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2D Fuzzy Constrained Fault-Tolerant Predictive Control of Nonlinear Batch Processes

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ABSTRACT Concerning actuator gain faults and strong system nonlinearity in batch processes, a design method of a kind of 2D fuzzy constrained model fault-tolerant predictive controller is proposed. Firstly, introduce two errors: state error and output error, after that the original system model is converted into a 2D Roesser fault model. Meanwhile, the design of iterative learning fault tolerant control under constraints has been transformed into the determination of the constrained update law. Subsequently, real-time on-line design of the fuzzy fault-tolerant update law that ensures the closed-loop system robustly asymptotic stability is presented by taking the appearance of linear matrix inequalities (LMIs) with constraint subject to the designed infinite optimization performance index and Lyapunov stability theory. Finally, taking a three-tank case as an example to compare with the 1D of the method proposed in this paper, the comparison results illustrated the 2D method of this paper has better control effects.

INDEX TERMS Actuator faults, fuzzy constrained iterative learning predictive fault-tolerant control, nonlinear batch processes, 2D T-S fuzzy model.

I. INTRODUCTION

As a typical model of production, batch processes [1], [2] have roughly two types of systematic descriptions: linear and nonlinear. So far, control of batch processes is mostly concerned with linear models [3], whereas, batch processes themselves are characteristic of strong nonlinearity in actual industrial processes, resulting in a significant problem where linear models do not match the actual processes well enough. Such a mismatch makes it hard to achieve optimal control performance in actual applications. Meanwhile, it is also difficult to process nonlinear systems. Concerning this problem, Japanese scholars Takagi and Sugeno put forward T-S (Takagi-Sugeno) fuzzy models for description of nonlinear systems in 1985 with a huge progress in the studies of fuzzy control of nonlinear systems [4]. Moreover, fuzzy control has been widely used so far [5]–[15]. Aiming at the problems existing in the current energy management system (EMS), the fuzzy plus filter energy management controller is designed by [7], and the improved dual-object optimization problem of fuzzy EMS is settled by the improved genetic algorithm (GA). For the nonlinear production process, [8] studied the output feedback control problem, [9] studied the interval time-varying delay problem and [10] proposed a model predictive control algorithm with neural networks for the control of temperature, which makes the control run more smoothly. Aiming at the uncertainty and interference problems, a multi-phase robust NMPC method is proposed by [11]. However, the above results are all based on the one-dimensional (1D) model, and the batch process itself has two-dimensional (2D) characteristics, with both time and batch directions. If only the time direction is considered, the system cannot be solved with the increase of batch performance. If only batches are considered, the problem of initial value uncertainty cannot be solved. Therefore, nonlinear research based on 2D is very necessary.

Industry plays a decisive role in human life, and the current research on nonlinear industrial production is rare, and it is under normal systems. However, in actual production, as the scale of production increases and the complexity of production steps increases, production equipment will be subjected to production operations under more complicated environmental conditions, which leads to a significant increase in the probability of system failure. In the system, an actuator fault is a common fault that it will affect the normal operation of the system and even endanger the personnel safety. It is necessary

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to ensure that the system can still operate stably under faults and to ensure certain control performance. Therefore, it is of extreme importance to research fault-tolerant control (FTC) technology.

There are some related results. The concept of [16]–[20] proposed combining iterative learning control (ILC) with FTC. For actuator faults, [21] proposes an improved infinite time domain linear quadratic tracking controller method, [22] proposed a new type of minimax linear quadratic (LQ) fault-tolerant tracking control. References [23]–[25] are some related results about fault diagnosis and FTC. However, the above studies are based on a linear model. Research on fault-tolerant control of nonlinear models has also been studied [26], [27]. In addition, some research on FTC of 2D system models has emerged [28]–[30]. 2D system control refers to the control method of feedback and ILC considering the repeatability and 2D characteristics of batch processes. Because it has better control performance, it has been paid attention and diffusely used in the control of batch processes [31]–[35].

However, the author finds that under the influence of interferences and faults, the present iterative learning reliable control method has the disadvantage that it can't solve the problem that the system state deviates, it means that the same control law is used all the time, and the state deviation will become larger and larger with time. In addition, there are fewer studies on input and output constraints [36], and in actual production, the constraint problem is also worth paying attention. The constraint refers to the input and output constraints, which means that the actual input and output values have a certain limit. If the constraint problem is not considered, the designed controller has drawbacks, and it is very likely that it cannot be changed because it reaches a saturated state, thereby affecting the system control performance and even not running smoothly.

The model predictive control (MPC) method is widely used because it can update the control law all the time and obtain the optimal control law at each moment to ensure the system good control performance [37], [38]. Currently, most of the MPC methods are based on the 1D model, that is, simply considering the time or batch direction. If only the time direction is considered, the control performance may decrease as the batch increases [39]–[41]; only considering the batch control problem, there is an initial value uncertainty in the batch direction [42]. In addition, the study finds that the 2D system model control effect is significantly better than the 1D system model design [43]. Currently, there are certain results for 2D systems. In [44], an ILC method based on 2D system design is designed, combining with single/multi-period prediction method, a single/multi-cycle generalized 2D prediction ILC scheme is proposed. The ILMPC strategy proposed in [45] has dynamic R parameters, integrating ILC and MPC into one whole, and achieving zero error tracking. So as to solve the multi-phase batch processing problem of input delay and actuator faults, [46] proposed a linear quadratic prediction FTC scheme. However, these are all studied under 2D linear systems. Since the batch itself has a strong nonlinearity, actuator faults often occur, and constraints and other factors cannot be ignored. It is very rare to study the faulty and constrained conditions in the 2D nonlinear batch process.

Therefore, to solve all the preceding existing problems and guarantee system control performance, a 2D fuzzy-constraint fault-tolerant predictive control method is put forward. Firstly, a 2D T-S spatial model where fuzzy states are built by nonlinear and 2D characteristics to further combine the errors of system state and the errors of output. Then the original model for system dynamics is transformed into a Roesser faulty system model in the predictive representation. The design of law for the iterative learning fault-tolerant control (ILFTC) under constraints is then to be converted into the determination of the constrained update law. On the basis of the designed infinite optimization performance index and Lyapunov stability theory for 2D systems, a real-time online fuzzy fault-tolerant update law that the robust asymptotic stability can be guaranteed under the closed-loop system is presented in the form of LMI constraint.

Note that the contribution is as follows. (1) This paper considers the nonlinear characteristics of batch processes under actuator faults and proposes a 2D controller based on T-S fuzzy model. The new model development and the corresponding optimization based on constrained fault tolerant MPC controller design are new. (2) The controller can be updated online to resist the worst impact caused by external disturbances, while optimal control can be obtained. That is, according to the given constraints and performance indicators, the optimal control law of the current time is solved and executed; before the start of the next time, the predicted output value of the model is corrected according to the measured output information, and then the corrected output information is utilized. Re-solve the optimal control law at this moment. In this way, the optimal control law for the entire time period can be solved by ''rolling'' step by step. Thereby achieving optimal control. (3) For nonlinear batch processes, the min-max optimization is first formulated to achieve the optimal control performance under various uncertainties. The 'max' refers to the maximum or 'worst case' value of the performance index, which is obtained by searching within the uncertainty range. The 'min' means minimizing the worst case value, which is obtained by searching the current and future control variables. For the MPC method, the 'min-max' problem is difficult to deal with in a limited range. This is to design future controller, which make performance metrics with 'worst-case' infinite time domain be minimized. Making use of LMI theory, the optimization problem of infinite time domain will be transformed into a convex optimization problem with LMI constraints. Then, in view of MPC principle, only the current control law is realized and the optimization problem is repeated via the new state information. Finally, taking a nonlinear three-tank as an example to certificate the practical value and effectiveness of the above idea raised.

II. PROBLEM FORMULATION

In a sense, the T-S fuzzy system is an alternative representation of the nonlinear system, which simplifies the nonlinear model and has good general approximation ability. In this paper, based on T-S fuzzy modeling method [47], the fuzzy model described by If-then rules is used to approximate a nonlinear batch process. Each rule of that represents the local linear input-output relationship of a nonlinear system. Each equation in the fuzzy model represents a "subsystem". In this paper, the previous steps are omitted, and the fuzzy model of the nonlinear batch process with actuator fault is directly given as shown in system (1):

$$\begin{cases} x(t+1,k)=\sum_{f=1}^{p} h_f(\tilde{z}(t,k))A_f x(t,k) \\ \qquad +\sum_{f=1}^{p} h_f(\tilde{z}(t,k))B_f u^F(t,k)+\omega(t,k) \\ y(t,k)=\sum_{f=1}^{p} C_f x(t,k) \\ x(0,k)=x_{0,k}, \quad 0<t<T_p, f=1,2,\ldots,p;\ g=1,2,\ldots,r \end{cases} \tag{1}$$

and

$$\begin{cases} |u(t,k)|\leq \bar{u} \\ |y(t,k)|\leq \bar{y} \end{cases}$$

where $x(t,k)$, $y(t,k)$, $u^F(t,k)$ and $\omega(t,k)$ denote the system state, output, control input and unknown perturbations separately; p represents the number of fuzzy rules; A_f, B_f, C_f represent the system's state matrix, input matrix, and output matrix separately under fuzzy rules f; $h_f(\tilde{z}(t,k)) = w_f(\tilde{z}(t,k))/\sum_{f=1}^{p} w_f(\tilde{z}(t,k))$ with $w_f(\tilde{z}(t,k)) = \prod_{g=1}^{r} M_{fg}(\tilde{z}_g(t,k))$ and $M_{fg}(\tilde{z}_g(t,k))$ represents the membership of the individual premise variables $\tilde{z}_g(t,k)$ to the fuzzy set M_{fg}, and from $\begin{cases}\sum_{f=1}^{p} w_f(\tilde{z}(t,k))>0 \\ w_f(\tilde{z}(t,k))\geq 0\end{cases}$, $\begin{cases}\sum_{f=1}^{p} h_f(\tilde{z}(t,k))=1 \\ h_f(\tilde{z}(t,k))\geq 0\end{cases}$ holds; t, k separately represent the time and batch in a batch; T_p represents the a batch total running time; $x(0,k)$ represents the initial state; $u^F = \alpha u$, that is the actuator actual output, where α represents the fault coefficient. $0 < \underline{\alpha} \leq \alpha \leq \bar{\alpha}$, $\underline{\alpha}(\underline{\alpha} < 1)$ and $\bar{\alpha}(\bar{\alpha} \geq 1)$ are known variables. when the fault factor is greater than 1, it indicates that actuator valves may be slack. When a value is entered, the actual output will be larger than the set output value. Anything less than 1 can be considered congestion. The reason why actuator fault is considered here is that it is the most typical type of fault among all fault types. Moreover, due to a jammed fault, the output of the actuator will be a constant value, and the entirely invalid fault will make the system control completely unavailable. Therefore, partial fault of the actuator has always been the focus of researchers.

A. BUILD EQUIVALENT 2D DESCRIPTION

Regarding the batch process described in system (1), a 2D ILC law is drawn as

$$\sum\nolimits_{ilc}:\begin{cases} u(t,k)=u(t,k-1)+u_\Delta(t,k) \\ u(t,0)=0 \end{cases}, \ t=0,\ 1,2,\ldots,T \tag{2}$$

where $u(t,0)$ represents the initial value of the iterative algorithm and $u_\Delta(t,k)\in R^m$ denotes the update law of the ILC to be designed. Define output tracking error as

$$e(t,k)\triangleq y_d-y(t,k) \tag{3}$$

where y_d is a constant value of the output expectation, does not change with time and batch, and thus has nothing to do with time and batch.

Note that

$$x_\Delta(t,k)=x(t,k)-x(t,k-1) \tag{4}$$

According to (1)–(4), we have

$$x_\Delta(t+1,k)=\sum_{f=1}^{p} h_f(\tilde{z}(t,k))(A_f x_\Delta(t,k)+B_f u_\Delta^F(t,k)) +\bar{w}(t,k) \tag{5}$$

For the system's output tracking error, $C_f = C$; $(f = 1,2,\ldots,p)$ is considered. This is reasonable since it is common in practical systems such as ref. [47], [48]. That we can have

$$\begin{aligned} e(t+1,k) &= y_d-y(t+1,k) \\ &= e(t+1,k-1) \\ &\quad -C\sum_{f=1}^{p} h_f(\tilde{z}(t,k))(A_f x_\Delta(t,k))+B_f u_\Delta^F(t,k)) \\ &\quad -C\bar{w}(t,k) \end{aligned} \tag{6}$$

where

$$\begin{aligned} \bar{w}(t,k) &= \tilde{w}(t,k)+\omega_\Delta(t,k), \\ \omega_\Delta(t,k) &= \omega(t,k)-\omega(t,k-1), \\ \tilde{w}(t,k) &= \sum_{f=1}^{p} h_{f\Delta}(\tilde{z}(t,k))\Big(A_f x(t,k-1)+B_f u^F(t,k-1)\Big), \end{aligned}$$
$$h_{f\Delta}(\tilde{z}(t,k)) = h_f(\tilde{z}(t,k))-h_f(\tilde{z}(t,k-1)).$$

Further, from (5) and (6), an equivalent 2D closed-loop model is drawn as:

$$\begin{cases} \hat{x}'(t,k)=\sum_{f=1}^{p} h_f(\tilde{z}(t,k))\bar{A}_f\hat{x}(t,k) \\ \qquad +\sum_{f=1}^{p} h_f(\tilde{z}(t,k))\bar{B}_f u_\Delta^F(t,k)+\bar{D}\bar{w}(t,k) \\ \hat{y}(t,k)\triangleq\begin{bmatrix} e(t+1,k) \\ e(t+1,k-1)\end{bmatrix}=C_y\hat{x}(t,k) \\ Z(t,k)\triangleq e(t+1,k-1)=\bar{C}\hat{x}(t,k) \end{cases} \tag{7}$$

where $\widehat{x}'(t,k) = \begin{pmatrix} x_h(t+1,k) \\ x_v(t,k+1) \end{pmatrix} = \begin{bmatrix} x_\Delta(t+1,k) \\ e(t+1,k) \end{bmatrix}$, $\bar{A}_f = \begin{bmatrix} A_f & 0 \\ -CA_f & I \end{bmatrix}$, $\widehat{x}(t,k) = \begin{bmatrix} x_h(t,k) \\ x_v(t,k) \end{bmatrix} = \begin{bmatrix} x_\Delta(t,k) \\ e(t+1,k-1) \end{bmatrix}$, $\bar{B}_f = \begin{bmatrix} B_f \\ -CB_f \end{bmatrix}$, $\bar{D} = \begin{bmatrix} I \\ -C \end{bmatrix}$, $C_y = \begin{bmatrix} -C & I \\ 0 & I \end{bmatrix}$, $\bar{C} = \begin{bmatrix} 0 & I \end{bmatrix}$; $x_h(t,k) \in R^{n_1}$ and $x_v(t,k) \in R^{n_2}$ indicate the abscissa and ordinate state component with the appropriate dimension vector, and $Z(t,k)$ is the controlled output. The system state is not only dependent on time but also related to batches, so the state can be broken down into time coordinates and batch coordinates. $x_h(t+1,k)$, $x_v(t,k+1)$ is a form of expression. The model is essentially an error model of system (1), which can equivalently denote its dynamic behavior. Therefore, the design of the system (7) update law $u_\Delta(t,k)$ is equal to the design of the FTC law.

Remark 1: A lot of the nonlinear researches on batch processes do not consider input and output constraints [9]. Aiming at this problem, the controller designed in this paper considers the constraints and makes the system run stably under faults and uncertainties. This paper is in view of the 2D system model, and the traditional fuzzy control is one-dimensional. The first difficulty of 2D system is to find the minimum upper bound of the optimal performance index. Due to the introduction of batches, it is more complicated than 1D and more difficult to deal with. The second one is that C is not taken as C_f, because different batches have different states. When the original system is transformed into an equivalent model, if C_f is selected, the batch error will not be expressed, so the common C is selected and regarded as C_f. Besides, this selection is quite reasonable, just as the description of that kind in [47].

Remark 2: The methods for nonlinear control include direct nonlinear control and T-S fuzzy model control. The former has some difficulties, while the latter is easy to express the dynamic characteristics of complex systems and has good approximation effect for nonlinear system. FTC is a control method that makes the system run stably even when the actuator fails. At present, it can be divided into passive FTC and active FTC. Active FTC is popular because it can readjust the controller structure or change the controller parameters in time according to the results of fault diagnosis and detection, so as to ensure the stable operation of the system after the failure. 2D is an own characteristic of the batch process because of its correlation with time and batch. MPC is a control method with real-time optimization and updating of control law. In this paper, in order to make the nonlinear batch process have good control performance in presence of faults, the advantages of the above control methods are combined to study its fault tolerant MPC control based on 2D T-S fuzzy in the case of failure.

B. OPTIMAL CONTROLLER DESIGN

Make $\widehat{x}'(t+m|t,k+n|k)$, $\widehat{y}(t+m|t,k+n|k)$ and $u_\Delta(t+m|t,k+n|k)$ defined as the predicted state, output, and manipulation action at time t in the k phase. Specially, $\widehat{x}(t|t,k|k) = \widehat{x}(t,k)$, $u_\Delta(t|t,k|k) = u_\Delta(t,k)$. In the infinite time range $[t,\infty)$, $[k,\infty)$, the "worst case' performance index of the system with uncertainty at time t in the k phase is denoted as

$$\begin{aligned} &\min_{u_\Delta(t+m|t,k+n|k),m,n=0,1,\ldots,\infty} \ \max_{\|\bar{w}(t,k)\|^2 \le \|Z(t,k)\|^2} J_\infty(t,k) \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} J(t+m|t,k+n|k) \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \begin{bmatrix} \widehat{x}(t+m|t,k+n|k) \\ u_\Delta(t+m|t,k+n|k) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \\ &\quad \times \begin{bmatrix} \widehat{x}(t+m|t,k+n|k) \\ u_\Delta(t+m|t,k+n|k) \end{bmatrix} \end{aligned} \tag{8}$$

limited by

$$\begin{cases} \widehat{x}'(t+m|t,k+n|k) = \sum_{f=1}^{p} h_f(\tilde{z}(t,k))\bar{A}_f\widehat{x}(t+m|t,k+n|k) \\ \qquad + \sum_{f=1}^{p} h_f(\tilde{z}(t,k))\bar{B}_f u_\Delta^F(t+m|t,k+n|k) \\ +\bar{D}\bar{w}(t+m|t,k+n|k) \\ \widehat{y}(t+m|t,k+n|k) \triangleq C_y\widehat{x}(t+m|t,k+n|k) \\ Z(t+m|t,k+n|k) \triangleq \bar{C}\widehat{x}(t+m|t,k+n|k) \end{cases} \tag{9}$$

$$\begin{cases} \|u_\Delta(t+m|t,k+n|k)\|_2 \le \bar{u}_\Delta \\ \left\|\widehat{y}(t+m|t,k+n|k)\right\|_2 \le \widehat{y} \end{cases} \tag{10}$$

where $\bar{u}_\Delta$ and $\widehat{y}$ are the variables boundaries of the control input and output, separately. And assume $\|\bar{w}(t,k)\| \le \|Z(t,k)\|$. This is a common notation, but it is considered more specifically in this article. A more common expression is $\|Z(t,k)\| \le \gamma \|\bar{w}(t,k)\|$ where γ describes the upper bound of sensitivity of 2D control system output to unknown disturbance signal $\bar{w}$, $\gamma > 0$.

Remark 3: In performance index (8), $\widehat{x}(t+m|t,k+n|k)^T Q\widehat{x}(t+m|t,k+n|k)$ represents the whole process state error and output error, and $u_\Delta(t+m|t,k+n|k)^T Ru_\Delta(t+m|t,k+n|k)$ shows the control power consumption. Therefore, the optimal control problem of the minimum and maximum quadratic performance indicators is essentially the optimal control that requires the minimum control input to obtain the minimum error in the case of maximum uncertainty. In additionally, The matrix Q, R are a weighting matrix of the process state and the output variable, separately, which is a symmetric positive definite condition which is proposed to ensure the stability of the optimal feedback system.

The iterative update law of 2D-ILC is denotes as

$$u_\Delta(t+m|t,k+n|k) = \sum_{f=1}^{p} h_f(\tilde{z}(t,k))K_f\widehat{x}(t+m|t,k+n|k) \tag{11}$$

The control goal is to minimize performance index $J_\infty(t,k)$ under uncertainties, input and output constraints by designing (11). It's shown in (9), (10).

Remark 4: Eq. (8) is a 'min-max' optimization problem, where 'max' is to search the largest or 'worst-case' value of J_∞ under uncertainties. Designing $u_\Delta(t+m|t,k+n|k)$ is to minimize a performance index with 'worst-case' infinite horizon (8). The worst-case here is the case with the minimum interference. Making use of LMI theory, that it may be converted into LMI constrains of convex optimization. And then, the control law $u_\Delta(t|t,k|k)$ is implemented at discrete-time t in phase k and the optimization problem is repeated at discrete-time $t+i$ in phase k.

Lemma 1 [12]: The system (9) is solvable if exists a function $V(\hat{x})$ and a scalar $0<\rho<1$ satisfy

(1) As to $\forall\hat{x}\in R^n$, $V(\hat{x})\geq 0$; and $V\left(\hat{x}\right)=0\Leftrightarrow\hat{x}=0$;

(2) When $\left\|\hat{x}\right\|\to\infty$, $V\left(\hat{x}\right)\to\infty$;

(3) For any boundary condition and allow actuator failures

$$\sum_{\substack{t+k=M_0+N_0+m+n+1\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\hat{x}(t+m|t,k+n|k)\right) \leq \rho \sum_{\substack{t+k=M_0+N_0+m+n\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\hat{x}(t+m|t,k+n|k)\right),$$
$$\forall T_0>0, K_0>0, m,n>0$$

where the minimum ρ meeting the above formula is referred to system's 2D convergence index (2D-CI).

Proof: According to condition (3), it is known to any $t,k,m,n>0$ that $V\left(\hat{x}(t+m|t,k+n|k)\right)<\sum_{n=0}^{k-1}\rho^{t+n}V(\hat{x}(0,k-n))+\sum_{m=0}^{t-1}\rho^{k+m}V(\hat{x}(t-m,0))$. Because $0<\rho<1$, the $\lim_{t,k\to\infty}V\left(\hat{x}(t+m|t,k+n|k)\right)=0, \forall\hat{x}_{0,tk}\in\mathbb{R}^{n_1+n_2}$ is established. According to the conditions (1) and (2), then $\lim_{t,k\to\infty}\hat{x}(t+m|t,k+n|k)=0, \forall\hat{x}_{0,tk}\in\mathbb{R}^{n_1+n_2}$. That is, the system is robustly asymptotically stable. The system (9) is solvable

Remark 5: It is easy to see from the proof of Lemma 1 that the 2D robust convergence index ρ quantifies the 2D robust stability of the system. $0<\rho<1$ shows that the system is 2D robustly asymptotically stable. The smaller the value of ρ, the better the convergence of the system. Therefore, the stability of the control system can be determined based on this when designing the controller. Therefore, the exponential convergence coefficient of this paper is ρ.

Pre-designed Lyapunov function that satisfies the following form

$$\begin{aligned} &V[\hat{x}(t+m|t,k+n|k)]\\ &\quad=\theta\cdot\hat{x}(t+m|t,k+n|k)^T\Lambda^{-1}\hat{x}(t+m|t,k+n|k)\\ &\quad\triangleq V_h\left(x_h(t+m|t,k+n|k)\right)+V_v\left(x_v(t+m|t,k+n|k)\right)\\ &\qquad\qquad m,n=0,\ldots,\infty \end{aligned}\tag{12}$$

where $\forall t,k\geq 0$, $\Lambda>0$. The following must be met for the model (8)–(9) to keep steady operation in the allowable range of faults:

(a) Inequality constraint for two-dimensional Lyapunov function:

$$\begin{aligned} &\hat{x}(t+m|t,k+n|k)^TQ\hat{x}(t+m|t,k+n|k)\\ &\quad=u_\Delta(t+m|t,k+n|k)^TRu_\Delta(t+m|t,k+n|k)\\ &\quad\leq V(\hat{x}(t+m|t,k+n|k))-V(\hat{x}'(t+m|t,k+n|k)) \end{aligned}\tag{13}$$

(b) For 2D system (10), suppose it has a limited set of initial conditions i.e., here exists positive integers m,n make

$$\hat{x}(t+m,k)=0,\quad m\geq l_1;\ \hat{x}(t,k+n)=0, n\geq l_2.\tag{14}$$

where $l_1<\infty$ and $l_2<\infty$, and are all positive integers, we call $\hat{x}(t+m,k)$ and $\hat{x}(t,k+n)$are the boundary of K and T of the current time and batch, separately.

Formula (13) is superposed from $m,n=0$ to $m,n=\infty$ to get the following form:

$$\max J_\infty(t,k)\leq lV[\bar{x}(t,k)]\leq\theta\tag{15}$$

It is obvious that condition (a), namely, equation (13), is the condition that must be satisfied to predict the solvability of the fault-tolerant control problem. (b) is the hypothesis that must be put forward to obtain the upper bound of performance indicators. Under the above conditions, the following theorem is given:

Theorem 1: Suppose R and Q are positive matrices, the problem of fault-tolerant MPC is solvable if there exists positive definite symmetric matrices $\Lambda=diag\{\Lambda^h,\Lambda^v\}$ and $\bar{Q}=\begin{bmatrix}\bar{Q}^h & 0\\ 0 & \bar{Q}^v\end{bmatrix}$, matrices $Y_f, Y_g, (f=1,2,\ldots,p; g=1,2,\ldots,r)$ scalars $\varepsilon_f,\varepsilon_g,\gamma,\theta>0$ and $0<\sigma<1, 0<\mu<1$, make the following hold (16)–(20), as shown at the top of the next page, and the robust update law is obtained by:

$$\begin{cases}K_f=Y_f\Lambda^{-1}\\ K_g=Y_g\Lambda^{-1}\end{cases},\quad f=1,2,\ldots,p;\ g=1,2,\ldots,r;$$

where

$$\psi_1'=\Lambda\overline{A}_f^T+Y_f^T\beta^T\overline{B}_f^T,\ \psi_2'=-\Lambda+\varepsilon_f\overline{B}_f\beta_0^2\overline{B}_f^T,$$
$$\psi_1=\Lambda\overline{A}_f^T+\Lambda\overline{A}_g^T+Y_g^T\beta^T\overline{B}_f^T+Y_f^T\beta^T\overline{B}_g^T,$$
$$\psi_2=-\Lambda+\varepsilon_f\overline{B}_f\beta_0^2\overline{B}_f^T+\varepsilon_g\overline{B}_g\beta_0^2\overline{B}_g^T,\ Y_{fg}^TR^{\frac{1}{2}}=\frac{Y_f^T+Y_g^T}{2}R^{\frac{1}{2}}.$$

Remark 6: There is an unknown matrix α_0, such that $\alpha=(I+\alpha_0)\beta$, $|\alpha_0|\leq\beta_0\leq I$, $\alpha_0\triangleq diag\left[\alpha_{01},\alpha_{02},\cdots,\alpha_{0m}\right]$. In Theorem 1, α is replaced by $(I+\alpha_0)\beta$. By considering the system model with actuator failure, an iterative learning predictive FTC method is proposed, and the sufficient conditions for system stability are obtained and expressed in the form of LMI in Theorem 1.

$$\begin{bmatrix} -\Lambda(\sigma,\mu)+(s-1)\overline{Q} & 0 & Y_f^T R^{\frac{1}{2}} & \Lambda^T Q^{\frac{1}{2}} & \psi_1' & \Lambda^T \overline{C}^T & Y_f^T \beta^T \\ 0 & -\gamma I_n & 0 & 0 & \overline{D}^T & 0 & 0 \\ * & * & -\theta I_n & 0 & 0 & 0 & 0 \\ * & * & * & -\theta I_n & 0 & 0 & 0 \\ * & * & * & * & \psi_2' & 0 & 0 \\ * & * & * & * & * & -\gamma I_n & 0 \\ * & * & * & * & * & * & -\varepsilon_f \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} -\Lambda(\sigma,\mu)-\overline{Q} & 0 & Y_{fg}^T R^{\frac{1}{2}} & \Lambda^T Q^{\frac{1}{2}} & \psi_1 & \Lambda^T \overline{C}^T & \frac{Y_g^T \beta^T}{2} & \frac{Y_f^T \beta^T}{2} \\ 0 & -\gamma I_n & 0 & 0 & \gamma^T \overline{D}^T & 0 & 0 & 0 \\ * & * & -\theta I_n & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\theta I_n & 0 & 0 & 0 & 0 \\ * & * & * & * & \psi_2 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma I_n & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_g & 0 \\ * & * & * & * & * & * & * & -\varepsilon_f \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} -1 & \widehat{x}(t|t,k|k)^T \\ * & -\Lambda \end{bmatrix} \le 0 \tag{18}$$

$$\begin{bmatrix} -\bar{u}_\Delta^2 I & Y_f \\ * & -\Lambda \end{bmatrix} \le 0 \tag{19}$$

$$\begin{bmatrix} -\bar{y}^2 \Lambda & \Lambda C_y^T \\ * & -I \end{bmatrix} \le 0 \tag{20}$$

Remark 7: 2D system is a special system, which contains two directions of time and batch. Therefore, the Lyapunov function (energy function in essence) is set as two directions, so the matrix in Lyapunov function must be a diagonal positive definite matrix. In addition, this paper does not consider the conservative problem, which will be the future work direction. Additionally, The Fuzzy weight $\sum_{f=1}^{p} h_f^2(\tilde{z}(t,k))$, $\sum_{f=1}^{p}\sum_{f<g}^{p} h_f(\tilde{z}(t,k))h_g(\tilde{z}(t,k))$ are both greater than 0, Therefore, it can be extracted in the theorem proving process without affecting the positive and negative of LMI, thus the problem becomes more convenient.

Proof: This part will first prove by the Theorem 1 condition (16) (17) that the system is asymptotically stable, that is, the formula (1) in Lemma 1, the proof process is the Eq. (21)–(24), then, from equations (22) and (25)–(29), it can be proved that (13) is established, and the system (9) convergence index is not higher than ρ. Moreover, a double sum of equation (29) is performed, and combining the assumption conditions of equation (14), equation (15) can be obtained. Namely, J has a minimum upper bound. Finally, the constraint conditions (18)–(20) are proved. The details is as follows:

Using the state-feedback control law (11), the system (9) can be represented as:

$$\begin{bmatrix} \widehat{x}'(t+m|t,k+n|k) \\ Z(t+m|t,k+n|k) \end{bmatrix} = \begin{bmatrix} \psi_3 & \bar{D} \\ \bar{C} & 0 \end{bmatrix} \begin{bmatrix} \widehat{x}(t+m|t,k+n|k) \\ \bar{w}(t+m|t,k+n|k) \end{bmatrix} \tag{21}$$

where

$$\psi_3 = \sum_{f=1}^{p} h_f^2(\tilde{z}(t,k))\left(\bar{A}_f + \bar{B}_f \alpha K_f\right) + 2\sum_{f=1}^{p}\sum_{f<g}^{p} h_f(\tilde{z}(t,k))h_g(\tilde{z}(t,k))\left(\frac{G_{fg}^{cF}+G_{gf}^{cF}}{2}\right),$$

$$G_{fg}^{cF} = \bar{A}_f + \bar{B}_f \alpha K_g.$$

Based on Schur lemma, (Eqs (16) and (17) are expansions of ψ_3, So the two are combined into one and become (22), some details can be found in reference [9])

$$\begin{bmatrix} -\Lambda^{-1}(\sigma,\mu) & 0 \\ 0 & -\gamma^{-1} \end{bmatrix} + \begin{bmatrix} \psi_3^T & \bar{C}^T \\ \bar{D}^T & 0 \end{bmatrix}\begin{bmatrix} \Lambda^{-1} & 0 \\ 0 & \gamma^{-1} \end{bmatrix}\begin{bmatrix} \psi_3 & \bar{D} \\ \bar{C} & 0 \end{bmatrix} + \frac{1}{\theta}\begin{bmatrix} \left(\sum_{f=1}^{p} h_f(\tilde{z}(t,k))K_f\right)^T R^{\frac{1}{2}} & Q^{\frac{1}{2}} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} R^{\frac{1}{2}}\left(\sum_{f=1}^{p} h_f(\tilde{z}(t,k))K_f\right) & 0 \\ Q^{\frac{1}{2}} & 0 \end{bmatrix} < 0 \tag{22}$$

Due to $\Lambda^{-1}(\sigma,\mu) = diag\left[\sigma\left(\Lambda^{h}\right)^{-1}, \mu\left(\Lambda^{v}\right)^{-1}\right]$, (22) can be shown as

$$\begin{bmatrix} -\Lambda^{-1} & 0 \\ 0 & -\gamma^{-1} \end{bmatrix} + \begin{bmatrix} \psi_3^T & \bar{C}^T \\ \bar{D}^T & 0 \end{bmatrix}\begin{bmatrix} \Lambda^{-1} & 0 \\ 0 & \gamma^{-1} \end{bmatrix}\begin{bmatrix} \psi_3 & \bar{D} \\ \bar{C} & 0 \end{bmatrix} + \frac{1}{\theta}\begin{bmatrix} \left(\sum_{f=1}^{p} h_f(\tilde{z}(t,k))K_f\right)^T R^{\frac{1}{2}} & Q^{\frac{1}{2}} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} R^{\frac{1}{2}}\left(\sum_{f=1}^{p} h_f(\tilde{z}(t,k))K_f\right) & 0 \\ Q^{\frac{1}{2}} & 0 \end{bmatrix} < 0 \tag{23}$$

Let (23) multiply left by $[\bar{x}^T(t+m|t, k+n|k)\bar{w}^T(t+m|t, k+n|k)]$ and right multiply by its transpose. Obviously, the system (9) evidently forms a retraction mapping form $\left[\widehat{x}^T(t+m|t,k+n|k)\ \ \bar{w}^T(t+m|t,k+n|k)\right]$ to $\left[\widehat{x}'^T(t+m|t,k+n|k)\ \ Z^T(t+m|t,k+n|k)\right]$.

As a result, it is seen that

$$\begin{aligned} &\widehat{x}^T(t+m|t,k+n|k)\Lambda^{-1}\widehat{x}(t+m|t,k+n|k) \\ &\quad + \bar{w}^T(t+m|t,k+n|k)\gamma^{-1}\bar{w}(t+m|t,k+n|k) \\ &\quad > \widehat{x}'^T(t+m|t,k+n|k)\Lambda^{-1}\widehat{x}'(t+m|t,k+n|k) \\ &\quad + Z(t+m|t,k+n|k)\gamma^{-1}Z(t+m|t,k+n|k) \end{aligned} \tag{24}$$

Because $Z^T(t+m|t,k+n|k)Z(t+m|t,k+n|k) \geq \bar{w}^T(t+m|t,k+n|k)\bar{w}(t+m|t,k+n|k)$, then $\widehat{x}'^T(t+m|t,k+n|k)\Lambda^{-1}\widehat{x}'(t+m|t,k+n|k) < \widehat{x}^T(t+m|t,k+n|k)\Lambda^{-1}\widehat{x}(t+m|t,k+n|k)\forall m,n \geq 0$.

$\left\{\widehat{x}^T(t+m|t,k+n|k)\Lambda^{-1}\widehat{x}(t+m|t,k+n|k)\right\}_{m=0}^{\infty}{}_{n=0}^{\infty}$ is a strict decreasing sequence whose lower boundary is 0. Therefore, $\lim_{m,n\to\infty}\widehat{x}(t+m|t,k+n|k) = 0$, thus the system (9) is stable.

Let (22) multiply left by $[\bar{x}^T(t+m|t,k+n|k)\bar{w}^T(t+m|t,k+n|k)]$ and right multiply by its transpose. We can obtain

$$\begin{aligned} &V(\widehat{x}'(t+m|t,k+n|k)) - V_{(\sigma,\mu)}(\widehat{x}(t+m|t,k+n|k)) \\ &\leq -J(t+m|t,k+n|k) \\ &= -\widehat{x}(t+m|t,k+n|k)^T\left[Q+K_f^TRK_g\right]\widehat{x}(t+m|t,k+n|k) \end{aligned}$$

Therefore, from $\left[Q+K_f^TRK_g\right] > 0$, the following equations hold

$$\begin{aligned} &x_h^T(t+m+1|t,k+n|k)\Lambda^{-h}x_h(t+m+1|t,k+n|k) \\ &\quad + x_v^T(t+m|t,k+n+1|k)\Lambda^{-v}x_v(t+m|t,k+n+1|k) \\ &\quad \leq \sigma x_h^T(t+m|t,k+n|k)\Lambda^{-h}x_h(t+m|t,k+n|k) \\ &\quad + \mu x_v^T(t+m|t,k+n|k)\Lambda^{-v}x_v(t+m|t,k+n|k) \end{aligned} \tag{25}$$

Thus, the following is drawn:

$$\begin{aligned} &V\left(\left(x_h^T(t+m+1|t,k+n|k)\ \ x_v^T(t+m|t,k+n+1|k)\right)^T\right) \\ &- V\left(\left(\sigma^{\frac{1}{2}}x_h^T(t+m+1|t,k+n|k)\mu^{\frac{1}{2}}x_v^T(t+m|t,\right.\right. \\ &\qquad\qquad \left.\left. k+n+1|k)\right)^T\right) < 0 \end{aligned}$$

When $0 < \rho = \max\{\sigma,\mu\} < 1$, it can get the following results

$$\begin{aligned} &V\left(\left(x_h^T(t+m+1|t,k+n|k)\ \ x_v^T(t+m|t,k+n+1|k)\right)^T\right) \\ &\qquad < \rho V\left(\widehat{x}(t+m|t,k+n|k)\right) \end{aligned} \tag{26}$$

In light of any integer $M_0 > 0, N_0 > 0, m, n > 0$ and pursuant to (25), the following inequality is drawn:

$$\begin{aligned} &V_h\left(x_h(M_0+m+1|M_0,N_0+n|N_0)\right) \\ &\quad + V_v\left(x_v(M_0+m|M_0,N_0+n+1|N_0)\right) \\ &\quad < \rho V\left(\widehat{x}(M_0+m|M_0,N_0+n|N_0)\right) \\ &\quad \cdots \\ &V_h\left(x_h(M_0+m+n+1|M_0,N_0|N_0)\right) \\ &\quad + V_v\left(x_v(M_0+m+n|M_0,N_0+1|N_0)\right) \\ &\quad < \rho V\left(\widehat{x}(M_0+m+n|M_0,N_0|N_0)\right) \end{aligned} \tag{27}$$

As far as the above inequality is concerned, get the following sum

$$\begin{aligned} &\sum_{\substack{t+k=M_0+N_0+m+n+1\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\widehat{x}(t+m|t,k+n|k)\right) \\ &< \sum_{\substack{t+k=M_0+N_0+m+n+1\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} \begin{array}{l} V\left(\widehat{x}(t+m|t,k+n|k)\right) \\ +V_v\left(x_v\left(M_0+m|M_0,N_0+n+1|N_0\right)\right) \\ +V_h\left(x_h\left(M_0+m+1|M_0,N_0|N_0\right)\right) \end{array} \\ &\leq \rho \sum_{\substack{t+k=M_0+N_0+m+n\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\widehat{x}(t+m|t,k+n|k)\right) \end{aligned} \tag{28}$$

For $V(\widehat{x}'(t+m|t,k+n|k)) - V(\widehat{x}(t+m|t,k+n|k)) < V(\widehat{x}'(t+m|t,k+n|k)) - V_{(\sigma,\mu)}(\widehat{x}(t+m|t,k+n|k))$, thus

$$\begin{aligned} &\Delta V(\widehat{x}(t+m|t,k+n|k) \\ &\quad = V(\widehat{x}'(t+m|t,k+n|k)) - V(\widehat{x}(t+m|t,k+n|k)) \\ &\quad < V(\widehat{x}'(t+m|t,k+n|k)) - V_{(\sigma,\mu)}(\widehat{x}(t+m|t,k+n|k)) \\ &\quad \leq -J(t+m|t,k+n|k) \\ &\quad = \widehat{x}(t+m|t,k+n|k)^TQ\widehat{x}(t+m|t,k+n|k) \\ &\qquad + u_\Delta(t+m|t,k+n|k)^TRu_\Delta(t+m|t,k+n|k) \end{aligned} \tag{29}$$

Therefore, the system (9) convergence index is not higher than ρ. In other words, the system is exponential convergent. Moreover, $\lim_{t+k\to\infty}\widehat{x}(t+m|t,k+n|k) \to 0$. Thus, Lemma 1 is met, indicating that the 2D predictive system (9) is fault-tolerant predictive optimal control.

Adding the formula (29) from $m, n = 0$ to ∞, it can be got (30), as shown at the top of the next page

Choose, $l = \max\{l_1, l_2\}$ then

$$\begin{aligned} &\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Delta V(\widehat{x}'(t+m|t,k+n|k)) \\ &\leq \sum_{n=0}^{\infty}[V_h(x_h(t+\infty+1|t,k+n|k)) - V_h(x_h(t|t,k+n|k))] \end{aligned}$$

$$
\begin{aligned}
&\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Delta V(\widehat{x}'(t+m|t,k+n|k)) \\
&=\sum_{n=0}^{\infty}\left(\begin{array}{l}
V_h(x_h(t+1|t,k+n|k))-V_h(x_h(t|t,k+n|k)) \\
+V_v(x_v(t|t,k+n+1|k))-V_v(x_v(t|t,k+n|k)) \\
+V_h(x_h(t+2|t,k+n|k))-V_h(x_h(t+1|t,k+n|k)) \\
+V_v(x_v(t+1|t,k+n+1|k))-V_v(x_v(t+1|t,k+n|k)) \\
+\ldots+V_h(x_h(t+\infty+1|t,k+n|k))-V_h(x_h(t+\infty|t,k+n|k)) \\
+V_v(x_v(t+\infty|t,k+n+1|k))-V_v(x_v(t+\infty|t,k+n|k))
\end{array}\right) \\
&=\sum_{n=0}^{\infty}[V_h(x_h(t+\infty+1|t,k+n|k))-V_h(x_h(t|t,k+n|k))] \\
&\quad+\sum_{m=0}^{\infty}[V_v(x_v(t+m|t,k+\infty+1|k))-V_v(x_v(t+m|t,k|k))] \\
&\leq-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}J(t+m|t,k+n|k)
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
&+\sum_{m=0}^{\infty}[V_v(x_v(t+m|t,k+\infty+1|k))-V_v(x_v(t+m|t,k|k))] \\
&\leq -l_1V_h(x_h(t|t,k|k))-l_2V_v(x_v(t|t,k|k)) \\
&\leq -lV(\widehat{x}(t,k))\leq-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}J(t+m|t,k+n|k)
\end{aligned}
$$

i.e.

$$
J_\infty(t,k)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}J(t+m|t,k+n|k)\leq lV[\widehat{x}(t,k)]\leq\theta \tag{31}
$$

where θ is the $J_\infty(t,k)$ upper bound. Thus, equation (15) holds.

The constrained three formulas (18), (19) and (20) are proved, respectively.

The state is in the unit circle to guarantee that the system (9) is stabilizing, thus $\widehat{x}(t|t,k|k)^T\Lambda^{-1}\widehat{x}(t|t,k|k)\leq 1$ holds. $\begin{bmatrix}-1 & \widehat{x}(t|t,k|k)^T\\ * & -\Lambda\end{bmatrix}\leq 0$ is obtained.

From $u_\Delta(t,k)=\sum_{f=1}^{p}h_f(\tilde{z}(t,k))K_f\widehat{x}(t,k)=\sum_{f=1}^{p}h_f(\tilde{z}(t,k))Y_f\Lambda^{-1}\widehat{x}(t,k)$ and $\|u_\Delta(t,k)\|_2\leq\bar{u}_\Delta$, $\begin{bmatrix}-\bar{u}_\Delta^2 I & \sum_{f=1}^{p}h_f(\tilde{z}(t,k))Y_f\\ * & -\Lambda\end{bmatrix}\leq 0$ is available, i.e., $\begin{bmatrix}-\bar{u}_\Delta^2 I & Y_f\\ * & -\Lambda\end{bmatrix}\leq 0$.

From $\widehat{y}(t,k)\triangleq C_y\widehat{x}(t,k)$ and $\left\|\widehat{y}(t,k)\right\|_2\leq\bar{\widehat{y}}$, $C_y^TC_y\Lambda\leq\bar{y}^2$ holds, i.e. $\begin{bmatrix}-\bar{y}^2\Lambda & \Lambda C_y^T\\ * & -I\end{bmatrix}\leq 0$ holds.

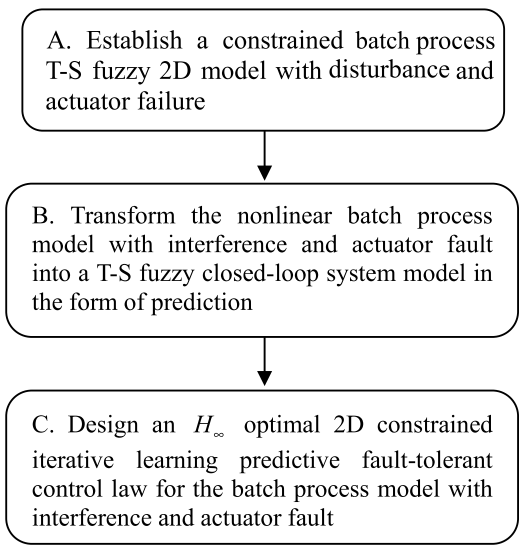
Remark 8: It can be seen from the proof of theorem 1 that

$$
\sum_{\substack{t+k=M_0+N_0+m+n+1\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\widehat{x}(t+m|t,k+n|k)\right) \leq\rho\sum_{\substack{t+k=M_0+N_0+m+n\\ M_0\leq t\leq M_0+m\\ N_0\leq k\leq N_0+n}} V\left(\widehat{x}(t+m|t,k+n|k)\right),
$$

$V(\bullet,\bullet)$ is convergent, so $\widehat{x}(t+m|t,k+n|k)\rightarrow 0$, the state contains state error and output tracking error. From the simulation result, Fig. 6 shows that the system error is infinitely zero after a certain point, which verifies that ILC is convergent in this scheme. In addition, the algorithm assumes that the system state tends to 0 after a certain point. As can be seen from the above, this algorithm is assumed to correspond to the simulation result in Fig. 6.

Remark 9: For most actual iterative processes, the parameter disturbances and external disturbances of the system often vary with time and batch. This kind of non-repeated parameter disturbance and external disturbance are the main reasons that affect the convergence of iterative learning system. In severe cases, the closed-loop control system may be unstable. Therefore, the H_∞ robust control method under 2D (time and batch) is more effective for unknown disturbances.

In order to better display the overall idea of this article, a brief description is made by the following flow chart



C. DESIGN ALGORITHM

Considering $\bar{w}(t,k) \neq 0$, in this condition, the H_∞ problem of system must be considered, and the controller is designed by using theorem 1. At this point, the convergence index t, the convergence index k and the 2D robust H_∞ "worst" performance index should be simultaneously considered as the decision variables to be optimized, and the nonlinear optimization problem should be solved. If the convergence index t, $k$are known, the following optimization algorithm is adopted:

Robust H_∞ optimization control algorithm:

For an upper bound on a given expected convergence index $0 < \sigma, \mu < 1$, solving the optimization problem as follow:

$$\underset{\Lambda, Y_f, Y_g, \varepsilon_f, \varepsilon_g}{Minimize} \ \gamma$$

Subject to (16) (17) (18) (19) (20)

If there is a feasible solution to the optimization problem, a new fuzzy constraint fault-tolerant iterative updating law can be designed.

III. CASE STUDIES

A. CASE ANALYSIS

In this article, each injection process of the three-tank is considered as a batch, which considered as a batch process. The three-tank can be conveniently transformed into single-input and-single-output, multi-input-and-multi-output, three-order, two-order and one-order models by opening and closing the junction valve and the leak valve. Odds may increase the chances of system faults under long-time and high-intensity equipment operation.

In order to better reflect the effectiveness of the proposed method, 1D fuzzy constrained fault-tolerant predictive control (referred to as 1D) is compared with 2D fuzzy constrained fault-tolerant predictive control (referred to as 2D). 1D method is only time-dependent and independent of batch. That is, it is impossible to solve problems such as system deviation and non-repetitive disturbance as the batch increases. To change this situation, a 2D method dealing with nonlinearity is put forward in this paper. The simulation is performed by a three-tank single-input single-output second-order model to illustrate that the method of this article has better control effect.

The model is shown as:

$$\begin{cases} \begin{bmatrix} \dot{h}_1 \\ \dot{h}_3 \end{bmatrix} = \frac{1}{S}\begin{bmatrix} -Q_{13} \\ Q_{13} - Q_{out} \end{bmatrix} + \frac{1}{S}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot Q_{in} \\ y = h_1 \end{cases} \tag{32}$$

where h_1 and h_3 represent the liquid level of tank 1 and tank 3 separately; Q_{in} represents the flow of fill pipe 1 as the manipulated variable; Q_{13} represents the flow of liquid from tank 1 to tank 3; Q_{out} represents the flow from tank 3 to bottom tank; $Q_{13} = az_1 S_n sgn(h_1 - h_3)\sqrt{2g|h_1 - h_3|}$; S denotes the bottom area of the tank and S_n denotes the cross sectional area of the flow valve, where, the upper bound of the water level of each tank is $H_{\max} = 0.6m$, $az_1 = 0.48$, $az_3 = 0.58$, $sgn(\cdot)$ represents a sign function, $S_n = 5 \times 10^{-5}m^2$, $S = 0.154m^2$. The process in which water is constantly filled in the tank can be regarded as a batch process repetition and each batch is assumed to operate in limited time interval. The initial value of each batch is given as $h_1 = h_3 = 0m$; the four working points including $M1 : 0.2 * H_{\max}$, $M_2 : 0.4 * H_{\max}$, $M_3 : 0.6 * H_{\max}$ and $M_4 : 0.8 * H_{\max}$ are set in the method of local linearization and $x = \begin{bmatrix} x_1(t,k) \\ x_2(t,k) \end{bmatrix} = \begin{bmatrix} h_1(t,k) \\ h_3(t,k) \end{bmatrix}$ and $u(t,k) = Q_{in}(t,k)$ are defined. Upon discretization, the fuzzy rules for the 2D T-S fuzzy discrete system are described as follows:

Rule 1: If $x_1(t,k) \in M_1$, then

$$x(t+1,k) = A_1 x(t,k) + B_1 u^F(t,k) + \omega(t,k)$$

Rule 2: If $x_2(t,k) \in M_2$, then

$$x(t+1,k) = A_2 x(t,k) + B_2 u^F(t,k) + \omega(t,k)$$

Rule 3: If $x_3(t,k) \in M_3$, then

$$x(t+1,k) = A_3 x(t,k) + B_3 u^F(t,k) + \omega(t,k)$$

Rule 4: If $x_4(t,k) \in M_4$, then

$$x(t+1,k) = A_4 x(t,k) + B_4 u^F(t,k) + \omega(t,k)$$

where

$$A_1 = \begin{bmatrix} 0.9951 & 0.0035 \\ 0.0025 & 0.9930 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.9944 & 0.0040 \\ 0.0029 & 0.9919 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 0.9863 & 0.0098 \\ 0.0071 & 0.9804 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 0.9807 & 0.0137 \\ 0.0100 & 0.9724 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 32.3885 \\ 0.0414 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 32.3763 \\ 0.0478 \end{bmatrix},$$
$$B_3 = \begin{bmatrix} 64.4900 \\ 0.2327 \end{bmatrix},$$
$$B_4 = \begin{bmatrix} 64.3072 \\ 0.3276 \end{bmatrix},$$

The membership function is as shown in figure 1:

Commonly used membership functions are bell-shaped, triangular, trapezoidal membership functions, and so on. At the beginning of the development of fuzzy control theory, most of them use the bell shaped membership function. In recent years, almost all of them have been changed to membership functions of triangles. This is because the calculation of the triangle membership function is simple, and the performance and the bell shape are almost no difference.

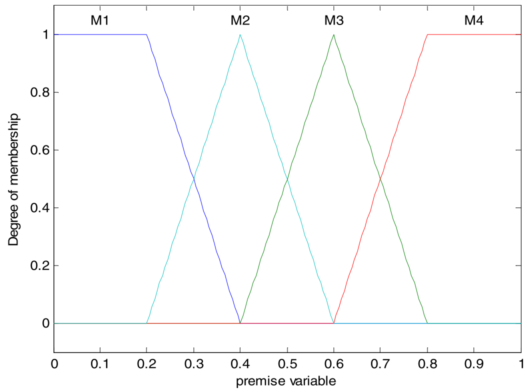


FIGURE 1. Membership function.

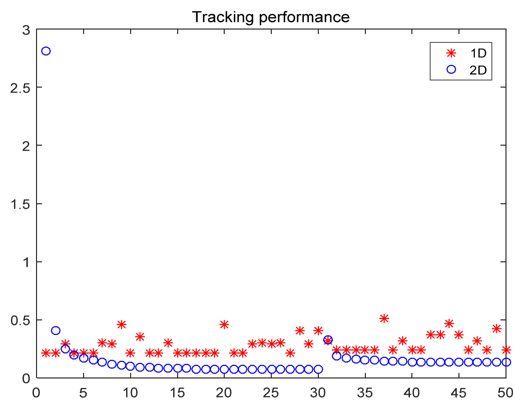


FIGURE 2. Tracking performance.

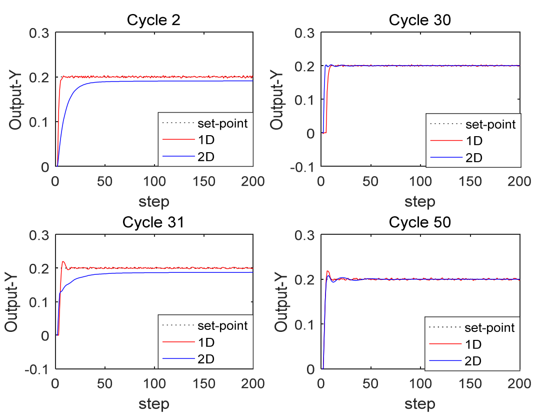


FIGURE 3. Output response comparison.

B. SIMULATION RESULT

In actual processes, the interference is common and most are not repetitive. In the initial batch, solve theorem 1 to draw the controller gain as:

$$\begin{aligned} K_{11} &= [\,-0.0045 \quad -0.0001 \quad -0.0098\,] \\ K_{12} &= [\,-0.0047 \quad -0.0002 \quad -0.0089\,] \\ K_{13} &= [\,-0.0033 \quad -0.0001 \quad -0.0059\,] \\ K_{14} &= [\,-0.0031 \quad -0.0001 \quad -0.0049\,] \end{aligned} \tag{33}$$

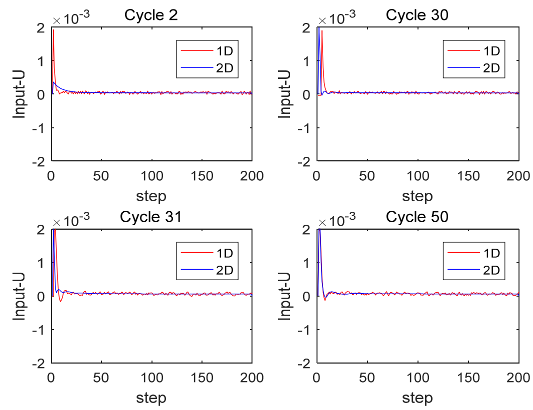


FIGURE 4. The system control input comparison.

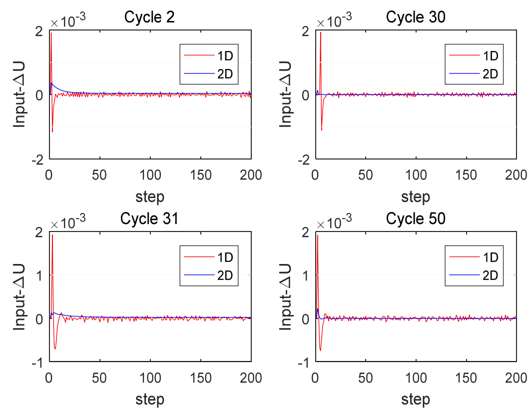


FIGURE 5. The system input increment comparison.

In that case, the non-repeated interference is designed to be $\omega = [\,0.04 \quad 0.02\,] * \Delta$, where, Δ is the random number of [0,1]. For illustrate the proposed method of this paper is better, a comparison diagram between the 1D control method and the 2D method is presented.

The relevant constraints are chosen as

$$\begin{aligned} \|u_{\Delta}(t,k)\|_2 &\leq 0.002 \\ \left\| \widehat{y}(t,k) \right\|_2 &\leq 0.3 \end{aligned} \tag{34}$$

Case 1: Constant failure under non-repeated interference

In this part, the above-mentioned non-repetitive interference is selected, and the fault constant value is 0.6. Assuming that all faults start happening in batch 31.

Fig.2 illustrates the tracking performance under non-repeated interference (The root-sum-squared-error is introduced by $DT(k) \triangleq \sqrt{\sum_{t=1}^{T_k} e^2(t,k)}$ to assess the control performance). Since the 1D method does not consider batch information, the system stability and performance are poor. The 2D method combines ILC and fuzzy predictive fault-tolerant control to optimize performance indicators. Though it is found that when faults happen, the tracking performance is poor, its tracking performance quickly gets better, which

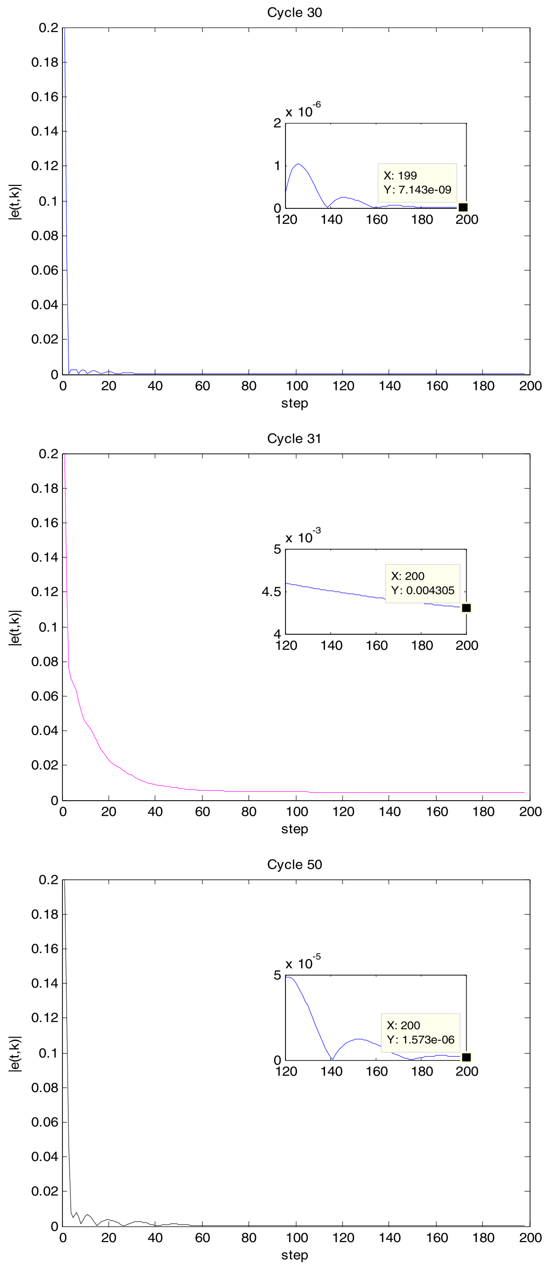


FIGURE 6. The system output tracking error response (Cycle 30, 31, 50).

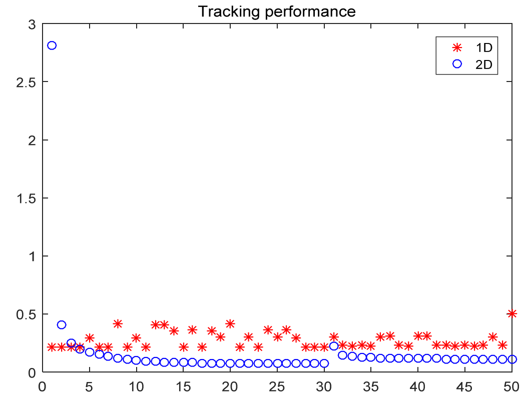


FIGURE 7. Tracking performance.

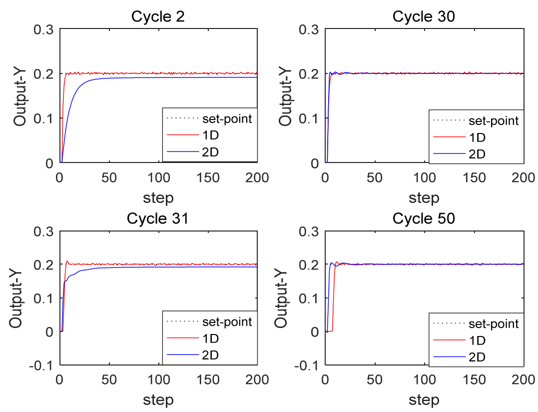


FIGURE 8. Output response comparison.

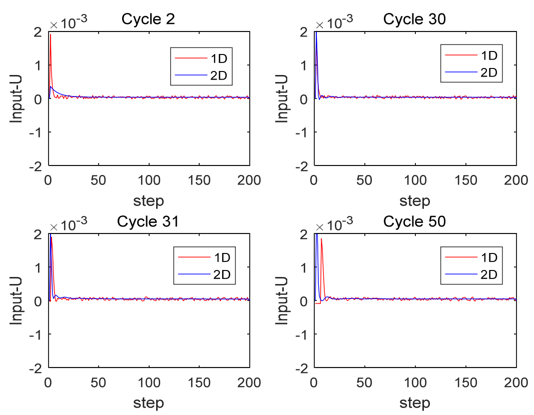


FIGURE 9. The system control input comparison.

even returns to the normal control level particularly after being influenced by faults.

Four diagrams in Fig.3 are the output responses before faults, in the batch before faults happen, when faults happen and after faults happen. It is found that, the proposed method's output is infinitely approximate to the set point as batch increases, while that of the 1D method fluctuates greatly. Fig.4 shows input response contrast diagrams. It is revealed that at the initial stage of each batch, both control methods have a greatly fluctuating input curve; whereas, As the batch increases, the 2D method input is superior to the traditional method.

Fig.5 shows the control increments. It is found that both control methods have great fluctuation beyond the range of constraint at the initial running stage of each batch; though both can make steady operation after running for a while, the 1D method still fluctuates while the 2D method almost approximates a constant value 0 in a smooth curve or small fluctuation. In conclusion, the 2D method shows better performance than the 1D method.

Fig.6 shows the system output tracking error response. According to the figure, in the 30th batch before the failure, the system output tracking error, although there are some

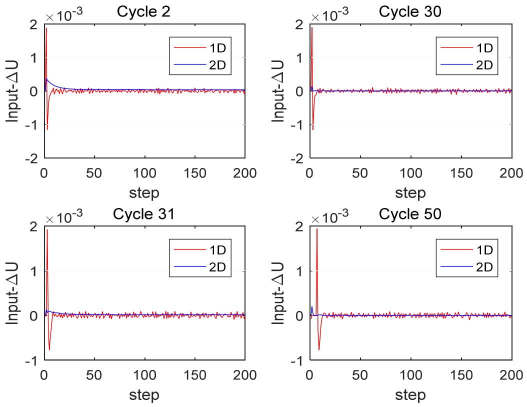


FIGURE 10. The system input increment comparison.

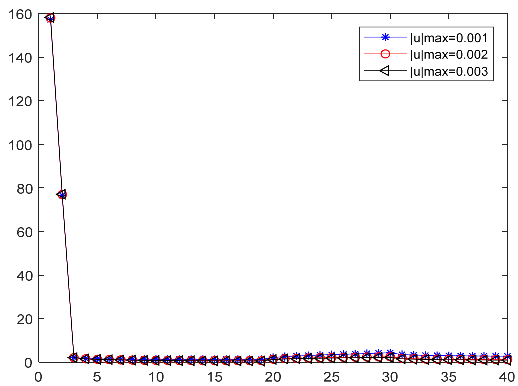


FIGURE 11. The system performance value under different input constraint.

fluctuations in the initial stage, after running for a period of time, is a curve that infinitely tends to zero. Explain that the system is convergent. In the 31th batch of the failure, the systematic error was significantly worse, but after several batches, the 50th batch, the error was significantly smaller, and it was still infinitely close to zero. But it is slightly worse than before the fault. Explain that the fault has an impact on the system control performance.

Case 2: Time-varying fault under non-repetitive interference

Since the faults are not always constant in actual production, they will be affected by the environment and process complexity. Therefore, this part considers the time-varying fault and selects it as $\alpha = 0.6 + 0.4\sin(t)$, $0 \le t \le 200$; and the interference is the same as Case 1.

It can be seen from Fig. 7–10 that under the non-repetitive interference and time-varying fault, the tracking performance of the method of this paper is better than the 1D method; the output is more stable than the 1D method except that the fault batch fluctuates greatly; the same is true for system input and input increments. In summary, the method of this paper is superior to the 1D method in the case of non-repetitive interference constant failure and time-varying failure.

Among them, due to the time-varying failure of the 31th batch, the stability and performance of the system are deteriorated, so the DT (k) of the proposed method is worse than the conventional lD method, while the subsequent batch, the performance is improved even more than the conventional method. In the 31th to 36th steps of Fig. 7, the maximum and minimum values of DT(k) in the 2D method are: 0.246, 0.125; the maximum and minimum values of the 1D method are: 0.318, 0.238.

Fig 11 shows the performance values for different batches at the same output for different inputs. As can be seen from the figure, when the input upper bound value is 0.003, the performance is optimal.

IV. CONCLUSION

With regard to actuator gain faults and batch processes with strong nonlinearity, a 2D fuzzy-constraint fault-tolerant predictive control strategy is designed in this article. According to this strategy, the batch process is converted into an 2D T-S spatial fault model with fuzzy state using its nonlinear and 2D characteristics. Meanwhile, the design of the ILFTC strategy is transformed into the determination of the constrained update law and system stability sufficient conditions are offered. The determined constraint update law guarantees that system can still make steady operation and exponential convergence under faults. Last but not least, the three-tank simulation proves the practical value and effectiveness of the above method. There are still factors such as time delay and multi-phase in batch process that affect production, the corresponding control under such complex situation is our future work.

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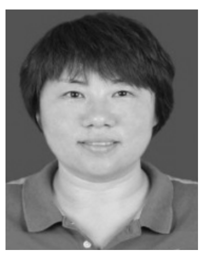

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