IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received July 23, 2019, accepted August 11, 2019, date of publication August 14, 2019, date of current version August 27, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2935304

A New Matrix Projective Synchronization and Its Application in Secure Communication

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This work was supported in part by the National Natural Science Foundation of China under Grant 61471158 and Grant 61571181.

ABSTRACT In this paper, a new matrix projective synchronization for chaotic (hyperchaotic) maps is proposed. The novel scheme called P-M synchronization is presented in this paper, since it combines two matrix projective synchronization schemes (one is based on the invertible matrix P, and the other is based on matrix M). Compared to the regular matrix projection synchronization, the required matrix of the proposed scheme don't change with different master systems. Under the framework of classical Lyapunov stability theory, a state feedback controller is selected to realize global synchronization. In addition, simulation results are reported, to highlight the capabilities of the P-M synchronization proposed in this paper. Finally, a speech secure communication scheme based on the P-M synchronization is implemented, implying the P-M synchronization can be applied into secure communication filed.

INDEX TERMS Chaotic dynamical systems, Lyapunov stability theory, matrix projective synchronization, secure communication, state feedback controller.

I. INTRODUCTION

Chaos theory is an attractive subject with some excellent characteristics, including sensitivity to initial values, intrinsic randomness, ergodicity, topological transitivity and positive Lyapunov exponent [1]–[3]. These characteristics make chaotic systems widely used in secure communications, data encryption, flow dynamics and so on [4]–[11].

Chaotic synchronization refers to two chaotic systems starting from different initial values. As time goes on, the orbit of one system will converge to the same value as the orbitals of the other system. The study of chaotic synchronization started in the 1990s. The idea of synchronization with two identical initial conditions was introduced by Pecora and Carroll [12], [13]. In 2015, Pal et al. proposed a design method for coupling to achieve target synchronization for a chaotic discrete dynamic system with two parameters not matching [14]. In 2016, Wu et al. proposed the Riemann-Liuerville type fractional logic map and the fractional Lorentz map [15]. Ouannas et al. studied discrete-time chaotic (hyperchaotic) systems and studied several new methods for simultaneous coexistence between different dimensional mappings [16]. Azarang et al. proposed a new four-term nonlinear fractional-order chaotic system [17]. In 2017, Megherbi et al. studied the pulse synchronization problem of fractional-order discrete-time chaotic systems [18]. Gam et al. extended the stability conditions of continuous chaotic systems to discrete chaotic systems [19]. Ouannas et al. studied the problem of reliable universal synchronization between two coupled chaotic discrete systems [20]. In 2018, Ouannas et al. proposed a new discrete-time system chaotic synchronization method [21]. In 2019, Berber pointed out that the problem of sequence synchronization has been extensively studied in direct sequence spread spectrum system and code division multiple access system based on chaos. All signals are expressed in discrete time domain. In order to represent finite and random discrete delay between sequences, the uniform distribution probability density function is expressed in discrete form [22].

In this paper, a new matrix projection synchronization, P-M synchronization, is proposed. Compared with the conventional matrix projection synchronization, if an invertible matrix P and an arbitrary matrix M are selected, in addition, as long as a matrix F is constructed, the eigenvalue of the matrix *(E-F)* is strictly less than 1, then for any two generalized chaotic systems, the P-M synchronization can be achieved. However, in regular matrix projection synchronization, the required matrix changes with different

The associate editor coordinating the review of this article and approving it for publication was Yi Fang.

master systems [23]–[29]. In order to improve matrix projection synchronization, this paper proposes a new matrix projection synchronization: the P-M synchronization.

The main purpose of this paper is to realize the synchronization of two generalized chaotic systems by the P-M synchronization and apply the synchronization system to chaotic secure communication. The paper is arranged as follows. In section 2, a new discrete state feedback controller and new matrix projective synchronization are introduced. Section 3 illustrates that the new matrix projective synchronization was applied into the coupled identical chaotic dynamical systems, the different dimension chaotic dynamical systems and two different chaotic dynamical systems of the same dimension, respectively. A secure communication scheme based on the synchronization was implemented in Section 4. In section 5, some discussions were proposed, and Section 6 summarized the conclusions of the presented paper.

II. P-M SYNCHRONIZATION CRITERIONS

The master(driver) and slave(response) chaotic maps considered in the presented paper are in the following forms

$$X(i+1) = DX(i) + \varphi(X(i)) \tag{1}$$

$$Y(i+1) = EY(i) + \phi(Y(i)) + U, \tag{2}$$

where $X(i) \in \mathbb{R}^s$ and $Y(i) \in \mathbb{R}^t$ are state vectors of the driver and response maps, respectively, $D \in \mathbb{R}^{s\times s}$ and $E \in \mathbb{R}^{t\times t}$ are the linear sections of the driver and response maps, respectively, map $\varphi : \mathbb{R}^s \to \mathbb{R}^s$ and $\phi : \mathbb{R}^t \to \mathbb{R}^t$ are the nonlinear sections of the driver and response maps, respectively, and $U \in \mathbb{R}^t$ is a vector controller. Many chaotic maps can be written under the form of the of the map (1), such as 2-D generalized Henon map, Fold map, 3-D generalized Henon map, Chen map, Arnold's cat map, and Wang map, etc.

Our aim is to realize synchronization between system (1) and system (2) for arbitrary matrix $D \in \mathbb{R}^{s\times s}$, $E \in \mathbb{R}^{t\times t}$, map $\varphi : \mathbb{R}^s \to \mathbb{R}^s$ and $\phi : \mathbb{R}^t \to \mathbb{R}^t$, and to determine the controller $U \in \mathbb{R}^t$, which stabilize the synchronization error. The synchronization error is that the difference between the states will converge to 0 as i goes to $+\infty$.

Definition 2.1 The drive maps (1) and response map (2) with state vectors $X(i)$ and $Y(i)$, respectively, achieve P-M synchronization if there exists a controller $U \in \mathbb{R}^t$, an invertible matrix $P \in \mathbb{R}^{t\times t}$ and a matrix $M \in \mathbb{R}^{t\times s}$ so that the synchronization error (3) satisfies the condition $\lim_{k\to\infty} \|e(i)\| = 0$, which indicates that the map (1) and map (2) can achieve complete synchronization.

$$e(i) = PY(i) - MX(i) \tag{3}$$

Furthermore, in order to achieve the goal synchronization, the controller U in map (2) is derived as follows:

$$U = -EY(i) - \phi(Y(i)) + QR, \tag{4}$$

where matrix Q is the inverse of matrix P, and the matrix R is derived as follows:

$$R = (E-F)\,e(i) + M\,(DX(i) + \varphi(X(i)))\,. \tag{5}$$

Theorem 2.2 The map, which is described by (3), can be stabilized by the controller (4) along with (5), provided that $F \in \mathbb{R}^{s\times s}$ is chosen such that the eigenvalues of the matrix $(E-F)$ are placed strictly inside the unit disk.

Proof. According that definition 2. 1, the error system between the drive map (1) and the response map (2) can be derived as follows:

$$\begin{aligned} e(i+1) = {} & (E-F)\,e(i) \\ & + Q\,(EY(i) - \phi\,(Y(i)) + U) + R. \end{aligned} \tag{6}$$

By substituting the controller (4) along with (5), the error system (6) reduces to

$$e(i+1) = (E-F)\,e(i)\,. \tag{7}$$

The Lyapunov function is constructed with the form $V(e(i)) = e^T(i)\,e(i)$, it follows that

$$\begin{aligned} \Delta V(e(i)) &= e^T(i+1)\,e(i+1) - e^T(i)\,e(i) \\ &= e^T(i)\,(E-F)^T(E-F)\,e(i) - e^T(i)\,e(i) \\ &= e^T(i)\left[(E-F)^T(E-F) - I\right]e(i)\,. \end{aligned}$$

Since the eigenvalues of the matrix $(E-F)$ are placed strictly inside the unit disk, so $\left((E-F)^T(E-F) - I\right)$ is a negative definite matrix, and we can obtain the fact $\Delta V(e(i)) < 0$.Thus, from the Lyapunov stability theory, the error system (7) is globally asymptotically stable, that is to say, $\lim_{k\to\infty} \|e(i)\| = 0$.

III. EXAMPLES BASED ON P-M SYNCHRONIZATION

In the section, we are going to validate the theoretical results illustrated above. Some typical chaotic maps such as the 2-D generalized Henon map, the 3-D generalized Henon map, and Wang map are considered [30]–[32]. The 2-D generalized Henon map can be described as follows:

$$\begin{cases} x_1(i+1) = 1 - ax_1^2(i) + x_2(i) \\ x_2(i+1) = bx_1(i)\,. \end{cases} \tag{8}$$

When $a = 1.4$ and $b = 0.3$, system (8) is chaotic system, and the chaotic attractor of the 2-D generalized Henon map with initial valuables $x_1(0) = 0.5$ and $x_2(0) = 0.6$ is shown in Fig.1. In addition, the linear section and the nonlinear section of map (8) are given by

$$D = \begin{bmatrix} 0 & 1 \\ b & 0 \end{bmatrix} \quad \text{and } \varphi(X(i)) = \begin{bmatrix} 1 - ax_1^2(i) \\ 0 \end{bmatrix}.$$

The 3-D generalized Henon map can be described as equation (9), when $a = 1.07$ and $b = 0.3$, system (8) is chaotic system and the chaotic attractor of the map with initial valuables $x_1(0) = 0.2$, $x_2(0) = 0.6$, $x_3(0) = -0.3$ is shown in Fig.2.

$$\begin{cases} x_1(i+1) = -ax_2(i) \\ x_2(i+1) = 1 - ax_2^2(i) + x_3(i) \\ x_3(i+1) = x_1(i) + bx_2(i) \end{cases} \tag{9}$$

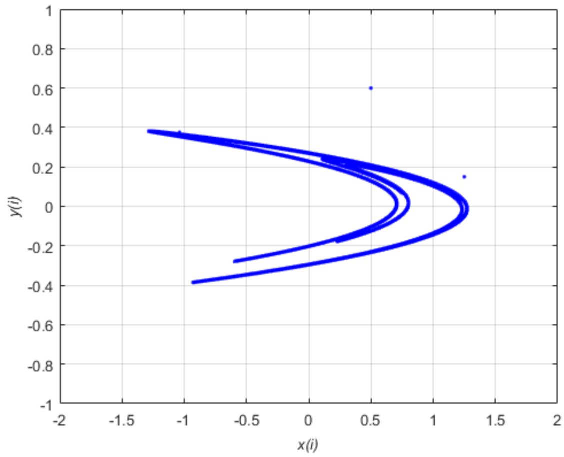


FIGURE 1. x-y phase space of the 2-D generalized Henon map.

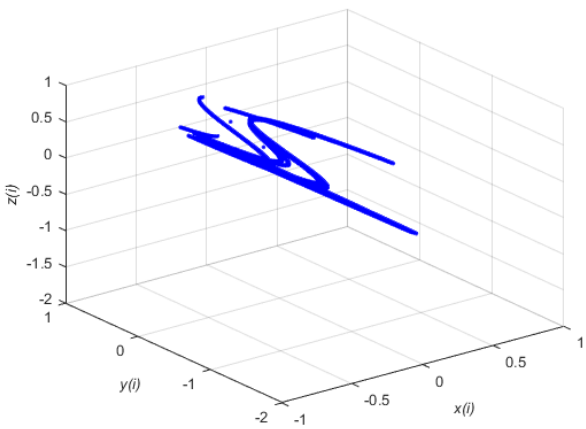


FIGURE 2. x-y-z phase space of the 3-D generalized Henon map.

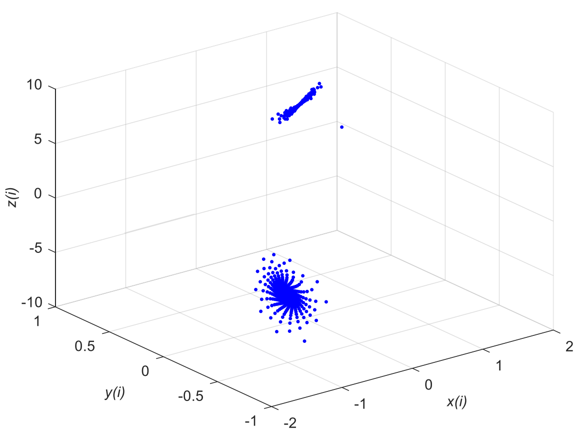


FIGURE 3. x-y-z phase space of Wang map.

In addition, the linear section and the nonlinear section of map (9) are given by

$$D = \begin{bmatrix} 0 & -a & 0 \\ 0 & 0 & 1 \\ 1 & b & 0 \end{bmatrix} \quad \text{and } \varphi\left(X\left(i\right)\right) = \begin{bmatrix} 0 \\ 1 - ax_2^2\left(i\right) \\ 0 \end{bmatrix}.$$

Wang map can be described as equation (10), when $a_1 = 0.5, a_2 = 1.3, a_3 - 1.9, a_4 = 0.2, a_5 = -0.9, a_6 = -0.6, a_7 = 2$, system (10) is chaotic system and the chaotic attractor of the map with initial valuables $x_1(0) = 1.3, x_2(0) = 0.5$ and $x_3(0) = 3$ is shown in Fig.3.

$$\begin{cases} x_1(i+1) = a_1x_2(i) - a_2x_1(i) \\ x_2(i+1) = a_3x_1(i) + x_2(i) + a_4x_3(i) \\ x_3(i+1) = a_5x_3(i) + a_6x_2(i)x_3(i) + a_7 \end{cases} \tag{10}$$

In addition, the linear section and the nonlinear section of map (10) are given by

$$D = \begin{bmatrix} -a_2 & a_1 & 0 \\ a_3 & 1 & a_4 \\ 0 & 0 & a_5 \end{bmatrix} and \varphi\left(X\left(i\right)\right) = \begin{bmatrix} 0 \\ 0 \\ a_6x_2\left(i\right)x_3\left(i\right) + a_7 \end{bmatrix}.$$

A. SYNCHRONIZATION OF THE IDENTICAL CHAOTIC MAP

The 3-D generalized Henon map is adapted to P-M Synchronization. The master system is map (9), and the slave system is given by:

$$\begin{cases} y_1(i+1) = -ay_2\left(i\right) + u_1 \\ y_2\left(i+1\right) = 1 - ay_2^2\left(i\right) + y_3\left(i\right) + u_2 \\ y_3\left(i+1\right) = y_1\left(i\right) + by_2\left(i\right) + u_3, \end{cases} \tag{11}$$

where $U = [u_1, u_2, u_3]^T$ is the vector controller; $a = 1.07$ and $b = 0.3$. In addition, the linear section and the nonlinear section of map (9) are given by

$$D = \begin{bmatrix} 0 & -a & 0 \\ 0 & 0 & 1 \\ 1 & b & 0 \end{bmatrix} \quad \text{and } \phi\left(Y\left(i\right)\right) = \begin{bmatrix} 0 \\ 1 - ay_2^2\left(i\right) \\ 0 \end{bmatrix}.$$

In this case, an invertible matrix P_1 and a matrix M_1 are selected as follows:

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}. \tag{12}$$

Then, the error system is defined as

$$\begin{cases} e_1\left(i\right) = y_1\left(i\right) - x_1\left(i\right) - 2x_2\left(i\right) - 3x_3\left(i\right) \\ e_2\left(i\right) = y_2\left(i\right) - 4x_1\left(i\right) - 5x_2\left(i\right) - 6x_3\left(i\right) \\ e_3\left(i\right) = y_3\left(i\right) - 7x_1\left(i\right) - 8x_2\left(i\right) - 9x_3\left(i\right). \end{cases} \tag{13}$$

If the state feedback controller U is selected as in equation (4) along with equation (5). Then, the matrix F_1 yields error system (15), and with eigenvalues of the matrix $(E - F_1)$ controlled in the unit disk.

$$F_1 = \begin{bmatrix} -0.1 & -a & 0 \\ 0 & 0 & 0.5 \\ 1 & 0.2 & 0 \end{bmatrix} \tag{14}$$

$$\begin{cases} e_1\left(i+1\right) = 0.1e_1\left(i\right) \\ e_2\left(i+1\right) = 0.5e_2\left(i\right) \\ e_3\left(i+1\right) = 0.1e_3\left(i\right). \end{cases} \tag{15}$$

According to Theorem2.2, synchronization is achieved between the identical chaotic system. Let the initial values

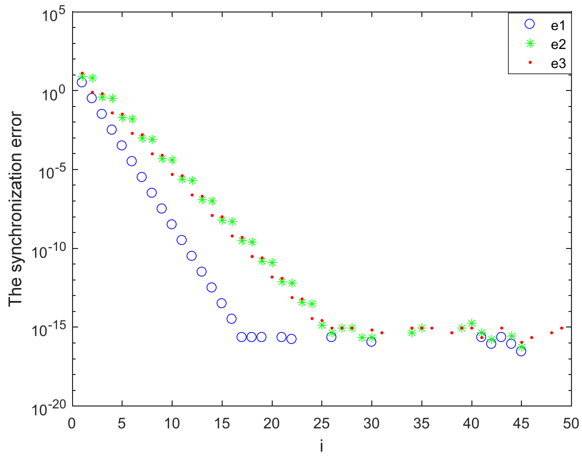


FIGURE 4. The synchronization error of the discrete-time system.

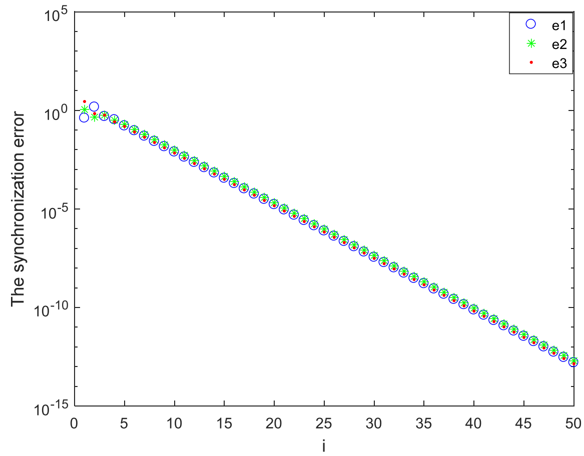


FIGURE 5. The synchronization error of the discrete-time system.

of the systems (9) and (11) be: $X_1(0) = (0.4, 0.7, 0.5)^T$, $Y(0) = (0.2, 0.3, 0.1)$. The simulation results of the synchronization error are shown in Fig.4. The synchronization error quickly approaches 0, and the transceiver system is synchronized.

B. SYNCHRONIZATION OF CHAOTIC MAPS WITH DIFFERENT DIMENSIONS

The 2-D generalized Henon map and the 3-D generalized Henon map are adapted to P-M Synchronization. The master system is map (8), and the slave system is map (11).

In this case, an invertible matrix P_2 and a matrix M_2 are selected as follows:

$$P_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}. \tag{16}$$

Then, the error system is defined as

$$\begin{cases} e_1(i) = 2y_1(i) - x_1(i) - 2x_2(i) \\ e_2(i) = 3y_2(i) - 3x_1(i) - 4x_2(i) \\ e_3(i) = 4y_3(i) - 5x_1(i) - 6x_2(i). \end{cases} \tag{17}$$

If the state feedback controller U is selected as in equation (4) along with equation (5). Then, the matrix F_2 yields error system (19), and with eigenvalues of the matrix $(E - F_2)$ controlled in the unit disk.

$$F_2 = \begin{bmatrix} -0.1 & -a & -0.5 \\ -0.3 & -0.3 & 1 \\ 0.7 & b & -0.2 \end{bmatrix} \tag{18}$$

$$\begin{cases} e_1(i+1) = 0.1e_1(i) + 0.5e_2(i) \\ e_2(i+1) = 0.3e_1(i) + 0.3e_3(i) \\ e_3(i+1) = 0.3e_1(i) + 0.2e_3(i) \end{cases} \tag{19}$$

According to Theorem2.2, synchronization is achieved between the identical chaotic system. Let the initial values of the systems (8) be:$X_2(0) = (0.4, 0.2)^T$,and the initial values of the system (11) remains the same. The simulation results of the synchronization error are shown in Fig.5. The synchronization error quickly approaches 0, and the transceiver system is synchronized.

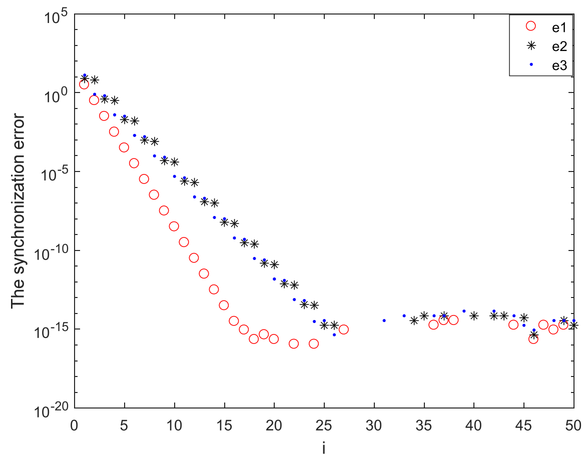


FIGURE 6. The synchronization error of the discrete-time system.

C. SYNCHRONIZATION OF THE DIFFERENT CHAOTIC MAP IN THE SAME DIMENSION

The 3-D generalized Henon map and Wang map are adapted to P-M Synchronization. The master system is map (10), and the slave system is map (11).

In this case, an invertible matrix P_1 and a matrix M_1 are selected as "(12)". Then, the error system is defined as "(13)". If the state feedback controller U is selected as in equation (4) along with equation (5). Then, the matrix F_1 yields error system (15), and with eigenvalues of the matrix $(E - F_1)$ controlled in the unit disk.

According to Theorem2.2, synchronization is achieved between the identical chaotic system. Let the initial values of the systems (9) and (11) be:$X(0) = (0.4, 0.7, 0.5)^T$, and the initial values of the system (11) remains the same. The simulation results of the synchronization error are shown in Fig.6. The synchronization error quickly approaches 0, and the transceiver system is synchronized.

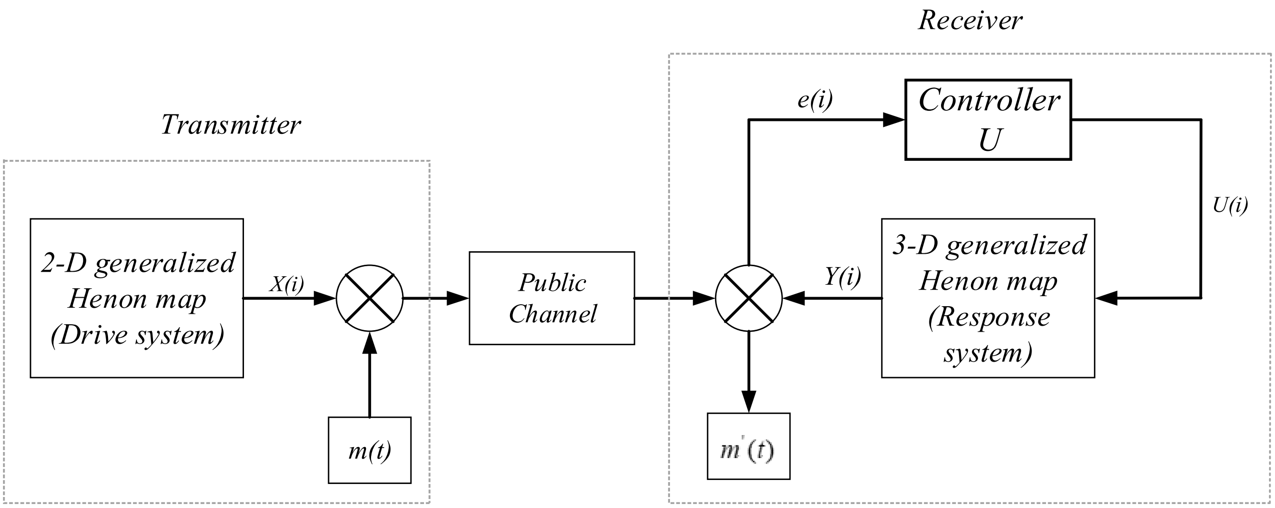


FIGURE 7. Discrete chaotic secure communication scheme.

It can be seen from Fig.4, Fig.5 and Fig.6 that three types of chaotic systems can achieve synchronization by the P-M synchronization. However, the convergence time in Fig.4 and Fig.5 is different, while the convergence time of Fig.4 is approximately the same as that of Fig.6. Since the P-M synchronization proposed in this paper is based on the construction method, and it only relies on an invertible matrix P, matrix M, and a matrix $(E-F)$ with an eigenvalue less than 1. If the three matrices are the same, the synchronization results are the same. If one of the matrices changes, the synchronization results will be different.

IV. A SECURE COMMUNICATION SCHEME

Synchronous chaotic communication can be classified into three major security technologies: chaotic masking technology, chaotic parameter modulation technology and chaotic keying technology. Chaos masking technology belongs to chaos analog communication, chaos parameter modulation and chaos keying technology belongs to chaos digital communication technology. This speech encryption scheme is mainly based on chaos masking, and the block diagram of encryption is shown in Fig.8.

In this section, a secure communication scheme based on P-M synchronization is presented. We will use the discrete time chaotic map (8) as master system, and map (11) is used as slave system. Since we wanted to show that the matrix projection map can synchronize chaotic systems of different dimensions. In addition, the three synchronization examples in Section 3 can be applied to the speech secure communication scheme. It's just that the expressions for E_n and $m'(t)$ are different. In this section, the second synchronization scheme was adopted for the speech secure communication.

Speech signal $m(t)$ and state variable $x_1(i)$, $x_2(i)$ are masked into the signal E_n, then, the signal E_n is sent to the receiver end via the public channel. At receiver end, the recovered signal $m'(t)$ can be obtained through the signal $m(t)$ and state variable $y_1(i)$.

$$E_n = m(t) + x_1(i) + 2x_2(i) \tag{20}$$

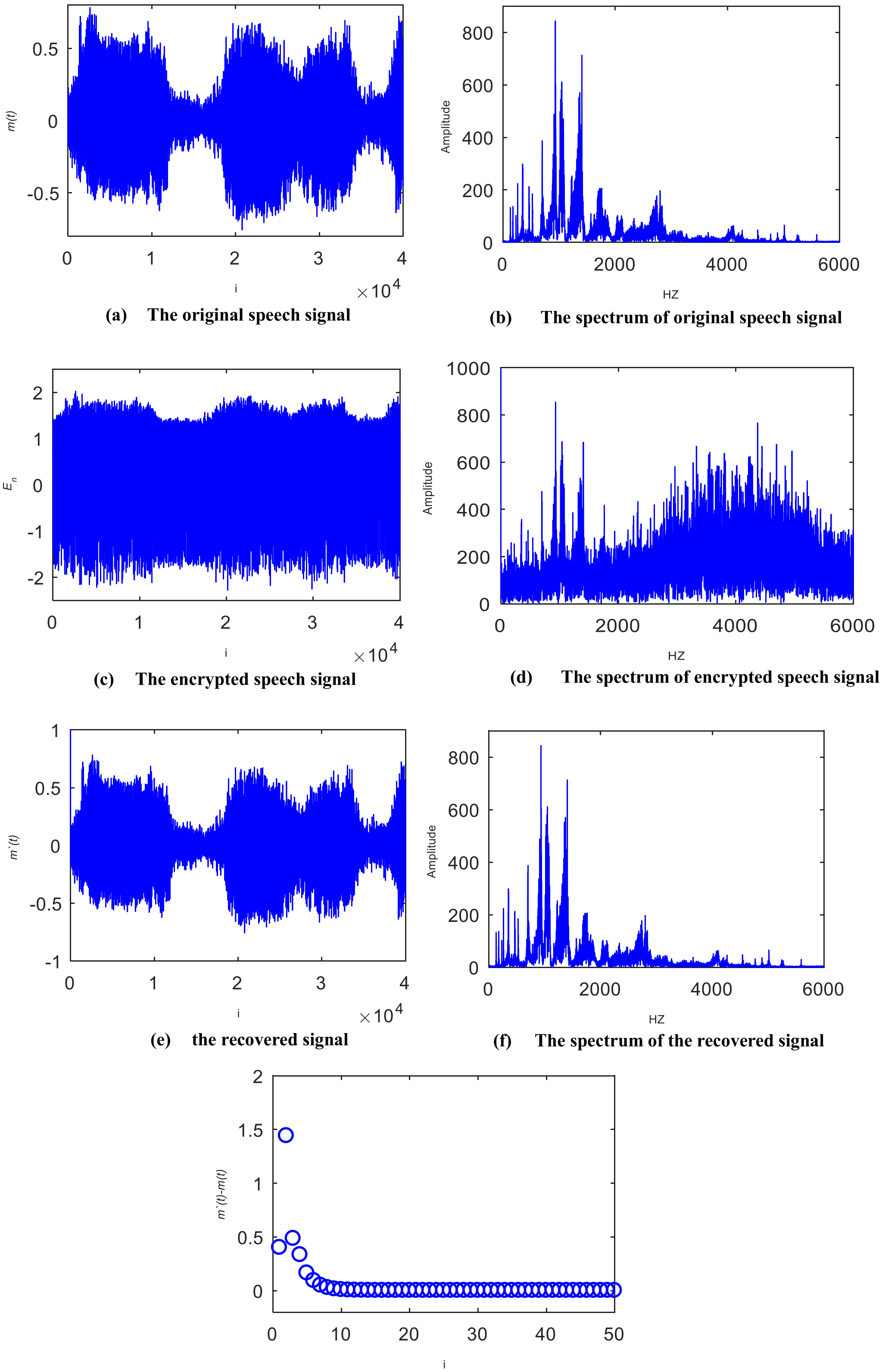
$$m'(t) = E_n - 2y_1(i) \tag{21}$$

Let the initial values of the systems (8) and (11) be: $X(0) = (0.4, 0.2)^T, Y(0) = (0.2, 0.3, 0.1)^T$. The original speech signal at the transmitting end, the spectrum of original speech signal, the speech signal after chaos masking, the spectrum of encrypted speech signal, the restored speech signal, the spectrum of restored speech signal, and the error of the recovered speech signal and the original speech signal are simulated, respectively. The simulation results are shown in Fig.8.

Fig.8 proves that P-M synchronization proposed in this paper can be applied into secure communication field from time domain and frequency domain, respectively. The robustness of this scheme is poor, since the chaotic system is sensitive to the initial value and system parameters. If the system parameters are changed, the output of the chaotic system will change. Furthermore, from the perspective of secure communication, if the system has better robustness, it will lose confidentiality. Finally, this scheme can resist the phase space reconstruction attack. Because mixed signal E_n is transmitted in the common channel, the phase space reconstructed from a series of values of E_n is not completely topologically equivalent to the real chaotic system.

V. DISCUSSION

In this section, some comparisons, between the proposed P-M synchronization method and similar synchronization method in the literature, will be made, aimed to focus on the difference between the presented paper and the available researches. First, compared to conventional matrix projection synchronization [23]–[29], the required matrices of the P-M synchronization don't change with different master systems. In order to improve matrix projection synchronization, this paper proposed a new matrix projection synchronization: the



(a) The original speech signal
(b) The spectrum of original speech signal
(c) The encrypted speech signal
(d) The spectrum of encrypted speech signal
(e) the recovered signal
(f) The spectrum of the recovered signal
(g) the error of the scheme

FIGURE 8. Speech signal encryption based on the scheme.

P-M synchronization. In addition, attention was centered on some interesting synchronization method based on control laws. Taking literature [21] for example, the author proposed a new type of synchronization in discrete time systems, which combines the inverse generalized synchronization (based on a functional relationship F) with matrix projective synchronization (based on a matrix M). However, it is for us difficult to construct an invertible functional relationship F: $\mathbb{R}^n \to \mathbb{R}^n$, which just right meet scheme requirements. Furthermore, it is not easy to find the right invertible function F and matrix M. Compared to the new type of synchronization, we only need, according to the controller (4) along with (5), to construct an invertible matrix $P \in \mathbb{R}^{t\times t}$ and an arbitrary matrix $M \in \mathbb{R}^{t\times s}$, both which fit the dimensions with the proposed method. Moreover, as long as a matrix F is constructed, the eigenvalue of the matrix $(E - F)$ is strictly less than 1, then for any two generalized chaotic systems, the P-M synchronization can be achieved. In the end, compared with other literatures, this paper proposed a speech secure communication scheme based on P-M synchronization.

VI. CONCLUSION

In this paper, we proposed a new type of matrix projective synchronization, called P-M synchronization. The technique exploits the state feedback controller and Lyapunov stability theory so that it is easy to synchronize the coupled identical chaotic dynamical systems, the different dimension chaotic dynamical systems and two different chaotic dynamical systems of the same dimension, respectively. Then, simulation results involving the2-D generalized Henon map, 3-D generalized Henon map, and Wang map are provided to highlight the capabilities of the proposed new matrix projective synchronization. Finally, A Secure communication scheme based on P-M synchronization was implemented, whose simulation results imply P-M synchronization can be applied in secure communications.

ACKNOWLEDGMENT

The authors would like to thank two researchers Chuanfu Wang and Chunlei Fan, both who come from Heilongjiang University. for technical support on database inquiry and paper review the simulation computing environment and computer equipment provided by Heilongjiang University and the Nanjing 321 innovative talents program.

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