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Boundary Control for a Suspension Cable System of a Helicopter With Saturation Nonlinearity Using Backstepping Approach

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ABSTRACT In this paper, the problems of vibration reduction and trajectory tracking are investigated for a suspension cable system of a helicopter in the presence of input saturation and external disturbances. First, an auxiliary system is proposed to compensate for the error between control input and actuator output which is caused by input saturation. Then, based on the introduced auxiliary system, an adaptive boundary control scheme is proposed to track a desired trajectory and restrain the vibration by using backstepping method. Under the designed control scheme, the uniform ultimate boundedness of closed-loop system is guaranteed. Moreover, the vibration amplitude and the trajectory tracking error will be guaranteed to converge ultimately to a small neighborhood of zero by selecting suitable parameters. The rationality and validity of designed control law is verified by a numerical simulation.

INDEX TERMS Vibration reduction, input saturation, adaptive boundary control, uniform ultimate boundedness, suspension cable system.

I. INTRODUCTION

As one of the most important skills of helicopter, helicopter suspension cable system has drawn much attention of many experts and scholars in the past decades and many nice research results have been achieved [1]–[7]. In the processing of helicopter lifting, if the vibration range exceeds the allowable limits, it may result in the damage of the suspension load and even threaten the life security of the pilot. Thus, the research for vibration attenuation is a meaningful and challenging topic. Some common and effective control strategies have been adopted to reduce vibration, such as input shaping [8], [9], delayed feedback control [10], [11], dynamic programming approach [12], [13]. However, input shaping and dynamic programming approach are open-loop control strategies which are sensitive to external disturbance. In addition, these methods are only suitable for the linearized system model which presents the stability of equilibrium points. Considering the process of helicopter lifting, the aforementioned approaches are not suitable to eliminate the vibration for the helicopter lifting system. Moreover, the distributed disturbance resulted from atmospheric turbulence and wind may render that the suspension cable can not keep tight all the time. Hence, representing a suspension cable system of a helicopter by a set of ordinary differential equations is not suitable under the circumstance. In general, a system with vibration is often represented by a distributed parameter system from a mathematical perspective. Therefore, in this paper, the suspension cable system of a helicopter will be modeled as a distributed parameter system, whose state variables are related with both time and space. For the distributed parameter system, boundary control is an effective control strategy to realize vibration reduction according to engineering experience. The boundary control scheme will be proposed to decrease the vibration of flexible cable for the suspension cable system of a helicopter.

Boundary control has received increasing attention and has been applied to many flexible structures [14]–[20]. A flexible string has been modeled as a distributed parameter system, where the adaptive boundary control schemes have been employed to reduce oscillation in [21]–[23]. For the flexible manipulator, the problems of uniform ultimate boundedness have been investigated in [24]–[26], where boundary controllers have been designed by using Lyapunov direct method. Under the proposed control strategies, the oscillation ranges have converged to a small neighborhood of zero ultimately.

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The boundary control strategy has been utilized to analyze the stability of closed-loop system for a flexible riser system in [27]–[29]. The vibration ranges have converged to a compact set with the proposed control laws by selecting proper design parameters. For the flexible cable suspended robot system, the anti-oscillation control problems have been investigated by using the feedback linearization control approach, where boundary controllers have been designed in [30]–[34]. Owing to actuator physical constraint, hence there may be an error between the control input and the actuator output. This phenomenon is called input saturation which is one of the most common and important non-smooth input nonlinearities in mechanical equipments. Input saturation can severely degrade the performance of closed-loop system, thus it will be considered for the suspension cable system of a helicopter in this paper.

As a common input nonlinearity, input saturation has been studied and some effective methods have been raised [25], [35]–[38]. In [39], an adaptive tracking controller has been introduced for a class of uncertain multi-input and multi-output nonlinear systems with input nonlinearities. To tackle with the problem of input nonlinearities, the method of designing auxiliary system has been used. Based on the backstepping approach, a dynamic surface control scheme has been proposed for a class of strict-feedback nonlinear systems with input saturation in [40]. The method of radial basis function neural network has been utilized to eliminate the effect of input saturation. The guaranteed transient performance has been investigated for the near space vehicle in the presence of actuator input saturation by employing the backstepping technique in [41], where the parameter adaptive strategy has been adopted to deal with the problem of input saturation. The vibration reduction control problem has been studied for a suspension cable system of a helicopter with input constraints in [42]. Another effective and simple auxiliary system has been applied to compensate for the effect of input nonlinearities. To the best of authors' knowledge, there are few vibration reduction and trajectory tracking control results for a suspension cable system of a helicopter with input saturation by using the backstepping method.

This paper investigates the vibration reduction and trajectory tracking control problems for the helicopter suspension cable system with input saturation and external disturbances by backstepping method. Firstly, an auxiliary system is proposed to tackle with the problem of input saturation. Then, based on the proposed auxiliary system and the Lyapunov theory, an adaptive boundary control is introduced to reduce the cable's vibration and follow a pre-given trajectory. Under the developed control scheme, the closed-loop system for the suspension cable system of a helicopter is proved to be uniformly ultimately bounded.

The main contributions of this paper are summarized as follows:

1) Different from the existed results, the vibration reduction and trajectory tracking control problems are investigated by employing backstepping strategy for a suspension cable system of a helicopter.
2) Different from the article [3], [6], [7], [42], an unilateral adaptive boundary control method is adopted to stabilize the closed-loop system. The unilateral control is more easy to implement than bilateral control which has been used in [3], [6], [7], [42].
3) A numerical simulation is developed to verify that the proposed control scheme in this paper is superior to the proportional-differential (PD) control and the introduced control strategy in [5].

The rest of this paper is listed as follows. Section II raises the problems that need to be solved and gives out some assumptions and lemmas which are needed for the stability analysis. An adaptive boundary control is introduced based on the proposed auxiliary system for the suspension cable system of a helicopter in Section III. Section IV shows the control performances by using the proposed control scheme in this paper, PD control and the applied control strategy in [5] by a numerical simulation, which is followed by the conclusion in Section V.

Notations. $\mathcal{R}$ is the set of real numbers. $\tanh(\cdot)$ represents the hyperbolic tangent function. $\ln(\cdot)$ denotes the nature logarithm of $(\cdot)$. $\min(a_1, a_2)$ is to acquire the minimum value between a_1 and a_2. Throughout the paper, for $\forall\, z \in [0, L]$, $t \in [0, +\infty)$, the abbreviations and physical meanings of system variables are given by TABLE 1.

TABLE 1. The abbreviations and physical meanings of system variables.

Symbol	Abbreviation	Physical Meaning
$\frac{\partial s(z,t)}{\partial t}$	$\dot{s}(z,t)$	The velocity of suspension cable
$\frac{\partial s(z,t)}{\partial z}$	$s'(z,t)$	The slope of suspension cable at the position z for time t
$\frac{\partial s^2(z,t)}{\partial z \partial t}$	$\dot{s}'(z,t)$	The change rate of the slope $s'(z,t)$ with respect to t
$\frac{\partial^2 s(z,t)}{\partial t^2}$	$\ddot{s}(z,t)$	The acceleration of suspension cable
$\frac{\partial^2 s(z,t)}{\partial z^2}$	$s''(z,t)$	The change rate of the slope $s'(z,t)$ with respect to z

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, consider a suspension cable system of a helicopter which is provided by Fig. 1. The longitudinal deflection of the helicopter suspension cable system is only considered in the whole paper.

In Fig. 1, $p(z, t)$ is the vibration amplitude in the local reference frame $x-y$, while $s(z, t) = s(0, t) - p(z, t)$ is the position of the suspension cable where $s(0, t)$ is the position of payload in the inertial coordinate system $X-Y$. According to Fig. 1, it derives that $p(0, t) = 0$. Considering the safety problem of a simple point hanging, a small segment of the suspension cable is tightly fixed to the payload, then, it has $s'(0, t) = 0$. Thus, a small swing angle is assumed in this paper, then the tension of the suspension cable T in the vertical direction can be approximated as $T = m_p g + \int_0^L \mu(z) dz g$ where m_p denotes the mass of payload, $\mu(z)$ represents the nonuniform mass

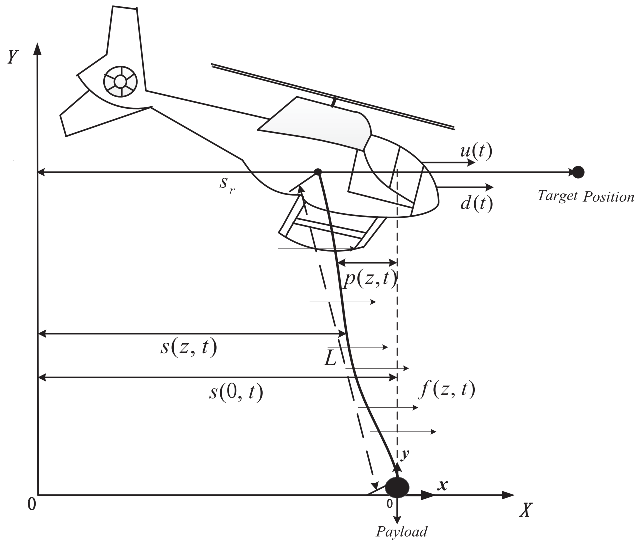

FIGURE 1. **A suspension cable system of a helicopter.**

per unit length of the suspension cable, g is the gravitational acceleration, and L represents the length of the suspension cable, moreover, L is supposed to be a constant. The system parameters for the suspension cable system of a helicopter are listed as follows: m_h denotes the mass of helicopter; c and c_h denote the damping coefficients of the suspension cable and the helicopter, respectively; $u(t)$ and $d(t)$ are the control input and external disturbance, respectively; $f(z,t)$ is the distributed disturbance which is resulted from the high speed airflow and wind.

According to the paper [5], the model for the suspension cable system is described as:

$$\mu(z)\ddot{s}(z,t) = Ts''(z,t) - c\dot{s}(z,t) + f(z,t) \tag{1}$$

for $\forall\ z \in (0, L)$ and $t \in [0, \infty)$, under the boundary conditions of the suspension cable system can be described by

$$m_h\ddot{s}(L,t) = -Ts'(L,t) + u(t) + d(t) - c_h\dot{s}(L,t) \quad for\ \forall\ t \in [0, +\infty). \tag{2}$$

The input saturation is considered in this paper, then the relationship between the desired control input $\nu(t)$ and the actual actuator output $u(t)$ can be written as:

$$u(t) = sat(\nu(t)) = \begin{cases} \overline{U}, & if \quad \nu(t) \geq \underline{U} \\ \nu(t), & if \quad \underline{U} < \nu(t) < \overline{U} \\ \underline{U}, & if \quad \nu(t) \leq \underline{U} \end{cases}$$

where $\overline{U} > 0$ and $\underline{U} < 0$ are the saturation levels of the actual control $u(t)$.

Then, (2) can be represented as:

$$m_h\ddot{s}(L,t) = -Ts'(L,t) + \nu(t) + \Delta u(t) + d(t), \quad - c_h\dot{s}(L,t), \quad for\ \forall\ t \in [0, +\infty) \tag{3}$$

where $\Delta u(t) = u(t) - \nu(t)$ is the error value between the actual control $u(t)$ and the designed control $\nu(t)$ which will be designed in the following paper.

Furthermore, define $x_1(t) = s(L,t)$ and $x_2(t) = \dot{s}(L,t)$. The boundary conditions of the helicopter suspension cable system can be rewritten as:

$$\dot{x}_1(t) = x_2(t), \tag{4}$$

$$\dot{x}_2(t) = m_h^{-1}\Big\{ - c_h x_2(t) - Tx_1'(t) + \nu(t) + \Delta u(t) + d(t)\Big\}, \ for\ \forall\ t \in [0, +\infty). \tag{5}$$

To realize the control targets of tracking trajectory and decreasing vibration for the suspension cable system of a helicopter, the following assumptions and lemmas are introduced.

Assumption 1 [3]: For $\forall\ z \in (0, L), t \in [0, \infty)$, there exist unknown positive constants $\bar{d}$ and D such that the following conditions hold:

$$\bar{d} \geq \Big\{|d(t)|\Big|Max_{t\geq 0}|d(t)|\Big\},$$
$$D \geq \Big\{|f(z,t)|\Big|Max_{t\geq 0}|f(z,t)|\Big\}.$$

Assumption 2 [43]: For the system parameter $\mu(z)$, the following conditions should be satisfied:

$$\mu_1 \leq \mu(z) \leq \mu_2, \quad \mu_3 \leq \mu'(z) \leq \mu_4,$$

where μ_1, μ_2, μ_3, and μ_4 are reasonable positive constants.

Besides, there exists a differentiable function $a(z)$, $\forall\ z \in [0, L]$, with respect to z render that

$$\underline{a} \leq n(z) \leq \bar{a},$$
$$a(z) + za'(z) > \psi,$$
$$a(z)\mu(z) + za'(z)\mu(z) + za(z)\mu'(z) > \psi$$

where $\underline{a}$, $\bar{a}$, and ψ are suitable positive constants.

Assumption 3 [3]: For the error $\Delta u(t)$ between designed control input $\nu(t)$ and actual control input $u(t)$, the following conclusion holds:

$$|\Delta u(t)| = |\nu(t) - u(t)| \leq \epsilon$$

where ϵ is a positive constant.

Moreover, in spite of considering input saturation, there exists a control scheme render that the control targets of tracking trajectory and decreasing vibration for the helicopter suspension cable system can be achieved.

Assumption 4: The reference trajectory of helicopter $r(t)$ satisfies the following conditions:

1) The first-order derivative of $r(t)$ is bounded, i.e., $|\dot{r}(t)| \leq \varpi_1$,
2) The second-order derivative of $r(t)$ is bounded, i.e., $|\ddot{r}(t)| \leq \varpi_2$

where ϖ_1 and ϖ_2 are positive constants.

Lemma 1 [44]: If the function $\omega(z,t) \in \mathcal{R}$ is continuous differentiable and $\omega(0,t) = 0$, then

$$\omega^2(z,t) \leq L\int_0^L [\omega'(z,t)]^2 dz, \quad \forall\ (z,t) \in [0, L] \times [0, \infty).$$

Lemma 2 [44]: For the functions $\omega_1(z,t), \omega_2(z,t) \in \mathcal{R}$, the following inequation holds:

$$\omega_1(z,t)\omega_2(z,t) \le \delta\omega_1^2(z,t) + \frac{1}{\delta}\omega_2^2(z,t), \quad \forall\,(z,t) \in [0,L]\times[0,\infty)$$

with δ being a positive constant.

Lemma 3 [45]: For $\forall\, o(t) \in \mathcal{R}$, the following conclusion can be drawn:

$$|o(t)| - o(t)\tanh\left(\frac{o(t)}{\varepsilon}\right) \le 0.2785\varepsilon \tag{6}$$

where ε is a positive constant.

Lemma 4 [46]: For a $\mathcal{C}^1$ function $W(t) > 0$, if the following conditions hold:

1) the initial value of $W(t)$ is bounded;
2) the function $W(t)$ has upper and lower bounds, i.e., $\Phi_1^2(x(t)) \le W(t) \le \Phi_2^2(x(t))$;
3) the first-order of the function $W(t)$ $\dot{W}(t) \le -Q_1\,W(t) + Q_2$,

then the solution $x(t)$ is uniformly ultimately bounded, with $\Phi_1(x(t))$ and $\Phi_2(x(t))$ being class $\mathcal{K}$ functions, Q_1 and Q_2 being positive constants.

III. CONTROL DESIGN AND STABILITY ANALYSIS

In the following section, first, by using backstepping method, a boundary control law will be designed for the helicopter suspension cable system subject to input saturation. An auxiliary system is introduced to compensate for the effect which is resulted from input saturation. Then, based on the proposed control scheme, the uniform ultimate boundedness of closed-loop will be investigated, moreover, the trajectory tracking error and the vibration amplitude will be guaranteed to converge ultimately to a small neighborhood of zero.

To demonstrate the process of control design clearly, the schematic diagram of control design is given by Fig. 2.

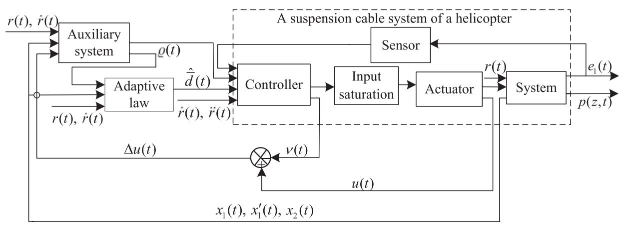

FIGURE 2. The schematic diagram of control design.

Since input saturation is considered, an auxiliary system is introduced to compensate for the error $\Delta u(t)$ between control input and actuator output. The format of the proposed auxiliary system can be described as follows:

$$\dot{\varrho}(t) = \frac{1}{m_h}\Big\{ - l\varrho(t) - \Delta u(t) + Tx_1'(t) + Tx_2(t) - T\dot{r}(t) + (kT+1)\xi_1(t)\Big\} \tag{7}$$

where l and k are positive constants.

Let $r(t)$ being the reference trajectory of helicopter, moreover, $e_1(t) = x_1(t) - r(t)$ and $e_2(t) = x_2(t) - \dot{r}(t)$, thus, $\dot{e}_1(t) = e_2(t)$ for $\dot{x}_1(t) = x_2(t)$.

Based on the aforementioned contents, define

$$\xi_1(t) = e_1(t), \tag{8}$$

$$\begin{aligned}\xi_2(t) &= \dot{e}_1(t) + \eta(t) + \varrho(t)\\ &= e_2(t) + \eta(t) + \varrho(t)\end{aligned} \tag{9}$$

with $\eta(t)$ being the virtual control which will be designed in the following section.

The process of backstepping technology can be described as follows:

Step 1: Invoking (8) and (9), the derivative of $\xi_1(t)$ with respect to time yields

$$\dot{\xi}_1(t) = \xi_2(t) - \eta(t) - \varrho(t). \tag{10}$$

Consider a Lyapunov function as follows:

$$V_{o1}(t) = \frac{\alpha}{2}\xi_1^2(t). \tag{11}$$

where α is a positive constant with α satisfying $\frac{L\bar{\alpha}\mu_2}{\min\{\alpha\mu_1,\alpha T\}} < 1$.

Invoking (10) and (11), the derivative of $V_{o1}(t)$ can be written as:

$$\begin{aligned}\dot{V}_{o1}(t) &= \alpha\xi_1(t)\dot{\xi}_1(t)\\ &= \alpha\xi_1(t)[\xi_2(t) - \eta(t) - \varrho(t)].\end{aligned} \tag{12}$$

Thus, the virtual control law $\eta(t)$ can be designed as follows:

$$\eta(t) = k\xi_1(t) + x_1'(t). \tag{13}$$

Substituting (13) into (12), it has

$$\dot{V}_{o1}(t) = -k\alpha\xi_1^2(t) + \alpha\xi_1(t)\xi_2(t) - \alpha\xi_1(t)x_1'(t) - \alpha\xi_1(t)\varrho(t). \tag{14}$$

Step 2: The adaptive control method which is a strong robustness and simple control method will be used to compensate for the effect of unknown external disturbance in the following section. Moreover, an adaptive control law can be designed as follows:

$$\dot{\hat{\bar{d}}}(t) = \beta^{-1}\alpha\xi_2(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right) - \beta^{-1}\gamma\hat{\bar{d}}(t), \tag{15}$$

$$\begin{aligned}\nu(t) = {}& c_h x_2(t) - \hat{\bar{d}}(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right) + l_1\varrho(t)\\ & - l_2\xi_2(t) + m_h\ddot{r}(t) - m_h\dot{\eta}(t) - l_3x_2(t)\\ & - Tx_2(t) + T\dot{r}(t) - (kT+1)\xi_1(t)\end{aligned} \tag{16}$$

where β, γ, ε, l_1, l_2, and l_3 are positive constants, $\hat{\bar{d}}(t)$ is the estimation of the unknown constant $\bar{d}$.

Invoking (5) and (7) yields

$$\begin{aligned}m_h\dot{\xi}_2(t) &= m_h\dot{x}_2(t) - m_h\ddot{r}(t) + m_h\dot{\eta}(t) + m_h\dot{\varrho}(t)\\ &= -Tx_1'(t) + \nu(t) + \Delta u(t) + d(t) - c_hx_2(t)\\ &\quad - l_1\varrho(t) - \Delta u(t) + Tx_1'(t) + Tx_2(t)\end{aligned}$$

$$
\begin{aligned}
&-T\dot{r}(t)+(kT+1)\xi_1(t)-m_h\ddot{r}(t)\\
&+m_h\dot{\eta}(t)\\
=&-l_1\varrho(t)+\nu(t)+d(t)-c_hx_2(t)+Tx_2(t)\\
&-T\dot{r}(t)+(kT+1)\xi_1(t)-m_h\ddot{r}(t)\\
&+m_h\dot{\eta}(t).
\end{aligned}\tag{17}
$$

Substituting (16) into (17), it follows that

$$
m_h\dot{\xi}_2(t)=-l_2\xi_2(t)-l_3x_2(t)+d(t)-\hat{\bar{d}}(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right).\tag{18}
$$

A Lyapunov function can be described as follows:

$$
V_{o2}(t)=V_{o1}(t)+\frac{\alpha m_h}{2}[\xi_2^2(t)+\varrho^2(t)]+\frac{\beta}{2}\tilde{\bar{d}}^2(t)\tag{19}
$$

where $\tilde{\bar{d}}(t)=\bar{d}-\hat{\bar{d}}(t)$.

Quoting (7), (14), (15), (18), and (19), it derives that

$$
\begin{aligned}
\dot{V}_{o2}(t)=&\dot{V}_{o1}(t)+\alpha m_h\xi_2(t)\dot{\xi}_2(t)+\alpha m_h\varrho_1(t)\dot{\varrho}_1(t)\\
&+\beta\tilde{\bar{d}}(t)\dot{\tilde{\bar{d}}}(t)\\
\le&-k\alpha\xi_1^2(t)+\alpha\xi_1(t)\xi_2(t)-\alpha\xi_1(t)x_1'(t)\\
&-\alpha\xi_1(t)\varrho(t)+\alpha\xi_2(t)\Big\{-l_2\xi_2(t)-l_3x_2(t)\\
&+d(t)-\hat{\bar{d}}(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right)\Big\}+\alpha\varrho(t)\Big\{-l\varrho(t)\\
&-\Delta u(t)+Tx_1'(t)+Tx_2(t)-T\dot{r}(t)\\
&+(kT+1)\xi_1(t)\Big\}-\alpha\tilde{\bar{d}}(t)\xi_2(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right)\\
&+\alpha\gamma\tilde{\bar{d}}(t)[\bar{d}-\tilde{\bar{d}}(t)].
\end{aligned}\tag{20}
$$

According to Lemma 2, Lemma 3, Assumption 1, and Assumption 3, the derivative of $V_{o2}(t)$ can be rewritten as:

$$
\begin{aligned}
\dot{V}_{o2}(t)\le&-\alpha k\xi_1^2(t)-\alpha l_2\xi_2^2(t)+\alpha\tilde{\bar{d}}(t)\xi_2(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right)\\
&-\alpha(l-\varsigma_1)\varrho^2(t)+\frac{\alpha}{\varsigma_1}[\Delta u(t)]^2+\alpha Tx_2(t)\varrho(t)\\
&-\alpha T\dot{r}(t)\varrho(t)+\alpha Tx_1'(t)\varrho(t)+\alpha kT\xi_1(t)\varrho(t)\\
&-\alpha\tilde{\bar{d}}(t)\xi_2(t)\tanh\left(\frac{\xi_2(t)}{\varepsilon}\right)+\alpha\gamma\varsigma_2\bar{d}^2\\
&-\alpha\gamma(1-\frac{1}{\varsigma_2})\tilde{\bar{d}}^2(t)+0.2785\alpha\varepsilon\bar{d}\\
\le&-\alpha k\xi_1^2(t)-\alpha l_2\xi_2^2(t)-\alpha(l-\varsigma_1)\varrho^2(t)\\
&-\alpha\gamma(1-\frac{1}{\varsigma_2})\tilde{\bar{d}}^2(t)+\alpha Tx_2(t)\varrho(t)-\alpha T\dot{r}(t)\varrho(t)\\
&+\alpha Tx_1'(t)\varrho(t)+\alpha kT\xi_1(t)\varrho(t)+\alpha\gamma\varsigma_2\bar{d}^2\\
&+\frac{\alpha}{\varsigma_1}\epsilon^2+0.2785\alpha\varepsilon\bar{d}
\end{aligned}\tag{21}
$$

where ς_1 and ς_2 are positive constants.

Theorem 1: For the helicopter suspension cable system (1) under boundary conditions (4) and (5) with external disturbances and input saturation, under the designed adaptive control law (16), the problem of uniform ultimate boundedness for the closed-loop system is investigated. Moreover, the following targets will be realized with the proposed control scheme:

1) Trajectory tracking error $\xi_1(t)$ will converge ultimately to a small neighborhood of zero, i.e.,

$$
\lim_{t\to+\infty}|\xi_1(t)|\le\sqrt{\frac{2\kappa_2}{\alpha\kappa_1}},\tag{22}
$$

2) The amplitude of oscillation $p(z,t)$ will remain ultimately in a compact $\Gamma=\left\{p(z,t)\middle||p(z,t)|\le\sqrt{\frac{2L\kappa_2}{\alpha T\kappa_1\kappa_3}}\right\}$, for $\forall z\in[0,L]$,

where

$$
\begin{aligned}
\kappa_1=&\min\Big\{k^2T-2k\pi_3T-2k\pi_5T-2k\pi_6T+2k,\\
&\frac{2l_2-T}{m_h},\ \frac{2l+T-2\varsigma_1}{m_h},\ \frac{\gamma(1-\frac{1}{\varsigma_2})}{\beta},\\
&\frac{2c-\frac{2}{\pi_1}+\frac{2\psi}{\alpha}}{\mu_2},\ \frac{\psi-\frac{2L\bar{a}}{\pi_7}}{\alpha},\ \frac{c}{\mu_2}\Big\},\\
\kappa_2=&\alpha\gamma\varsigma_2\bar{d}^2+\frac{\alpha}{\varsigma_1}\epsilon^2+0.2785\alpha\varepsilon\bar{d}+(\alpha\pi_1L+L^2\bar{a}\pi_7)\\
&\times D^2+[\alpha T\pi_2+\alpha T\pi_4+\alpha kT\pi_5]\varpi_1^2,\\
\kappa_3=&1-\frac{L\bar{a}\mu_2}{\min\{\alpha\mu_1,\alpha T\}}
\end{aligned}
$$

with π_1, π_2, π_3, π_4, π_5, π_6, and π_7 being positive constants.

Proof: Under the adaptive control law (16), the stability of the closed-loop system will be analyzed in the following section. A Lyapunov function is introduced as follows:

$$
E(t)=V_{o2}(t)+E_1(t)+E_2(t)\tag{23}
$$

with

$$
\begin{aligned}
E_1(t)=&\frac{\alpha}{2}\int_0^L\mu(z)\dot{s}^2(z,t)dz\\
&+\frac{\alpha T}{2}\int_0^L[s'(z,t)]^2dz,
\end{aligned}\tag{24}
$$

$$
E_2(t)=\int_0^L za(z)\mu(z)\dot{s}(z,t)s'(z,t)dz.\tag{25}
$$

Invoking (1), (24), and Lemma 2, considering $s'(0,t)=0$, the derivative of $E_1(t)$ can be described as:

$$
\begin{aligned}
\dot{E}_1(t)=&\alpha\int_0^L\mu(z)\dot{s}(z,t)\ddot{s}(z,t)dz+\alpha T\int_0^L[s'(z,t)\\
&\times\dot{s}'(z,t)]dz\\
=&\alpha\int_0^L\dot{s}(z,t)[Ts''(z,t)+f(z,t)-c\dot{s}(z,t)]dz\\
&+\alpha T\int_0^L[s'(z,t)\dot{s}'(z,t)]dz\\
\le&\alpha Tx_2(t)x_1'(t)-\alpha\int_0^L(c-\frac{1}{\pi_1})\dot{s}^2(z,t)dz\\
&+\alpha\pi_1LD^2.
\end{aligned}\tag{26}
$$

Consider $\alpha T x_2(t)x_1'(t) = \frac{\alpha T}{2}\xi_2^2(t) - \frac{\alpha T}{2}x_2^2(t) - \frac{\alpha T}{2}[x_1'(t)]^2 - \frac{\alpha T}{2}\dot{r}^2(t) - \frac{\alpha k^2 T}{2}\xi_1^2(t) - \frac{\alpha T}{2}\varrho^2(t) + \alpha T x_2\dot{r}(t) - \alpha k T x_2(t)\xi_1(t) - \alpha T x_2(t)\varrho(t) + \alpha T x_1'(t)\dot{r}(t) + \alpha k T\xi_1(t)\dot{r}(t) + \alpha T\varrho(t)\dot{r}(t) - \alpha k T\xi_1(t)x_1'(t) - \alpha T x_1'(t)\varrho(t) - \alpha k T\xi_1(t)\varrho(t)$, Assumption 1, Assumption 4, and Lemma 2, (26) can be rewritten as

$$\begin{aligned}\dot{E}_1(t) \leq{}& \frac{\alpha T}{2}\xi_2^2(t) - \alpha T\Big(\frac{1}{2} - \frac{1}{\pi_2} - \frac{k}{\pi_3}\Big)x_2^2(t)\\ &- \alpha T\Big(\frac{1}{2} - \frac{1}{\pi_4} - \frac{k}{\pi_6}\Big)[x_1'(t)]^2 - \alpha k T\Big(\frac{k}{2} - \pi_3\\ &- \pi_5 - \pi_6\Big)\xi_1^2 - \frac{\alpha T}{2}\varrho^2(t) - \alpha T x_2(t)\varrho(t)\\ &+ \alpha T\dot{r}(t)\varrho(t) - \alpha T x_1'(t)\varrho(t) - \alpha k T\xi_1(t)\varrho(t)\\ &- \alpha\int_0^L (c - \frac{1}{\pi_1})\dot{s}^2(z,t)dz + \alpha\pi_1 L D^2\\ &+ [\alpha T\pi_2 + \alpha T\pi_4 + \alpha k T\pi_5]\varpi_1^2. \end{aligned} \tag{27}$$

According to (1), (25), Assumption 1, Assumption 2, and Lemma 2, the derivative of $E_2(t)$ with respect to time is given by

$$\begin{aligned}\dot{E}_2(t) ={}& \int_0^L za(z)\mu(z)[\ddot{s}(z,t)s'(z,t) + \dot{s}(z,t)\\ &\times\dot{s}'(z,t)]dz\\ ={}& \int_0^L za(z)[Ts''(z,t) - c\dot{s}(z,t) + f(z,t)]\\ &\times s'(z,t)dz + \int_0^L za(z)\mu(z)\dot{s}(z,t)\dot{s}'(z,t)dz\\ \leq{}& -\frac{T}{2}\int_0^L [a(z) + za'(z) - \frac{2L\bar{a}}{\pi_7}][s'(z,t)]^2 dz\\ &+ L\mu_2\bar{a}x_2^2(t) - \frac{1}{2}\int_0^L [a(z)\mu(z) + za'(z)\mu(z)\\ &+ za(z)\mu'(z)]\dot{s}^2(z,t)dz - c\int_0^L za(z)\dot{s}(z,t)\\ &\times s'(z,t)dz + L^2\bar{a}\pi_7 D^2. \end{aligned} \tag{28}$$

Quoting (21), (27), (28), and Assumption 2 yields

$$\begin{aligned}\dot{E}(t) ={}& \dot{V}_{o2}(t) + \dot{E}_1(t) + \dot{E}_2(t)\\ \leq{}& -\alpha k\Big\{\frac{kT}{2} - \pi_3 T - \pi_5 T - \pi_6 T + 1\Big\}\xi_1^2\\ &- \alpha(l_2 - \frac{T}{2})\xi_2^2(t) - \alpha(l + \frac{T}{2} - \varsigma_1)\varrho^2(t)\\ &- \alpha\gamma(1 - \frac{1}{\varsigma_2})\tilde{\bar{d}}^2(t) - \alpha\int_0^L (c - \frac{1}{\pi_1})\dot{s}^2(z,t)dz\\ &- \frac{T}{2}\int_0^L [\psi - \frac{2L\bar{a}}{\pi_7}][s'(z,t)]^2 dz - c\int_0^L zn(z)\\ &\times\dot{s}(z,t)s'(z,t)dz + \alpha T x_2(t)\varrho(t) - \alpha T\dot{r}(t)\varrho(t)\\ &+ \alpha T x_1'(t)\varrho(t) + \alpha\gamma\varsigma_2\bar{d}^2 + \frac{\alpha}{\varsigma_1}\epsilon^2 + 0.2785\alpha\varepsilon\bar{d}\\ &- \alpha\Big\{T\Big(\frac{1}{2} - \frac{1}{\pi_2} - \frac{k}{\pi_3}\Big) - \frac{L\mu_2\bar{a}}{\alpha}\Big\}x_2^2(t)\\ &- \alpha T\Big(\frac{1}{2} - \frac{1}{\pi_4} - \frac{k}{\pi_6}\Big)[x_1'(t)]^2\\ &- \frac{1}{2}\int_0^L \psi\dot{s}^2(z,t)dz + (\alpha\pi_1 L + L^2\bar{a}\pi_7)D^2\\ &+ [\alpha T\pi_2 + \alpha T\pi_4 + \alpha k T\pi_5]\varpi_1^2\\ \leq{}& -\alpha k\Big\{\frac{kT}{2} - \pi_3 T - \pi_5 T - \pi_6 T + 1\Big\}\xi_1^2\\ &- \alpha(l_2 - \frac{T}{2})\xi_2^2(t) - \alpha(l + \frac{T}{2} - \varsigma_1)\varrho^2(t)\\ &- \alpha\gamma(1 - \frac{1}{\varsigma_2})\tilde{\bar{d}}^2(t) - \alpha\int_0^L (c - \frac{1}{\pi_1} + \frac{\psi}{2\alpha})\\ &\times\dot{s}^2(z,t)dz - \frac{T}{2}\int_0^L [\psi - \frac{2L\bar{a}}{\pi_7}][s'(z,t)]^2 dz\\ &- c\int_0^L za(z)\dot{s}(z,t)s'(z,t)dz + \alpha\gamma\varsigma_2\bar{d}^2\\ &+ \frac{\alpha}{\varsigma_1}\epsilon^2 + 0.2785\alpha\varepsilon\bar{d} + (\alpha\pi_1 L + L^2\bar{a}\pi_7)D^2\\ &+ [\alpha T\pi_2 + \alpha T\pi_4 + \alpha k T\pi_5]\varpi_1^2\\ \leq{}& -\kappa_1 E(t) + \kappa_2 \end{aligned} \tag{29}$$

with

$$\begin{aligned} k^2T - 2k\pi_3 T - 2k\pi_5 T - 2k\pi_6 T + 2k &> 0,\\ 2l_2 - T &> 0,\\ 2l + T - 2\varsigma_1 &> 0,\quad 1 - \frac{1}{\varsigma_2} > 0,\\ c - \frac{1}{\pi_1} + \frac{\psi}{\alpha} &> 0,\\ \psi - \frac{2L\bar{a}}{\pi_7} &> 0,\\ T\Big(\frac{1}{2} - \frac{1}{\pi_2} - \frac{k}{\pi_3}\Big) - L\mu_2\bar{a} &\geq 0,\\ \frac{1}{2} - \frac{1}{\pi_4} - \frac{k}{\pi_6} &\geq 0. \end{aligned}$$

The inequation (29) means that the closed-loop system is uniformly ultimately bounded. The specific results will be given in the following section.

Multiplied (29) by $e^{\kappa_1 t}$, it follows that

$$\dot{E}(t)e^{\kappa_1 t} \leq -\kappa_1 E(t)e^{\kappa_1 t} + \kappa_2 e^{\kappa_1 t}. \tag{30}$$

Then,

$$\frac{d(E(t)e^{\kappa_1 t})}{dt} \leq \kappa_2 e^{\kappa_1 t}. \tag{31}$$

Computing inequation (31), and considering κ_1 and κ_2 are positive constants, it has

$$\begin{aligned} E(t) &\leq \Big(E(0) - \frac{\kappa_2}{\kappa_1}\Big)e^{-\kappa_1 t} + \frac{\kappa_2}{\kappa_1}\\ &\leq E(0)e^{-\kappa_1 t} + \frac{\kappa_2}{\kappa_1}. \end{aligned} \tag{32}$$

Since $E_2(t)$ is a cross-term, the relationship between $E_2(t)$ and $E_1(t)$ should be given firstly. The rigorous analysis will be shown in the following section.

$$|E_2(t)| \leq \frac{L\bar{a}\mu_2}{2}\Big(\int_0^L \dot{s}^2(z,t)dz + \int_0^L [s'(z,t)]^2 dz\Big) \leq \frac{L\bar{a}\mu_2}{\min\{\alpha\mu_1, \alpha T\}}E_1(t). \quad (33)$$

Then,

$$E_1(t) + E_2(t) \geq (1 - \frac{L\bar{a}\mu_2}{\min\{\alpha\mu_1, \alpha T\}})E_1(t). \quad (34)$$

Further, since $V_{o2}(t)$ is positive definite and $\kappa_3 = 1 - \frac{L\bar{a}\mu_2}{\min\{\alpha\mu_1,\alpha T\}}$, one obtains

$$E(t) = V_{o2}(t) + E_1(t) + E_2(t) \geq V_{o2}(t) + \kappa_3 E_1(t). \quad (35)$$

Due to $\kappa_3 > 0$, thus

$$\frac{\alpha}{2}\xi_1^2(t) \leq V_{o2}(t) \leq E(t) \leq E(0)e^{-\kappa_1 t} + \frac{\kappa_2}{\kappa_1}. \quad (36)$$

Further,

$$\lim_{t\to+\infty} |\xi_1(t)| \leq \sqrt{\frac{2\kappa_2}{\alpha\kappa_1}}. \quad (37)$$

Invoking (24), according to the Lemma 1 and $p'(z,t) = -s'(z,t)$, it has

$$\frac{\alpha T}{2}[p(z,t)]^2 \leq \frac{\alpha TL}{2}\int_0^L [p'(z,t)]^2 dz = \frac{\alpha TL}{2}\int_0^L [s'(z,t)]^2 dz \leq LE_1(t). \quad (38)$$

Invoking (35) and (38), for $\forall z \in [0, L]$, it follows that

$$\frac{\alpha T}{2}[p(z,t)]^2 \leq LE_1(t) \leq L\frac{E(t)}{\kappa_3}, \quad (39)$$

then

$$|p(z,t)| \leq \sqrt{\frac{2L}{\alpha T\kappa_3}E(t)} \leq \sqrt{\frac{2L}{\alpha T\kappa_3}(E(0)e^{-\kappa_1 t} + \frac{\kappa_2}{\kappa_1})}. \quad (40)$$

Therefore, when $t \to +\infty$, one has $|p(z,t)| \leq \sqrt{\frac{2L\kappa_2}{\alpha T\kappa_1\kappa_3}}$ i.e., the amplitude of oscillation $p(z,t)$ will remain ultimately in a compact set $\Gamma = \Big\{p(z,t)\Big| |p(z,t)| \leq \sqrt{\frac{2L\kappa_2}{\alpha T\kappa_1\kappa_3}}\Big\}$.

This concludes the proof. $\diamondsuit$

IV. NUMERICAL SIMULATION

In this section, consider a suspension cable system of a helicopter with input saturation and external disturbances which is depicted by (1), (4), and (5). The external disturbances are given by: $d(t) = (1.2 + 0.4\sin(0.1t) + 0.2\sin(0.2t) + 0.4\sin(0.1t)) \times 10^4$ and $f(z,t) = (1.5 + 0.6\sin(0.2\pi t) + 0.4\sin(0.4\pi t) + 0.2\sin(0.6\pi t))z$. The parameters of the suspension cable system of a helicopter are provided by TABLE 2.

TABLE 2. Parameters of the suspension cable system of a helicopter.

Symbol	Description	Value
L	Length of the cable	20m
m_h	Mass of the helicopter	1.5×10^4kg
m_p	Mass of the payload	2.88×10^3kg
c	Damping of the suspension cable	1.0N s/m
c_h	Damping of the helicopter	0.1×10^4N s/m
T	The tension of the suspension cable	0.5×10^4 N
$\mu(z)$	The nonuniform mass per unit length	$(0.1z + 5)$kg/m

The desired trajectory is chosen as follows [47]:

$$r(t) = \frac{s_r}{2} + \frac{\iota_1}{2}\ln\frac{e^{(\iota_2 t-\iota_3)} + e^{(-\iota_2 t+\iota_3)}}{e^{(\iota_2 t-\iota_3-s_r/\iota_1)} + e^{(-\iota_2 t+\iota_3+s_r/\iota_1)}} \quad (41)$$

where ι_1, ι_2, and ι_3 are positive constants, s_r is the distance between initial position and target position of helicopter.

The saturation levels of the actual actuator output $u(t)$ are $\overline{U} = 1.0 \times 10^4$ and $\underline{U} = -2.0 \times 10^4$.

The designed parameters are given as follows: $l = 0.1$, $k = 100$, $\alpha = 0.005$, $\beta = 1$, $\gamma = 0.1$, $\varepsilon = 0.1$, $l_1 = 1.0 \times 10^4$, $l_2 = 1.0 \times 10^4$, and $l_3 = 4.0 \times 10^5$. The desired trajectory parameters are listed as: $\iota_1 = 2.5$, $\iota_2 = 1$, $\iota_3 = 2$, and $s_r = 100$m.

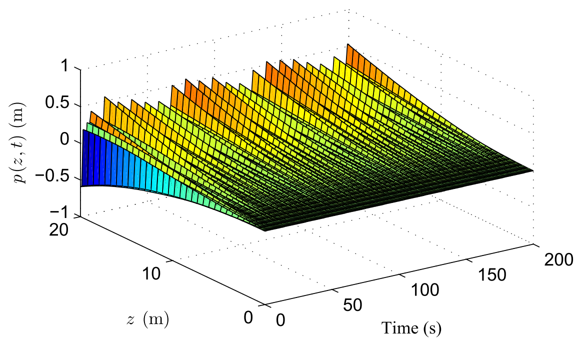

FIGURE 3. The longitudinal vibration amplitude $p(z,t)$ of the suspension cable without control.

When control input $u(t) = 0$, the response curve of vibration amplitude $p(z,t)$ is presented by Fig. 3. Based on the introduced adaptive boundary control scheme which is described by (7), (15), and (16), the simulation results of closed-loop system for the helicopter suspension cable system are given by Figs. 4−6. Fig. 4 and Fig. 5 denote the vibration amplitude $p(z,t)$ of the helicopter suspension cable system and the trajectory tracking of helicopter under the proposed adaptive boundary control. It shows that the proposed adaptive control scheme is efficient to restrain the vibration of cable and track the given trajectory. Fig. 6 shows the designed adaptive boundary control input $v(t)$ and actual actuator output $u(t)$. From Fig. 6, it is clearly seen that consider the phenomenon of input saturation is meaningful.

To illustrate the effectiveness of proposed control scheme which is represented by (7), (15), and (16), a PD control law

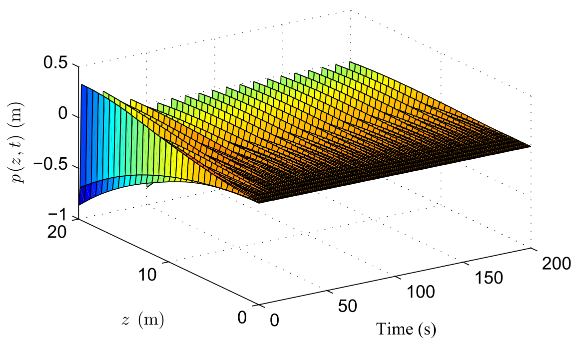

FIGURE 4. The longitudinal vibration amplitude $p(z, t)$ of the suspension cable with proposed adaptive boundary control scheme.

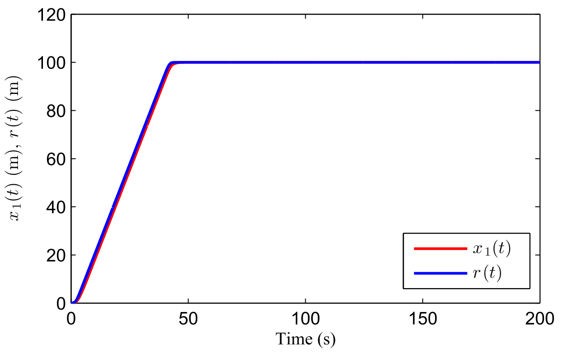

FIGURE 5. The curve trajectory tracking with proposed adaptive boundary control scheme.

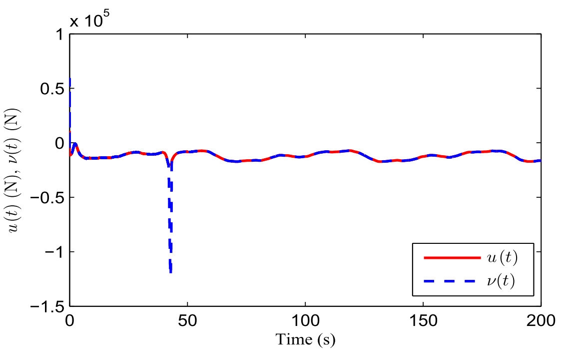

FIGURE 6. The designed adaptive boundary control input $\nu(t)$ and actual adaptive boundary control control input $u(t)$.

is presented as:

$$\nu(t) = -s_1[s(L, t) - r(t)] - s_2[\dot{s}(L, t) - \dot{r}(t)] \quad (42)$$

where s_1 and s_2 are positive constants.

The design parameters of PD control law are chosen as: $s_1 = 1.0 \times 10^4$ and $s_2 = 1.0 \times 10^5$. The design parameters remain unchanged for desired trajectory $r(t)$.

Under the PD controller (42), Fig. 7 and Fig. 8 represent the vibration amplitude $p(z, t)$ of the helicopter suspension cable system and the trajectory tracking curve of helicopter.

Compare Fig. 4 with Fig. 7, it is obvious that the control performance by using the proposed adaptive boundary control is better than the PD control scheme (42). From Fig. 5

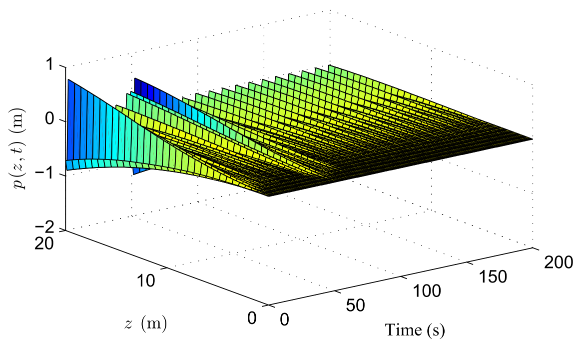

FIGURE 7. The longitudinal vibration amplitude $p(z, t)$ of the suspension cable with proposed PD control scheme.

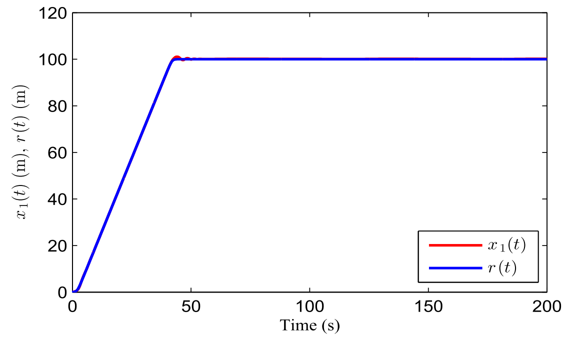

FIGURE 8. The curve trajectory tracking with proposed PD control scheme.

and Fig. 8, it can be seen that the good control performances can be achieved by utilizing two control strategies.

Considering the PD control strategy is a linear method, it will be more persuasive to use a nonlinear control method as a comparison. Thus, under the auxiliary system (7) and adaptive law (15), another nonlinear control approach which has been used in article [5] will be adopted to check the effectiveness of the proposed control method in this paper.

With the proposed control scheme in paper [5], Fig. 9 and Fig. 10 are simulation results of the vibration amplitude $p(z, t)$ and the trajectory tracking curve, respectively.

According to Fig. 4 and Fig. 9, it is obvious that the control performances by using the proposed adaptive boundary control in this paper are better than the proposed control scheme in [5]. Meanwhile, combining Fig. 5 and Fig. 10, it can be drawn that the position of helicopter $x_1(t)$ all can track the given reference trajectory $r(t)$ accurately by using both the proposed control scheme in this paper and the control strategy in [5].

Moreover, in Fig. 7 and Fig. 9, the longitudinal vibration amplitudes $p(z, t)$ of the suspension cable become big at about 40s resulted from the motion of helicopter. However, in Fig. 4, the longitudinal vibration amplitudes $p(z, t)$ of the suspension cable is not affected by the motion of helicopter by using the control approach in this paper.

On the whole, the control performances by utilizing the proposed control strategy in this paper are superior to the

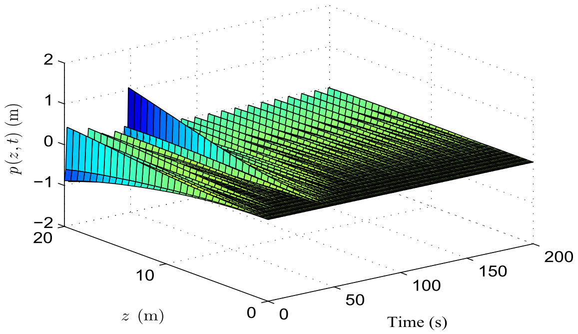

FIGURE 9. The longitudinal vibration amplitude $p(z, t)$ of the suspension cable with control strategy in [5].

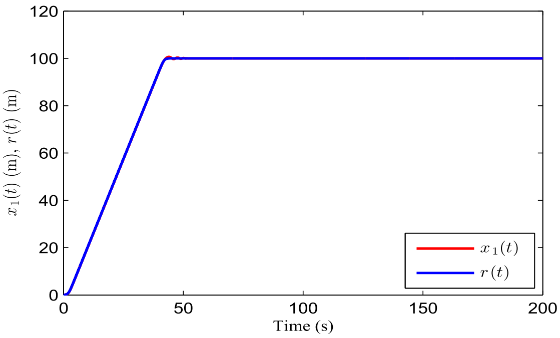

FIGURE 10. The curve trajectory tracking with control strategy in [5].

control performances by using both PD control and the introduced control in [5].

According to the obtained simulation results, rationality and validity of the designed adaptive boundary control scheme are validated.

V. CONCLUSION

A boundary control scheme has been adopted to investigate the problems of vibration reduction and trajectory tracking for the suspension cable system of a helicopter with input saturation and external disturbances. To tackle with the problem of actuator saturation, an auxiliary system has been given. Based on the proposed auxiliary system, an adaptive boundary control law has been designed by employing backstepping method. With the introduced control approach, the vibration amplitude and the trajectory tracking error have converged ultimately to a bounded compact set by choosing suitable design parameters. The effectiveness of the proposed adaptive boundary control scheme has been verified by a numerical simulation. The problem of vibration reduction will be investigated for the planar model of a helicopter suspension cable system in the future study. Moreover, the problem of varying cable length which is a meaningful topic will be considered in the future research.

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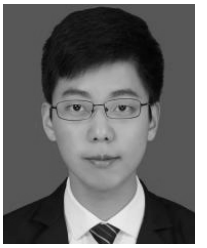

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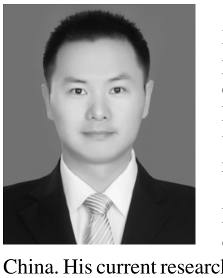

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