c2e} + \frac{1}{b}\phi \delta_t^e. \end{cases} \quad (38)$$

In what follows, we extend our design of one-bit CT control systems to the more realistic case of one-bit DT control systems. It follows from (35) that we can similarly use  $\Delta\Sigma$ -Q in (20) to convert the input signals  $\hat{w}_k$  to binary signals  $\delta_k$ ,

$$\hat{w}_k = \begin{bmatrix} \hat{u}_k \\ \hat{u}_k^{cy} \\ \hat{u}_k^{ce} \\ \hat{e}_k \end{bmatrix} \xrightarrow{\Delta\Sigma-Q} \delta_k = \begin{bmatrix} \delta_k^u \\ \delta_k^{cy} \\ \delta_k^{ce} \\ \delta_k^e \end{bmatrix}. \quad (39)$$

where  $\hat{e}_k = r - \hat{y}_k^p$  and  $\hat{u}_k = \hat{u}_k^{ce} + \hat{u}_k^{cy}$ . The one-bit DT control subsystem counterparts of (33) and (34) are then expressed as

$$\hat{\mathcal{P}}_k : \begin{cases} \rho \hat{x}_k^{p1} = ha\hat{x}_k^{p1} + hb\phi \delta_k^u \\ \rho \hat{x}_k^{p2} = h\hat{x}_k^{p1} \\ \hat{y}_k^p = \hat{x}_k^{p2} \end{cases} \quad (40)$$

$$\hat{C}_k^y : \begin{cases} \rho \hat{x}_k^{c1y} = -h\bar{b}\phi \delta_k^{cy} \\ \hat{u}_k^{cy} = \hat{x}_k^{c1y} + \bar{a}\hat{y}_k^p \end{cases} \quad (41)$$

$$\hat{C}_k^e : \begin{cases} \rho \hat{x}_k^{c1e} = h\frac{k_0}{b}\phi \delta_k^e \\ \rho \hat{x}_k^{c2e} = h\hat{x}_k^{c1e} + h\frac{k_1}{b}\phi \delta_k^e - h\bar{b}\phi \delta_k^{ce} \\ \hat{u}_k^{ce} = \hat{x}_k^{c2e} + \frac{1}{b}\phi \delta_k^e. \end{cases} \quad (42)$$

Next, the simulations and experimental results for the DC motor control system with the one-bit GPI controller are presented in the following section.

**V. RESULTS**

The effectiveness of the proposed formulated one-bit DT control system embedded with the  $\Delta\Sigma$ -Q is illustrated via both a numerical simulation and the real-time implementation of a GPI controller for position control of the DC motor.

**A. GPI DESIGN REQUIREMENT**

The physical parameters for the DC motor are  $\tau_s = 0.023$  sec and  $\ell = 27.3$ . The parameters  $a$  and  $b$  in (31) are accordingly obtained such that  $a = -43.4783$  and  $b = 1.182 \times 10^3$ . The desired response is chosen to satisfy  $\zeta_r = 5$  and  $\omega_n = 42.8$  (rad/sec). As a result, we obtain the GPI controller gains:  $k_0 = 3.3556 \times 10^6$ ,  $k_1 = 1.5680 \times 10^6$ ,  $k_2 = 1.8684 \times 10^5$ ,  $k_3 = 856$  and accordingly  $\bar{a} \approx -128$ .

Substituting the system parameters and GPI controller gains into (31) and (32), respectively, gives  $A$ ,  $B$  and  $C$  in (3), in which  $A$  has five eigenvalues:  $-812.5$  (single),  $-4.3$  (double) and  $-423.7$  (double). Thus, the sampling period  $h$  for the formulated DT control scheme has to satisfy  $0 < h \leq h_M$ , with  $h_M = 2/812.5 = 2.5$  m sec, where the eigenvalues of  $A_d$  are all within the unit circle. From a practical point of view,  $h$  is chosen to be considerably smaller than  $h_M$  (roughly speaking,  $0 < h \leq h_M/5$ ) to

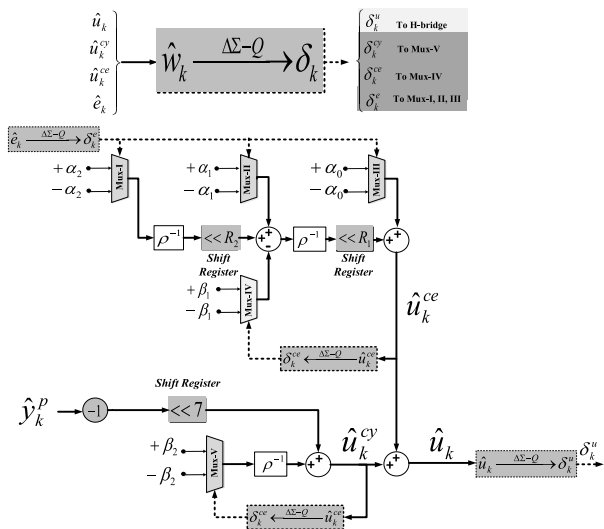


FIGURE 1. Realization of one-bit GPI controller.

avoid high noise or overheating the motor. The reference signal to be tracked is a square wave in which the amplitude alternates between  $\pi$  and  $-\pi$  every 10 sec. Using a numerical simulation, we find that this gives  $\|\mathbf{w}_k\|_\infty = 10.21$ . Hence,  $\phi$  is set to  $\phi = 12$  as per Remark 2.

**B. HARDWARE SAVINGS**

The one-bit GPI realization is shown in Fig. 1, and the other scaled parameters are given in Table 1. It is straightforward to see that  $\alpha_2$  is  $\mathcal{O}(h^2)$  and  $\alpha_1, \beta_1,$  and  $\beta_2$  are all  $\mathcal{O}(h)$  (see Table 1). Therefore, as  $h \rightarrow 0$ , these coefficients, i.e.,  $\alpha_1, \alpha_2, \beta_1,$  and  $\beta_2$ , tend to zero. Representing parameters with very small values demands a long FWL. To alleviate the effects related to sensitivity or overflow, scaling parameters,  $2^{R_1}$  and  $2^{R_2}$  where  $R_1, R_2 \in \mathbb{N}$ , are embedded in the controller realization to scale the sampling period  $h$  and prevent the drawbacks associated with small values [15]. In this work, we let  $R_1 = 0$  and  $R_2 = 8$  at  $h = 1/5k$  sec and  $R_1 = 1$  and  $R_2 = 12$  at  $h = 1/20$  m sec. Additionally,  $\bar{a} \approx -128$  can be replaced by a *sign* converter and a shift register with 7-bits to the right.

Note that the one-bit GPI controller uses  $\Delta\Sigma$ -Qs to convert  $\hat{\mathbf{w}}_k$  to  $\delta_k$  (see Fig. 1). Both the GPI and one-bit GPI controllers are described in the VHSIC Hardware Description Language (VHDL) and then compiled by an Altera DE2-115 FPGA by means of Quartus II. Fig. 2 shows the experimental prototype of the one-bit control DC motor system. Table 2 shows the hardware consumption for both the conventional GPI and one-bit GPI controllers. We successfully design a *multiplier-less* one-bit GPI controller with a high cost savings in terms of hardware with approximately 75% of the DSP elements. As an immediate advantage of one-bit processing, the six multipliers in the conventional GPI controller are replaced by six simple multiplexers in the one-bit GPI controller. Furthermore, the conventional GPI controller requires PWM

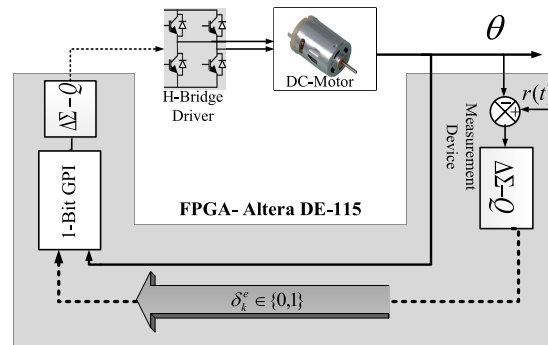


FIGURE 2. The experimental prototype of one-bit control systems.

TABLE 1. One-bit GPI parameters.

Controller parameter	Value
$\alpha_0$	$\phi/b$
$\alpha_1$	$h\phi k_1 2^{R_1}/b$
$\alpha_2$	$h^2\phi k_0 2^{R_2}/b$
$\beta_1$	$h\phi\bar{b}$
$\beta_2$	$-h\phi\bar{b}$

TABLE 2. Hardware Resource Consumption.

General Hardware Resources		
Controller	Multiplier	Multiplexers
Standard GPI	6	0
One-bit GPI	0	6

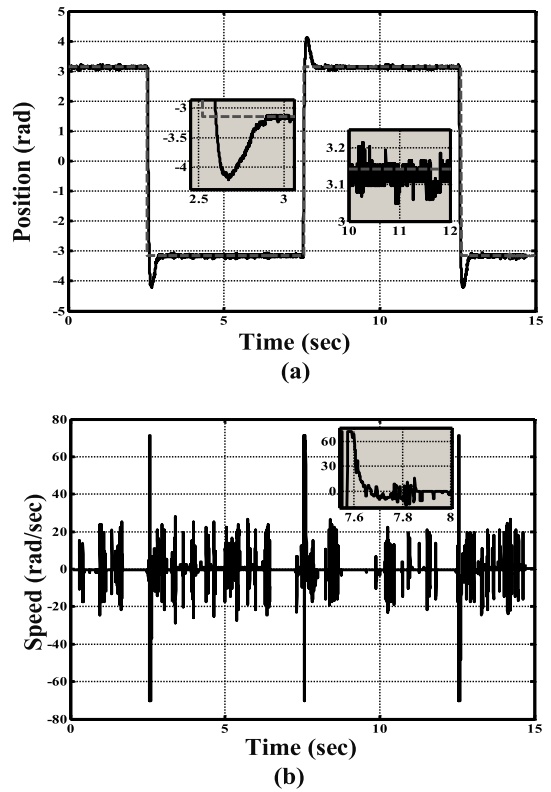
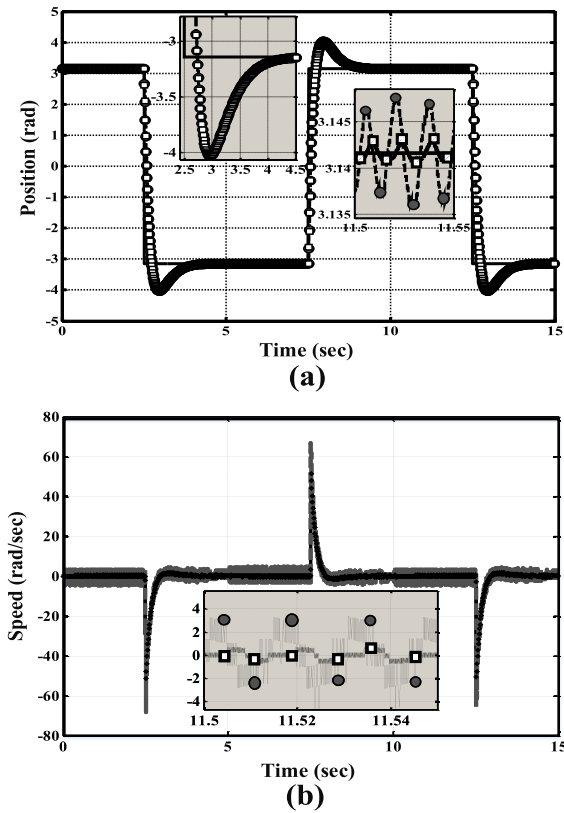
FPGA Hardware Resources		
Controller	DSP - LE	DSP 18 x 18
Standard GPI	48	24
One-bit GPI	12	0

to convert the control signals to switch signals to drive the switch components of the H-bridge. In the one-bit GPI controller, the  $\Delta\Sigma$ -Q replaces the conventional PWM to convert  $\hat{u}_k$  to  $\delta_k^e$  for the same purpose, i.e., control the switch components in the H-bridge (LMD18200). This results in a significant reduction of the overall implementation costs.

**C. SIMULATION RESULTS**

To verify the effectiveness of the one-bit GPI controller in a real-time implementation, a conventional GPI controller is utilized and compared with the one-bit GPI controller with two sampling periods, i.e.,  $5\text{ kHz}$  and  $20\text{ kHz}$ . The comparison is carried out for both types of GPI controllers in terms of performance. Therefore, some specifications of the transient response, e.g., the settling time  $t_s$ , peak time  $t_p$  and overshoot  $M_p$  are considered in this comparison.

Fig. 3 shows the simulation results at sampling frequencies of  $5\text{ kHz}$  and  $20\text{ kHz}$ . For one-bit GPI controllers, we can easily notice the significant reduction in the free oscillations (limit cycles) in both the position and speed responses by increasing the sampling frequency from  $5\text{ kHz}$  to  $20\text{ kHz}$ . Nevertheless, both the conventional and one-bit GPI controllers



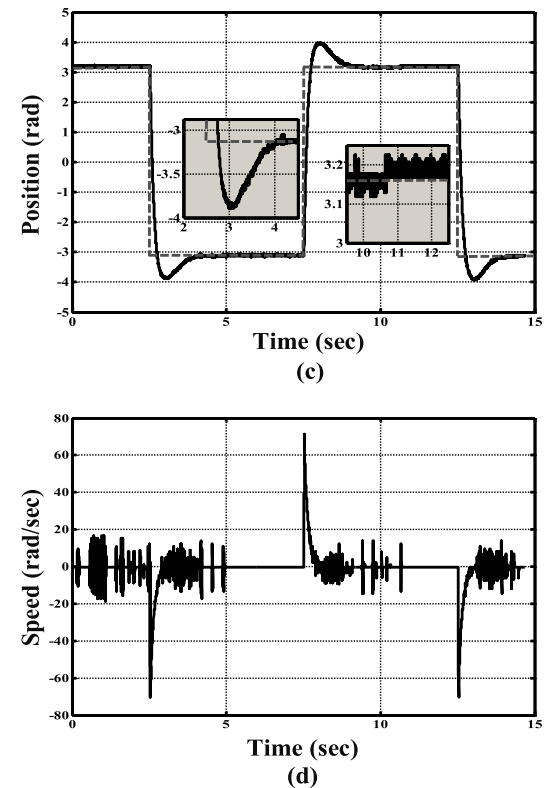
**FIGURE 3.** Simulation results of (a) position control at sampling frequency 5 kHz (dashed line with circle points) and 20 kHz (solid line with square points) and (b) speed response at 5 kHz (gray line with circle points) and 20 kHz (solid line with square points).

exhibit similar specifications during the transient response with  $t_p = 0.4$  sec,  $t_s = 1.7$  sec and  $M_p = 25\%$ .

#### D. EXPERIMENTAL RESULTS

The control system from the signal measurement (shaft position) to the power converter is fully implemented using the FPGA platform. The output of the system, i.e., the shaft position, is estimated using a quadrature encoder attached to the shaft of the DC motor. The results of tracking the reference signal for the GPI controllers with sampling frequencies of 5 kHz and 20 kHz are shown in Fig. 4.

In accordance with Fig. 4(a)-(b), the one-bit GPI controller performs satisfactorily in the tracking performance of the position reference with a 5 kHz sampling frequency. Notice that the one-bit GPI controller exhibits different transient response characteristics than the desired transient response characteristics. As shown in Fig. 4(a), there is a reduction in both the peak time ( $t_p = 0.2$  sec) and settling time ( $t_s = 0.4$  sec) and a higher overshoot with  $M_p = 35\%$ . This implies that the one-bit GPI controller yields high dither (noise) in the system bandwidth, as depicted in the measured angular speed response of the one-bit GPI controller in Fig. 4(b). As shown in Fig. 4(c)-(d), the one-bit GPI controller successfully achieves the desired response with transient response specifications of  $t_p = 0.4$  sec,  $t_s = 1.7$  sec and  $M_p = 25\%$  using a sampling frequency of 20 kHz.



**FIGURE 4.** The results of experiment for (a) position control and (b) speed response for one-bit GPI at 5 kHz. The results of experiment for (c) position control and (d) speed response for one-bit GPI at 20 kHz.

*Remark 3:* From realization and implementation point of view, it is worth noting that proportional and integral (PI) control strategy can be considered as a special



case of the GPI, see [27]–[29]. Hence, the methodology we consider in this work for designing one-bit GPI controller, can be adopted to implement a one-bit PI control strategy.

*Remark 4:* Some pioneering works show that for a given sampling period  $h$ , the PCMs such as  $\Delta\Sigma$ -Q significantly reduce the network bandwidth (data rate) by transmitting a single bit every sample time (i.e., 1 bit/h) over the communication network [9], [33]. Although in this work, we restrict our analysis to the one-bit control system with traditional time-triggered mechanisms (TTMs), i.e., periodic sampling, it is of interest to consider one of the new event-triggered mechanisms (ETMs) [9], [34]–[36]. More precisely, the  $\Delta\Sigma$ -Q can be effectively applied to periodic-event-triggered control, where the event triggering conditions are synchronized to a periodic clock such that data are transmitted and the control law is executed periodically only if the event takes place. Thus, it is important to consider the performance and stability analysis of the control system using an event-triggered  $\Delta\Sigma$ -Q, which will be investigated thoroughly in subsequent work.

## VI. CONCLUSION

In this paper, a generalized one-bit control system has been presented. We have applied ECB-SMC to study the stability and performance of the proposed system motivated by the fact that one-bit control system is a differential inclusion in the sense of Filippov solutions. We show that the binary signals in a one-bit control system are equivalent to their counterpart signals in the conventional control system during ideal sliding motion. The feasibility and effectiveness of the use of  $\Delta\Sigma$ -Qs in both CT and DT control systems, where a multiplier-less one-bit GPI controller structure for realizing Euler's discretization, is demonstrated through a numerical simulation and a real-time implementation using an example of a DC motor. We successfully show that the one-bit control system achieves the desired steady and transient response specifications, while offering significantly high costs savings in terms of hardware resources compared to the standard control system. The results of the simulation and experiment demonstrate that the one-bit control system effectively controls the system and achieves the desired specifications while consuming fewer hardware resources than the standard control system.

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