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Consensus Control of Multiple AUVs Recovery System Under Switching Topologies and Time Delays

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ABSTRACT The consensus control of multiple AUVs recovery system is the key to complete the multi-AUV recovery mission. This paper proposes consensus control of multiple AUVs recovery system under switching communication topologies and time delays including input delay and communication delay. Consensus control algorithm for a multi-AUV recovery system is assumed to guarantee all the members to reach a common motion state and desired point. The mothership is regarded as the point which all AUVs have to catch up with, the recovery problem of multiple AUVs can be seen as the consensus problem of multi-agents system. A randomly changing consensus stability criteria and stabilization conditions are derived by a suitable Lyapunov-Krasovskii functional for the Markovian switching recovery system with time delays, and then a consensus controller design method is derived. Finally, the correctness of the proposed method is proved by simulation experiments.

INDEX TERMS Consensus control, multiple AUVs, recovery system, switching topologies, time delays.

I. INTRODUCTION

Similar to the spurt development of Unmanned Aerial Vehicles (UAVs), Autonomous Underwater Vehicles (AUVs) are also entering the fast lane of development. At present, there are hundreds of AUVs in the world, which are active in various fields such as marine science, marine engineering, underwater security and underwater operations. Recovery is an essential part of the AUV operation process, and successful recovery is the basis for AUV to work normally under water. Unsuccessful recovery may cause damage to the AUV, causing the whole system to fail and even the AUV to be lost. Extensive cooperation among unrelated individuals is unique to humans, who often work together to achieve what they are unable to execute alone. It is important to focus on the collective behavior that emerges as the result of the interactions among individuals, groups, and even societies [1], [2]. Similarly, with the maturity and development of AUV technology, single AUV can no longer meet the task requirements. Therefore, it is an inevitable direction for AUV development that multiple AUVs work together to perform tasks. At the same time, it also puts forward new requirements for the recovery of AUV and promotes the application of cluster intelligence, formation control and other technologies. Multiple AUVs are characterized by distribution, dynamics, adaptability, intelligence, coordination, generalization and stratification, which can effectively reduce costs, expand capabilities, improve efficiency and detect probabilities. During the recovery process, when the mothership sails along the desired trajectory, multiple AUVs transmit information through the sonar to ensure that mothership and AUVs can coordinate and maintain a consistent state. However, there are few studies on recovery, especially multiple AUVs recovery system, therefore, the studies of consensus control of multiple AUVs recovery system have practical significance.

Consensus control algorithm for a multi-AUV recovery system is assumed to guarantee all the members to reach a common motion state and desired points. During recovery

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process, the mothership is regarded as the point which all AUVs have to catch up with, the recovery problem of multiple AUVs can be seen as the consensus problem of multi-agents system similarly. Consensus algorithms can be viewed as the primary method of solving the problem for the recovery system. Because of the uncertain, dynamical, and adversarial underwater environments and the multiple AUVs have highly complex nonlinear dynamics and complicated interaction behaviors, the problem of multi-AUV consensus control is well-known challenging [3]. A large number of studies have been conducted on consensus control of multiple AUVs over the past decades [4]–[15]. The consensus is defined as that all agents maintain the common state in positions, velocities and/or attitudes.

From the existing literature review, it has been investigated that graph theory analysis has great advantages for dealing with consensus control problem because it can simplify communication topology and optimize information model of multi-AUV system. It chooses edges and vertexes to delegate the each independent AUV and their communication topological relations, respectively. Consequently, deployment of graph theory is the appropriate preference for the consensus tracking control of multi-AUV system in complex oceanic environment [16].

Multiple AUVs can effectively carry out a recovery task when they are successfully exchanging information. However, due to the complex and varied underwater environment and the influence of communication distance, the information affected by the actual sea conditions may be delayed or interrupted. The status information of multi-AUVs cannot be received in time, so coordination of the system should be guaranteed in the case of input delay and communication delay. Communication can be disconnected because of limited communication during the recovery process, in order to maintain the connection, communication topology switching is necessary. The stochastic switching topologies ensure that all agents can receive interconnection information, which contributes to the stable convergence under the limited communication conditions.

[17] discusses linear/nonlinear consensus problems for a network of dynamic agents with fixed and switching topologies. Reference [18] considers a leader-following consensus problem of second-order multi-agent systems with fixed and switching topologies as well as non-uniform time-varying delays. Reference [19] uses time-domain (Lyapunov theorems) and frequency-domain (the Nyquist stability criterion) approaches to study leaderless and leader-following consensus algorithms with communication and input delays under a directed network topology in both the first-order and second-order cases. Reference [20] investigates a novel consensus protocol without using the neighbors velocity information. The stochastic switching topology and the random communication delay which exist in the switching signal as well as the position information exchanges are dominated by two mutually independent Markov chains. Reference [21] considers the problem of leader-following consensus stability and also stabilization for multi-agent systems with interval time-varying delays. Stochastic consensus problems for linear time-invariant multi-agent systems over Markovian switching networks with time-varying delays and topology uncertainties are dealt with in [22]. Reference [23] aims at extending the deterministic averaging method to the stochastic case which includes communication white noises and Markovian switching network topology. A condition for consensus for a networked system based on linear matrix inequalities that takes into account the joint effect of time-varying delays and switching network topology, [24] proposes a new approach for the analysis of consensus of multi-agent systems subject to time-varying delayed control inputs and switching topology. [25] establishes general and applicable results for uniform stability, uniform asymptotic stability and exponential stability of the systems by using the impulsive control theory and some comparison arguments. Reference [26] divides the communication topology into two different switching parts and then investigates a consensus algorithm to solve the coordinate control problems of leaderless multi-AUVs with double independent Markovian switching communication topologies and time-varying delays among the underwater sensors. In [27], the system is affected by data processing and communication time-delays that are assumed to be asynchronous, the consensus of a saturated second-order multi-agent system with non-switching dynamics that can be represented by a directed graph is presented. Reference [28] studies the switching laws designed to maintain the stability of delayed switched nonlinear systems with both stable and unstable modes. The addressed time delays include finite and infinite delays. Reference [29] concerns with exploring the theoretically and technically research outcomes for the conflict resolution of multiple unmanned aerial vehicles (UAVs) by using the Internet of Things (IoT) technologies. [30] establishes some efficient criteria for finite-time consensus of a class of nonsmooth opinion dynamics over a digraph. Reference [31] investigates the problem of output tracking control for a class of delayed switched linear systems via state-dependent switching and dynamic output feedback control. Reference [32] proposes coordinated control protocols with or without time delay for the coordination control problem of multiple AUVs under switching communication topologies based on discrete information.

The consensus problem for multiple AUVs recovery system under switching communication topology and time delays including input delay and communication delay has not been investigated yet. In this paper, leader-following consensus criteria is used to solve multi-AUV recovery problem, consensus control algorithm for a multi-AUV recovery system is assumed to guarantee all the members to reach a common motion state and desired points. The mothership is regarded as the point which all AUVs have to catch up with, the recovery problem of multiple AUVs can be seen as the consensus problem of multi-agents system similarly, multiple AUVs consensus control protocol under switching communication topologies and time delay including input delay and communication delay is designed during the recovery process. Firstly, the single AUV nonlinear mathematical model is transformed into a second-order integral model via state feedback linearization by [21]. Secondly, a randomly changing consensus stability criteria and stabilization conditions are derived by a suitable Lyapunov-Krasovskii functional for the Markovian switching recovery system with times delays, and then a consensus controller design method is derived. Finally, the correctness of the proposed method is proved by simulation experiments.

Notation: $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrix. $\mathbb{C}_{n,h} = \mathbb{C}\left([-h, 0], \mathbb{R}^n\right)$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into $\mathbb{R}^n$. For symmetric matrices X and Y, $X > Y$ means that the matrix $X-Y$ is positive definite, $X \geq Y$ means that the matrix $X - Y$ is nonnegative. $X^{\perp}$ denotes a basis for the null-space of X. I_n, 0_n and $0_{m\times n}$ denotes $n \times n$ identity matrix, $n \times n$ and $n \times m$ zero matrices, respectively. $E\{\cdot\}$ stands for the mat-hematical expectation operator. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $\lambda_{\max}(\cdot)$ means the largest eigenvalues of a given square matrix. $diag\{\cdots\}$ denotes the block diagonal matrix. $*$ represents the elements below the main diagonal of a symmetric matrix.

II. PROBLEM FORMULATION

The basic introduction of graph theory and the kinematic and dynamic models of the AUV that moves in the horizontal plane are given in this section, and transform the AUV model into a linear model.

A. GRAPH THEORY

The interaction topology of a network of underwater vehicles can be described by the weighted directed graph. Suppose a system with n nodes(vehicles). Let $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$ be a weighted directed graph with a node set $\mathcal{V} = \{1, 2, \cdots, n\}$, an edge set $\varepsilon = \{(i,j)\,|i,j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$, and an adjacency matrix $\mathcal{A} = \left[a_{ij}\right] \subset \mathbb{R}^{N\times N}$ of the digraph $\mathcal{G}$ is the matrix with nonnegative elements, where $a_{ij} > 0$ if $(i,j) \in \varepsilon$, while $a_{ij} = 0$ if $(i,j) \notin \varepsilon$. Moreover, $a_{ii} = 0$ where $i = \{1, 2, \cdots, n\} \in v$. The digraph $\mathcal{G}$ is said to be undirected if $a_{ij} = a_{ji}$. A set of neighbors of underwater vehicle i is denoted by $\mathcal{N}_i = \{j \in V : (i,j) \in \varepsilon\}$. The in-degree matrix is defined as $\mathcal{D} = diag\left(d_1^{in}, d_2^{in}, \ldots, d_n^{in}\right), i \in (1, \ldots, n)$, where d_i^{in} and d_i^{out} are the in-degree and out-degree of the node $d_i^{in} = \sum_{j=1}^{n} a_{ij}$ and $d_i^{out} = \sum_{j=1}^{n} a_{ji}$. The $\mathcal{G}$ is said to be balanced if $d_i^{in} = d_i^{out}$ for all $i = 1, 2, \ldots, n$.

B. THE AUV MODEL

The dynamic model of AUV can be described by the 6 degrees of freedom (DOFs) according to the earth-fixed $\{E\}$ and body-fixed $\{G\}$ coordinate systems as shown the red vehicle in Fig. 2. In this paper, the type of AUV is common torpedo-like, which is symmetrical in plane and vertical. As rolling has little influence on translational motion, the roll speed is ignored in this paper. The symbols and parameters adopted conform to the system recommended by the International Pool Conference(ITTC) and the Society of Shipbuilding and Marine Engineering (SNAME) Terminology Bulletin. The coordinate system satisfies the right-hand Cartesian rectangular coordinate system (see Table 1) [33]. The mothership model is the same as the AUV model since the roll speed is ignored as shown the yellow vehicle in Figure 2.

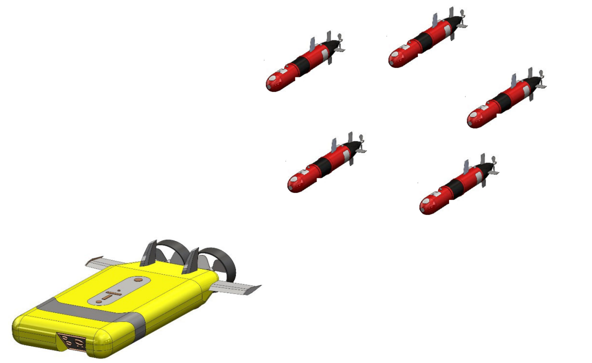


FIGURE 1. Multiple AUVs recovery system is assumed to guarantee all the members to reach a common motion state and desired point. The mothership is regarded as the point which all AUVs have to catch up with.

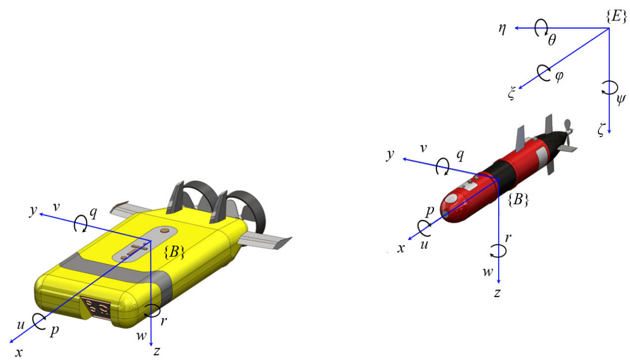


FIGURE 2. The dynamic model of mothership(yellow) and AUV(red) with the 6 degrees of freedom(DOFs) under earth-fixed and body-fixed coordinate system.

TABLE 1. The name and symbol in earth-fixed and body-fixed coordinate system.

Earth-fixed frame		Body-fixed frame	
Symbol	Description	Symbol	Description
x	longitudinal coordinate	u	surge
y	transverse coordinate	v	sway
z	vertical coordinate	w	heave
θ	pitch angle	q	pitch
φ	yaw angle	r	yaw

The Kinematics equation and Kinetics equation are as follows

$$\dot{\eta} = J(\eta)\, v \tag{2.1}$$

$$M\dot{v} + C(v)\, v + D(v)\, v + g(\eta) = \tau \tag{2.2}$$

where $\eta = \left[x\ y\ z\ \theta\ \varphi\right]^T \in \mathbb{R}^5$ represents the states of position and Euler angles respectively, $J(\eta)$ is the Jacobian matrix from body-fixed frame to earth-fixed frame. $v_i = [u_i, v_i, w_i, q_i, r_i]^T \in \mathbb{R}^5$ represents the states of velocities. Matrix M denotes inertia matrix, $C(v)$ denotes Coriolis and centripetal matrix and $D(v)$ denotes damping matrix. $g(\eta)$ is a vector of generalized gravitational and buoyancy forces and moments. τ is the control input. Further details regarding the model parameters can be found in [33] and [34].

The model of AUV is a nonlinear and coupling model, the feedback linear method can be used to translate the nonlinear model into the following linear model in [32]. The derivation process is as follows.

The mathematical model of AUV now can be represented as

$$\begin{bmatrix} \dot{\eta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & -M^{-1} \end{bmatrix} \begin{bmatrix} J(\eta)v \\ W(v)v \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}\gamma(\xi) \end{bmatrix} u_\tau \quad (2.3)$$

Then the nonlinear model of AUV can be described as

$$\begin{aligned} \dot{\xi} &= p(\xi) + q(\xi)u_\tau \\ \mu &= r(\xi) \end{aligned} \quad (2.4)$$

where $\dot{\xi} = [\eta^T, v^T]^T$,

$$\begin{aligned} p(\xi) &= \begin{bmatrix} I & 0 \\ 0 & -M^{-1} \end{bmatrix} \begin{bmatrix} J(\eta)v \\ W(v)v \end{bmatrix} \\ &= [p_i(\xi)]^T, \quad i = 1, 2, \ldots, 10 \\ q(\xi) &= \begin{bmatrix} 0 \\ M^{-1}\gamma(\xi) \end{bmatrix} \\ &= [q_{ij}(\xi)]_{10\times 10}, \quad i, j = 1, 2, \ldots, 10 \\ r(\xi) &= \eta \end{aligned}$$

Lemma 1: The system can be linear by feedback linearization method, which is a system as Eq. (3.1), if the conditions can be satisfied:

(1) The dimension of input is same as output;

(2) The system has relative degree, $\rho_1, \rho_2, \ldots, \rho_n$;

(3) The sum of relative degree is same as the dimension of the system.

Combining $q_{ij}(\xi)$ in the Eq. (2.4), the matrix $\Gamma(\xi)$ can be determine with the property of Lie derivative. $\Gamma(\xi)$ is following as (2.5), as shown at the top of the next page.

According to the Eq. (2.4), the specific numerical $q_{ij}(\xi)$ can be computed, so $\Gamma(\xi)$ is nonsingular matrix, and the relative degree of the system is followed as

$$\rho_1 = 2, \quad \rho_2 = 2, \ \rho_3 = 2, \ \rho_4 = 2, \ \rho_5 = 2 \quad (2.6)$$

So $\rho_1 + \rho_2 + \rho_3 + \rho_4 + \rho_5 = 10$, which is same as the dimension of the system. According to Lemma 1, the nonlinear model of AUV can be linear by feedback linearization method, and the transformation of the new coordinates is

$$\begin{aligned} x &= \begin{bmatrix} r_1(\xi) & r_2(\xi) & r_3(\xi) & r_4(\xi) & r_5(\xi) \end{bmatrix} \\ v &= \begin{bmatrix} L_p r_1(\xi) & L_p r_2(\xi) & L_p r_3(\xi) & L_p r_4(\xi) & L_p r_5(\xi) \end{bmatrix} \end{aligned} \quad (2.7)$$

The control input in the new linearization system can be defined as

$$u = T(\xi) + \Gamma(\xi)u_\tau \quad (2.8)$$

where $T(\xi) = [L_p^2 r_1(\xi) \ L_p^2 r_2(\xi) \ L_p^2 r_3(\xi) \ L_p^2 r_4(\xi)] \ L_p^2 r_5(\xi)$, because the expression of $T(\xi)$ is too complex, it won't be written out here. Therefore, the actual control input can be obtained by $u_\tau = \Gamma^{-1}(\xi)(u_i - T(\xi))$.

Combining with the Eq. (2.7) and (2.8), the feedback linearization dynamic model of AUV's standard second-order integral form can be obtained:

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \quad (2.9)$$

where $x_i \in \mathbb{R}^5$, $v_i \in \mathbb{R}^5$, $v_i \in \mathbb{R}^5$, $u_i \in \mathbb{R}^5$.

C. LEMMA

In this section, various lemmas that play important roles in the subsequent analysis are introduced.

Lemma 2 [35]: Let $\otimes$ denotes the notation of Kronecker product. Then, its properties are easily established:

(1) $(\alpha A) \otimes B = A \otimes (\alpha B)$

(2) $(A + B) \otimes C = A \otimes C + B \otimes C$

(3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

Lemma 3 [36]: For any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, scalar γ, vector function $\chi : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, then

$$-\gamma \int_0^\gamma x^T(s) M x(s)\, ds \leq -\left(\int_0^\gamma x(s)\, ds\right)^T M \left(\int_0^\gamma x(s)\, ds\right)$$

Lemma 4: [37]: Let $\zeta \in \mathbb{R}^n$, $\Phi = \Phi^T \in R^{n\times n}$, and $\gamma \in R^{m\times n}$ such that rank $\gamma < n$. The following statements are equivalent:

(1) $\zeta^T \Phi \zeta < 0, \quad \forall \gamma\zeta = 0, \ \zeta \neq 0$

(2) $\Upsilon^{\perp^T} \Phi \Upsilon^{\perp} < 0$

(3) $\exists F \in R^{n\times m} : \Phi + F\gamma + \gamma^T F^T < 0$

III. CONSENSUS CONTROL UNDER SWITCHING TOPOLOGIES AND TIME DELAYS

As mentioned previously, the following will introduce consensus control algorithm for a multi-AUV recovery system under switching topologies and time delays. The mothership and the multi-AUV operate in the form of a stochastically switching topology. It is assumed the interconnection topology is Markovian switching, the Markov chain describing the switching process has a stationary probability distribution. Since the graph is allowed to be time-varying, we suppose that there are M possible different graphs, and the network topologies switch among them.

A. DESIGN PROCEDURE

In this section, consensus control algorithm for a multi-AUV recovery system is assumed to guarantee all the members to reach a common motion state and desired points. The mothership is regarded as the point which all AUVs have to catch up with, the recovery problem of multiple AUVs can be seen as the consensus problem of multi-agents system similarly.

$$\Gamma(\xi) = L_{qi}L_{P}r_j(\xi) = \begin{bmatrix} q_{6,1}\cos\psi\cos\theta & -q_{7,2}\sin\psi & q_{8,3}\cos\psi\sin\theta & q_{8,4}\cos\psi\sin\theta & -q_{7,5}\sin\psi \\ q_{6,1}\sin\psi\cos\theta & q_{7,2}\cos\psi & q_{8,3}\sin\psi\sin\theta & q_{8,4}\sin\psi\sin\theta & q_{7,5}\cos\psi \\ -q_{6,1}\sin\theta & 0 & q_{8,3}\cos\theta & q_{8,4}\cos\theta & 0 \\ 0 & 0 & q_{9,3} & q_{9,4} & 0 \\ 0 & q_{10,2}/\cos\theta & 0 & 0 & q_{10,5}/\cos\theta \end{bmatrix} \tag{2.5}$$

Definition 1 [38]: In a system consisting of AUVs, the motion state vector of the i-th AUV at time t is $x_i(t)$, the mothership's motion state is $x_m(t)$. If the system satisfies the following formula, the recycling system can achieve consistency, and ensure that the AUVs can continuously and stably follow the mothership.

$$\lim_{t\to\infty}\|x_i(t)-x_m(t)\|=0$$

According to dynamics (2.9), consider the consensus algorithm for the dynamics of the mothership is described as:

$$\dot{x}_m(t)=v_m(t)$$

The dynamics of the group of AUVs under multiple independent topologies defined as follows:

$$\dot{x}_i(t)=v_i(t)$$
$$\dot{v}_i(t)=u_i(t)$$

Consider a multi-AUV recovery system consisting of AUVs, each AUV is regarded as a node in a digraph G, employing the standard double-integrator dynamic of AUV in Eq. (2.9), the consensus control can be designed as follows based on the consensus algorithm.

$$\begin{aligned} u_i(t) &= -K\sum_{j\in N_i}a_{ij}(t)\left(\left(p_i(t)-p_j(t)\right)+\left(v_i(t)-v_j(t)\right)\right) \\ &\quad -Kc_i(t)\left(\left(p_i(t)-p_m(t)\right)+\left(v_i(t)-v_m(t)\right)\right) \end{aligned} \tag{3.1}$$

Due to the limited communication, which easily results in the time delay. The consensus algorithm with the time delay is given by

$$\begin{aligned} u_i(t) &= -K\sum_{j\in N_i}a_{ij}(t)\left(\left(p_i(t-\tau_1)-p_j(t-\tau_1-\tau_2)\right)\right. \\ &\quad \left.+\left(v_i(t-\tau_1)-v_j(t-\tau_1-\tau_2)\right)\right) \\ &\quad -Kc_i(t)\left(\left(p_i(t-\tau_1)-p_m(t)\right)+\left(v_i(t-\tau_1)-v_m(t)\right)\right) \end{aligned} \tag{3.2}$$

where u_i is the control input for i-th AUV which uses only the state information from its neighboring agents. K is a protocol gain; $p_i(t)\in\mathbb{R}^n$ and $v_i(t)\in\mathbb{R}^n$ ($p_j(t)\in\mathbb{R}^n$ and $v_j(t)\in\mathbb{R}^n$) are the position and velocity of the i-th AUV (j-th AUV) at time t, respectively. $p_m(t)\in\mathbb{R}^n$ and $v_m(t)\in\mathbb{R}^n$ are the position and velocity of the mothership; $a_{ij}(t)$ is the communication weight between i-th AUV and j-th AUV, where $a_{ij}(t)>0$ if i-th AUV is connected with j-th AUV at time t, otherwise $a_{ij}(t)=0$; $c_m(t)$ is the communication weight between the mothership and i-th AUV, which is described by diagonal matrix $\mathcal{C}$, $\mathcal{C}=diag\{c_1,c_2,\cdots c_N\}\in\mathbb{R}^{N\times N}$, where $c_{mi}(t)>0$ if i-th AUV is connected with mothership at time t, otherwise $c_{mi}(t)=0$; τ_1 and $\tau_2(t)$ represent input and communication time-delays, respectively. τ_1 is constant, $\tau_2(t)$ is an interval time-varying continuous function satisfying, $0<\tau_1+\tau_2(t)\le h$ and $\dot{\tau}_2(t)\le h_p$.

The state of mothership and i-th AUV can be described as

$$x_m(t)=\begin{bmatrix}p_m^T(t) & v_m^T(t)\end{bmatrix}^T\in\mathbb{R}^{10n}$$
$$x_i(t)=\begin{bmatrix}p_i^T(t) & v_i^T(t)\end{bmatrix}^T\in\mathbb{R}^{10n}$$

The above recovery problem is transformed into an error analysis problem, the system state error vector of i-th AUV relative to the motion state information of mothership is defined as $\varepsilon_i(t)=x_i(t)-x_m(t)$. Let's define $\varepsilon(t)=\left(\varepsilon_1^T(t),\varepsilon_2^T(t),\cdots,\varepsilon_N^T(t)\right)^T$, $\varepsilon(t)=\left[\varepsilon_p^T(t)\ \varepsilon_v^T(t)\right]^T$, $\varepsilon_p(t)$ and $\varepsilon_v(t)$ represent the state error of position and velocity in the group, respectively. Then, the system can be rewritten as the matrix form by

$$\dot{\varepsilon}(t)=(I_n\otimes A)\,\varepsilon(t)-B\otimes K\varepsilon(t-\tau_1)+C\otimes K\varepsilon(t-\mu) \tag{3.3}$$

where

$$\begin{aligned} \mu &= \tau_1+\tau_2 \\ A &= \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\in\mathbb{R}^{10\times10} \\ B &= \begin{bmatrix}0 & 0\\ \mathcal{D}+\mathcal{C} & \mathcal{D}+\mathcal{C}\end{bmatrix}\in\mathbb{R}^{10\times10} \\ C &= \begin{bmatrix}0 & 0\\ \mathcal{A} & \mathcal{A}\end{bmatrix}\in\mathbb{R}^{10\times10} \\ \mathcal{A} &= \left[a_{ij}\right]\in\mathbb{R}^{5\times5} \\ \mathcal{C} &= diag\{c_1,c_2,\cdots c_N\}\in\mathbb{R}^{5\times5} \\ \mathcal{D} &= diag\left\{\sum_{j\in N_1}a_{1j},\cdots,\sum_{j\in N_N}a_{Nj}\right\}\in\mathbb{R}^{5\times5} \end{aligned}$$

The mothership and the multi-AUVs operate in the form of a stochastically switching topology. The common probability space for all random variables in the paper is denoted by $(\Omega,\mathcal{F},\mathcal{P})$, Ω is the space of elementary events, $\mathcal{F}$ is the underlying σ-field on Ω, $\mathcal{P}$ is a probability measure on $\mathcal{F}$.

ρ_t is a right-continuous Markov process representing the topology switching process of the agents, which takes values in a given finite set $\mathbb{M} = \{1, 2, \ldots, M\}$ with generator $\prod = \{\pi_{kl}\}$. The transition probability can be described as [39].

$$\Pr\left\{\rho_{t+\delta} = l \mid \rho_t = k\right\} = \begin{cases} \pi_{kl}\delta + o(\delta) & l \neq k \\ 1 + \pi_{kk}\delta + o(\delta) & l = k \end{cases}$$

where $\delta > 0$, $\lim_{\delta \to 0^+} (o(\delta)/\delta) = 0$ and the transition probability rates from mode k at time t to mode l at time $t + \delta$ satisfy $\pi_{kl} \geq 0$ for $k, l \in \mathbb{M}$, $k, l \in \mathbb{M}$, $k \neq l$, and $\pi_{kk} = -\sum_{l \neq k} \pi_{kl}$.

In this paper, a model with the consensus algorithm (3.3) and Markovian switching interconnection topology are considered as

$$\dot{\varepsilon}(t) = (I_n \otimes A)\,\varepsilon(t) - B(\rho_t) \otimes K(\rho_t)\,\varepsilon(t - \tau_1) + C(\rho_t) \otimes K(\rho_t)\,\varepsilon(t - \mu) \quad (3.4)$$

where

$$\mathcal{A}(\rho_t) = \left[a_{ij}^{\rho_t}\right] \in \mathbb{R}^{5\times5}$$
$$\mathcal{C}(\rho_t) = diag\left\{c_1^{\rho_t}, c_2^{\rho_t}, \cdots c_N^{\rho_t}\right\} \in \mathbb{R}^{5\times5}$$
$$\mathcal{D}(\rho_t) = diag\left\{\sum_{j\in\mathcal{N}_1} a_{1j}^{\rho_t}, \cdots, \sum_{j\in\mathcal{N}_N} a_{Nj}^{\rho_t}\right\} \in \mathbb{R}^{5\times5}$$
$$B(\rho_t) = \begin{bmatrix} 0 & 0 \\ \mathcal{D}(\rho_t) + \mathcal{C}(\rho_t) & \mathcal{D}(\rho_t) + \mathcal{C}(\rho_t) \end{bmatrix} \in \mathbb{R}^{10\times10}$$
$$C(\rho_t) = \begin{bmatrix} 0 & 0 \\ \mathcal{A}(\rho_t) & \mathcal{A}(\rho_t) \end{bmatrix} \in \mathbb{R}^{10\times10}$$

System (3.4) is the consensus control of the multiple AUVs recovery system with Markovian switching topologies and time delays, which reflects a stochastic communication process between the mothership and AUVs or among AUVs. Communication is described by a Markov stochastic process, ρ_t is defined as the Markov process taking values on state space $\mathbb{M} = \{1, 2, \ldots, \mathrm{M}\}$. The matrices $B(\rho_t)$ and $C(\rho_t)$ would change randomly from one mode to another mode via a Markov jump process, the control gains vary from mode to mode in Markov switching topologies. The protocol gain $K(\rho_t)$ is to guarantee the consensus of the recovery system.

Definition 2 [40]: A Markovian system (3.4) is said to be stochastically stable if for any finite $\phi \in \mathbb{C}_{n,h}$, which denotes a vector valued initial function, and the initial condition of the mode $\rho_0 \in \mathbb{M}$, the following condition is satisfied

$$\lim_{t\to\infty} \mathbb{E}\left\{\int_0^t x^T(s)\,x(s)\,ds \,|\phi, \rho_0\right\} < \infty$$

B. CONVERGENCE ANALYSIS

In this section, we shall propose stabilization criteria for system (3.4). For simplicity of matrix representation, $e_i \in \mathbb{R}^{5Nn\times5Nn}$ $(i = 1, 2, \ldots 5)$ are defined as block entry matrices. The notations of several matrices are defined as:

$$\xi(t) = \left[\varepsilon^T(t)\ \varepsilon^T(t-\tau_1)\ \varepsilon^T(t-\mu)\ \varepsilon^T(t-h)\ \dot{\varepsilon}^T(t)\right]$$

$$\Theta(\rho_t) = \left[I_n \otimes A\ \ B(\rho_t) \otimes K(\rho_t)\ \ C(\rho_t) \otimes K(\rho_t)\ \ 0\ \ -I_n\right]$$

$$\Xi_1 = e_1\left(I_N \otimes P^k\right)e_5^T + e_5\left(I_N \otimes P^k\right)e_1^T + e_1\sum_{l=1}^{M}\left(\pi_{kl}\left(I_N \otimes P^l\right)\right)e_1^T$$

$$\Xi_2 = e_1\left(I_N \otimes (Q_1 + Q_2 + Q_3)\right)e_1^T - e_2\left(I_N \otimes Q_1\right)e_2^T - \left(1 - h_p\right)e_3\left(I_N \otimes Q_2\right)e_3^T - e_4\left(I_N \otimes Q_3\right)e_4^T$$

$$\begin{aligned}\Xi_3 = {} & e_5\left(I_N \otimes h^2R\right)e_5^T - (e_1 - e_2)(I_N \otimes R)(e_1 - e_2)^T \\ & - (e_2 - e_3)(I_N \otimes S)(e_2 - e_3)^T \\ & - (e_3 - e_4)(I_N \otimes S)^T(e_3 - e_4)^T \\ & - (e_2 - e_3)(I_N \otimes R)(e_2 - e_3)^T \\ & - (e_3 - e_4)(I_N \otimes R)(e_3 - e_4)^T\end{aligned}$$

$$\begin{aligned}\Xi_4 = {} & e_1\left(I_N \otimes (hW_1 + U_1)\right)e_1^T + e_2\left(I_N \otimes (U_2 - U_1)\right)e_2^T \\ & + e_3\left(I_N \otimes (U_3 - U_2)\right)e_3^T - e_4\left(I_N \otimes U_3\right)e_4^T \\ & + e_5\left(I_N \otimes hW_2\right)e_5^T\end{aligned}$$

$$\begin{aligned}\Omega(\rho_t) = {} & \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 \\ = {} & e_1\left(I_N \otimes P^k\right)e_5^T + e_5\left(I_N \otimes P^k\right)e_1^T \\ & + e_1\Bigg(\left(I_N \otimes (Q_1 + Q_2 + Q_3)\right) - (I_N \otimes R) \\ & + I_N \otimes (hW_1 + U_1) + \sum_{l=1}^{M}\left(\pi_{kl}\left(I_N \otimes P^l\right)\right)\Bigg)e_1^T \\ & + e_2\left((I_N \otimes (U_2 - U_1)) - (I_N \otimes Q_1) - (I_N \otimes S)\right)e_2^T \\ & + e_3\left((I_N \otimes (U_3 - U_2)) - \left(1 - h_p\right)(I_N \otimes Q_2)\right)e_3^T \\ & + e_4\left((I_N \otimes S)^T + (I_N \otimes R) - (I_N \otimes U_3) - (I_N \otimes Q_3)\right)e_4^T \\ & + e_5\left(\left(I_N \otimes h^2R\right) + (I_N \otimes hW_2)\right)e_5^T \end{aligned} \quad (3.5)$$

Theorem 1: For given the gains $K(\rho_t)$ and the scalars h, h_p, the AUVs in the recovery system (3.4) converge to the state of the mothership stochastically, if there exist positive definite matrices $P^k \in \mathbb{R}^{n\times n}$, $W_1, W_2 \in \mathbb{R}^{n\times n}$, $Q_1, Q_2, Q_3 \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, any symmetric matrices $U_1, U_2, U_3 \in \mathbb{R}^{n\times n}$ and any matrix $S \in \mathbb{R}^{n\times n}$, satisfying the following LMIs for $k \in \mathbb{M}$:

$$[\Theta(\rho_t)]^{\perp^T}\,\Omega(\rho_t)\,[\Theta(\rho_t)]^{\perp} < 0 \quad (3.6)$$

$$\begin{bmatrix} R & S \\ * & R \end{bmatrix} \geq 0 \quad (3.7)$$

$$\begin{bmatrix} W_1 & U_1 \\ * & W_2 \end{bmatrix} > 0 \begin{bmatrix} W_1 & U_2 \\ * & W_2 \end{bmatrix} > 0$$
$$\begin{bmatrix} W_1 & U_3 \\ * & W_2 \end{bmatrix} > 0 \quad (3.8)$$

where $\Omega(\rho_t)$ and $\Theta(\rho_t)$ are defined in Eq. (3.5)

Proof: For each $k \in \mathbb{M}$, $V(\varepsilon(t), k)$ is the following Lyapunov-Krasovskii stochastic functional

$$\begin{aligned}
V(\varepsilon(t),k) &= \sum_{i=1}^{4} V_i(\varepsilon(t),k)\\
V_1 &= \varepsilon^T(t)\left(I_N \otimes P^k\right)\varepsilon(t)\\
V_2 &= \int_{t-\tau_1}^{t} \varepsilon^T(s)(I_N\otimes Q_1)\varepsilon(s)ds\\
&\quad + \int_{t-\tau_1-\tau_2}^{t} \varepsilon^T(s)(I_N\otimes Q_2)\varepsilon(s)ds\\
&\quad + \int_{t-h}^{t} \varepsilon^T(s)(I_N\otimes Q_3)\varepsilon(s)ds\\
V_3 &= h\int_{t-h}^{t}\int_{s}^{t} \dot{\varepsilon}^T(\theta)(I_N\otimes R)\dot{\varepsilon}(\theta)d\theta ds\\
V_4 &= \int_{t-h}^{t}\int_{s}^{t}\left(\varepsilon^T(\theta)(I_N\otimes W_1)\varepsilon(\theta)\right)d\theta ds\\
&\quad + \int_{t-h}^{t}\int_{s}^{t}\left(\dot{\varepsilon}^T(\theta)(I_N\otimes W_2)\dot{\varepsilon}(\theta)\right)d\theta ds
\end{aligned} \tag{3.9}$$

There exist some real matrices $P^k = \left(P^k\right)^T$, $W = W^T$, $Q = Q^T$ and $R = R^T$, to satisfy the condition, we assume each matrix variable to be positive definite.

By use of the weak infinitesimal operator $\mathbb{L}$ [21], [41], the $\mathbb{L}V(\varepsilon(t), k)$ is calculated as

$$\begin{aligned}
\mathbb{L}V(\varepsilon(t),k) &= \lim_{\delta\to 0^+}\frac{1}{\delta}[\mathbb{E}\{V(\chi(t+\delta),\rho_{t+\delta})|\chi(t),\rho_t=k\}\\
&\quad - V(\chi(t),\rho_t=k)]\\
\mathbb{L}V(\varepsilon(t),k) &= \mathbb{L}V_1+\mathbb{L}V_2+\mathbb{L}V_3+\mathbb{L}V_4
\end{aligned} \tag{3.10}$$

The term $\mathbb{L}V_1$ is given by

$$\begin{aligned}
&\mathbb{L}V_1\\
&= 2\varepsilon^T(t)\left(I_N\otimes P^k\right)\dot{\varepsilon}(t)+\varepsilon^T(t)\left(\sum_{l=1}^{M}\pi_{kl}\left(I_N\otimes P^l\right)\right)\varepsilon(t)\\
&= \xi^T(t)\,\Xi_1^k\xi(t)
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
&\mathbb{L}V_2\\
&\le \varepsilon^T(t)((I_N\otimes Q_1)+(I_N\otimes Q_2)+(I_N\otimes Q_3))\varepsilon(t)\\
&\quad -\varepsilon^T(t-\tau_1)(I_N\otimes Q_1)\varepsilon(t-\tau_1)\\
&\quad -\left(1-h_p\right)\varepsilon^T(t-\mu)(I_N\otimes Q_2)\varepsilon(t-\mu)\\
&\quad -\varepsilon^T(t-h)(I_N\otimes Q_3)\varepsilon(t-h)\\
&= \xi^T(t)\,\Xi_2\xi(t)
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
&\mathbb{L}V_3\\
&= \dot{\varepsilon}^T(t)\left(I_N\otimes h^2R\right)\dot{\varepsilon}(t) - h\int_{t-\tau_1}^{t}\dot{\varepsilon}^T(s)(I_N\otimes R)\dot{\varepsilon}(s)ds\\
&\quad - h\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}^T(s)(I_N\otimes R)\dot{\varepsilon}(s)ds\\
&\quad - h\int_{t-h}^{t-\mu}\dot{\varepsilon}^T(s)(I_N\otimes R)\dot{\varepsilon}(s)ds
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
&\mathbb{L}V_3\\
&\le \dot{\varepsilon}^T(t)\left(I_N\otimes h^2R\right)\dot{\varepsilon}(t)-\varepsilon^T(t)(I_N\otimes R)\varepsilon(t)\\
&\quad +\varepsilon^T(t-\tau_1)(I_N\otimes R)\varepsilon(t-\tau_1)\\
&\quad -\frac{h-\tau_1}{\mu-\tau_1}\left(\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\right)^T(I_N\otimes R)\left(\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\right)\\
&\quad -\frac{h-\tau_1}{h-\mu}\left(\int_{t-h}^{t-\mu}\dot{\varepsilon}(s)ds\right)^T(I_N\otimes R)\left(\int_{t-h}^{t-\mu}\dot{\varepsilon}(s)ds\right)\\
&= \dot{\varepsilon}^T(t)\left(I_N\otimes h^2R\right)\dot{\varepsilon}(t)-\varepsilon^T(t)(I_N\otimes R)\varepsilon(t)\\
&\quad +\varepsilon^T(t-\tau_1)(I_N\otimes R)\varepsilon(t-\tau_1)\\
&\quad -\sum_{i=1}^{N}\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}^T\begin{bmatrix}\frac{1}{1-\alpha_t}R & 0\\ 0 & \frac{1}{\alpha_t}R\end{bmatrix}\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}
\end{aligned} \tag{3.14}$$

where $\alpha_t = (h-\mu)/(h-\tau_1)$, the inequalities in inequality (3.14) come from the Lemma (3).

$$\begin{aligned}
&-\sum_{i=1}^{N}\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}^T\begin{bmatrix}-\sqrt{\frac{\alpha_t}{1-\alpha_t}} & 0\\ 0 & \sqrt{\frac{1-\alpha_t}{\alpha_t}}\end{bmatrix}\begin{bmatrix}R & S\\ * & R\end{bmatrix}\\
&\quad\times\begin{bmatrix}-\sqrt{\frac{\alpha_t}{1-\alpha_t}} & 0\\ 0 & \sqrt{\frac{1-\alpha_t}{\alpha_t}}\end{bmatrix}\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}\le 0
\end{aligned} \tag{3.15}$$

$$\begin{aligned}
\mathbb{L}V_3 &\le \dot{\varepsilon}^T(t)\left(I_N\otimes h^2R\right)\dot{\varepsilon}(t)-\varepsilon^T(t)(I_N\otimes R)\varepsilon(t)\\
&\quad +\varepsilon^T(t-\tau_1)(I_N\otimes R)\varepsilon(t-\tau_1)\\
&\quad -\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}^T\begin{bmatrix}I_N\otimes R & I_N\otimes S\\ * & I_N\otimes R\end{bmatrix}\\
&\quad\times\begin{bmatrix}\int_{t-\mu}^{t-\tau_1}\dot{\varepsilon}(s)ds\\ \int_{t-h}^{t-\mu}\dot{\varepsilon}(s)\,ds\end{bmatrix}
\end{aligned} \tag{3.16}$$

Lastly, for $\mathbb{L}V_4$, it is calculated as

$$\begin{aligned}
\mathbb{L}V_4 &= \varepsilon^T(t)(I_N\otimes hW_1)\varepsilon(t)+\dot{\varepsilon}^T(t)(I_N\otimes hW_2)\dot{\varepsilon}(t)\\
&\quad -\int_{t-h}^{t}\left(\varepsilon^T(s)(I_N\otimes W_1)\varepsilon(s)\right)ds\\
&\quad -\int_{t-h}^{t}\left(\dot{\varepsilon}^T(s)(I_N\otimes W_2)\dot{\varepsilon}(s)\right)ds
\end{aligned} \tag{3.17}$$

Inspired by the work of [42], the following three zero equalities with any symmetric matrices U_1 and U_2 are considered

$$\begin{aligned}
0 &= \varepsilon^T(t)(I_N\otimes U_1)\varepsilon(t)\\
&\quad -\varepsilon^T(t-\tau_1)(I_N\otimes U_1)\varepsilon(t-\tau_1)\\
&\quad -2\int_{t-\tau_1}^{t}\left(\varepsilon^T(s)(I_N\otimes U_1)\dot{\varepsilon}(s)\right)ds\\
0 &= \varepsilon^T(t-\tau_1)(I_N\otimes U_2)\varepsilon(t-\tau_1)\\
&\quad -\varepsilon^T(t-\mu)(I_N\otimes U_2)\varepsilon(t-\mu)\\
&\quad -2\int_{t-\mu}^{t-\tau_1}\left(\varepsilon^T(s)(I_N\otimes U_2)\dot{\varepsilon}(s)\right)ds\\
0 &= \varepsilon^T(t-\mu)(I_N\otimes U_3)\varepsilon(t-\mu)
\end{aligned}$$

$$
\begin{aligned}
&-\varepsilon^T(t-h)(I_N\otimes U_3)\varepsilon(t-h)\\
&-2\int_{t-h}^{t-\mu}\left(\varepsilon^T(s)(I_N\otimes U_3)\dot\varepsilon(s)\right)ds
\end{aligned}\tag{3.18}
$$

$$
\begin{aligned}
\mathbb{L}V_4 &= \xi^T(t)\,\Xi_4\xi(t)-\int_{t-\tau_1}^{t}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}^T\\
&\quad\times\begin{bmatrix}I_N\otimes W_1 & I_N\otimes U_1\\ * & I_N\otimes W_2\end{bmatrix}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}ds\\
&\quad-\int_{t-\mu}^{t-\tau_1}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}^T\begin{bmatrix}I_N\otimes W_1 & I_N\otimes U_2\\ * & I_N\otimes W_2\end{bmatrix}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}ds\\
&\quad-\int_{t-h}^{t-\mu}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}^T\begin{bmatrix}I_N\otimes W_1 & I_N\otimes U_3\\ * & I_N\otimes W_2\end{bmatrix}\begin{bmatrix}\varepsilon(s)\\ \dot\varepsilon(s)\end{bmatrix}ds\\
&= \xi^T(t)\,\Xi_4\xi(t)-\sum_{i=1}^{N}\int_{t-\tau_1}^{t}\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}^T\begin{bmatrix}W_1 & U_1\\ * & W_2\end{bmatrix}\\
&\quad\times\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}ds\\
&\quad-\sum_{i=1}^{N}\int_{t-\mu}^{t-\tau_1}\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}^T\begin{bmatrix}W_1 & U_2\\ * & W_2\end{bmatrix}\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}ds\\
&\quad-\sum_{i=1}^{N}\int_{t-h}^{t-\mu}\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}^T\begin{bmatrix}W_1 & U_3\\ * & W_2\end{bmatrix}\begin{bmatrix}\varepsilon_i(s)\\ \dot\varepsilon_i(s)\end{bmatrix}ds\\
&\quad\begin{bmatrix}W_1 & U_1\\ * & W_2\end{bmatrix}>0,\quad\begin{bmatrix}W_1 & U_2\\ * & W_2\end{bmatrix}>0,\quad\begin{bmatrix}W_1 & U_3\\ * & W_2\end{bmatrix}>0
\end{aligned}\tag{3.19}
$$

Then, an upper bound of $\mathbb{L}V_4$ is

$$\mathbb{L}V_4\le\xi^T(t)\,\Xi_4\xi(t)\tag{3.20}$$

From Eq. (3.10), (3.11), (3.12), (3.16), (3.19)

$$
\begin{aligned}
&\mathbb{L}V(\varepsilon(t),k)\\
&\le\xi(t)^T\Big(e_1\left(I_N\otimes P^k\right)e_5^T+e_5\left(I_N\otimes P^k\right)e_1^T\\
&\quad+e_1\big(I_N\otimes(Q_1+Q_2+Q_3)-(I_N\otimes R)\\
&\quad+I_N\otimes(hW_1+U_1)+\sum_{l=1}^{M}\left(\pi_{kl}\left(I_N\otimes P^l\right)\right)\big)e_1^T\\
&\quad+e_2\left((I_N\otimes(U_2-U_1))-(I_N\otimes Q_1)-(I_N\otimes S)\right)e_2^T\\
&\quad+e_3\left((I_N\otimes(U_3-U_2))-\left(1-h_p\right)(I_N\otimes Q_2)\right)e_3^T\\
&\quad+e_4\left((I_N\otimes S)^T+(I_N\otimes R)-(I_N\otimes U_3)-(I_N\otimes Q_3)\right)e_4^T\\
&\quad+e_5\left(\left(I_N\otimes h^2R\right)+(I_N\otimes hW_2)\right)e_5^T\Big)\xi(t)\\
&=\xi(t)^T\,\Omega(\rho_t)\,\xi(t)
\end{aligned}\tag{3.21}
$$

λ_Ω and λ_P is defined as follows

$$
\begin{aligned}
\lambda_\Omega&=\max_{\rho t\in\mathbb{M}}\left\{\lambda_{\max}\left(\Omega^{\rho_t}\right)\right\}<0\\
\lambda_P&=\max_{k\in\mathbb{M}}\left\{\lambda_{\max}\left(-P^k\right)\right\}<0
\end{aligned}
$$

From inequality (3.21), it is easy to prove that

$$\mathbb{L}V(\varepsilon(t),k)\le\lambda_\Omega\xi(t)^T\xi(t)\le\lambda_\Omega\varepsilon^T(t)\,\varepsilon(t)\tag{3.22}$$

By applying the S-procedure, the $\mathbb{L}V(\varepsilon(t),k)$ has new upper bound as

$$\mathbb{E}\{\mathbb{L}V(\varepsilon(t),k)\}\le\mathbb{E}\left\{\xi(t)^T\,\Omega(\rho t)\,\xi(t)\right\}\tag{3.23}$$

Then the state error and a delay-dependent stability condition for the system (3.4) can be rewritten as

$$\mathbb{E}\{\Theta(\rho t)\,\xi(t)\}=0\tag{3.24}$$

$$
\begin{aligned}
&\mathbb{E}\left\{\xi(t)^T\,\Omega(\rho t)\,\xi(t)\right\}<0\text{ subject to}\\
&\qquad\mathbb{E}\{\Theta(\rho t)\,\xi(t)\}=0
\end{aligned}\tag{3.25}
$$

It follows from Dynkin's formula that

$$
\begin{aligned}
&\mathbb{E}\{\mathbb{L}V(\varepsilon(t),k)\}-\mathbb{E}\{\mathbb{L}V(\varepsilon(0),\rho_0)\}\\
&\qquad=\mathbb{E}\left\{\int_0^t\mathbb{L}V(\varepsilon(s),k)\,ds\right\}\\
&\qquad\le\lambda_\Omega\mathbb{E}\left\{\int_0^t\varepsilon^T(s)\,\varepsilon(s)\,ds\right\}
\end{aligned}
$$

$$\mathbb{E}\left\{\int_0^t\varepsilon^T(s)\,\varepsilon(s)\,ds\right\}\le-\frac{V(\varepsilon(0),\rho_0)}{\lambda_\Omega}\tag{3.26}$$

$$-\lambda_P\mathbb{E}\left\{\varepsilon^T(t)\,\varepsilon(t)\right\}\le\mathbb{E}\{V(\varepsilon(t),k)\}\tag{3.27}$$

Above all inequalities, it is easy to obtain

$$\mathbb{E}\left\{\varepsilon^T(t)\,\varepsilon(t)\right\}\le-\frac{\lambda_\Omega\mathbb{E}\left\{\int_0^t\varepsilon^T(t)\,\varepsilon(t)\,ds\right\}+V(\varepsilon(0),\rho_0)}{\lambda_P}\tag{3.28}$$

By the Gronwall-Bellman Lemma [43], we know that

$$
\begin{aligned}
&\mathbb{E}\left\{\int_0^t\varepsilon^T(s)\,\varepsilon(s)\,ds\,|\phi,\rho_0\right\}\\
&\qquad\le\frac{\left[1-\exp\left(-\frac{\lambda_\Omega}{\lambda_P}t\right)\right]V(\varepsilon(0),\rho_0)}{\lambda_\Omega}
\end{aligned}\tag{3.29}
$$

Then, there exists a scalar γ as $t\to\infty$, such that

$$
\begin{aligned}
&\lim_{t\to\infty}\mathbb{E}\left\{\int_0^t\varepsilon^T(s)\,\varepsilon(s)\,ds\,|\phi,\rho_0\right\}\\
&\qquad\le\frac{V(\varepsilon(0),\rho_0)}{\lambda_\Omega}\le\gamma\sup_{-h\le s\le 0}\|\phi(s)\|^2
\end{aligned}\tag{3.30}
$$

From the Lyapunov method, Eq. (3.30) and Definition 2, it can be concluded that the system (3.4) is stochastically stable. Finally, by utilizing Lemma 4, the condition (3.25) is equivalent to the following inequality

$$[\Theta(\rho_t)]^{\perp^T}\,\Omega(\rho_t)\,[\Theta(\rho_t)]^{\perp}<0\tag{3.31}$$

From the inequality (3.31), if the LMIs (3.6) satisfy, then stability condition (3.25) holds. This completes our proof.

Theorem 1 provides that the consensus criterion for system (3.4) in the framework of LMIs with the known communication topology consensus protocol gain, a consensus controller design method for system (3.4) based on the results of

Theorem 1 will be derived. To design the consensus protocol gain, the following zero equalities are introduced:

$$0=2\left(\varepsilon\left(t\right)\left(I_n\otimes P^k\right)+\dot{\varepsilon}\left(t\right)\left(I_n\otimes P^k\right)\right)\Theta\left(\rho_t\right)\xi\left(t\right) \quad (3.32)$$

Before deriving this, the notations of several matrices are defined for simplicity:

$$\begin{aligned}
\bar{\Theta}\left(\rho_t\right) &= \left[I_n\otimes A\otimes\left(P^k\right)^1 \quad B\left(\rho_t\right)\otimes Y\left(\rho_t\right)\right.\\
&\qquad\left.\times C\left(\rho_t\right)\otimes Y\left(\rho_t\right) \quad 0 \quad -I_n\otimes\left(P^k\right)^1\right]\\
\eta &= \left[I_N \;\; 0 \;\; 0 \;\; 0 \;\; I_N\right]^T\\
\bar{\Xi}_1 &= e_1\left(I_N\otimes\bar{P}^k\right)e_5^T+e_5\left(I_N\otimes\bar{P}^k\right)e_1^T\\
&\quad+e_1\sum_{l=1}^{M}\left(\pi_{kl}\left(I_N\otimes\bar{P}^l\right)\right)e_1^T\\
\bar{\Xi}_2 &= e_1\left(I_N\otimes\left(\bar{Q}_1+\bar{Q}_2+\bar{Q}_3\right)\right)e_1^T-e_2\left(I_N\otimes\bar{Q}_1\right)e_2^T\\
&\quad-\left(1-h_p\right)e_3\left(I_N\otimes\bar{Q}_2\right)e_3^T-e_4\left(I_N\otimes\bar{Q}_3\right)e_4^T\\
\bar{\Xi}_3 &= e_5\left(I_N\otimes h^2\bar{R}\right)e_5^T-\left(e_1-e_2\right)\left(I_N\otimes\bar{R}\right)\left(e_1-e_2\right)^T\\
&\quad-\left(e_2-e_3\right)\left(I_N\otimes\bar{S}\right)\left(e_2-e_3\right)^T\\
&\quad-\left(e_3-e_4\right)\left(I_N\otimes\bar{S}\right)^T\left(e_3-e_4\right)^T\\
&\quad-\left(e_2-e_3\right)\left(I_N\otimes\bar{R}\right)\left(e_2-e_3\right)^T\\
&\quad-\left(e_3-e_4\right)\left(I_N\otimes\bar{R}\right)\left(e_3-e_4\right)^T\\
\bar{\Xi}_4 &= e_1\left(I_N\otimes\left(h\bar{W}_1+\bar{U}_1\right)\right)e_1^T+e_2\left(I_N\otimes\left(\bar{U}_2-\bar{U}_1\right)\right)e_2^T\\
&\quad+e_3\left(I_N\otimes\left(\bar{U}_3-\bar{U}_2\right)\right)e_3^T-e_4\left(I_N\otimes\bar{U}_3\right)e_4^T\\
&\quad+e_5\left(I_N\otimes h\bar{W}_2\right)e_5^T\\
\bar{\Omega}\left(\rho_t\right) &= \bar{\Xi}_1+\bar{\Xi}_2+\bar{\Xi}_3+\bar{\Xi}_4 \qquad (3.33)
\end{aligned}$$

Theorem 2: For given scalars 0 < h and hp, the agents in the system (3.4) converge to the state of mothership stochastically, if there exist positive definite matrices $\bar{P}^k \in \mathbb{R}^{n\times n}$, $\bar{W}_1, \bar{W}_2 \in \mathbb{R}^{n\times n}$, $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3 \in \mathbb{R}^{n\times n}$, $\bar{R} \in \mathbb{R}^{n\times n}$, any symmetric matrices $\bar{U}_1, \bar{U}_2, \bar{U}_3 \in \mathbb{R}^{n\times n}$ and any matrix $\bar{S} \in \mathbb{R}^{n\times n}$, $Y\left(\rho_t\right) \in \mathbb{R}^{l\times n}$ satisfying the following LMIs for $k \in \mathbb{M}$:

$$\bar{\Omega}\left(\rho_t\right)+\eta\bar{\Theta}\left(\rho_t\right)+\left(\eta\bar{\Theta}\left(\rho_t\right)\right)^T<0 \quad (3.34)$$

$$\begin{bmatrix}\bar{R} & \bar{S}\\ * & \bar{R}\end{bmatrix}\geq 0 \quad (3.35)$$

$$\begin{bmatrix}\bar{W}_1 & \bar{U}_1\\ * & \bar{W}_2\end{bmatrix}>0 \begin{bmatrix}\bar{W}_1 & \bar{U}_2\\ * & \bar{W}_2\end{bmatrix}>0$$

$$\begin{bmatrix}\bar{W}_1 & \bar{U}_3\\ * & \bar{W}_2\end{bmatrix}>0 \quad (3.36)$$

where

$$\begin{aligned}
&\bar{Q}_1=\left(P^k\right)^{-1}Q_1\left(P^k\right)^{-1}, \quad \bar{Q}_2=\left(P^k\right)^{-1}Q_2\left(P^k\right)^{-1},\\
&\bar{Q}_3=\left(P^k\right)^{-1}Q_3\left(P^k\right)^{-1}\\
&\bar{R}=\left(P^k\right)^{-1}R\left(P^k\right)^{-1}\\
&\bar{S}=\left(P^k\right)^{-1}S\left(P^k\right)^{-1}\\
&\bar{W}_1=\left(P^k\right)^{-1}W_1\left(P^k\right)^{-1}, \quad \bar{W}_2=\left(P^k\right)^{-1}W_2\left(P^k\right)^{-1}\\
&\bar{U}_1=\left(P^k\right)^{-1}U_1\left(P^k\right)^{-1}, \quad \bar{U}_2=\left(P^k\right)^{-1}U_2\left(P^k\right)^{-1},\\
&\bar{U}_3=\left(P^k\right)^{-1}U_3\left(P^k\right)^{-1}
\end{aligned}$$

$\bar{\Theta}\left(\rho_t\right)$, η and $\bar{\Omega}\left(\rho_t\right)$ are defined in Eq. (3.33).

Then, the system (3.4) under the consensus protocol gains $K\left(\rho_t\right)=Y\left(\rho_t\right)P^k$ are stochastically stable.

Proof: With the same Lyapunov Krasovskii functional candidate in Eq. (3.10), by using a similar method in Eq. (3.11)-(3.20), and considering the zero equation in Eq. (3.32), a sufficient condition guaranteeing stability for the system (3.4) can be

$$\Omega\left(\rho_t\right)+\eta\left(I_n\otimes P^k\right)\Theta\left(\rho_t\right)+\left(\eta\left(I_n\otimes P^k\right)\Theta\left(\rho_t\right)\right)^T<0 \quad (3.37)$$

where $\Theta\left(\rho_t\right)$, η and $\Omega\left(\rho_t\right)$ are defined in Eq. (3.5) and Eq. (3.33), respectively.

Also, to obtain the consensus protocol gain, pre- and post-multiplying inequality (3.37) by matrix

$$diag\left\{\underbrace{\left(I_n\otimes\left(P^k\right)^{-1}\right),\ldots,\left(I_n\otimes\left(P^k\right)^{-1}\right)}_{5}\right\}$$

lead to LMIs (3.34). This completes our proof.

IV. NUMERICAL EXAMPLES

In this section, to testify the correctness of the consensus control protocol in this paper, two numerical examples are presented. Then, the numerical examples are illustrated the result of Theorem 2. The formation of multiple AUVs recovery system has six vehicles, which consist of one mothership and five AUVs. To illustrate the random information between the AUVs, the communication topologies set includes four communication topologies.

Example 1: Communication topologies set includes four communication topologies as shown in Fig. 3. The initial value of position and attitude of the mothership is $p_m=\left(50 \;\; 0 \;\; 0 \;\; -\pi/9 \;\; \pi/2\right)$, and the initial value of velocity of the mothership is $v_m=\left(2 \;\; 0 \;\; 0 \;\; 0 \;\; 0\right)$. The initial value of x is randomly distributed in the $[0, 50]$, y is randomly distributed in the $[0, 50]$, z is randomly distributed in the $[-25, 0]$. Θ is in the interval $[-\pi/9, \pi/9]$, φ is in the interval $[0, 2\pi]$. The time delay is set as $\tau_2\left(t\right)=0.14\left(1+\sin\left(2t\right)\right)$, $\tau_1=0.1$, $h_p=0.28$.

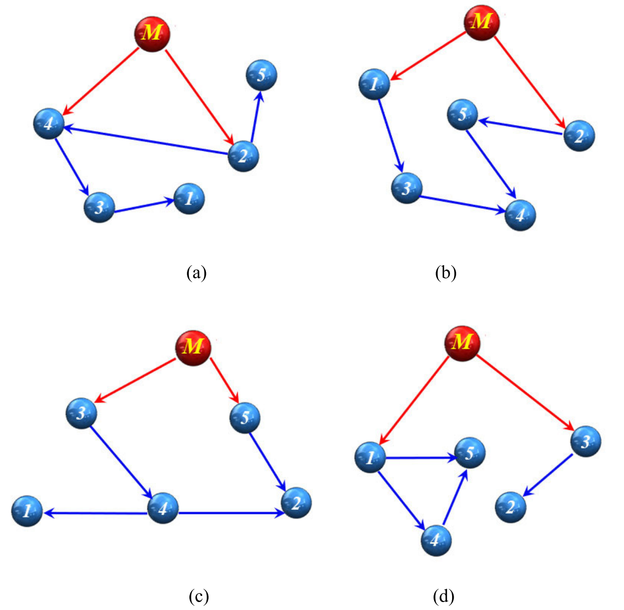


FIGURE 3. Structure of Communication Topology Set. The mothership transmits information to the AUVs under four switched communication topologies. The structure of communication topology is randomly switched under communication constraints.

From Fig. 3, the parameters of the matrix $\mathcal{A}$ and $\mathcal{C}$ at switching interconnection topologies are

$$\mathcal{A}_1=\begin{bmatrix}0&0&0&0&0\\0&0&0&1&1\\1&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\end{bmatrix}\quad \mathcal{A}_2=\begin{bmatrix}0&0&1&0&0\\0&0&0&0&1\\0&0&0&1&0\\0&0&0&0&0\\0&0&0&1&0\end{bmatrix}$$
$$\mathcal{C}_1=diag\begin{bmatrix}0&1&0&1&0\end{bmatrix}\quad \mathcal{C}_2=diag\begin{bmatrix}1&1&0&0&0\end{bmatrix}$$
$$\mathcal{A}_3=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&1&0\\1&1&0&0&0\\0&1&0&0&0\end{bmatrix}\quad \mathcal{A}_4=\begin{bmatrix}0&0&0&1&1\\0&0&0&0&0\\0&1&0&0&0\\0&0&0&0&1\\0&0&0&0&0\end{bmatrix}$$
$$\mathcal{C}_3=diag\begin{bmatrix}0&0&1&0&1\end{bmatrix}\quad \mathcal{C}_4=diag\begin{bmatrix}1&0&1&0&0\end{bmatrix}$$

The original velocity of AUV is zero. The recovery path is a helical curve, and it expressed as:

$$x(t)=50\cos(0.02\pi t)$$
$$y(t)=50\sin(0.02\pi t)$$
$$z(t)=-0.03t$$

The Markov process taking values in $\mathbb{M}$ with generator Π are as follows

$$\Pi=\begin{bmatrix}-0.7&0.3&0.4&0\\0.3&-0.9&0&0.6\\0&0.6&-0.6&0\\0.4&0&0.2&-0.6\end{bmatrix}$$

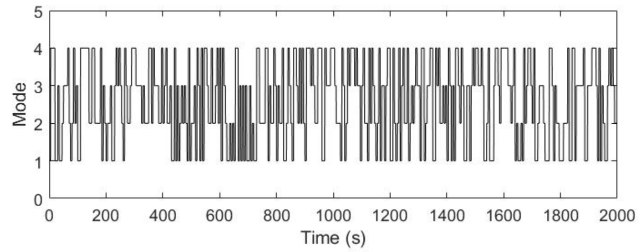


FIGURE 4. The Switching Mode. The structure of communication topology is randomly switched under four modes in 2000 seconds.

The consensus protocol gains $K(\rho_t)$ and the maximum bound of the time-delay h by Theorem 2 are

$$K_1=0.085\quad K_2=0.95\ K_3=1.12\ K_4=0.042\ h=0.38$$

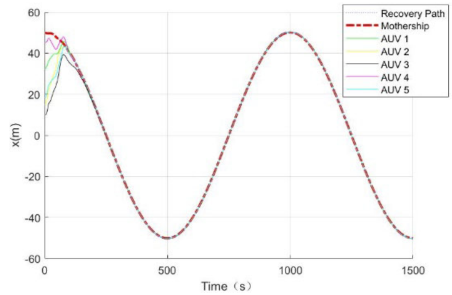
The switching mode chosen from the set randomly is shown in Figure 4.

The simulation results are shown in Figs. 4-7. Fig. 5 describes the position and attitude states of AUVs during recovery process under switching topologies and time delays. The initial states of five AUVs are randomly distributed. The mothership tracks the recovery path, and the AUVs follow the mothership in 1500 seconds. The position and the attitude states converge to the mothership in 500 seconds, and then keep consistent with the mothership in the remaining 1000 seconds.

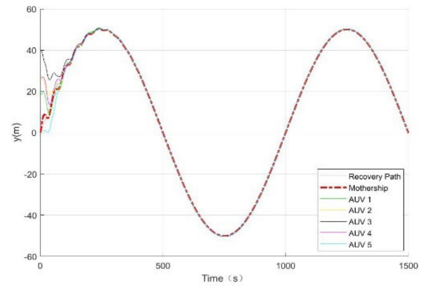
The velocity states of AUVs during recovery process are shown in Fig. 6, it's shown that the velocity states converge to a fixed value in 500 seconds, and then keep consistent with the mothership in the remaining 1000 seconds. Due to random topological transformation, the reference motion state information of AUVs changes frequently, position, attitude and velocity have large oscillations and some variability during the adjustment process. Therefore, the AUVs oscillates around the recovery trajectories during the recovery trajectories tracking process. Since the control input needs to be rotated by the feedback linearization method, the convergence speed is slow.

Fig. 7 describes the three-dimensional trajectories of all AUVs, which describes the whole recovery process more intuitively. It shows that the mothership tracks the recovery trajectory, and five AUVs converge to the mothership and then keep consistent with the mothership. The multi-AUV recovery mission is completed.

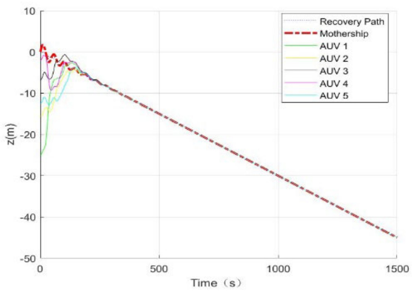
Example 2: Communication topology set includes four communication topologies as shown in Fig. 8. The initial value of position and attitude of the mothership is $p_m=\begin{pmatrix}60\ 0\ 0\ -\pi/18\ \pi/2\end{pmatrix}$, and the initial value of velocity of the mothership is $v_m=\begin{pmatrix}3\ 0\ 0\ 0\ 0\end{pmatrix}$. The initial value of x is randomly distributed in the $[0,60]$, y is randomly distributed in the $[0,60]$, z is randomly distributed in the $[-40,0]$. Θ is in the interval $[-\pi/18,\pi/18]$, φ is in the interval $[0,2\pi]$. The time delay is set as $\tau_2(t)=0.14(1+\cos(2t))$, $\tau_1=0.1$, $h_p=0.28$.



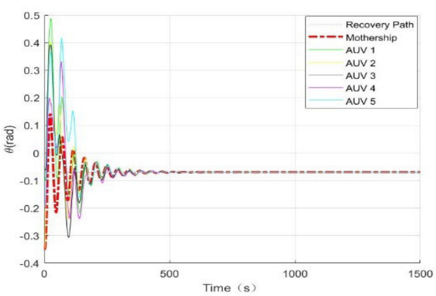
(a) Longitudinal coordinate of AUVs



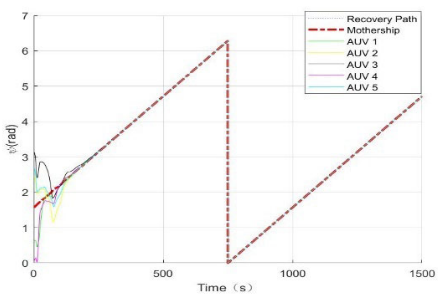
(b) Transverse coordinate of AUVs



(c) Vertical coordinate of AUVs

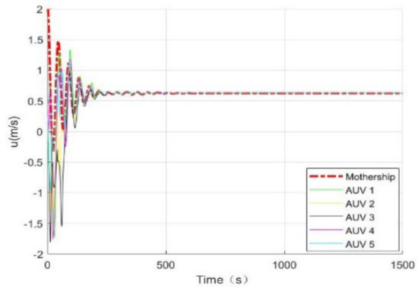


(d) Pitch angle of AUVs

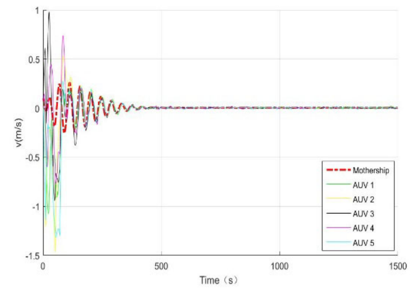


(e) Yaw angle of AUVs

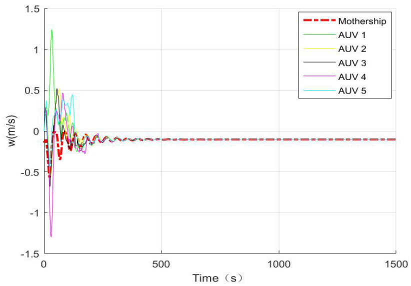
FIGURE 5. Position and Attitude States of AUVs. The position and the attitude states converge to the mothership in 500 seconds.



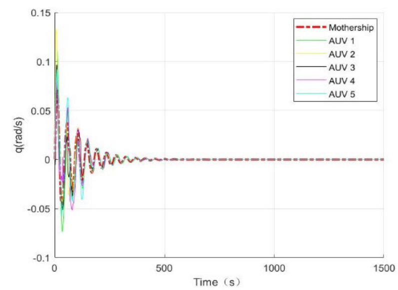
(a) Surge of AUVs



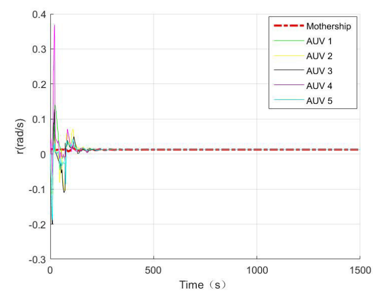
(b) Sway of AUVs



(c) Heave of AUVs



(d) Pitch of AUVs



(e) Yaw of AUVs

FIGURE 6. Velocity States of AUVs. The velocity states converge to a fixed value in 500 seconds.

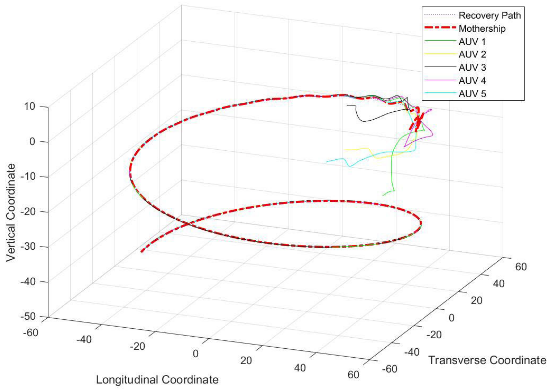


FIGURE 7. Three-dimensional Trajectories of Recovery Process. The AUVs converge to the desired helical curve.

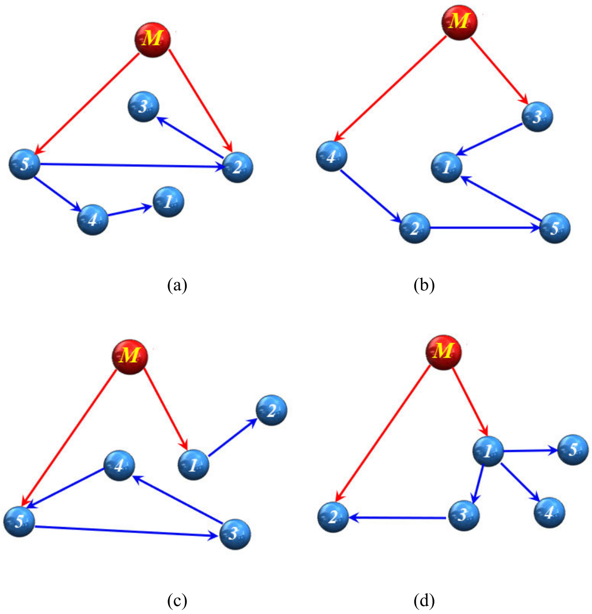


FIGURE 8. Structure of Communication Topology Set. The mothership transmits information to the AUVs under four switched communication topologies. The structure of communication topology is randomly switched under communication constraints.

From Fig. 8, the parameters of the matrix $\mathcal{A}$ and $\mathcal{C}$ at switching interconnection topology are

$$\mathcal{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad \mathcal{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{C}_1 = diag\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \mathcal{C}_2 = diag\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathcal{A}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathcal{A}_4 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{C}_3 = diag\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathcal{C}_4 = diag\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

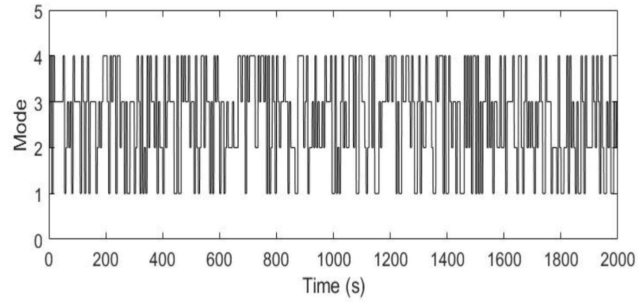


FIGURE 9. The Switching Mode. The structure of communication topology is randomly switched under four modes in 2000 seconds.

The original velocity of AUV is zero. The recovery path is a helical curve, and it expressed as:

$$\begin{aligned} x(t) &= 60\cos(0.02\pi t) \\ y(t) &= 60\sin(0.02\pi t) \\ z(t) &= -0.05t \end{aligned}$$

The Markov process taking values in $\mathbb{M}$ with generator Π are as follows

$$\Pi = \begin{bmatrix} -0.5 & 0.2 & 0.3 & 0 \\ 0.3 & -0.7 & 0 & 0.4 \\ 0 & 0.5 & -0.5 & 0 \\ 0.2 & 0 & 0.2 & -0.4 \end{bmatrix}$$

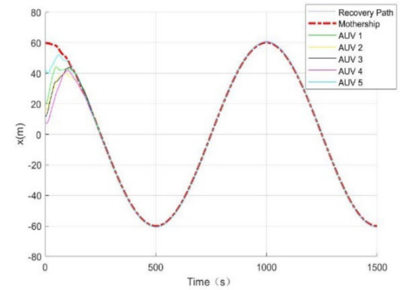
The consensus protocol gains $K(\rho_t)$ and the maximum bound of the time-delay h by Theorem 2 are

$$K_1 = 0.065 \quad K_2 = 1.73\ K_3 = 1.21\ K_4 = 0.023\ h = 0.38$$

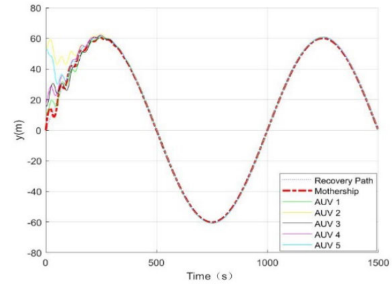
The switching mode chosen from the set randomly is shown in Figure 9.

The simulation results are shown in Figs.9-12. The position, attitude and velocity states of AUVs during recovery process are shown in Fig. 10 and Fig. 11, Fig. 12 describes the three-dimensional trajectories of all AUVs. The initial states of five AUVs are randomly distributed. From the above pictures, the mothership tracks the recovery path, and the AUVs follow the mothership in 1500 seconds. The position and the attitude states converge to the mothership and the velocity states converge to a fixed value in 500 seconds, and then keep consistent with the mothership in the remaining 1000 seconds. Three-dimensional trajectories describes the whole recovery process more intuitively. The mothership tracks the recovery trajectory, and five AUVs converge to the mothership and then keep consistent with the mothership. The multi-AUV recovery mission is completed.

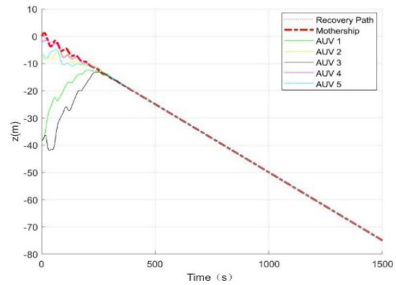
Based on the simulation results above, it is easy to derive that under switching topologies and time delays, the mothership transmits information to the AUVs under four switched communication topologies, the structure of communication topology is randomly switched under communication constraints. The position, attitude, velocity and convergence time of AUVs depend on the attitude, velocity, and recovery trajectories of the mothership and the initial position



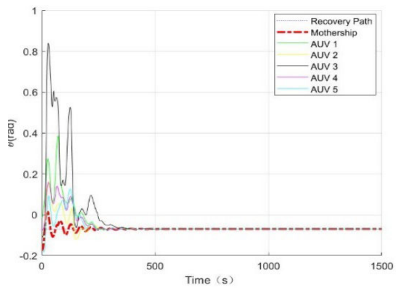
(a) Longitudinal coordinate of AUVs



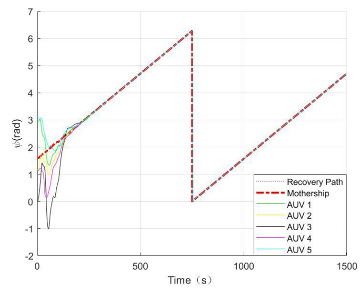
(b) Transverse coordinate of AUVs



(c) Vertical coordinate of AUVs

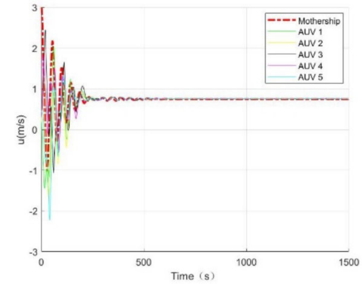


(d) Pitch angle of AUVs

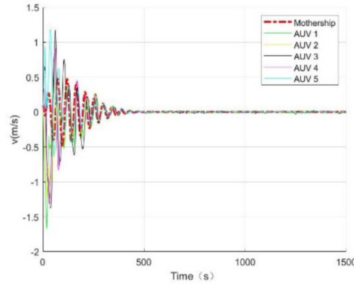


(e) Yaw angle of AUVs

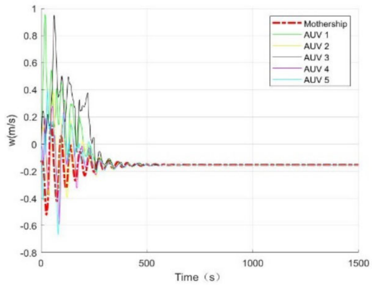
FIGURE 10. Position and Attitude States of AUVs. The position and the attitude states converge to the mothership in 500 seconds.



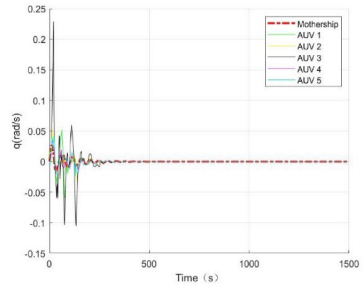
(a) Surge of AUVs



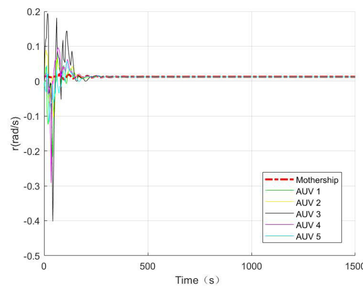
(b) Sway of AUVs



(c) Heave of AUVs



(d) Pitch of AUVs



(e) Yaw of AUVs

FIGURE 11. Velocity States of AUVs. The velocity states converge to a fixed value in 500 seconds.

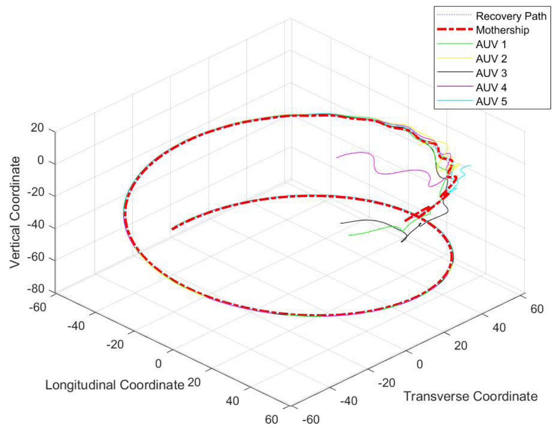


FIGURE 12. Three-dimensional Trajectories of Recovery Process. The AUVs converge to the desired helical curve.

of AUVs. Consensus control is used to adjust the parameters of TABLE I, the mothership tracks the recovery trajectory, and five AUVs converge to the mothership and then keep consistent with the mothership. The multi-AUV recovery mission is completed when the states of all AUVs are consistent with that of the mothership.

V. CONCLUSION

In this paper, consensus control is applied to multi-AUV recovery system for the first time. Switching communication topologies and time delays are introduced in consideration of underwater complexity. The mothership is regarded as the point which all AUVs have to catch up with, the recovery problem of multiple AUVs can be seen as the consensus problem of multi-agents system similarly. Consensus control of multiple AUVs recovery system under switching communication topologies and time delays including input delay and communication delay is designed.

The research progress of multi-agent is introduced in section I. Section II establishes the dynamic model of mothership and AUV, the single AUV nonlinear mathematical model is transformed into a second-order integral model via state feedback linearization, in addition, graph theory and related lemmas are introduced. In section III, a randomly changing consensus stability criteria and stabilization conditions are derived by a suitable Lyapunov-Krasovskii functional for the Markovian switching recovery system with time delays, and then a consensus controller design method is derived. Two numerical examples are used to illustrate the effectiveness of the proposed control methods in section IV. It is easy to derive that under switching topologies and time delays, the multi-AUV recovery mission is completed when the states of all AUVs are consistent with that of the mothership.

Above all, consensus control of multiple AUVs recovery system under switching communication topologies and time delays is designed. It should be pointed out that the algorithm designed is aimed at AUV dynamic model under the condition of limited communication, which cannot be applied to unmanned aerial vehicles and other unmanned vehicle systems, therefore, the algorithm is not universally applicable.

The proposed method can be extended to discrete-time consensus control of multiple AUVs. This work will be carried out in our future research. With current control laws, the recovery system is not immune to external disturbances in the state information. Noise in the state information and its mitigation strategies also will be considered in the future research. In addition, we will apply the designed controller to the actual AUV system.

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