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Stabilization of Linear Systems Over Networks With Limited Communication Capacity

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ABSTRACT In this paper, we discuss the modeling and control problem for networked systems with access constraints and packet dropouts. Due to the limitation of communication capacity, only the limited number of sensors and actuators are allowed to gain access to network medium according to a stochastic access protocol, and packet dropouts may happen in both backward and forward channels, which can be described by i.i.d Bernoulli processes. Under consideration of the access constraints and packet dropouts, the networked control system can be modelled as a Markov jump system. In such a framework, to solve the controller synthesis problem, the time-varying Kalman filter is first designed. Then, by using the theory of Markovian jump systems and dynamic programming, an optimal controller is designed such that the networked system is exponentially mean-square stable while minimizing the quadratic cost. Finally, two illustrative examples are given to demonstrate the effectiveness of the proposed results.

INDEX TERMS Network-access constraints, packet dropouts, Markov process, stochastic optimal control.

I. INTRODUCTION

Networked control systems (NCSs) are spatially distributed control systems in which the components, like sensors, actuators, and controllers, are connected through a network medium [1]–[7]. In recent years, NCSs have attracted much research interest due to their advantages in practical applications, such as lower cost, high reliability, reduced system power requirement, as well as simpler installation and maintenance. The presence of limited bandwidth network has also given rise to communication constraints for modeling, analysis, and control of NCSs, for example, transmission delays, packet dropouts, quantization effects, which are potential sources of poor performance and instability, see, e.g., [8]–[11] and references therein. Aside from these communication constraints, another fundamental limitation is the so-called medium access constraint, i.e., limitations on the number of sensors and actuators that can be connected to the controller simultaneously for limited network bandwidth [12]–[15]. In such a situation, in order to meet certain performance requirements, the control synthesis of NCSs involves not only stabilizing controller design but also a channel-access policy that determines how to efficiently utilize the limited network resource.

Recently, the control problem of NCSs with medium access constraints has attracted considerable research interests, see in [16]–[18]. To name a few, in [16], the medium access sequence was first considered in a stabilization problem, and the issue was further studied in [17] and [18]. The communication and control co-design problem was solved by first designing a static access protocol and afterwards a controller in [17]. The authors of [18] extended the results to include a ZOH strategy, in which the static access sequences are all obtained by iterative algorithms that guarantee the structural properties of networked systems. Essentially, static access protocols are off-line designed and easily realizable, but they are not adaptable to situations with stochastic disturbances. In contrast, dynamic access protocols can be designed by on-line means and used to handle systems with coupled dynamics robustly. As a consequence, more and more research attention has been paid to dynamic access protocols. For example, [19] and [20] addressed the integrated design of controller and communication sequences for networked systems with dynamic access scheduling via a single-packet and multiple-packet transmission policy, respectively. However, such dynamic access protocols are very conservative and spend too many system resources.

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Apparently, all of the aforementioned results for networked systems subject to access constraints are based on an essential premise, i.e., the sensors and actuators are time-driven and hence schedulable. However, in many networked systems, the medium access status of actuators and sensors is governed by random communication protocols, see e.g., [21], or driven by events featuring certain stochastic processes (e.g., a Markov chain), which could be exemplified by the fire monitoring and extinguishing system of an outdoor timber depositary [22]. In this case, the control synthesis methods for networked systems, which are based on channel-access scheduling technique, become inapplicable. Motivated by this, in this paper, we are interested in investigating controller design of NCSs in which the channel-access status is determined by a stochastic access protocol [23], [24]. At any time instant, a subset of sensors and actuators is allowed to communicate with controller for transmitting measured data and receiving control signal. At the next time, another subset of sensors and actuators is provided with access to the channels. The switching between two successive subsets of sensors and actuators is governed by the stochastic access protocols, which are described by two vector-valued Markov process.

The packet dropout is another important communication constraint considered in this work. Due to node failures and network congestion, data packets might often be lost randomly in networked systems, which is one of the major causes of deterioration in system performance. Up to now, there have been generally two methods for modeling the packet dropout in networked systems, i.e., a binary switching sequence method and a Markovian jump system method. It is more popular that packet dropouts are treated as a binary switching sequence which obeys a Bernoulli distributed white sequence and takes on values of zero and one with certain probability, see e.g., [25]–[29]. To mention some, [26] investigated the robust stability of uncertain discrete-time linear systems subject to input and output quantization and packet loss. In [28], the active resilient control problem was studied for the singular networked control systems with both external disturbances and missing data, which was modeled as Bernoulli distributed white sequence. Reference [29] discussed the variance-constrained state estimation problem for a class of networked multi-rate systems with network-induced probabilistic sensor failures and measurement quantization. The second method is to use a discrete-time linear system with Markovian jumping parameter to represent random packet dropout model of the network. Reference [30] adopted H_∞ norm to analyze the effect of packet dropouts in the feedback loop of a control system. Similar studies along this line can be seen in [31]–[33]. For example, in [31] the mean square stability of event-driven networked systems was discussed in the presence of the Markovian packet losses and quantization, and [33] presented the design of a sliding mode controller for networked control systems subject to successive Markovian packet dropouts.

In response to the above discussion, the modeling and control problem is investigated for networked systems with both a stochastic access protocol and packet dropouts in this paper. The channel-access status of the sensors and actuators is determined by random access protocols, which are described by two independent Markov process with known transition probability matrices. Similar to [25]–[28], we use an i.i.d. Bernoulli process to describe the random packet loss in each channel. Our contribution will be on the comprehensive integration of the optimal controller design and the random access mechanisms for the actuators and sensors. In particular, we will provide a systematic optimal controller design technique which is dependent on the stochastic access protocol and packet dropouts.

The rest of this paper is organized as follows. Section 2 is the problem description and preliminaries. In Section 3, we solve the optimal estimation problem with partial observations. The procedure of optimal controller design is proposed in Section 4. Two illustrative examples are provided in Section 5 to demonstrate the effectiveness of the proposed results. Finally, Section 6 concludes the paper and discusses future research directions.

Notation: The notation used throughout the paper is fairly standard. R^n denotes the n-dimensional Euclidean space and $S > 0\,(\geq 0)$ means that S is real symmetric and positive definite (semi-definite). C_w^v is the combinatorial number that v elements are selected from a total of w elements. X^T and X^{-1} represent the transpose and the inverse of matrix X, and $\mathrm{tr}\,(X)$ denotes the trace of X. $\mathrm{E}\,[\xi]$ stands for the mathematical expectation of the stochastic variable ξ, and $\Pr\{\zeta\}$ means the occurrence probability of the event ζ. Z denotes the set of nonnegative integers. Define the sets $\mathrm{T} = \{0, 1, 2, \cdots, T-1\}$, $\mathrm{N}_1 = \{1, 2, \cdots, N_1\}$ and $\mathrm{N}_2 = \{1, 2, \cdots, N_2\}$, where T, N_1 and N_2 are positive integer.

II. PROBLEM FORMULATION

The framework of NCSs considered in the paper is depicted in Fig. 1, where the plant is a linear time-invariant system described by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + \omega(k), \\ y(k) &= Cx(k) + \upsilon(k), \end{aligned} \tag{1}$$

where $x = [x_1 \cdots x_n]^T \in R^n$ is the system state, $u = [u_1 \cdots u_m]^T \in R^m$ is the control input actually executed by

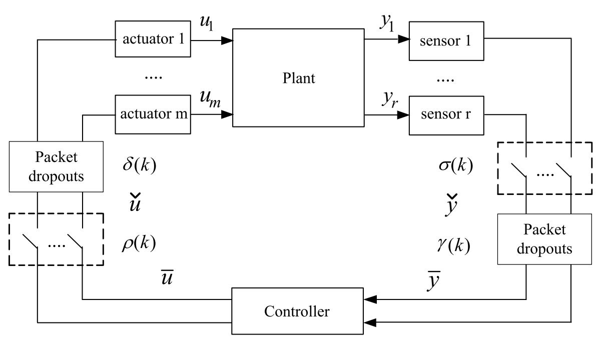

FIGURE 1. A basic NCS configuration.

the actuators, and $y = [y_1 \cdots y_r]^T \in R^r$ is the plant's outputs measured by the sensors. $\omega(k) \in R^n$ and $\upsilon(k) \in R^r$ are zero-mean Gaussian white noise processes with covariances $H > 0$ and $W > 0$ respectively. The unknown initial state x_0 is Gaussian, with known expected value $\bar{x}_0$ and covariance $P_0 \geq 0$. A, B and C are known real constant matrices with appropriate dimensions.

It is assumed that there are r sensors and m actuators in an NCS. For the bandwidth limitation, r sensors share q output channels, $1 \leq q < r$, i.e., the controller can only communicate with q sensors at any time k. Therefore, there are $N_1 = C_r^q$ (N_1 is a natural number) possible channel-access modes for the sensors. At the actuators' side, only p actuators, $1 \leq p < m$, can access the channels to execute certain control actions on the plant, and another p actuators will be assigned the channels in the next sampling period. Similarly, there are $N_2 = C_m^p$ (N_2 is also a natural number) possible channel-access modes for the actuators. Furthermore, packet dropouts may randomly occur during transmission process. Without loss of generality, we assume that the plant's outputs and control signals are transmitted in separate packets to the controller and actuators via multiple communication channels.

A. MEDIUM ACCESS CONSTRAINTS

For the medium access constraints, let the binary-valued function $\sigma_s(k)$ denote channel-access status of sensor s at time k, where $s = 1, \cdots, m$. When sensor s is accessing the channel, i.e., $\sigma_s(k) = 1$, the sensor takes the measurement and sends it to the controller; Otherwise, $\sigma_s(k) = 0$, the measured data is not transmitted, and the controller considers the s-th measurement to be zero. Let $M_\sigma(k) = diag\{\sigma_1(k), \cdots, \sigma_r(k)\}$ denote the random protocol sequence that assigns channel access to r sensors at time k. Then we have

$$\breve{y}(k) = M_\sigma(k)\, y(k). \tag{2}$$

Assume that $M_\sigma(k)$ can be modeled by a Markov process taking matrix values in a finite set $\mathrm{M}_\sigma = \left\{M_\sigma^1, \cdots, M_\sigma^{N_1}\right\}$ with the following conditional probability:

$$\Pr\left\{M_\sigma(k+1) = M_\sigma^h \left| M_\sigma(k) = M_\sigma^s\right.\right\} = \lambda_{sh},$$
$$\Pr\left\{M_\sigma(k) = M_\sigma^s\right\} = \lambda_s(k),$$

where $\lambda = [\lambda_{sh}]$ is the transition probability matrix, and $\lambda_{sh} > 0$, $s, h \in \mathrm{N}_1$, represents the transition probability from mode s to mode h, satisfying $\sum_{h=1}^{N_1} \lambda_{sh} = 1$. $\lambda_s(0)$ is the initial probability of mode s, while $\lambda_s(k)$ is the probability of mode s at time k, and $\lambda(k) = [\lambda_1(k) \cdots \lambda_{N_1}(k)]^T$ obeys the recursion $\lambda(k+1) = \lambda^T \lambda(k)$. For simplicity, let $\tau(k)$ be the indicator of the Markov process, $\tau(k) = s$ if $M_\sigma(k) = M_\sigma^s$.

Similarly, the channel-access status of m actuators is denoted by the random protocol sequence $M_\rho(k) = diag\{\rho_1(k), \cdots, \rho_m(k)\}$. It is also assumed that $M_\rho(k)$ is generated by a Markov process that takes values in another finite set $\mathrm{M}_\rho = \left\{M_\rho^1, \cdots, M_\rho^{N_2}\right\}$ with transition probability matrix $\pi = [\pi_{ij}]$, where $\pi_{ij} = \Pr\left\{M_\rho(k+1) = M_\rho^j \left| M_\rho(k) = M_\rho^i\right.\right\}$, $\sum_{j=1}^{N_2} \pi_{ij} = 1$, $i, j \in \mathrm{N}_2$. Let $\theta(k)$ be the indicator of the Markov process, $\theta(k) = i$ if $M_\rho(k) = M_\rho^i$. We thus have

$$\breve{u}(k) = M_\rho(k)\, \bar{u}(k), \tag{3}$$

where $\breve{u}(k)$ expresses the obtained control vector under the random protocol sequence, and $\bar{u}(k)$ denotes the control vector actually generated by the controller.

B. PACKET DROPOUTS

Random packet dropouts in backward and forward channels, can be modeled as independent Bernoulli processes. At the sensors' side, let the binary-valued random variable $\gamma_s(k)$, $s = 1, \cdots, r$, denote the packet-loss of the s-th channel at time k. When the packet is successfully transmitted to the controller, $\gamma_s(k) = 1$; If the packet is dropped during transmission, we have $\gamma_s(k) = 0$. Assume that any packet can be lost according to an i.i.d. Bernoulli process with the following probability distribution

$$\Pr\{\gamma_s(k) = 1\} = \mathrm{E}[\gamma_s(k)] = \bar{\gamma}_s,$$
$$\Pr\{\gamma_s(k) = 0\} = 1 - \mathrm{E}[\gamma_s(k)] = 1 - \bar{\gamma}_s,$$

where $\bar{\gamma}_s \in [0, 1]$ is a known constant to denote the s-th channel's arrival probability of the measurement data packet. In general, different channels are also independent of each other as their expected values could be different, i.e., $\bar{\gamma}_s \neq \bar{\gamma}_h$, for $s \neq h$. Considering the effect of packet loss, the plant's output packet actually arriving at the controller can be represented as

$$\bar{y}(k) = N_\gamma(k)\, \breve{y}(k) = N_\gamma(k)\, M_\sigma^{\tau(k)} y(k), \tag{4}$$

where $N_\gamma(k) = diag\{\gamma_1(k), \cdots, \gamma_r(k)\}$ is the probability matrix that denotes the successful transmission of measured data.

Similarly, at the actuators' side, we take the binary-valued random variable $\delta_i(k)$, $i = 1, \cdots, m$, as the indicator of packet dropout in the i-th channel, with $\delta_i(k) = 1$ indicating that the packet is successfully transmitted while $\delta_i(k) = 0$ indicating that it is lost during transmission. Assume that $\delta_i(k)$ is governed by another i.i.d. Bernoulli sequence with

$$\Pr\{\delta_i(k) = 1\} = \mathrm{E}[\delta_i(k)] = \bar{\delta}_i,$$
$$\Pr\{\delta_i(k) = 0\} = 1 - \mathrm{E}[\delta_i(k)] = 1 - \bar{\delta}_i,$$

where $\bar{\delta}_i \in [0, 1]$. Based on the above discussions, the control vector actually received by the actuators can be expressed as follows:

$$u(k) = N_\delta(k)\, M_\rho^{\theta(k)} \bar{u}(k), \tag{5}$$

where $N_\delta(k) = diag\{\delta_1(k), \cdots, \delta_m(k)\}$ is the probability matrix that denotes the successful transmission of control vector.

We assume that the current modes of the Markov process $\tau(k)$ and $\theta(k)$ are available at each time k. This assumption is critical, since it allows us to avoid the more difficult and generally unsolved "dual control" problem. For the packet loss, the controller always receives acknowledgments from the actuators that inform the controller whether the control packets are successfully delivered to their respective actuators at the previous time instant. These acknowledgments are often implemented in common network protocols such as the TCP. It is noteworthy that the actuator acknowledgments have one-step time delay. Let I_k denote the information sets available to the controller at time k. Then we have

$$\mathrm{I}_k = \left\{\bar{y}(t), \tau(t), \theta(t), N_\gamma(t), N_\delta(t-1), \bar{u}(t-1), \bar{x}_0 \,|t = 0, 1, \cdots, k\right\}.$$

It is clear that $\mathrm{I}_k \subset \mathrm{I}_{k+1} \subset \mathrm{I}_T$.

C. SYSTEM MODELING

By combining (1), (4) and (5), and taking into account that $M_\rho^{\theta(k)}$ and $N_\delta(k)$ are both diagonal matrices, it is obtained

$$\begin{aligned} x(k+1) &= Ax(k) + B_{\theta(k)} N_\delta(k)\bar{u}(k) + \omega(k), \\ \bar{y}(k) &= N_\gamma(k) C_{\tau(k)} x(k) + \upsilon(k), \end{aligned} \tag{6}$$

where $B_{\theta(k)} = BM_\rho^{\theta(k)}$, $C_{\tau(k)} = M_\sigma^{\tau(k)}C$. From the derivation it is clear to know that $\left\{B_{\theta(k)}; k \in Z\right\}$ and $\left\{C_{\tau(k)}; k \in Z\right\}$ are two independent Markov processes with the transition probability π_{ij} and λ_{sh}. The following definition is needed for the sequel development.

Definition 1[34]: A system in the form of (6) is said to be exponentially mean-square stable if for any initial condition (x_0, τ_0, θ_0) with $\omega(k) = 0$, there exists constants $\alpha > 0$ and $0 < \beta < 1$ such that,

$$\mathrm{E}\left[\|x(k)\|^2 \,|x_0, \tau_0, \theta_0\right] \le \alpha\beta^k \|x_0\|^2, \quad \forall k \ge 0.$$

The purpose of this paper is to provide a systematic framework incorporating the design of state estimator and optimal controller for the presented networked systems with access constraints and partial observations. After systems analysis, our problem is then to derive a control policy $\bar{u}(0), \bar{u}(1), \ldots, \bar{u}(T-1)$, so as to render the systems in (6) exponentially mean-square stable and minimize the following quadratic cost function

$$\begin{aligned} &J(\bar{x}_0, P_0, \theta_0, \bar{u}) \\ &\quad = \mathrm{E}\Big[x^T(T) Q(T) x(T) \\ &\qquad + \sum_{k=0}^{T-1} \Big(x^T(k) Q(k) x(k) \\ &\qquad + \bar{u}^T(k) N_\delta(k) R(k) N_\delta(k) \bar{u}(k)\Big) \,|\bar{x}_0, P_0, \theta_0, \bar{u}\Big], \end{aligned} \tag{7}$$

where $Q(T) \ge 0$, $Q(k) \ge 0$, $R(k) > 0$. The minimal is denoted by $J^*(\bar{x}_0, P_0, \theta_0)$.

III. MAIN RESULTS

A. OPTIMAL STATE ESTIMATION

In this section, we will construct an optimal state estimator for the networked system (6) under deterministic acknowledgment. Let $\hat{x}(k\,|k)$ be the optimal estimate of $x(k)$ based on all available data, $e(k\,|k)$ be the estimation error, and $P(k\,|k)$ be the estimation error covariance, i.e.,

$$\hat{x}(k\,|k) = \mathrm{E}\left[x(k)\,|\mathrm{I}_k\right], \tag{8}$$

$$e(k\,|k) = x(k) - \hat{x}(k\,|k), \tag{9}$$

$$P(k\,|k) = \mathrm{E}\left[e(k) e^T(k)\,|\mathrm{I}_k\right]. \tag{10}$$

Theorem 1: For the system (6), the optimal state estimator is given by

$$\begin{aligned} \hat{x}(k+1\,|k+1) = \hat{x}(k+1\,|k) + L(k+1)\big(\bar{y}(k+1) \\ - N_\gamma(k+1) C_{\tau(k+1)} \hat{x}(k+1\,|k)\big), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \hat{x}(k+1\,|k) &= A\hat{x}(k\,|k) + B_{\theta(k)} N_\delta(k)\bar{u}(k), \quad \hat{x}(0\,|0) = \bar{x}_0, \\ L(k+1) &= P(k+1\,|k) C_{\tau(k+1)}^T N_\gamma(k+1) \\ &\quad \times \big(N_\gamma(k+1) C_{\tau(k+1)} P(k+1\,|k) \\ &\quad \times C_{\tau(k+1)}^T N_\gamma(k+1) + W\big)^{-1}, \end{aligned}$$

with

$$\begin{aligned} &e(k+1\,|k) = x(k+1) - \hat{x}(k+1\,|k) = Ae(k\,|k) + \omega(k), \\ &P(k+1\,|k) = e(k+1\,|k) e^T(k+1\,|k) = AP(k\,|k) A^T + H, \\ &P(k+1\,|k+1) \\ &\qquad = P(k+1\,|k) - L(k+1) N_\gamma(k+1) \\ &\qquad\quad \times C_{\tau(k+1)} P(k+1\,|k). \end{aligned}$$

Proof: Note that the current modes of the Markov process $\tau(k+1)$, $\theta(k+1)$, and the random matrices $N_\gamma(k+1)$, $N_\delta(k)$ are known when the $\hat{x}(k+1\,|k+1)$ is calculated. The theorem can be proved in the similar way as the standard Kalman filter for time-varying linear systems, see, e.g., [35], and references therein.

Remark 1: Based on the Theorem 1, one can know that the optimal estimator is independent of the control input $\bar{u}(k)$. As a consequence, the separation principle still holds true for the networked system (6) with a perfect acknowledgment mechanism, which means that the optimal controller and state estimator can be designed independently. Moreover, the optimal controller is a linear function of the state estimation.

B. STOCHASTIC OPTIMAL CONTROL

In this section, we will solve the optimal control problem in finite-time horizon, and then derive the optimal control law to minimize the quadratic cost function (7). Subsequently, we will prove that the optimal control sequence can also render the system (6) exponentially mean-square stable. To prove Theorem 2, we need some preliminary results presented as follow.

Lemma 1[36]: Let $G, F \in R^{n\times n}$. The following equalities are true:

$$\mathrm{E}\left[e(k\,|k)\hat{x}^T(k\,|k)\,|\mathrm{I}_k\right] = 0, \tag{12}$$

$$\begin{aligned} \mathrm{E}\left[x^T(k) Gx(k)\,|\mathrm{I}_k\right] &= \hat{x}^T(k\,|k) G\hat{x}(k\,|k) \\ &\quad + \mathrm{tr}\left(GP(k\,|k)\right), \quad \forall G \ge 0, \end{aligned} \tag{13}$$

$$\mathrm{E}\left[e^T(k|k)Fe(k|k)|\mathrm{I}_k\right] = \mathrm{tr}\left\{FE\left[e(k|k)e^T(k|k)|\mathrm{I}_k\right]\right\} = \mathrm{tr}(FP(k|k)). \quad (14)$$

Theorem 2: For the system (6), the optimal control law that minimizes the quadratic cost function (7) is given by

$$\bar{u}(k) = K_{\theta(k)}(k)\hat{x}(k|k), \quad (15)$$

and the optimal cost is

$$\begin{aligned} &J^*(\bar{x}_0, P_0, \theta_0) \\ &= \sum_{i=1}^{N_2}\left[\bar{x}_0^T\pi_i(0)S_i(0)\bar{x}_0 + \mathrm{tr}(\pi_i(0)S_i(0)P_0)\right] \\ &\quad + \sum_{k=0}^{T-1}\mathrm{tr}\left(\sum_{i=1}^{N_2}(H\pi_i(k)\bar{S}_i(k+1))\right) \\ &\quad + \sum_{k=0}^{T-1}\mathrm{tr}\left\{\left(\sum_{i=1}^{N_2}A^T\pi_i(k)\right.\right. \\ &\quad \left.\left.\times\bar{S}_i(k+1)A + Q(k) - S_{\theta(k)}(k)\right)\mathrm{E}[P(k|k)]\right\}, \end{aligned} \quad (16)$$

where

$$\begin{aligned} &K_{\theta(k)}(k) \\ &= -\left(\sum_{I\subset\Omega_m}p_IN_I\left(R(k)+B_{\theta(k)}^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\right)N_I\right)^{-1} \\ &\quad\times\bar{N}_\delta B_{\theta(k)}^T\bar{S}_{\theta(k)}(k+1)A, \end{aligned} \quad (17)$$

$$\begin{aligned} &S_{\theta(k)}(k) \\ &= Q(k) + A^T\bar{S}_{\theta(k)}(k+1)A - A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta \\ &\quad\times\left(\sum_{I\subset\Omega_m}p_IN_I\left(R(k)+B_{\theta(k)}^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\right)N_I\right)^{-1} \\ &\quad\times\bar{N}_\delta B_{\theta(k)}^T\bar{S}_{\theta(k)}(k+1)A, \end{aligned} \quad (18)$$

and

$$\begin{aligned} &\bar{S}_{\theta(k)}(k+1) = \sum_{j=1}^{N_2}\pi_{\theta(k)j}S_j(k+1) > 0, \\ &S_{\theta(T)}(T) = Q(T), \quad \theta(k)\in N_2, \\ &\bar{N}_\delta = diag\left\{\bar{\delta}_1,\cdots,\bar{\delta}_m\right\}, \quad k = T-1,\cdots,1,0. \end{aligned}$$

Proof: Define the following optimal value function $V(\theta(k), x(k), k)$:

$$\begin{aligned} &V(\theta(T), x(T), T) \\ &= \mathrm{E}\left[x^T(T)Q(T)x(T)|\mathrm{I}_T\right], \end{aligned} \quad (19)$$

$$\begin{aligned} &V(\theta(k), x(k), k) \\ &= \min_{\bar{u}(k)}\left\{\mathrm{E}\left[x^T(k)Q(k)x(k)+\bar{u}^T(k)N_\delta(k)R(k)N_\delta(k)\bar{u}(k)\right.\right. \\ &\quad \left.\left.+V(\theta(k+1), x(k+1), k+1)|\mathrm{I}_k\right]\right\}. \end{aligned} \quad (20)$$

It is clear that $J^*(\bar{x}_0, P_0, \theta_0) = V(\theta(0), x(0), 0)$. It is claimed that the optimal value function can be written as follows:

$$V(\theta(k), x(k), k) = \mathrm{E}\left[x^T(k)S_{\theta(k)}(k)x(k)|\mathrm{I}_k\right] + \alpha(k), \quad k = T,\cdots,1,0, \quad (21)$$

where

$$\begin{aligned} \alpha(T) &= 0, \\ \alpha(k) &= \mathrm{E}[\alpha(k+1)|\mathrm{I}_k] + \mathrm{tr}\left(H\bar{S}_{\theta(k)}(k+1)\right) \\ &\quad + \mathrm{tr}\left(\left(A^T\bar{S}_{\theta(k)}(k+1)A\right.\right. \\ &\quad \left.\left.+Q(k) - S_{\theta(k)}(k)\right)P(k|k)\right). \end{aligned}$$

The proof of Theorem 2 is developed by deducing that (19), (20) and (21) are equivalent, in which the mathematical induction is employed.

When $S_{\theta(T)}(T) = Q(T)$, $\theta(T)\in N_2$, and $\alpha(T) = 0$, the claim of (21) is true for $k = T$. We suppose that the claim in (21) is true for $k+1$,

$$\begin{aligned} &V(\theta(k+1), x(k+1), k+1) \\ &= \mathrm{E}\left[x^T(k+1)S_{\theta(k+1)}(k+1)x(k+1)|\mathrm{I}_{k+1}\right] + \alpha(k+1). \end{aligned} \quad (22)$$

Then, the value function at time k is

$$\begin{aligned} &V(\theta(k), x(k), k) \\ &= \min_{\bar{u}(k)}\left\{\mathrm{E}\left[x^T(k)Q(k)x(k) + \bar{u}^T(k)N_\delta(k)R(k)N_\delta(k)\bar{u}(k)\right.\right. \\ &\quad \left.\left.+V(\theta(k+1), x(k+1), k+1)|\mathrm{I}_k\right]\right\} \\ &= \min_{\bar{u}(k)}\left\{\mathrm{E}\left[x^T(k)Q(k)x(k)\right.\right. \\ &\quad \left.+\bar{u}^T(k)N_\delta(k)R(k)N_\delta(k)\bar{u}(k)|\mathrm{I}_k\right] \\ &\quad +\mathrm{E}\left[\mathrm{E}\left[x^T(k+1)S_{\theta(k+1)}(k+1)x(k+1)|\mathrm{I}_{k+1}\right]\right. \\ &\quad \left.\left.+\ \alpha(k+1)|\mathrm{I}_k\right]\right\}. \end{aligned} \quad (23)$$

By considering the smoothing property of the conditional expectations

$$\mathrm{E}[V(k+1)|\mathrm{I}_k] = \mathrm{E}[\mathrm{E}[V(k+1)|\mathrm{I}_{k+1}]|\mathrm{I}_k], \quad (24)$$

substituting (24) into (23) leads to

$$\begin{aligned} &V(\theta(k), x(k), k) \\ &= \min_{\bar{u}(k)}\left\{\mathrm{E}\left[x^T(k)Q(k)x(k) + \bar{u}^T(k)N_\delta(k)R(k)N_\delta(k)\right.\right. \\ &\quad \left.\times\bar{u}(k)|\mathrm{I}_k\right] + \mathrm{E}\left[x^T(k+1)S_{\theta(k+1)}(k+1)x(k+1)|\mathrm{I}_k\right] \\ &\quad \left.+\mathrm{E}[\alpha(k+1)|\mathrm{I}_k]\right\}. \end{aligned} \quad (25)$$

Consider that $\theta(k)$ and $N_\delta(k)$ are uncorrelated random variables. According to (6), it is obtained

$$\begin{aligned} &\mathrm{E}\left[x^T(k+1)S_{\theta(k+1)}(k+1)x(k+1)|\mathrm{I}_k\right] \\ &= \mathrm{E}\left[\left(Ax(k) + B_{\theta(k)}N_\delta(k)\bar{u}(k) + \omega(k)\right)^TS_{\theta(k+1)}(k+1)\right. \\ &\quad \left.\times\left(Ax(k) + B_{\theta(k)}N_\delta(k)\bar{u}(k) + \omega(k)\right)|\mathrm{I}_k\right] \\ &= \mathrm{E}\left[x^T(k)A^TS_{\theta(k+1)}(k+1)Ax(k) + 2\bar{u}^T(k)N_\delta(k)B_{\theta(k)}^T\right. \\ &\quad \times S_{\theta(k+1)}(k+1)Ax(k) + 2x^T(k)A^TS_{\theta(k+1)}(k+1)\omega(k) \\ &\quad +\bar{u}^T(k)\times N_\delta(k)B_{\theta(k)}^TS_{\theta(k+1)}(k+1)B_{\theta(k)}N_\delta(k)\bar{u}(k) \\ &\quad +2\bar{u}^TN_\delta(k)B_{\theta(k)}^T\times S_{\theta(k+1)}(k+1)\omega(k) \\ &\quad \left.+\omega^T(k)S_{\theta(k+1)}(k+1)\omega(k)|\mathrm{I}_k\right]. \end{aligned} \quad (26)$$

From the definition of the mathematical expectation, for any measurable functions g and f, the following equation holds

$$E\left[g\left(\omega\left(k\right)\right)f\left(\theta\left(k+1\right)\right)|\mathrm{I}_k\right] = E\left[g\left(\omega\left(k\right)\right)|\mathrm{I}_k\right]\sum\nolimits_{j=1}^{N}\pi_{\theta(k)j}f\left(j\right). \quad (27)$$

Based on (27) and Lemma 1, taking the mathematical expectation on (26) can obtain

$$\begin{aligned}
&\mathrm{E}\left[x^T\left(k+1\right)S_{\theta(k+1)}\left(k+1\right)x\left(k+1\right)|\mathrm{I}_k\right]\\
&= \hat{x}^T\left(k\,|k\right)A^T\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right)\\
&\quad+\mathrm{tr}\left(A^T\bar{S}_{\theta(k)}\left(k+1\right)AP\left(k\,|k\right)\right)\\
&\quad+2\bar{u}^T\left(k\right)\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right)\\
&\quad+2\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)x^T\left(k\right)|\mathrm{I}_k\right]A^T\bar{S}_{\theta(k)}\left(k+1\right)\right\}+\bar{u}^T\left(k\right)\\
&\quad\times\left(\sum\nolimits_{I\subset\Omega_m}p_IN_IB^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}N_I\right)\bar{u}\left(k\right)\\
&\quad+2\bar{N}_\delta\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)\bar{u}^T\left(k\right)|\mathrm{I}_k\right]B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)\right\}\\
&\quad+\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)\omega^T\left(k\right)|\mathrm{I}_k\right]\bar{S}_{\theta(k)}\left(k+1\right)\right\}, \qquad (28)
\end{aligned}$$

where I represents the set of channels in which the data are transmitted successfully, and I is a subset of the index set $\Omega_m = \{1, 2, \ldots, m\}$ with 2^m possible values. $p_I = \prod\limits_{i\in I}\bar{\delta}_i\prod\limits_{i\notin I}\left(1-\bar{\delta}_i\right)$ denotes the probability of $N_\delta(k)$ taking valves in N_I, and N_I is a diagonal matrix with diagonal elements

$$(N_I)_{ii} = \begin{cases}1, & \text{if } i\in I\\ 0, & \text{if } i\notin I\end{cases}.$$

By exploiting the independence of $\omega\left(k\right)$, we can know that $x\left(k\right)$ and $\omega\left(k\right)$, $\bar{u}\left(k\right)$ and $\omega\left(k\right)$ are uncorrelated, then we have

$$\mathrm{E}\left[\omega\left(k\right)x^T\left(k\right)\right]=0,\quad \mathrm{E}\left[\omega\left(k\right)\bar{u}^T\left(k\right)\right]=0. \qquad (29)$$

It follows from (29) that

$$\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)x^T\left(k\right)|\mathrm{I}_k\right]A^T\bar{S}_{\theta(k)}\left(k+1\right)\right\}=0, \qquad (30)$$

$$\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)\bar{u}^T\left(k\right)|\mathrm{I}_k\right]B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)\right\}=0, \qquad (31)$$

$$\mathrm{tr}\left\{\mathrm{E}\left[\omega\left(k\right)\omega^T\left(k\right)|\mathrm{I}_k\right]\bar{S}_{\theta(k)}\left(k+1\right)\right\}=\mathrm{tr}\left(H\bar{S}_{\theta(k)}\left(k+1\right)\right). \qquad (32)$$

Substituting (30), (31), and (32) into (28), one has

$$\begin{aligned}
&\mathrm{E}\left[x^T\left(k+1\right)S_{\theta(k+1)}\left(k+1\right)x\left(k+1\right)|\mathrm{I}_k\right]\\
&= \hat{x}^T\left(k\,|k\right)A^T\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right)\\
&\quad+\mathrm{tr}\left(A^T\bar{S}_{\theta(k)}\left(k+1\right)AP\left(k\,|k\right)\right)\\
&\quad+2\bar{u}^T\left(k\right)\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}A\hat{x}\left(k\,|k\right)\\
&\quad+\bar{u}^T\left(k\right)\left(\sum\nolimits_{I\subset\Omega_m}p_I\right.\\
&\quad\left.\times N_IB^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}N_I\right)\bar{u}\left(k\right)\\
&\quad+\mathrm{tr}\left(H\bar{S}_{\theta(k)}\left(k+1\right)\right). \qquad (33)
\end{aligned}$$

Considering (33) and taking the mathematical expectation on (25), one has

$$\begin{aligned}
&V\left(\theta\left(k\right),x\left(k\right),k\right)\\
&=\min_{\bar{u}(k)}\left\{\hat{x}^T\left(k\,|k\right)\left(Q\left(k\right)+A^T\bar{S}_{\theta(k)}\left(k+1\right)A\right)\hat{x}\left(k\,|k\right)\right.\\
&\quad+\mathrm{tr}\left(Q\left(k\right)P\left(k\,|k\right)\right)\\
&\quad+\mathrm{tr}\left(A^T\bar{S}_{\theta(k)}\left(k+1\right)AP\left(k\,|k\right)\right)\\
&\quad+\mathrm{tr}\left(H\bar{S}_{\theta(k)}\left(k+1\right)\right)+2\bar{u}^T\left(k\right)\bar{N}_\delta\\
&\quad\times B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right)+\bar{u}^T\left(k\right)\\
&\quad\times\left(\sum\nolimits_{I\subset\Omega_m}p_IN\left(R\left(k\right)+B^T_{\theta(k)}\right.\right.\\
&\quad\left.\left.\left.\times\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}\right)N_I\right)\bar{u}\left(k\right)+\mathrm{E}\left[\alpha\left(k+1\right)|\mathrm{I}_k\right]\right\}.
\end{aligned} \qquad (34)$$

Since the control input $\bar{u}\left(k\right)$ is unconstrained, let

$$\frac{\partial V\left(\theta\left(k\right),x\left(k\right),k\right)}{\partial\bar{u}\left(k\right)}=0,$$

which yields

$$\begin{aligned}
&2\bar{N}B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right)\\
&\quad+2\left(\sum\nolimits_{I\subset\Omega_m}p_IN_I\left(R\left(k\right)+B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}\right)N_I\right)\\
&\quad\times\bar{u}\left(k\right)=0.
\end{aligned}$$

Because of $R\left(k\right)>0$, It is clear that

$$\left(R\left(k\right)+B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}\right)>0,$$

so that the optimal control law of (15) can be obtained, which is a linear function of the state estimator. Substituting (15) into the optimal value function $V\left(\theta\left(k\right),x\left(k\right),k\right)$, one has

$$\begin{aligned}
&V\left(\theta\left(k\right),x\left(k\right),k\right)\\
&=\hat{x}^T\left(k\,|k\right)\left(Q\left(k\right)+A^T\bar{S}_{\theta(k)}\left(k+1\right)A\right)\hat{x}\left(k\,|k\right)\\
&\quad+\mathrm{tr}\left(\left(Q\left(k\right)\;+A^T\;\bar{S}_{\theta(k)}\left(k+1\right)A\right)P\left(k\,|k\right)\right)\\
&\quad+\mathrm{tr}\left(H\bar{S}_{\theta(k)}\left(k+1\right)\right)+\mathrm{E}\left[\alpha\left(k+1\right)|\mathrm{I}_k\right]\\
&\quad-\hat{x}^T\left(k\,|k\right)A^T\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}\bar{N}_\delta\\
&\quad\times\left(\sum\nolimits_{I\subset\Omega_m}p_IN_I\left(R\left(k\right)+B^T_{\theta(k)}\right.\right.\\
&\quad\left.\left.\times\bar{S}_{\theta(k)}\left(k+1\right)B_{\theta(k)}\right)N_I\right)^{-1}\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}\left(k+1\right)A\hat{x}\left(k\,|k\right).
\end{aligned} \qquad (35)$$

By using (13) of Lemma 1, we can rewrite $V(\theta(k), x(k), k)$ as

$$\begin{aligned}
&V(\theta(k), x(k), k)\\
&= \mathrm{E}\left[x^T(k)\left(Q(k) + A^T\bar{S}_{\theta(k)}(k+1)A\right)x(k)\,|\mathrm{I}_k\right]\\
&\quad + \mathrm{tr}\left(H\bar{S}_{\theta(k)}(k+1)\right)\\
&\quad + \mathrm{E}\left[\alpha(k+1)\,|\mathrm{I}_k\right] - \mathrm{E}\left[x^T(k)A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\right.\\
&\quad \times \left(\sum\nolimits_{I\subset\Omega_m} p_I N_I\right.\\
&\quad \times \left.\left(R(k) + B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\right)N_I\right)^{-1}\\
&\quad \left.\times \bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)Ax(k)\,|\mathrm{I}_k\right]\\
&\quad + \mathrm{tr}\Big(A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\Big(\sum\nolimits_{I\subset\Omega_m} p_I N_I\Big(R(k)+B^T_{\theta(k)}\\
&\quad \times \bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\Big)N_I\Big)^{-1}\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)AP(k\,|k)\Big)\\
&= \mathrm{E}\Big[x^T(k)\Big(Q(k)+A^T\bar{S}_{\theta(k)}(k+1)A - A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\\
&\quad \times \Big(\sum\nolimits_{I\subset\Omega_m} p_I N_I\left(R(k)+B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\right)N_I\Big)^{-1}\\
&\quad \times \bar{N}_\delta B^T_{\theta(k)} \times \bar{S}_{\theta(k)}(k+1)A\Big)x(k)\,|\mathrm{I}_k\Big]\\
&\quad + \mathrm{tr}\left(H\bar{S}_{\theta(k)}(k+1)\right) + \mathrm{E}\left[\alpha(k+1)\,|\mathrm{I}_k\right]\\
&\quad + \mathrm{tr}\Big(A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\Big(\sum\nolimits_{I\subset\Omega_m} p_I N_I\Big(R(k)+B^T_{\theta(k)}\\
&\quad \times \bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\Big)N_I\Big)^{-1}\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)AP(k\,|k)\Big).
\end{aligned} \tag{36}$$

Let

$$\begin{aligned}
&S_{\theta(k)}(k)\\
&= Q(k)+A^T\bar{S}_{\theta(k)}(k+1)A - A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\\
&\quad \times \Big(\sum\nolimits_{I\subset\Omega_m} p_I N_I\left(R(k)+B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\right)N_I\Big)^{-1}\\
&\quad \times \bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)A,\\
&\alpha(k)\\
&= \mathrm{tr}\left(H\bar{S}_i(k+1)\right) + \mathrm{E}\left[\alpha(k+1)\,|\mathrm{I}_k\right]\\
&\quad + \mathrm{tr}\Big(A^T\bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\bar{N}_\delta\Big(\sum\nolimits_{I\subset\Omega_m} p_I N_I\Big(R(k)+B^T_{\theta(k)}\\
&\quad \times \bar{S}_{\theta(k)}(k+1)B_{\theta(k)}\Big)N_I\Big)^{-1}\bar{N}_\delta B^T_{\theta(k)}\bar{S}_{\theta(k)}(k+1)AP(k\,|k)\Big)\\
&= \mathrm{E}\left[\alpha(k+1)\,|\mathrm{I}_k\right] + \mathrm{tr}\left(H\bar{S}_{\theta(k)}(k+1)\right)\\
&\quad + \mathrm{tr}\left(\left(A^T\bar{S}_{\theta(k)}(k+1)A + Q(k) - S_{\theta(k)}(k)\right)P(k\,|k)\right).
\end{aligned}$$

We can obtain

$$V(\theta(k), x(k), k) = \mathrm{E}\left[x^T(k)S_{\theta(k)}(k)x(k)\,|\mathrm{I}_k\right] + \alpha(k),$$

which indicates that (21) is also satisfied at time k for all of $x(k)$ if and only if the matrices $S_{\theta(k)}(k)$ satisfy (18). From $J^*(x_0, P_0, \theta_0) = V(\theta(0), x(0), 0)$, we can obtain the optimal value function

$$\begin{aligned}
&J^*(x_0, P_0, \theta_0)\\
&= \mathrm{E}\left[x^T(0)S_{\theta(0)}(0)x(0)\right] + \alpha(0)\\
&= \sum\nolimits_{i=1}^{N_2}\left[\bar{x}_0^T\pi_i(0)S_i(0)\bar{x}_0 + \mathrm{tr}\left(\pi_i(0)S_i(0)P_0\right)\right]\\
&\quad + \sum\nolimits_{k=0}^{T-1}\mathrm{tr}\left(\sum\nolimits_{i=1}^{N_2}\left(H\pi_i(k)\bar{S}_i(k+1)\right)\right)\\
&\quad + \sum\nolimits_{k=0}^{T-1}\mathrm{tr}\Big\{\Big(\sum\nolimits_{i=1}^{N_2}A^T\pi_i(k)\bar{S}_i(k+1)A\\
&\quad + Q(k) - S_{\theta(k)}(k)\Big)\mathrm{E}\left[P(k\,|k)\right]\Big\}.
\end{aligned}$$

This completes the proof.

On the basis of Theorem 2, we can then prove that the optimal control law (15) can render the system (6) exponentially mean-square stable.

Theorem 3: The optimal control sequence (15) renders the system (6) exponentially mean square stable when the system (6) has partial observations with the following assumption:

(1) $\omega(k) = 0$, $\upsilon(k) = 0$, $k \in T$;
(2) $\mathrm{tr}\Big(P(k\,|k)A^TC^T_{\tau(k+1)}N_\gamma(k+1)K^T(k+1)\bar{S}_{\theta(k)}$
$\times (k+1)K(k+1)N_\gamma(k+1)C_{\tau(k+1)}A\Big) \le 0$;
(3) $\mathrm{tr}\left(S_{\theta(k)}P(k\,|k)\right) = 0$.

Proof: Define the Lyapunov function

$$V(k) = \hat{x}^T(k\,|k)S_{\theta(k)}(k)\hat{x}(k\,|k),$$

which is positive definite when $\hat{x}(k\,|k) \neq 0$. From (11) we can obtain

$$\begin{aligned}
&\hat{x}(k+1\,|k+1)\\
&= A\hat{x}(k\,|k) + B_{\theta(k)}N_\delta(k)\bar{u}(k) + L(k+1)\\
&\quad \times \left(\bar{y}(k+1) - N_\gamma(k+1)C_{\tau(k+1)}\left(A\hat{x}(k\,|k)\right.\right.\\
&\quad \left.\left. + B_{\theta(k)}N_\delta(k)\bar{u}(k)\right)\right)\\
&= A\hat{x}(k\,|k) + B_{\theta(k)}N_\delta(k)\bar{u}(k) - L(k+1)N_\gamma(k+1)C_{\tau(k+1)}\\
&\quad \times \left(A\hat{x}(k\,|k) + B_{\theta(k)}N_\delta(k)\bar{u}(k)\right)\\
&\quad + L(k+1)\left(N_\gamma(k+1)C_{\tau(k+1)}\right.\\
&\quad \left. \times \left(Ax(k) + B_{\theta(k)}N_\delta(k)\bar{u}(k) + \omega(k)\right) + \upsilon(k+1)\right)\\
&= A\hat{x}(k\,|k) + B_{\theta(k)}N_\delta(k)\bar{u}(k)\\
&\quad + L(k+1)N_\gamma(k+1)C_{\tau(k+1)}Ae(k\,|k)\\
&\quad + L(k+1)N_\gamma(k+1)C_{\tau(k+1)}\omega(k) + L(k+1)\upsilon(k+1).
\end{aligned} \tag{37}$$

Then, we calculate the difference by using (37) and the first and second assumptions of the Theorem 3.

$$\begin{aligned}
&\mathrm{E}\left[V(k+1)\,|\mathrm{I}_k\right] - V(k)\\
&= E\left[\hat{x}^T(k+1\,|k+1)S_{\theta(k+1)}(k+1)\hat{x}(k+1\,|k+1)\,|\mathrm{I}_k\right]\\
&\quad - \hat{x}^T(k\,|k)S_{\theta(k)}(k)\hat{x}(k\,|k)\\
&= \hat{x}^T(k\,|k)A^T\bar{S}_{\theta(k)}(k+1)A\hat{x}(k\,|k)\\
&\quad - \hat{x}^T(k\,|k)S_{\theta(k)}(k)\hat{x}(k\,|k)
\end{aligned}$$

$$
\begin{aligned}
&+\bar{u}^T(k)\left(\sum\nolimits_{I\subset\Omega_m} p_I N_I B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) B_{\theta(k)} N_I\right)\bar{u}(k)\\
&+2\bar{u}^T(k)\bar{N}_\delta B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) A\hat{x}(k|k)\\
&+\mathrm{tr}\Big(P(k|k)A^T C_{\tau(k+1)}^T\\
&\times N_\gamma(k+1)L^T(k+1)\bar{S}_{\theta(k)}(k+1)\\
&\times L(k+1)N_\gamma(k+1)C_{\tau(k+1)}A\Big)\\
&\le -\hat{x}^T(k|k)\Big(K_{\theta(k)}^T\left(\sum\nolimits_{I\subset\Omega_m} p_I N_I R(k) N_I\right)\\
&\times K_{\theta(k)}+Q(k)\Big)\hat{x}(k|k). \qquad (38)
\end{aligned}
$$

Let

$$F_{\theta(k)}=K_{\theta(k)}^T\left(\sum\nolimits_{I\subset\Omega_m} p_I N_I R(k) N_I\right)K_{\theta(k)}+Q(k),$$

and hence

$$
\begin{aligned}
&\mathrm{E}[V(k+1)|\mathrm{I}_k]\\
&\quad\le V(k)-\hat{x}^T(k|k)F_{\theta(k)}\hat{x}(k|k)\\
&\quad\le\left(1-\lambda_{\min}\left(F_{\theta(k)}\right)\left(\lambda_{\max}\left(S_{\theta(k)}\right)\right)^{-1}\right)V(k)\\
&\quad<\left(1-\mu\sigma^{-1}\right)V(k)=\beta V(k), \qquad (39)
\end{aligned}
$$

where $0<\mu<\lambda_{\min}\left(F_{\theta(k)}\right)$, $\sigma>\lambda_{\max}\left(S_{\theta(k)}\right)$. Apparently, $\mu<\sigma$, $0<\beta<1$. Similar to (39), one has

$$\mathrm{E}[V(k)|\mathrm{I}_{k-1}]<\beta V(k-1). \qquad (40)$$

Applying the conditional expectations and smoothing property, we can obtain

$$
\begin{aligned}
\mathrm{E}[V(k)|\mathrm{I}_{k-2}]&=\mathrm{E}[\mathrm{E}[V(k)|\mathrm{I}_{k-1}]|\mathrm{I}_{k-2}]\\
&<\beta\mathrm{E}[V(k-1)|\mathrm{I}_{k-2}]<\beta^2V(k-2).
\end{aligned}
$$

Continuing the process, we finally obtain

$$\mathrm{E}[V(k)]<\beta^kV(0).$$

Considering (13) and using the third assumption of Theorem 3, it is obtained

$$
\begin{aligned}
&\mathrm{E}\left[x^T(k)S_{\theta(k)}(k)x(k)\right]\\
&=\hat{x}^T(k|k)S_{\theta(k)}(k)\hat{x}(k|k)+\mathrm{tr}\left(S_{\theta(k)}(k)P(k|k)\right)\\
&=\mathrm{E}[V(k)], \qquad (41)
\end{aligned}
$$

so

$$
\begin{aligned}
\mathrm{E}\left[x^T(k)S_{\theta(k)}(k)x(k)\right]&<\beta^k\hat{x}^T(0|0)S_{\theta_0}(0)\hat{x}(0|0)\\
&=\beta^kx_0^TS_{\theta_0}(0)x_0,
\end{aligned}
$$

then

$$
\begin{aligned}
&\mathrm{E}\left[x^T(k)x(k)\right]\\
&<\beta^k\left(\lambda_{\min}\left(S_{\theta(k)}(k)\right)\right)^{-1}x_0^TS_{\theta_0}x_0\\
&\le\lambda_{\max}\left(S_{\theta_0}(0)\right)\left(\lambda_{\min}\left(S_{\theta(k)}(k)\right)\right)^{-1}\beta^kx_0^Tx_0\\
&=\alpha\beta^kx_0^Tx_0,
\end{aligned}
$$

where $\alpha=\lambda_{\max}\left(S_{\theta_0}(0)\right)\left(\lambda_{\min}\left(S_{\theta(k)}(k)\right)\right)^{-1}$. Therefore, the system (6) is exponentially mean-square stable according to Definition 1. The proof is thus completed.

Remark 2: If there is no packet dropout during data transmission in both backward and forward channels, the system in (6) can be rewritten as follows:

$$
\begin{aligned}
x(k+1)&=Ax(k)+B_{\theta(k)}\bar{u}(k)+\omega(k),\\
\bar{y}(k)&=C_{\tau(k)}x(k)+\upsilon(k). \qquad (42)
\end{aligned}
$$

It is obvious that the system (42) is a normal Markovian jump system. In view of the previous results, we can derive the following corollaries immediately by using similar lines of the proofs of Theorem 1 and Theorem 2.

Corollary 1: Given the system (42), the optimal state estimator is given by

$$
\begin{aligned}
\hat{x}(k+1|k+1)=\hat{x}(k+1|k)+L(k+1)(\bar{y}(k+1)\\
-C_{\tau(k)}\hat{x}(k+1|k)), \qquad (43)
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{x}(k+1|k)&=A\hat{x}(k|k)+B_{\theta(k)}\bar{u}(k),\\
L(k+1)&=P(k+1|k)C_{\tau(k+1)}^T\Big(C_{\tau(k+1)}P(k+1|k)\\
&\quad\times C_{\tau(k+1)}^T+W\Big)^{-1},
\end{aligned}
$$

with

$$
\begin{aligned}
e(k+1|k)&=x(k+1)-\hat{x}(k+1|k)\\
&=Ae(k|k)+\omega(k),\\
P(k+1|k)&=e(k+1|k)e^T(k+1|k)\\
&=AP(k|k)A^T+H,\\
P(k+1|k+1)&=P(k+1|k)-L(k+1)\\
&\quad\times C_{\tau(k+1)}P(k+1|k).
\end{aligned}
$$

Corollary 2: Given the system (42), the optimal control sequence that minimizes the quadratic cost function

$$
\begin{aligned}
J(x_0,P_0,\theta_0,\bar{u})=\mathrm{E}\Big[&x^T(T)Q(T)x(T)\\
&+\sum\nolimits_{k=0}^{T-1}\Big(x^T(k)Q(k)x(k)\\
&+\bar{u}^T(k)R(k)\bar{u}(k)\Big)|x_0,P_0,\theta_0,\bar{u}\Big], \qquad (44)
\end{aligned}
$$

is given by

$$\bar{u}(k)=K_{\theta(k)}\hat{x}(k|k), \qquad (45)$$

with the optimal cost

$$
\begin{aligned}
&J^*(x_0,P_0,\theta_0)\\
&=\sum\nolimits_{i=1}^{N_2}\left[\bar{x}_0^T\pi_i(0)S_i(0)\bar{x}_0+\mathrm{tr}\left(\pi_i(0)S_i(0)P_0\right)\right]\\
&\quad+\sum\nolimits_{k=0}^{T-1}\mathrm{tr}\left(\sum\nolimits_{i=1}^{N_2}\left(H\pi_i(k)\bar{S}_i(k+1)\right)\right)\\
&\quad+\sum\nolimits_{k=0}^{T-1}\mathrm{tr}\Big[\Big(\sum\nolimits_{i=1}^{N_2}A^T\pi_i(k)\bar{S}_i(k+1)A\\
&\quad+Q(k)-S_{\theta(k)}(k)\Big)P(k|k)\Big], \qquad (46)
\end{aligned}
$$

where

$$K_{\theta(k)}(k) = -\left(R(k) + B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) B_{\theta(k)}\right)^{-1} \times B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) A, \tag{47}$$

$$\begin{aligned} S_{\theta(k)}(k) = {} & Q(k) + A^T \bar{S}_{\theta(k)}(k+1) A \\ & - A^T \bar{S}_{\theta(k)}(k+1) B_{\theta(k)} \Big(R(k) \\ & + B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) B_{\theta(k)}\Big)^{-1} \\ & \times B_{\theta(k)}^T \bar{S}_{\theta(k)}(k+1) A, \end{aligned} \tag{48}$$

and

$$\bar{S}_{\theta(k)}(k+1) = \sum\nolimits_{j=1}^{N_2} \pi_{\theta(k)j} S_j(k+1) > 0,$$
$$S_{\theta(T)}(T) = Q(T), \theta(k) \in N_2, \quad k = T-1, \cdots, 1, 0.$$

IV. NUMERICAL EXAMPLE

To demonstrate the effectiveness of the proposed results, two numerical examples are given.

Example 1: Consider the quadruple-tank process proposed in [37] and the schematic diagram of the process is shown in Fig. 2. The target of the networked controller is to remotely control the levels in Tank 1 and Tank 2 with two pumps. The process inputs are v_1 and v_2 that are input voltages to the pumps, and the outputs are y_1 and y_2 that are voltages from level measurement devices. The linearized state-space equation of the quadruple-tank process is given by

$$\dot{x} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u,$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x.$$

where A_i is the cross-section of Tank i, with the parameter values $A_1 = A_3 = 28cm^2$ and $A_2 = A_4 = 32cm^2$, α_i is the cross-section of outlet hole i, with the parameter values $\alpha_1 = \alpha_3 = 0.071cm^2$ and $\alpha_2 = \alpha_4 = 0.057cm^2$. The voltage applied to Pump i is v_i and the corresponding flow is $k_i v_i$. The parameters $\gamma_1, \gamma_2 \in (0, 1)$ are determined from the values set prior to an experiment. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1-\gamma_1) k_1 v_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted by g, with the parameter values $g = 981cm/s^2$. The time constants are, respectively, $T_1 = 63s^{-1}$, $T_2 = 91s^{-1}$, $T_3 = 39s^{-1}$, $T_4 = 56s^{-1}$. The measured parameter is $k_c = 0.5V/cm$.

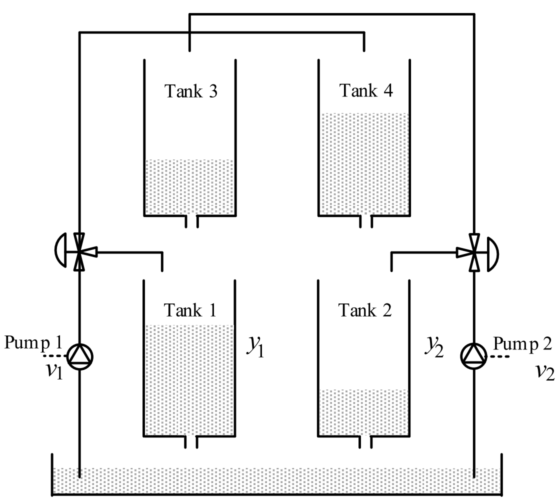

FIGURE 2. Schematic diagram of the quadruple-tank process.

We take sampling period as 2s, then the discrete-time dynamics are governed by the following parameters:

$$A = \begin{bmatrix} 0.9688 & 0 & 0.0492 & 0 \\ 0 & 0.9783 & 0 & 0.0347 \\ 0 & 0 & 0.9500 & 0 \\ 0 & 0 & 0 & 0.9460 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.1639 & 0.0024 \\ 0.0001 & 0.1243 \\ 0 & 0.0933 \\ 0.0061 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5000 & 0 \\ 0 & 0.5000 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T.$$

Our objective is to derive an optimal state estimator and feedback controller to minimize the quadratic cost function in (7) and render the system in (6) exponentially mean-square stable. Assuming that packet arrival probabilities of the backward and forward channels are

$$\bar{N}_\delta = \begin{bmatrix} 0.85 & 0 \\ 0 & 0.82 \end{bmatrix}, \quad \bar{N}_\gamma = \begin{bmatrix} 0.78 & 0 \\ 0 & 0.86 \end{bmatrix}.$$

We consider the worst-case scenario that $p = 1$ and $q = 1$, i.e., only one actuator and one sensor can access to channels at any time. Then, the actuators and sensors access sequences set are

$$\{M_\rho^1, M_\rho^2\} = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\},$$
$$\{M_\sigma^1, M_\sigma^2\} = \left\{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right\}.$$

In the simulation, we assume the following transition matrices for the Markov chains in (2) and (3), respectively

$$\lambda = \begin{bmatrix} 0.7775 & 0.2225 \\ 0.2253 & 0.7747 \end{bmatrix}, \quad \pi = \begin{bmatrix} 0.6975 & 0.3025 \\ 0.3150 & 0.6850 \end{bmatrix}.$$

We take the expected value of initial state x_0 as $\bar{x}_0 = [1 -1\ 3\ 4]^T$ with covariance $P_0 = 0.2I_{4\times4}$. The Gaussian white noise terms in (1) are $\omega(\cdot) \sim N(0, 0.02I_{4\times4})$ and $\upsilon(\cdot) \sim N(0, 0.02I_{2\times2})$. We formulate the optimal control problem with the state weighting matrix $Q = 4I_{2\times2}$, and the control weighting matrix $R = I_{2\times2}$. According to Theorem 2, the optimal control sequence is obtained, which minimizes the quadratic cost function in (7). The activation mode sequences generated for the sensors and the actuators

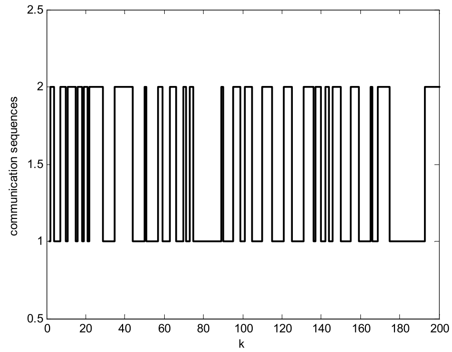

FIGURE 3. Activation mode sequence of sensors.

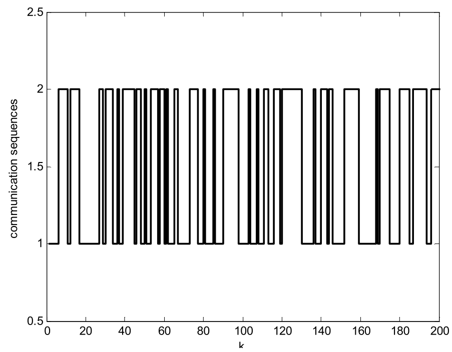

FIGURE 4. Activation mode sequence of actuators.

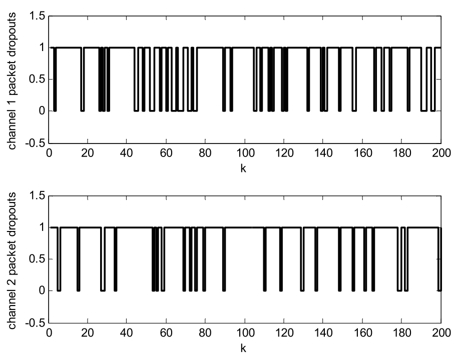

FIGURE 5. Packet dropouts of sensor channels.

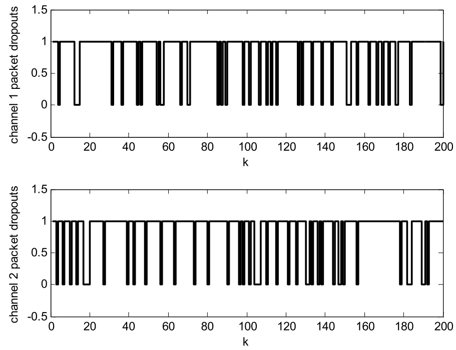

FIGURE 6. Packet dropouts of actuator channels.

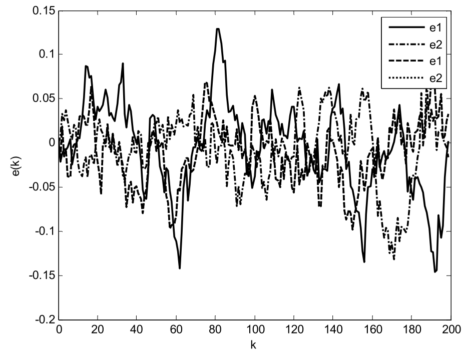

FIGURE 7. Dynamics of estimation errors.

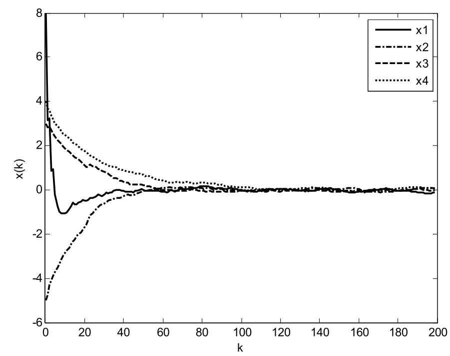

FIGURE 8. State trajectories.

are shown in Fig. 3 and Fig. 4, where '1' and '2' in the y-axis denote the working modes. The data packet dropouts of the sensors and actuators are shown in Fig.5 and Fig.6, where '1' and '0' in the y-axis denote cases of the packet arrival and packet loss, respectively. The state estimation error of Kalman filter is shown in Fig.7. The state trajectories are depicted in Fig.8, which have indicated that the quadruple-tank process is stable and that our control goal is achieved.

Under the same initial conditions, we take the packet arrival probabilities of the backward and forward channels as

$$\bar{N}_\delta = \bar{N}_\gamma = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix},$$

where the value of parameters α varies from 0.5 to 1. Simulation curve of the quadratic performance index with different arrival probability of the measurement and control data packet is provided in Fig.9, which indicates that control performance can be improved with the increase of data packet arrival rate.

In order to analyze the relationship of the sensors' channel access probability and the control performance, we choose the sensors' channel access probability matrix as follows:

$$\lambda = \begin{bmatrix} \varsigma & 1-\varsigma \\ \varsigma & 1-\varsigma \end{bmatrix},$$

where the value of parameters ς varies from 0.01 to 1. Fig.10 gives the quadratic performance index of networked system with different ς. Similarly, we select the actuators'

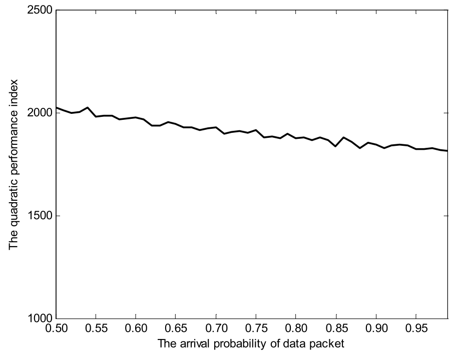

FIGURE 9. The quadratic performance index with different data packet's arrival probability.

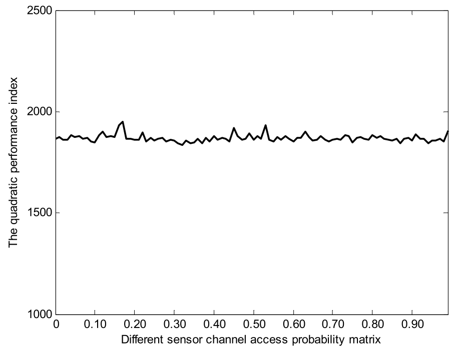

FIGURE 10. The quadratic performance index with different sensor channel access probability matrix.

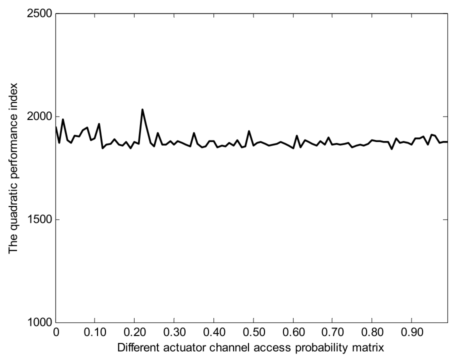

FIGURE 11. The quadratic performance index with different actuator channel access probability matrix.

channel access probability matrix as

$$\pi = \begin{bmatrix} \varphi & 1-\varphi \\ \varphi & 1-\varphi \end{bmatrix},$$

where the value of parameters φ varies from 0.01 to 1. Fig.11 gives the quadratic performance index under different φ. It can be seen from Fig.10 and Fig.11 that the channel access probability matrices only determine the currently selected communication channel of sensors or actuators, but have little effect on the control performance of the networked control system.

Example 2: Consider an unstable plant given by the following discrete-time state-space description:

$$A = \begin{bmatrix} 0.1600 & -1.2005 \\ -1.1042 & -0.8890 \end{bmatrix}, \quad B = \begin{bmatrix} -1.5350 & 1.8918 \\ -1.2902 & -1.6869 \end{bmatrix},$$
$$C = \begin{bmatrix} 1.0000 & 0.0000 \\ 0.0000 & 1.0000 \end{bmatrix}.$$

The eigenvalues of A are 0.9007 and -1.6297, thus the system is open-loop unstable. In the simulations, the parameters of packet arrival probabilities and access sequences are same with Example 1. The expected value of initial condition of the simulation is $\bar{x}_0 = [7-6]^T$ with covariance $P_0 = 0.2I_{2\times2}$ and the Gaussian white noise terms in (1) are $\omega(\cdot) \sim N(0, 0.02I_{2\times2})$ and $\upsilon(\cdot) \sim N(0, 0.02I_{2\times2})$. The cost function given in (7) is defined by $Q = 4I_{2\times2}$ and $R = I_{2\times2}$ respectively. According to Theorem 2, the optimal control sequence is obtained, and the state trajectories are depicted in Fig.12.

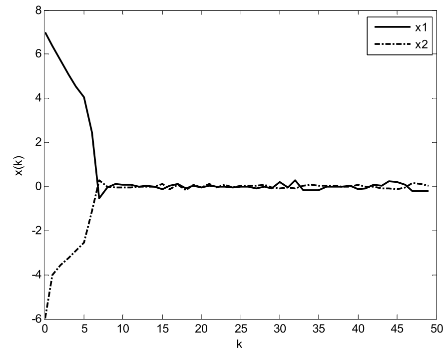

FIGURE 12. State trajectories.

For a comparison with the existing results, we use the control scheme of [38] under the same initial conditions. The state trajectories are depicted in Fig.13, and a summary of performance index is also given in Table 1. For the sake

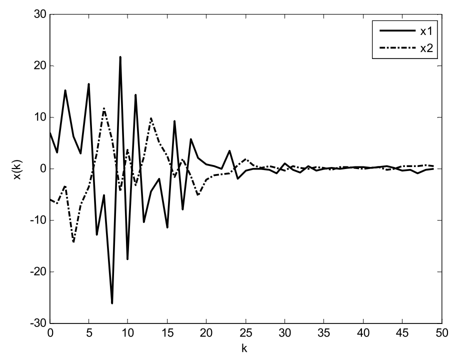

FIGURE 13. State trajectories in [38].

TABLE 1. Comparison of quadratic performance costs.

Control scheme	State index	Control index	The quadratic performance index
Our result	0.3722e+03	0.0040e+03	0.3758e+04
Guo [38]	3.7047e+03	0.7767e+03	4.4814e+04

of comparison, the performance index in the Table 1 does not take into account the impact of packet dropouts, which is corresponding to [38]. It can be seen from Table 1 that the results in this paper not only guarantee the networked system is exponentially mean square stable, but also have better performance than that of [38].

V. CONCLUSION

Motivated by applications where observation and control are performed over a multichannel network, this paper has investigated the modelling and control problem for networked systems with two-sided network access constraints and packet dropouts. We implement two independent Markov random access protocols to assign channel access to the sensors and actuators, and model packet dropouts as i.i.d Bernoulli processes where control packet acknowledgement is always available to the controller. In such a framework, we first compute optimal state estimator by utilizing the time-varying Kalman filter. Then, we provide an optimal controller design methodology to satisfy the quadratic cost function and guarantee the mean square exponential stability of NCSs based on the theory of Markovian jump systems and dynamic programming. Simulation results are given to demonstrate the effectiveness of the proposed method.

It is noted that the results presented in this paper are all for the finite horizon case. Future work will involve extending the controller synthesis framework of NCSs to the infinite horizon field. In particular, we will study the convergence of Modified Algebraic Riccati Equations for the controller and estimator respectively, in the presence of random access protocol and packet dropouts. Another interesting issue worthy of investigation is the case where the channel-access status of the nodes is governed by random access protocols, with taking into simultaneous consideration other network effects such as network-induced delays, data quantization, etc.

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