IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received July 18, 2019, accepted August 11, 2019, date of publication August 19, 2019, date of current version September 5, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2936286

High-Accuracy and Low-Complexity DOA Estimation Algorithm for Transmit-Only Diversity Bistatic MIMO Radar

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This work was supported in part by the Chongqing Development and Reform Commission for ''High dynamic range light intensity sensor chip development and testing project'' under Grant1020708020170011, in part by the National Nature Science of Foundation of China under Grant 61501065, Grant 61701054, in part by the Fundamental Research Funds for the Central Universities of China under Grant 106112017CDJQJ168817, Grant 2018CDXYTX0009, and Grant 2018CDYJSY0055.

ABSTRACT This paper proposes TOD-MUSIC algorithm to improve the Direction of Arrival (DOA) estimation accuracy under low Signal-to-Noise Ratio (SNR) in transmit-only diversity bistatic Multiple-Input Multiple-Output (MIMO) radar. Firstly, the TOD-MUSIC algorithm uses Toeplitz matrix reconstruction to make it possible to process the correlated sources. Then it uses the linear operation to reduce the computational complexity. Finally, the physical properties of spectral peak (which are not continuous functions and approach a larger value at the DOAs) is utilized to make the spectral peak more obvious. It is proved that the TOD-MUSIC algorithm can effectively improve the DOA estimation accuracy. We reveal that the TOD-MUSIC algorithm can not only process correlated sources, but also reduce calculation amount by solving the DOA estimation ambiguity problem under the under-sampling. The simulation results show that: the TOD-MUSIC algorithm makes the DOA estimation accuracy improved by 2° at low SNR, approaching to MUSIC algorithm at high SNR.

INDEX TERMS Bistatic MIMO radar, DOA, TOD-MUSIC, MUSIC.

I. INTRODUCTION

Transmit-only diversity bistatic MIMO radar uses emitter to transmit multiple signals, as well as the backscattered signals of targets are received by receiver, which has received extensive attention in recent years [1]–[5]. The Direction of Arrival (DOA), Direction of Departure (DOD) estimation and Doppler frequency are the core issues of MIMO radar [6], [7]. Many methods have been proposed for DOA estimation, which can be divided into traditional methods, subspace decomposition methods, and compressive sensing methods. Traditional methods include Conventional Beam Forming (CBF) Method [8], Maximum Entropy Method (MEM) [9], and Minimum Variance Method (MVM) [10], etc.

The associate editor coordinating the review of this article and approving it for publication was Chow-Yen-Desmond Sim.

The compressive sensing methods for DOA estimation accuracy have a high performance at low SNR. It is a sparse representation problem over a redundant dictionary, which is also a Non-deterministic Polynomial-time hard (NP hard) problem [11]. In order to solve this problem, some effective algorithms are proposed, including Matching Tracking (MP) [11], Orthogonal Matching Tracking (OMP) [12], and Compressed Sample Matching Tracking (CSMP) [13], etc. However, the performance of these algorithms is greatly reduced in dealing with adjacent signal sources. Thus the reference [14] proposes the Focused Orthogonal Matching Tracking (FOMP) algorithm based on the OMP algorithm, which can effectively distinguish two adjacent signal sources. In addition, the FOMP algorithm was extended to the three-dimensional algorithm by the reference [15] to propose the Three Dimensional Focused Orthogonal Matching Tracking (3D-FOMP) algorithm.

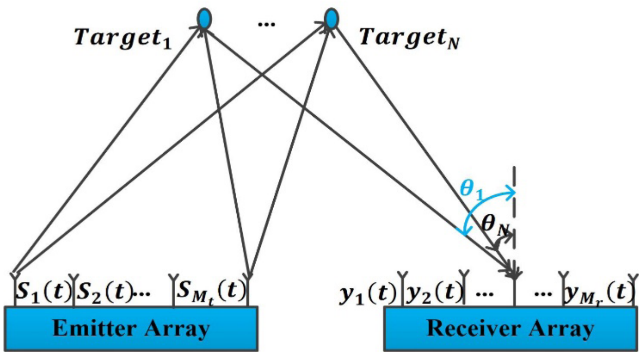

FIGURE 1. Bistatic MIMO radar model.

The subspace decomposition has been widely used due to its high accuracy [16]–[18]. The subspace decomposition method can be divided into Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) algorithm [19]–[21] and Multiple Signal Classification (MUSIC) algorithm [22]–[25]. ESPRIT algorithm does not need peak search. The amount of calculation is reduced, while more space resources or time resources are consumed [19], [21]. MUSIC algorithm does not cause a decrease in the freedom degree of receiving array, but it has the disadvantage of lowering the DOA estimation accuracy at low SNR [1], [23], [25].

Therefore, the TOD-MUSIC algorithm is proposed to solve the problem in the transmit-only diversity bistatic MIMO radar. The algorithm principle is organized as follows: firstly, Toeplitz matrix reconstruction is used to pre-processes the covariance matrix and make it possible to process the correlated sources. Secondly, in the far-field narrow-band condition, the incident signal sources are statistically independent, so the columns of direction matrix are full rank and the N rows of the direction matrix are linearly independent. Thus linear operations can be used instead of eigenvalue decomposition to reduce the computational complexity of the algorithm. Finally, the physical properties of spectral peak (which are not continuous functions and approach a larger value at the DOAs) are utilized to improve the DOA estimation accuracy.

The remaining of the paper is organized as follows: the transmit-only diversity bistatic MIMO radar channel model is introduced in Section 2.In Section 3, the principle of TOD-MUSIC is introduced, and it is proved that it can effectively improve the DOA estimation accuracy. The flow of the algorithm is introduced in Section 4. In Section 5, five comparative simulation were done on the MUSIC algorithm and the TOD-MUSIC algorithm, including de-correlating simulation, angle accuracy simulation, sampling simulation, and SNR simulation and complexity analysis. Some conclusions are provided in Section 6.

II. THE TRANSMIT-ONLY DIVERSITY BISTATIC MIMO RADAR CHANNEL MODEL

In Fig. 1, the emitter and receiver array elements spacing are d_t and d_r respectively; The number of emitter and receiver array elements are M_t and M_r respectively; the number of targets are N; The direction vectors of the emitter and receiver array are $A_t(\theta)$ and $A_r(\theta)$ respectively.

$$A_t(\theta) = [a_t(\theta_1)\, a_t(\theta_2) \cdots a_t(\theta_N)]^T, \tag{1}$$
$$A_r(\theta) = [a_r(\theta_1)\, a_r(\theta_2) \cdots a_r(\theta_N)]^T, \tag{2}$$

where,

$$a_t(\theta_t) = [1 e^{-j2\pi \frac{d_t}{\lambda} sin\theta_t} \cdots e^{-j2\pi(M_t-1)\frac{d_t}{\lambda} sin\theta_t}]^T, \tag{3}$$
$$a_r(\theta_r) = [1 e^{-j2\pi \frac{d_r}{\lambda} sin\theta_r} \cdots e^{-j2\pi(M_r-1)\frac{d_r}{\lambda} sin\theta_r}]^T. \tag{4}$$

The targets are assumed to consist of multiple small scatterers distributed in a one-dimensional region and considered by a MIMO radar model that transmit-only diversity. The targets have N independent isotropic scatterers, each scatterer is modeled as a zero mean, unit variance, independent and identically distributed circular symmetric complex Gaussian random variable [26]–[29]. Thus, the targets reflection can be expressed as a diagonal array (Equation 5).

$$\lambda = \frac{1}{\sqrt{N}} diag\{\lambda_0, \lambda_1, \cdots, \lambda_{N-1}\}, \tag{5}$$

the response vector from the signal emitted by the k^{th} emitter element to the independent scatterer is expressed as

$$g_k = \left[e^{-j2\pi f_c \tau_{k,1}^t} e^{-j2\pi f_c \tau_{k,2}^t} \cdots e^{-j2\pi f_c \tau_{k,N}^t}\right], \tag{6}$$

where $\tau_{k,n}^t$ represents the transmission delay between the k^{th} emitter element and the n^{th} scatterer, and the response vector of the n^{th} scatterer to the receiver array is expressed as

$$k_n = \left[e^{-j2\pi f_c \tau_{1,n}^r} e^{-j2\pi f_c \tau_{2,n}^r} \cdots e^{-j2\pi f_c \tau_{r,n}^r}\right], \tag{7}$$

where $\tau_{i,n}^r$ represents the transmission delay between the n^{th} scatterer and the i^{th} receive element. The direction matrix of emitter is defined as

$$H_t = \left[g_1 \quad g_2 \quad \cdots g_k \cdots \quad g_{M_t}\right] \in C^{N \times M_t}, \tag{8}$$

the direction matrix of receiver is defined as

$$H_r = [k_1 \quad k_2 \quad \cdots k_i \cdots \quad k_N]\, C \in^{M_r \times N}, \tag{9}$$

therefore, the statistical MIMO radar channel matrix is defined as [30]–[33]

$$H = H_r \lambda H_t C \in^{M_r \times M_t}. \tag{10}$$

In order to make the targets irradiated from different directions, the column vectors of emission direction matrix should satisfy the orthogonality condition (Equation 11).

$$\langle g_k, g_l \rangle = 0, \quad k \neq l; k, l = 1, \cdots, M_t. \tag{11}$$

Equation (11) shows that any two unequal g_k should satisfy the orthogonal relationship. Thus, in order to reduce the complexity of model, the orthogonality between the response vectors introduced by adjacent emitter elements is considered as [28], [34].

$$\langle g_k, g_{k+1} \rangle = 0, \quad k = 1, \cdots, M_t. \tag{12}$$

The target scatterers are assumed to be a uniform array that is parallel to the emitter and receiver arrays [31]–[33]. In this case, the response vector of scatterer array by the k^{th} array element transmitting is expressed as

$$g_k = \left[1 \quad e^{-j2\pi\Delta/\lambda_c sin\theta_k^t} \cdots e^{-j2\pi(N-1)\Delta/\lambda_c sin\theta_k^t}\right]^T, \tag{13}$$

where θ_k^t represents the angle of arrival from the k^{th} emitter element to the target, and Δ is the element spacing of scatterer array. Thus the equation (12) can be expressed as

$$\langle g_k, g_{k+1}\rangle = \sum\nolimits_{i=0}^{N-1} e^{j2\pi\left(sin\theta_{k+1}^t - sin\theta_k^t\right)} \frac{i\Delta}{\lambda_c} = 0, \tag{14}$$

$N \to \infty$, equation (14) can be expressed as

$$\frac{d_t\Delta}{\lambda_c R_t} \geq \frac{1}{N}, \tag{15}$$

where R_t represents the distance between the emitter array and the target. The receiving array is assumed to be a uniform linear array of $d_r = \lambda_c/2$. Moreover, the echo signals from the target scatterers are assumed to be unresolved by the receiving array. Thus the receiving direction matrix can be expressed as

$$H_r = a_r(\theta_r)\, 1_N^T, \tag{16}$$

where 1_N^T epresents an all-one vector of $N\times 1$ dimensions. Thus the channel matrix of a MIMO radar that only transmits diversity can be expressed as [32], [33]

$$H_r = a_r(\theta_r)\, \alpha^T, \tag{17}$$

where $\alpha = (1_N^T \lambda H)^T \in C^{M_t\times 1}$ is random fading vector. From the orthogonality of the emission vector g_k, α is zero-mean, unit-variant, independent and identically distributed circularly symmetric complex Gaussian random variables [27], [28].

Equation (17) is proved by reference [34] that is extreme situation without angular expansion at the receiving array elements. Thus the transmit-only diversity bistatic MIMO radar model is shown in Fig. 2.

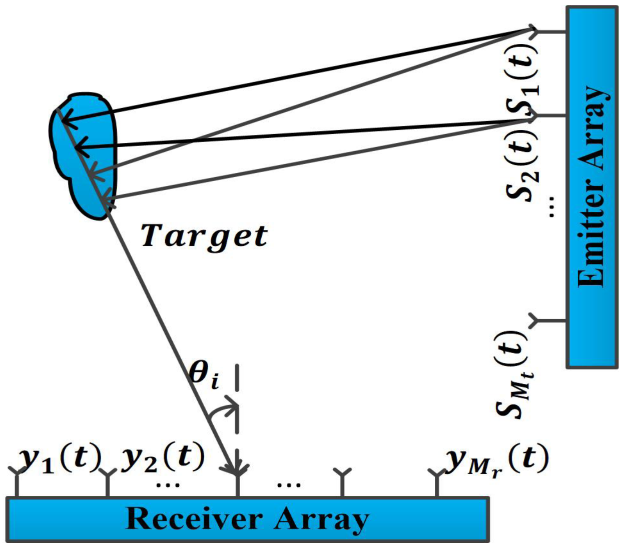

FIGURE 2. Transmit-only diversity bistatic MIMO radar model.

III. THE PRINCIPLE OF TOD-MUSIC ALGORITHM

Under additive white Gaussian noise, the spatial covariance matrix of receiver elements of transmit-only diversity bistatic MIMO radar can be expressed as

$$R_{yy} = E\left\{y(n)\, y^H(n)\right\} = APA^H + \sigma_v^2 I_{M_r}, \tag{18}$$

the *(k, l)*th element of matrix P is expressed as

$$P_{k,l} = E\left\{\alpha_l^H R_{ss}\alpha_k\right\}, \tag{19}$$

where R_{ss} is the covariance matrix of emitter signal $s(n)$,

$$R_{ss} = E\left\{s(n)\, s^H(n)\right\}, \tag{20}$$

the emitter signal is assumed to be a quadrature signal, so

$$R_{ss} = \sigma_s^2 I_{M_t}, \tag{21}$$

and the target fading should satisfies the independent condition of equation (22) and equation (23).

$$E\left\{\alpha_{k,p}^*\alpha_{l,q}\right\} = 0, \quad l \neq k, \tag{22}$$

$$\mathrm{E}\left\{\alpha_l^H\alpha_k\right\} = \begin{cases} E\left\{\|\alpha_l\|^2\right\}, & l = k \\ 0, & l \neq k, \end{cases} \tag{23}$$

therefore,

$$P_{k,l} = E\left\{\alpha_l^H \Lambda \alpha_k\right\} = \begin{cases} \sigma_s^2 M_t, & l = k \\ 0, & l \neq k, \end{cases} \tag{24}$$

$$P = \sigma_s^2 M_t I_L, \tag{25}$$

equation (25) was brought into equation (18), the covariance matrix of receiver elements is expressed as

$$R_{yy} = A\sigma_s^2 M_t I_L A^H + \sigma_v^2 I_{M_r}. \tag{26}$$

The Toeplitz matrix R_T is defined to approximate the covariance matrix R_{yy},

$$\min_{R_T\in S_T} \left\| R_T - R_{yy}\right\|, \tag{27}$$

where S_T is the Toeplitz matrix set, $\|\cdot\|$ is the measure distance. Equation (27) is found to be the minimum and the optimal solution R_t of R_T can be expressed as

$$R_t = Toep\left[Z_{1T}, Z_{2T}\cdots Z_{(2N-1)T}\right], \tag{28}$$

$$Z_{iT} = (N-i+1)^{-1}\sum\nolimits_{j=1}^{N-i+1} r_{(j(j+i-1))T}, \quad i=1,\cdots,N, \tag{29}$$

where $r_{(j(j+i-1))T}$ represents the j^{th} row and $(j+i-1)^{th}$ column of R_T [35].

The incident signal sources are far-field narrow-band and statistically independent, so the columns of direction matrix are full rank and the N rows of the direction matrix are linearly independent. In engineering applications, since the direction matrix of the antenna array is unknown, the covariance matrix is generally used instead of the direction matrix, and the matrix R_t can be divided into R_1 and R_2 [36]–[39].

$$R_t = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \tag{30}$$

where R_1 is $N \times M_r$ dimension, R_2 is $(M_r - N) \times M_r$ dimension. The M rows of R_1 are linearly independent, So a $N \times (M_r - N)$ dimensional propagation operator matrix P is defined to satisfy equation (31),

$$R_2 = P^H R_1, \tag{31}$$

considering the influence of noise, the minimum value of equation (32) is generally solved, and an estimated value $\hat{P}$ of P is obtained [36], [37]

$$\xi(P) = \left\| R_2 - P^H R_1 \right\|_F, \tag{32}$$

$$\hat{P} = \left(R_1 R_1^H \right)^{-1} R_1 R_2^H. \tag{33}$$

A new matrix $\hat{Q}^H$ is defined as

$$\hat{Q}^H = \left[\hat{P}^H, -I_{M_r - N} \right], \tag{34}$$

according to equations (30), (33), and (34), equation (35) can be expressed as [37]

$$\hat{Q}^H A_r(\theta) = 0, \tag{35}$$

the column vectors of the matrix $\hat{Q}^H$ are not mutually orthogonal. In order to improve the DOA estimation performance, the standard orthogonal matrix Q_0 can be used instead of $\hat{Q}^H$,which is defined as [36], [37]

$$Q_0 = \hat{Q} \left(\hat{Q}^H \hat{Q} \right)^{-1/2}. \tag{36}$$

Therefore, an important equation can be expressed as

$$Q_0^H A_r(\theta) = 0, \tag{37}$$

a new spatial spectrum function is defined by equation (37),

$$P_{MUSIC}(\theta) = \frac{1}{A_r^H(\theta) Q_0 Q_0^H A_r(\theta)}, \tag{38}$$

the first derivative of $P_{MUSIC}(\theta)$ is defined as

$$d(\theta) = \lim_{\Delta\theta \to 0} \frac{P_{MUSIC}(\theta + \Delta\theta) - P_{MUSIC}(\theta)}{\Delta\theta}, \tag{39}$$

for the convenience of calculation, taking $\Delta\theta$ a smaller value, $y(\theta)$ is defined to replace $d(\theta)$,

$$y(\theta) = \frac{P_{MUSIC}(\theta + \Delta\theta) - P_{MUSIC}(\theta)}{\Delta\theta}. \tag{40}$$

When θ_0 is the DOA of the source, It can be known from equation (37) and (38) that $P_{MUSIC}(\theta_0)$ approaches a very larger value, $P_{MUSIC}(\theta_0 + \Delta\theta)$ and $P_{MUSIC}(\theta_0 - \Delta\theta)$ are much smaller than $P_{MUSIC}(\theta_0)$, thus equation (41) approaches a very larger value, equation (42) approaches a larger negative value.

$$y(\theta_0 - \Delta\theta) = \frac{P_{MUSIC}(\theta_0) - P_{MUSIC}(\theta_0 - \Delta\theta)}{\Delta\theta}, \tag{41}$$

$$y(\theta_0) = \frac{P_{MUSIC}(\theta_0 + \Delta\theta) - P_{MUSIC}(\theta_0)}{\Delta\theta}. \tag{42}$$

Thus it is possible to determine the DOA of the source according to whether the $y(\theta)$ function is abruptly positive changed, and then abruptly negative changed (breakpoint). Similarly, if the $P_{MUSIC}(\theta)$ function is also a breakpoint at a minimum value, the minimum value can be judged based on the first abruptly negative changed and the post abruptly positive changed. However, observing the waveform of the spatial spectral function $P_{MUSIC}(\theta)$, it is found that the $P_{MUSIC}(\theta)$ will only generate breakpoint in the DOA, and will not occur at other angles. In addition, $P_{MUSIC}(\theta)$ only produces abruptly changed in the DOA, which is proved by the physical meaning of $P_{MUSIC}(\theta)$ and equation (37), (38) and (40).Therefore, the DOA can be judged only based on the $y(\theta)$ function abruptly changed.

When θ_1 and θ_2 are two DOAs, the TOD-MUSIC algorithm whether increasing the estimate angular accuracy of DOA is theoretically analyzed. The MUSIC algorithm DOA estimation accuracy is defined as

$$\varphi_{MUSIC}(\theta) = \frac{P(\theta_2) - P(\theta_1)}{180}. \tag{43}$$

Similarly, the TOD-MUSIC algorithm is

$$\varphi_{TOD-MUSIC}(\theta) = \frac{y(\theta_2) - y(\theta_1)}{180}. \tag{44}$$

Equation (40) was brought into equation (44), we can get

$$\begin{aligned} &\varphi_{TOD-MUSIC}(\theta) \\ &= \frac{\frac{P_{MUSIC}(\theta_2+\Delta\theta) - P_{MUSIC}(\theta_2)}{\Delta\theta} - \frac{P_{MUSIC}(\theta_1+\Delta\theta) - P_{MUSIC}(\theta_1)}{\Delta\theta}}{180} \\ &= \frac{\frac{P_{MUSIC}(\theta_2+\Delta\theta) - P_{MUSIC}(\theta_1+\Delta\theta)}{\Delta\theta} - \frac{P_{MUSIC}(\theta_2) - P_{MUSIC}(\theta_1)}{\Delta\theta}}{180} \\ &= \frac{\frac{\varphi_{MUSIC}(\theta_2) - \varphi_{MUSIC}(\theta_1)}{\Delta\theta}}{180} \approx \varphi'_{MUSIC}(\theta). \end{aligned} \tag{45}$$

It can be known from (45) that the angular accuracy of TOD-MUSIC algorithm is approximately equivalent to the first derivative of MUSIC algorithm. In addition, the physical property of spatial spectrum function (which are not continuous functions and approach a larger value at the DOAs) are utilized by the TOD-MUSIC algorithm. Which makes the peaks at DOA more obvious and improves the angular accuracy of DOA.

IV. THE TOD-MUSIC ALGORITHM

Step1: Calculating covariance matrix R_{yy} according to the transmit-only diversity bistatic MIMO radar channel model and equation (26).

Step2: Calculating the Toeplitz matrix R_t according to equation (28) and (29).

Step3: Defining the propagation operator matrix P, then calculating the estimated value $\hat{P}$ of P according to equation (33).

Step4: Defining an orthogonal propagation operator matrix Q_0 according to equation (34) and (36).

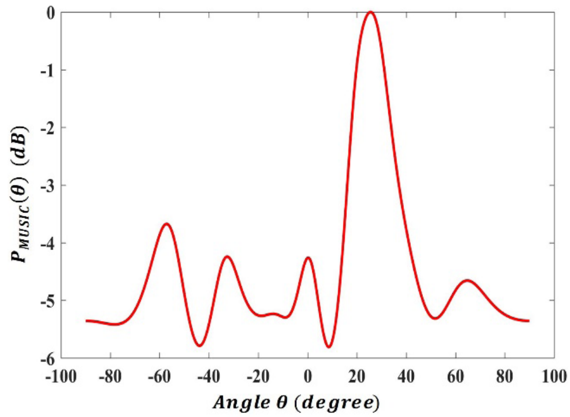

FIGURE 3. MUSIC algorithm for correlated sources ($P_{MUSIC}(\theta)$ (dB) represents the spatial spectrum function, unit is dB).

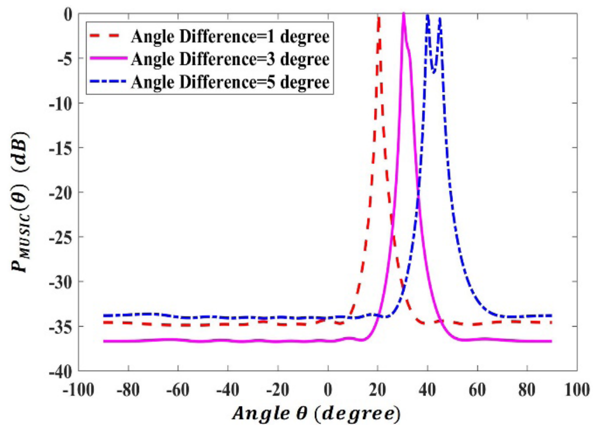

FIGURE 5. MUSIC with angular differences ($P_{MUSIC}(\theta)$ (dB) represents the spatial spectrum function, unit is dB).

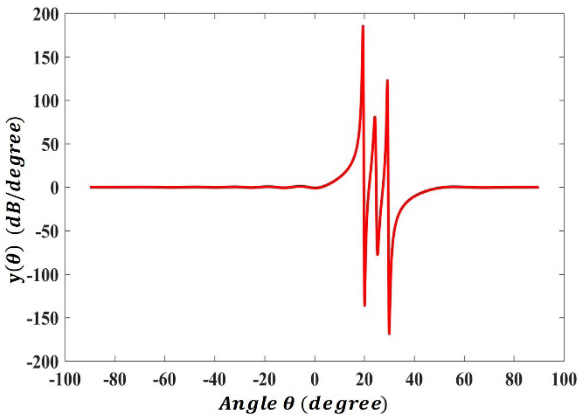

FIGURE 4. TOD-MUSIC algorithm for correlated sources ($y(\theta)$ $(dB/degree)$ represents the first derivative of spatial spectrum function, unit is $dB/degree$).

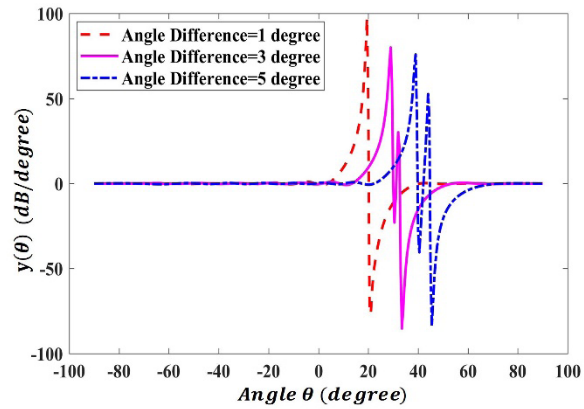

FIGURE 6. TOD-MUSIC with angular differences ($y(\theta)$ $(dB/degree)$ represents the first derivative of spatial spectrum function, unit is $dB/degree$).

Step5: Defining new spatial spectrum function $P_{MUSIC}(\theta)$ according to equation (37) and (38).

Step6: Defining the approximate first derivative y (θ) of the $P_{MUSIC}(\theta)$ according to equation (40), where the abrupt change of y (θ) is the DOA.

V. SIMULATION ANALYSIS

A. SIMULATION 1: DE-CORRELATION

$M_r = 10, d_r = \lambda/2, L = 1024, SNR=5dB$, the noise is Gaussian white noise. The signals reflected by the scatterers in the transmit-only diversity bistatic MIMO radar were simulated by the correlated far-field narrowband signals, the DOAs is 20°, 25°, and 30°. The simulation is performed by MUSIC [17] and TOD-MUSIC algorithm respectively, and the results are shown in Fig. 2 and Fig. 3.

Fig. 3 shows that the DOA of correlated sources cannot be detected by the MUSIC algorithm. Because the sources are correlate, the vector of the signal subspace diverges into the noise subspace, causing the loss of the covariance matrix rank. Fig. 4 shows that the DOA of correlated sources can be accurately detected by the TOD-MUSIC algorithm, because the TOD-MUSIC algorithm uses Toeplitz preprocessing on the covariance matrix. That is, the diagonal elements on the covariance matrix are averaged to obtain a new covariance matrix. The rank of the new matrix is independent of the correlate of the source, and the decorrelation of the source is achieved. Thus, the correlated sources can be processed by the TOD-MUSIC algorithm in the transmit-only diversity bistatic MIMO radar.

B. SIMULATION 2: ANGLE ACCURACY

$M_r = 10, d_r = \lambda/2$, $L=1024$, $SNR=0dB$, the signals reflected by the scatterers in the transmit-only diversity bistatic MIMO radar were simulated by the uncorrelated far-field narrowband signals, Respectively MUSIC algorithm and TOD-MUSIC algorithm experiment three times, the DOAs are (20°, 21°), (30°, 33°) and (40°, 45°). The angle differences are 1°, 3°, and 5°, respectively, and the results are shown in Fig. 5 and Fig. 6.

Fig. 5 shows the MUSIC algorithm completely fails when the angle difference is equal to 1° and 3° at $SNR=0dB$ and $L = 1024$,only one source can be estimated. When the angle

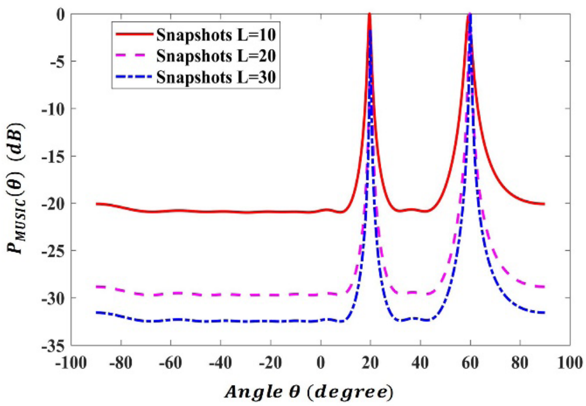

FIGURE 7. MUSIC algorithm for different snapshots (P_{MUSIC} (θ) (dB) represents the spatial spectrum function, unit is dB).

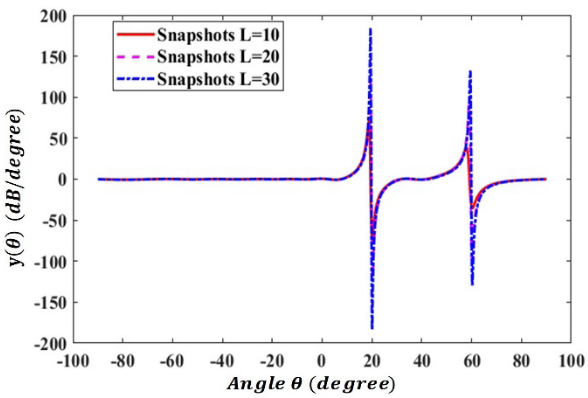

FIGURE 8. TOD-MUSIC algorithm for different snapshots (y (θ) $(dB/degree)$ represents the first derivative of spatial spectrum function, unit is $/degree$).

difference is equal to 5°, two sources can be estimated, but the peaks are not very obvious. As can be seen from Fig. 6, under the same conditions, the TOD-MUSIC algorithm also completely fails when the angle difference is 1°, but for the angle difference of 3° and 5°, two peaks are found. The reason is that the spatial correlation of sources are strong when the DOA of sources are close to each other. However, the TOD-MUSIC algorithm first uses Toeplitz matrix reconstruction and then peak is evaluated first derivative, so that the peak is more obvious and the DOA estimation accuracy is improved. Comparing Fig. 5 with Fig. 6, The TOD-MUSIC algorithm is applied to the transmit-only diversity bistatic MIMO radar, which makes the DOA estimation accuracy is improved by 2°.

C. SIMULATION 3: SAMPLING SIMULATION

DOAs are 20° and 60°, $SNR=5dB$, and snapshots L are equal to 10, 20, and 30 respectively. Other simulation conditions are the same as those of simulation 2. The simulation is performed by the MUSIC and TOD-MUSIC algorithm, and the results are shown in Fig. 7 and Fig. 8.

As can be seen from Fig. 7 and Fig.8, when the snapshots L are 10, 20 and 30, the DOA can be estimated by the MUSIC algorithm, and as the L increases, the estimation accuracy also increases. The DOA also can be estimated by the TOD-MUSIC algorithm, and the accuracy of the estimation is also higher than the MUSIC algorithm. As L increases, the accuracy of the estimation does not substantially change (the three curves in Fig. 8 coincide). The reason is that the Toeplitz structure of the covariance matrix of the receiving array is destroyed when the number of sampling snapshots is low. However, The TOD-MUSIC algorithm first restores the Toeplitz structure of the covariance matrix, and full uses of the physical properties of the spatial spectrum (which are not continuous functions and approach a larger value at the DOAs). Thus comparing Fig. 7 with Fig. 8, the DOA estimation ambiguity problem caused by under-sampling is solved by the TOD-MUSIC algorithm.

D. SIMULATION 4: SNR SIMULATION

The Root Mean Square Error (RMSE) of the DOA estimate is defined as

$$RMSE = \frac{1}{M}\sum_{j=1}^{M}\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\theta_{i,j}-\beta_{i,j})^2}, \tag{46}$$

where M is the number of Monte Carlo experiments, N is the number of sources, and $\theta_{i,j}$ is the estimated value of $\beta_{i,j}$ in the j^{th} Monte Carlo experiment. The Normalized Probability of Success (NPS) of the DOA estimate is defined as

$$\text{NPS} = \frac{T_s}{T_t}, \tag{47}$$

where T_s is the number of successful simulations, and T_t is the total number of Monte Carlo experiments. $|\theta_{i,j}-\beta_{i,j}| \le \varepsilon$ represents the success of DOA estimation.

DOAs are 20° and 30°, other simulation conditions are the same as those of simulation 2. For NPS, SNR from -10dB to 10dB, step is 2dB, ε is 0.8 based on experiences, and T_t is 500. For RMSE, SNR from 0dB to 30dB, step is 5dB and M is 500. In order to better compare the performance of each algorithm, the CRB algorithm proposed in reference [40] was added. The performance of NPS and RMSE were experimented and the results are shown in Fig. 9 and Fig. 10. Moreover, in order to verify the performance of TOD-MUSIC algorithm in terms of the effect of correlation of signals, in Fig. 10, it is especially compared with the FOMD algorithm in the reference [13].

In Fig.9, the NPS of MUSIC and TOD-MUSIC algorithm gradually increases and finally approaches 1 with the increase of SNR, but The NPS of TOD-MUSIC algorithm is higher than the MUSIC algorithm at less than 0dB. The NPS of TOD-MUSIC algorithm is approached to 1 at -2dB, while the MUSIC algorithm is approached to 1 at 0dB. Which shows that the performance of TOD-MUSIC algorithm is higher than the MUSIC algorithm.

In Fig.10, the RMSE of MUSIC, FOMD and TOD-MUSIC algorithm gradually decreases with the increase of SNR and all higher than the CRB in reference [40]. The RMSE of

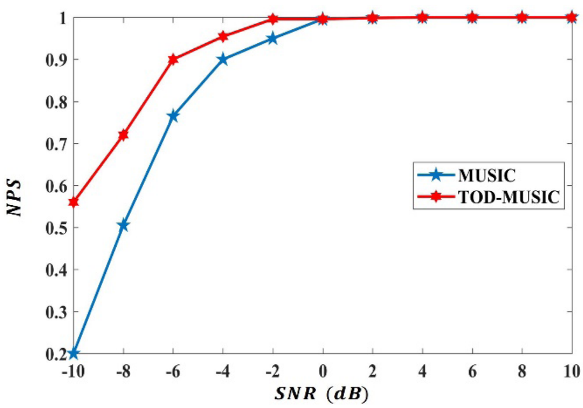

FIGURE 9. NPS vs. SNR for angular differences (*NPS* represents the normalized probability of success of the DOA estimate).

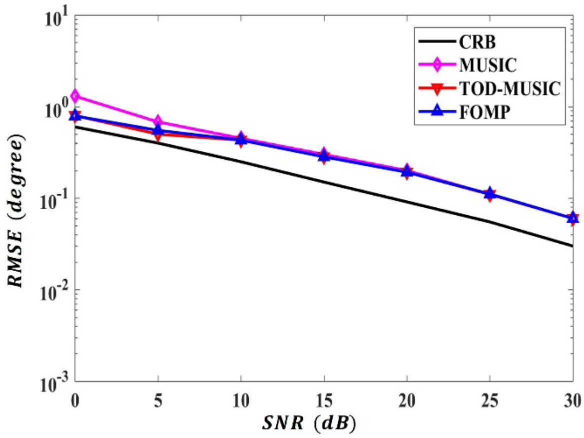

FIGURE 10. RMSE vs. SNR for angular difference (*RMSE(degree)* represents the root mean square error of the DOA estimate, unit is degree).

TOD-MUSIC and FOMD algorithms are similar, which is lower than MUSIC algorithm when SNR is lower than 10dB, approaching to MUSIC algorithm when SNR is higher than 10dB. The reason is that the covariance matrix of the MUSIC algorithm does not satisfy the Toeplitz structure at low SNR, resulting in low estimation accuracy for DOA. However, TOD-MUSIC algorithm can effectively improve the DOA estimation accuracy at low SNR by using the Toeplitz matrix reconstruction and the first derivative of spatial spectral function. Similarly, the FOMP algorithm based on the OMP algorithm can effectively distinguish two adjacent signal sources and also have a high performance at low SNR for DOA estimation accuracy.

In Fig.9, MUSIC algorithm and TOD-MUSIC algorithm performance are similar at $SNR=0dB$, while the Fig.10 is $SNR=10dB$. Because from 0dB to 10 dB, the value of ε is less than 0.8, but is closer to 0.8. It is closer to 0 at more than 10dB. In order to more accurately evaluate the performance of two algorithms, the results in Fig. 10 should be focused. By simulation 2 and simulation 4, we can get the conclusion that the TOD-MUSIC algorithm is applied to the transmit-only diversity bistatic MIMO radar, which makes the DOA estimation accuracy is improved by 2° at low SNR, approaching to the MUSIC algorithm at high SNR.

TABLE 1. Music algorithm computational analysis.

COVARIANCE MATRIX	EIGENVALUE DECOMPOSITION	SEARCH
$O(M_r{}^2L)$	$O(4/3\,M_r{}^3)$	$O(180/\Delta\theta\,M_r{}^2)$

TABLE 2. TOD-music algorithm computational analysis.

COVARIANCE MATRIX	TOEPLITZ	SEARCH	LINEAR PROCESS
$O(M_r{}^2L)$	$O(M_r)$	$O(180/\Delta\theta\,M_r{}^2)$	$O(NM_r{}^2)$

E. SIMULATION 5: COMPLEXITY ANALYSIS

The computational complexity of the MUSIC algorithm mainly includes the construction of $M_r \times M_r$ dimension covariance matrix, the eigenvalue decomposition of covariance matrix and the one dimension spectral peak search. The computational complexity of TOD-MUSIC algorithm mainly includes the construction of $M_r \times M_r$ dimension covariance matrix, the preprocessing of Toeplitz matrix, linear processing and one dimension spectral peak search. The calculations of MUSIC and TOD-MUSIC are shown in Table 1 and Table 2, where N is the number of sources, L is the number of snapshots, and $\Delta\theta$ is the angular scan interval.

Comparing Tables 1 with 2, the calculations of MUSIC algorithm is approximately M_r times that of the TOD-MUSIC algorithm under the same N, L and $\Delta\theta$. Because linear processing is used by the TOD-MUSIC algorithm to instead of eigenvalue decomposition, and the amount of calculation is effectively reduced. Under the same M_r, N and $\Delta\theta$, the calculations of MUSIC and TOD-MUSIC algorithm increase linearly with L, but $N < M_r$, the calculations of MUSIC algorithm is higher than the TOD-MUSIC algorithm. Although the amount of calculation is greatly reduced by the TOD-MUSIC algorithm, the spatial complexity is improved. Because the covariance matrix R_t is introduced by the Toeplitz process, the propagation operator matrix P is introduced by the linear processing and the first derivation increases the spatial complexity of $180/\Delta\theta$. With the rapid development of integrated circuits, the computational complexity is generally considered first, and the spatial complexity is considered then. Thus TOD-MUSIC algorithm is low complexity algorithm.

VI. CONCLUSION

In this paper, The TOD-MUSIC algorithm is proposed to the transmit-only diversity bistatic MIMO radar, improving the DOA estimation accuracy by 2° at low SNR, approaching to MUSIC algorithm at high SNR. The DOA estimation ambiguity problem caused by under-sampling is solved by the TOD-MUSIC algorithm, which not only can process correlated sources, but also reduces calculated amount.

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