IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 22, 2019, accepted July 29, 2019, date of publication August 13, 2019, date of current version September 5, 2019.

*Digital Object Identifier 10.1109/ACCESS.2019.2935067*

# Formation Control of Leader-Following Multi-UUVs With Uncertain Factors and Time-Varying Delays

**ZHEPING YAN, XIAOLI PAN, ZEWEN YANG, AND LIDONG YUE**
College of Automation, Harbin Engineering University, Harbin 150001, China

Corresponding author: Xiaoli Pan (panxiaoli006@hrbeu.edu.cn)

This work was supported in part by the National Nature Science Foundation of China under Grant 51679057, and in part by the Heilongjiang Province Outstanding Youth Fund under Grant J2016JQ0052.

**ABSTRACT** This paper is concerned with the multi-UUVs formation control problem and proposes the control protocol with an additional function when considering environmental disturbances. Firstly, the leader-following configuration is adopted to discuss the consensus problem for multi-UUVs system, the leader's behavior directs the follower's motion trajectory and the follower UUVs could, in turn, communicate with each other, respectively. Secondly, three types of coordination control protocols are proposed: the control protocol is designed without additional function and time delay; the control protocol with additional function and without time delay; the control protocol with additional function and time-varying delay. Sufficient consensus conditions are analyzed and derived by using the Lyapunov-Krasovskii functional theory, algebraic graph theory and matrix theory. Finally, two simulation experiments are given to illustrate the effectiveness of the proposed formation control methods.

**INDEX TERMS** Multiple unmanned underwater vehicles, leader-following consensus, uncertain factors, time-varying delays.

## I. INTRODUCTION

In the past decades, with the development of artificial intelligence and new engineering techniques, the cooperation and coordination problems of multi-Unmanned Underwater Vehicles (multi-UUVs) system have been rapidly developed and widely studied due to its broad applications in military and commercial fields, especially in ocean-graphic observation, ocean exploration, plane crash searches, trajectory tracking, etc. Compared with a single UUV, multi-UUVs formation equipped with more sensors can accomplish more complex and larger-scale deep-sea missions with efficient and stable communications. It is worth mentioning that the researches on motion control of a single UUV are getting deeper in recent years, there are several control methods such as robust control [1], adaptive control [2] and fuzzy control approaches [3] in ocean engineering applications, but the above algorithms cannot well applied to the formation problem of multi-UUVs system. The basic problem of formation control is the consensus algorithm, it is noteworthy that there are two types of consensus control algorithms: leaderless consensus [4] and leader-following consensus [5]–[8]. The final states of the leaderless consensus of multi-UUVs are deterministic which are related to the initial position and velocity states of every UUV. In this paper, we deal with the latter case, the leader-following consensus is preferred in complex underwater environment not only because of its energy-saving and scalability, but also because this method does not require the idealized hypothesis that the limited information communication of the leader UUV is accessible to all follower UUVs, the leader UUV only need to pass information to a portion of the followers and the final consensus is achieved only by using the neighboring information of each UUVs.

It should be pointed out that the leader-following consensus problem has been seen as a more important issue which has been greatly studied on multi-agent systems in recent years [9]–[12], some leader-following consensus algorithms have been used in UUVs systems [13]–[15]. Taking note of the fact that the limited communication distance underwater,

The associate editor coordinating the review of this article and approving it for publication was Nagarajan Raghavan.

communication delay or packet loss should be considered [16]–[18], and time-varying delays have been concerned in some researches [19]–[22]. However, most of the above-mentioned results didn't consider uncertain interference in UUVs systems. Some algorithms under external uncertain disturbances were applied to a single UUV [1], [23]–[25]. In UUVs systems, uncertain interference mainly refer to the influences of the environment on the position and velocity states measured by sensors equipped with UUVs. External disturbances are widespread in multi-UUVs systems which often have significant negative effects on the UUVs system performances. For example, wind, ocean waves and ocean currents usually increase a negative impact on speed and depth measurements of UUVs. Therefore, it is necessary to investigate the consensus problem of multi-UUVs formation with uncertain factors.

Inspired by the consensus algorithms of multi-UUVs, in this paper, the leader-follower cooperative architecture is used to realize formation control of the multi-UUVs system. There will be creating an additional nonlinear factor during the movement of UUV in the complicated ocean environment. Obviously, the additional nonlinear factor is related to the states of each UUV at various times. So we focus on the leader-following consensus formation control problem of multi-UUV system with uncertain nonlinear factors and time-varying delays.

Motivated by the discussions and the existing literatures above, the main contributions of this paper can be emphasized as follows:

(i) Different from previous works, we consider the leader-following consensus problem of multi-UUVs with an additional factor which is emerged in UUV's own movement and external interference in a complex underwater environment.

(ii) The communication of information between multi-UUVs through underwater acoustic sensors will cause time delay due to the influence of underwater environment and communication distance which are hard to estimate, so we consider the multi-UUVs system with time-varying delays. Based on the Lyapunov-Krasovskii functional theory, sufficient conditions are derived for the leader-following consensus of the multi-UUVs system with time-varying delays.

The structure of the paper is as follows: Section II introduces some preliminaries on graph theory and the model of a UUV. Section III gives the main results for the formation control of leader-following multi-UUVs with uncertain factors and time-varying delays in detail. Section IV presents two numerical simulations to show the effectiveness of the proposed theoretical results. Section V gives the conclusions and our future work.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### A. GRAPH THEORY FOR MULTI -UUVS SYSTEM

Assume that the communication topology between UUVs is denoted by a directed graph $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{A})$, in which $\mathcal{V} = \{1, \cdots, n\}$ is a node set, $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and $\mathcal{A} = \left[a_{ij}\right] \in R^{n\times n}$is the adjacency matrix. Note that $a_{ji} > 0$ if and only if $(i, j) \in \varepsilon$, which represents that UUV $j$ can get the information of UUV $i$ and UUV $i$ is a neighbor of UUV $j$, and $a_{ij} = 0$ otherwise. The neighbor set of UUV $i$ is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \varepsilon\}$. The Laplacian matrix is defined as $L = \left[l_{ij}\right]_{n\times n}$ with

$$l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}, \quad l_{ij} = -a_{ij},\ i \neq j.$$

Suppose that the leader is represented by the node UUV L, generally, the leader has no neighbors and the matrix between the leader and the followers is defined by $D = diag\,\{a_{10}, a_{20}, \cdots a_{n0}\}$ with $a_{i0} > 0$ if UUV $i$ can receive information from UUV L, and $a_{i0} = 0$ otherwise.

### B. THE UUV MODEL WITH FIVE DEGREES OF FREEDOM

The 6 DOF motion equations of UUV with the body and earth fixed can be written in a vectorial setting according to Fossen [26]. In the case of irrotational ocean currents, and ignores the roll speed, the nonlinear model of UUV can be written with five degrees of freedom:

$$\begin{cases} \dot{\eta} = J(\eta)\,\tilde{v}, \\ M\dot{\tilde{v}} + C(\tilde{v})\,\tilde{v} + D(\tilde{v})\,\tilde{v} + g(\eta) = T, \end{cases} \tag{1}$$

where $\eta = [x, y, z, \theta, \psi]^T$ denotes the position vector of the UUV, $\tilde{v} = [u, v, \omega, q, r]^T$ is the velocity vector, $J(\eta)$ is the rotational transformation matrix, $M$ is the system inertia matrix, $C(\tilde{v})$ is the Coriolis and centripetal matrix, $D(\tilde{v})$ is the damping matrix, $g(\eta)$ is the vector of weight and buoyancy forces, $T$ is the vector of control inputs. The specific forms of $J(\eta), M, C(\tilde{v}), D(\tilde{v}), g(\eta)$ are defined in Ref. [26].

Let $g(\eta) = \mathbf{0}$, and other parameters are given by

$$J(\eta) = \begin{bmatrix} J_1(\eta) & \mathbf{0} \\ \mathbf{0} & J_2(\eta) \end{bmatrix},$$

$$M = \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} \\ 0 & 0 & m_{33} & m_{34} & 0 \\ 0 & 0 & m_{43} & m_{44} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} \end{bmatrix},$$

$$C(\tilde{v}) = \begin{bmatrix} 0 & 0 & 0 & c_{14} & -c_{15} \\ 0 & 0 & 0 & 0 & c_{25} \\ 0 & 0 & 0 & -c_{25} & 0 \\ -c_{14} & 0 & c_{25} & 0 & 0 \\ c_{15} & -c_{25} & 0 & 0 & 0 \end{bmatrix},$$

$$D(\tilde{v}) = -diag\left\{X_u, Y_v, Z_w, M_q, N_r\right\},$$

$$T = [X, Y, Z, M, N]^T,$$

where

$$J_1(\eta) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi & \cos\psi\sin\theta \\ \sin\psi\cos\theta & \cos\psi & \sin\psi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$J_2(\eta) = diag\,\{1, 1/\cos\theta\},$$

$m_{11} = m - X_{\dot{u}}, m_{22} = m - Y_{\dot{v}}, m_{33} = m - Z_{\dot{w}}, m_{44} = I_y - M_{\dot{q}}$, $m_{55} = I_z - N_{\dot{r}}$, $m_{25} = -Y_{\dot{r}}$, $m_{34} = -Z_{\dot{q}}$, $m_{43} = -M_{\dot{w}}$, $m_{52} = N_{\dot{v}}$; $c_{14} = m\omega + Z_{\dot{\omega}}\omega$, $c_{15} = -mv - Y_{\dot{v}}v$, $c_{25} = mu + X_{\dot{u}}u$.

In order to study the coordination control of multi-UUVs formation, the above nonlinear mathematical model of the i-th UUV can be changed into a standard double integrator dynamic model [4]:

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \tag{2}$$

where $x_i \in R^5$, $v_i \in R^5$, $u_i \in R^5$, $i = 1, 2, \cdots, n$.

*Remark 1:* We know multi-UUVs formation problem could be described as all UUVs can reach the common velocity states and desired position states through transmitting information. From formula (2), we ignore the strong coupling parameters of single UUV model, then the formation control problem of multi-UUVs can be seen as a leader-following consensus control problem when the system has a leader, furthermore, when there are many UUVs in system, the leader one can be seen as a point in three dimensions, that is, ignoring the angles of leader UUV.

### C. THE UUV MODEL WITH UNCERTAIN FACTORS

There will be creating an additional nonlinear function when UUV is operating in a complex underwater environment due to UUV's movements and external interferences. Consider the delayed multi-UUVs system consisting of $n$ followers and one leader, and the followers have the following dynamics:

$$\begin{cases} \dot{\tilde{x}}_i(t) = \tilde{v}_i(t), i = 1, 2, \cdots, n, \\ \dot{\tilde{v}}_i(t) = u_i(t) + f(\tilde{x}_i(t - \tau(t)), \tilde{v}_i(t - \tau(t)), t), \end{cases} \tag{3}$$

where $\tilde{x}_i = [x_i, y_i, z_i, \theta_i, \psi_i]^T \in R^5$, $\tilde{v}_i = [u_i, v_i, \omega_i, q_i, r_i]^T \in R^5$, $(i = 1, 2, \cdots, n)$ are the position and velocity of the ith UUV, respectively. $f : R^5 \times R^5 \times R^+ \to R^5$ is a continuously differentiable nonlinear function with time-varying delays. $f(\tilde{x}_i, \tilde{v}_i, t) = (f_1(\tilde{x}_i, \tilde{v}_i, t), \cdots, f_5(\tilde{x}_i, \tilde{v}_i, t))^T$, $i = 1, 2, \cdots, n$, $u_i \in R^5$ is the control input of the ith UUV to be designed.

The leader's dynamics can be described by

$$\begin{cases} \dot{\tilde{x}}_0(t) = \tilde{v}_0(t), \\ \dot{\tilde{v}}_0(t) = f(\tilde{x}_0(t - \tau(t)), \tilde{v}_0(t - \tau(t)), t), \end{cases} \tag{4}$$

where $\tilde{x}_0 = [x_0, y_0, z_0, \theta_0, \psi_0]^T$, $\tilde{v}_0 = [u_0, v_0, \omega_0, q_0, r_0]^T$ are the position and velocity of the leader UUV, respectively,

$$f(\tilde{x}_0, \tilde{v}_0, t) = (f_1(\tilde{x}_0, \tilde{v}_0, t), \cdots, f_5(\tilde{x}_0, \tilde{v}_0, t))^T. \tag{5}$$

For our analysis, we give the following Assumptions and Lemmas.

*Assumption 1:* The interconnection graph $G$ for followers we adopt has a directed spanning tree, and the leader at least sends its information to the root node of follower graph.

*Assumption 2:* The nonlinear continuous Lipschitz function $f(\cdot, \cdot, t) = (f_1(\cdot, \cdot, t), \cdots, f_n(\cdot, \cdot, t))^T$ described in (3) and (4) satisfies the following inequality:

$$\|f(x_1, v_1, t) - f(x_2, v_2, t)\| \le \|G\| \|x_1 - x_2\| + \|J\| \|v_1 - v_2\|, \tag{6}$$

where $x_1, v_1, x_2, v_2 \in R^n$, $G = (g_{ik}) \in R^{n \times n}$ and $J = (j_{ik}) \in R^{n \times n}$ are constant matrices.

*Remark 2:* To achieve consensus for multi-agents systems, the Lipschitz condition in Assumption 2 is commonly used in many literatures [27], [28].

*Assumption 3:* The time-varying delay function $\tau(t)$ is continuously differentiable and satisfies $0 \le \tau(t) \le h$, $\dot{\tau}(t) < \kappa < 1$, where $h \ge 0$, $\kappa \ge 0$.

*Lemma 1 [29] (Schur Complement):* Let $S_1, S_2, S_3$ be given matrices such that $S_1 > 0$. Then

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix} > 0, \tag{7}$$

if and only if $S_3 - S_2^T S_1^{-1} S_2 > 0$.

*Lemma 2 [30] (Young's Inequality):* Foy any $x_1, x_2 \in R^n$, $R$ is a positive semi-definite matrix, the following inequality holds

$$x_1^T R x_2 \le \rho x_1^T R x_1 + \frac{1}{4\rho} x_2^T R x_2, \tag{8}$$

where $\rho > 0$.

*Lemma 3 [4]:* Assume that $p \in R^n$, $q \in R^m$ and $M \in R^{n \times m}$ are defined on the interval $\Omega$. Then, for any matrices $X \in R^{n \times m}$, $Y \in R^{n \times m}$ and $Z \in R^{n \times m}$, the following holds

$$-2 \int_\Omega p^T(t) M q(t)\, dt \le \int_\Omega \begin{bmatrix} p(t) \\ q(t) \end{bmatrix}^T \begin{bmatrix} X & Y - M \\ (Y - M)^T & Z \end{bmatrix} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix} dt, \tag{9}$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0.$$

*Definition 1:* The leader-following consensus of the multi-UUVs system (3) and (4) are said to be achieved if the input control satisfying:

$$\lim_{t\to\infty} \left\| \tilde{x}_i^1(t) - \tilde{x}_0^1(t) \right\| = l, \quad \lim_{t\to\infty} \left\| \tilde{x}_i^1(t) - \tilde{x}_j^1(t) \right\| = l,$$
$$\lim_{t\to\infty} \left\| \tilde{x}_i^2(t) - \tilde{x}_0^2(t) \right\| = 0, \quad \lim_{t\to\infty} \|\tilde{v}_i(t) - \tilde{v}_0(t)\| = 0,$$

where

$$\tilde{x}_i^1 = [x_i, y_i, z_i]^T, \quad \tilde{x}_i^2 = [\theta_i, \psi_i]^T, \ i, j = 1, 2, \cdots, n,$$

$l$ is the preset distance between each UUV in three-dimensional space.

The goal of this paper is to present a consensus protocol to ensure that the states of the follower UUVs can converge to that of the leader UUV. To achieve leader-following consensus, for the ith follower UUV, the following control protocol with time-varying delay is designed:

$$\begin{aligned} u_i(t) = k_1 &\sum_{j\in N_i} a_{ij}\left(\tilde{x}_j(t-\tau(t)) - \tilde{x}_i(t-\tau(t))\right) \\ &+ k_1 a_{i0}\left(\tilde{x}_0(t-\tau(t)) - \tilde{x}_i(t-\tau(t))\right) \\ &+ k_2 \sum_{j\in N_i} a_{ij}\left(\tilde{v}_j(t-\tau(t)) - \tilde{v}_i(t-\tau(t))\right) \\ &+ k_2 a_{i0}\left(\tilde{v}_0(t-\tau(t)) - \tilde{v}_i(t-\tau(t))\right), \end{aligned} \tag{10}$$

where $k_1$ and $k_2$ are positive gains to be designed, $\tau(t)$ is the time-varying delay.

Let

$$\bar{x}_i(t) = \tilde{x}_i(t) - \tilde{x}_0(t), \quad \bar{v}_i(t) = \tilde{v}_i(t) - \tilde{v}_0(t),$$

we have the error dynamics as follows:

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{v}_i(t), \\ \dot{\bar{v}}_i(t) = f\left(\tilde{x}_i(t-\tau(t)), \tilde{v}_i(t-\tau(t)), t\right) \\ \qquad -f\left(\tilde{x}_0(t-\tau(t)), \tilde{v}_0(t-\tau(t)), t\right) \\ \qquad -k_1 \sum_{i=1}^{n} h_{ij}\bar{x}_i(t-\tau(t)) - k_2 \sum_{i=1}^{n} h_{ij}\bar{v}_i(t-\tau(t)), \end{cases} \tag{11}$$

where

$$\begin{aligned} &[h_{ij}]_{n\times n} \\ &= H = L + D \\ &= \begin{bmatrix} \sum_{j=1}^{n} a_{1j} + a_{10} & -a_{12} & \cdots & -a_{1,n} \\ -a_{21} & \sum_{j=1}^{n} a_{2j} + a_{20} & \cdots & -a_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ -a_{n,1} & -a_{n,2} & \cdots & \sum_{j=1}^{n} a_{n,j} + a_{n,0} \end{bmatrix}. \end{aligned} \tag{12}$$

Set

$$\begin{aligned} \varepsilon(t) &= \left[\bar{x}^T(t), \bar{v}^T(t)\right]^T, \\ \bar{x}(t) &= \left[\bar{x}_1^T(t), \bar{x}_2^T(t), \cdots, \bar{x}_n^T(t)\right]^T, \\ \bar{v}(t) &= \left[\bar{v}_1^T(t), \bar{v}_2^T(t), \cdots, \bar{v}_n^T(t)\right]^T. \end{aligned}$$

The system (11) can be recast in the following compact form:

$$\begin{aligned} \dot{\varepsilon}(t) = (A\otimes I_5)\,\varepsilon(t) + (B\otimes I_5)\,\varepsilon(t-\tau(t)) \\ + F\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right), \end{aligned} \tag{13}$$

where

$$A = \begin{bmatrix} \mathbf{0} & I_n \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ -k_1 H & -k_2 H \end{bmatrix},$$

$$\begin{aligned} &F_i\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \\ &= f\left(\tilde{x}_i(t-\tau(t)), \tilde{v}_i(t-\tau(t)), t\right) \\ &\quad -f\left(\tilde{x}_0(t-\tau(t)), \tilde{v}_0(t-\tau(t)), t\right), \quad i = 1, 2, \cdots, n. \\ &F\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \\ &= \begin{bmatrix} \mathbf{0} \\ F_1\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \\ \vdots \\ F_n\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0} \\ \tilde{F}\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \end{bmatrix}. \end{aligned}$$

## III. MAIN RESULTS

In this section, the leader-following multi-UUVs consensus problem with uncertain factor and time-varying delay is investigated, and three consensus control protocols will be proposed, we have the following results:

### A. LEADER-FOLLOWING CONSENSUS OF MULTI-UUVS WITHOUT UNCERTAIN FACTOR AND TIME-VARYING DELAYS

Firstly, we consider the case of $\tau(t) = 0$ and

$$\tilde{F}\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \equiv \mathbf{0}.$$

For the following matrix form

$$\dot{\varepsilon}(t) = ((A+B)\otimes I_5)\,\varepsilon(t), \tag{14}$$

we have:

*Theorem 1:* Let Assumption 1 hold, for the case of $\tau(t) = 0$ and $\tilde{F}\left(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t\right) \equiv \mathbf{0}$, the leader-following consensus of multi-UUVs system with (3) and (4) can be achieved under control protocol (10), the consensus gains $k_1, k_2$ are chosen as

$$\begin{aligned} k_1 &> \max\left\{\alpha^2, \frac{2\alpha^2\lambda_{\max}(W)}{\alpha\beta - \lambda_{\max}\left(M^2\right)}\right\}, \\ k_2 &> \max\left\{\alpha^2\beta, \frac{2\alpha^2\beta\lambda_{\max}(W)}{\alpha\beta - \lambda_{\max}\left(M^2\right)}\right\}, \end{aligned} \tag{15}$$

where $\alpha > 0$, $\beta > 0$, $M = W - \alpha\beta WH - H^T W$.

*Proof:* Firstly, following Lyapunov theorem, for matrix $H$, there must exist a positive definite matrix $W = W^T \in R^{n\times n}$ satisfies $WH + H^T W = I_n$, then we consider the following Lyapunov function candidate:

$$V(t) = \varepsilon^T(t)\,Q\varepsilon(t), \tag{16}$$

where

$$Q = \begin{bmatrix} k_1 W & \alpha W \\ \alpha W & W \end{bmatrix} \otimes I_5 = E \otimes I_5$$

is positive definite and symmetric matrix with $k_1 > \alpha^2$, we also assume that $k_2 = \beta k_1, \beta > 0$, then the derivative of $V(t)$ satisfies

$$\dot{V}(t) = \varepsilon^T(t)\left(Q(A+B) + (A+B)^T Q\right)\varepsilon(t). \tag{17}$$

Let $P = Q(A+B) + (A+B)^T Q$, then

$$P = \left(\begin{bmatrix} k_1 W & \alpha W \\ \alpha W & W \end{bmatrix} \otimes I_5\right)\begin{bmatrix} \mathbf{0} & I_n \\ -k_1 H & -k_2 H \end{bmatrix} + \begin{bmatrix} \mathbf{0} & I_n \\ -k_1 H & -k_2 H \end{bmatrix}^T \left(\begin{bmatrix} k_1 W & \alpha W \\ \alpha W & W \end{bmatrix} \otimes I_5\right),$$

after convenient calculation, we get

$$P = \begin{bmatrix} -k_1\alpha I_n & k_1 M \\ k_1 M^T & 2\alpha W - k_1\beta I_n \end{bmatrix} \otimes I_5, \tag{18}$$

and $M = W - \alpha\beta WH - H^T W$.

Note that $-k_1\alpha I_n < 0$, Let

$$2\alpha W - k_1\beta I_n - k_1 M^T(-1/k_1\alpha) I_n k_1 M < 0,$$

from Lemma 1, we can get that matrix $P$is negative definite if $k_1, k_1$ satisfying (15), then we have $\dot{V}(t) < 0$. The proof is completed.

### B. LEADER-FOLLOWING CONSENSUS OF MULTI-UUVS WITH UNCERTAIN FACTOR

In the case of $\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \neq \mathbf{0}$. consider the time-varying delay $\tau(t) = 0$, for the following matrix form:

$$\dot{\varepsilon}(t) = ((A+B)\otimes I_5)\varepsilon(t) + F(\tilde{x}(t), \tilde{v}(t), t), \tag{19}$$

we have:

*Theorem 2:* Let Assumption 1 and 2 hold, for the case of $\tau(t) = 0$, the leader-following consensus of multi-UUVs system with (3) and (4) can be achieved under control protocol (10), the consensus gains $k_1, k_2$ are chosen as

$$k_1 > \max\{k_{11}, k_{12}\}, \quad k_2 > \max\{k_{21}, k_{22}\} \tag{20}$$

where

$$\begin{aligned} k_{11} &= \frac{(3\alpha+1)\rho_0\lambda_{\max}(W)}{\alpha}, \\ k_{21} &= \frac{(3\alpha+1)\beta\rho_0\lambda_{\max}(W)}{\alpha} \\ k_{12} &= \frac{2\alpha^2\lambda_{\max}(W) + \frac{\alpha(\alpha+3)}{2}\rho_0\lambda_{\max}(W)}{\alpha\beta - 2\lambda_{\max}(M^2)}, \\ k_{22} &= \frac{2\alpha^2\beta\lambda_{\max}(W) + \frac{\alpha(\alpha+3)}{2}\beta\rho_0\lambda_{\max}(W)}{\alpha\beta - 2\lambda_{\max}(M^2)}, \\ &\alpha > 0, \quad \beta > 0, \quad \rho_0 > 0, \\ M &= W - \alpha\beta WH - H^T W. \end{aligned} \tag{21}$$

*Proof:* Take the positive definite function in Theorem 1 as the Lyapunov function candidate:

$$V(t) = \varepsilon^T(t) Q\varepsilon(t). \tag{22}$$

Then the derivative of $V(t)$ along (19) gives

$$\begin{aligned} \dot{V}(t) &= 2\varepsilon^T(t)(E\otimes I_5)(((A+B)\otimes I_5)\varepsilon(t) \\ &\quad + F(\tilde{x}(t), \tilde{v}(t), t)) \\ &= \varepsilon^T(t) P\varepsilon(t) + \bar{x}^T(t)(\alpha W\otimes I_5) \\ &\quad \cdot (f(\tilde{x}(t), \tilde{v}(t), t) - \mathbf{1}_n \otimes f(\tilde{x}_0(t), \tilde{v}_0(t), t)) \\ &\quad + \bar{v}^T(t)(W\otimes I_5)(f(\tilde{x}(t), \tilde{v}(t), t) \\ &\quad - \mathbf{1}_n \otimes f(\tilde{x}_0(t), \tilde{v}_0(t), t)). \end{aligned} \tag{23}$$

By Assumption 2 and Lemma 2, we have

$$\begin{aligned} &\bar{x}^T(t)(\alpha W\otimes I_5)(f(\tilde{x}(t), \tilde{v}(t), t) \\ &\quad -\mathbf{1}_n \otimes f(\tilde{x}_0(t), \tilde{v}_0(t), t)) \\ &\le \alpha\lambda_{\max}(W)\sum_{i=1}^n \bar{x}_i^T(t)(f(\tilde{x}_i(t), \tilde{v}_i(t), t) \\ &\quad - f(\tilde{x}_0(t), \tilde{v}_0(t), t)) \\ &\le \alpha\lambda_{\max}(W)\sum_{i=1}^n \rho_0\|\bar{x}_i(t)\|(\|\bar{x}_i(t)\| + \|\bar{v}_i(t)\|) \\ &\le \alpha\lambda_{\max}(W)\sum_{i=1}^n \rho_0\left(\frac{3}{2}\|\bar{x}_i(t)\|^2 + \frac{1}{2}\|\bar{v}_i(t)\|^2\right) \\ &= \frac{3}{2}\rho_0\alpha\lambda_{\max}(W)\bar{x}^T(t)\bar{x}(t) \\ &\quad + \frac{1}{2}\rho_0\alpha\lambda_{\max}(W)\bar{v}^T(t)\bar{v}(t) \end{aligned} \tag{24}$$

and similarly, we have

$$\begin{aligned} &\bar{v}^T(t)(W\otimes I_5)(f(\tilde{x}(t), \tilde{v}(t), t) \\ &\quad - \mathbf{1}_n \otimes f(\tilde{x}_0(t), \tilde{v}_0(t), t)) \\ &\le \frac{1}{2}\rho_0\lambda_{\max}(W)\bar{x}^T(t)\bar{x}(t) \\ &\quad + \frac{3}{2}\rho_0\lambda_{\max}(W)\bar{v}^T(t)\bar{v}(t). \end{aligned} \tag{25}$$

Combining (23)-(25), we have

$$\dot{V}(t) \le \varepsilon^T(t)\tilde{P}\varepsilon(t), \tag{26}$$

and

$$\tilde{P} = \begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{bmatrix} \otimes I_5,$$

where

$$\begin{aligned} \tilde{P}_{11} &= -k_1\alpha I_n + \frac{(3\alpha+1)}{2}\rho_0\lambda_{\max}(W) I_n, \\ \tilde{P}_{12} &= k_1 M, \\ \tilde{P}_{21} &= k_1 M^T, \\ \tilde{P}_{22} &= 2\alpha W - k_1\beta I_n + \frac{(\alpha+3)}{2}\rho_0\lambda_{\max}(W) I_n. \end{aligned}$$

Similar to the proof of Theorem 1, the matrix $M$ satisfies (21). Let

$$-k_1\alpha I_n + \frac{(3\alpha+1)}{2}\rho_0\lambda_{\max}(W) I_n < -\frac{1}{2}k_1\alpha I_n < 0,$$

From Lemma 1, we obtain

$$2\alpha W - k_1\beta I_n + \frac{(\alpha+3)}{2}\rho_0\lambda_{\max}(W) I_n - k_1 M\left(1\Big/ -k_1\alpha + \frac{(3\alpha+1)}{2}\rho_0\lambda_{\max}(W)\right) k_1 M^T < 0.$$

For convenient calculation, we can get that matrix $\tilde{P}$ is negative definite if $k_1, k_2$ satisfying (20), then we have $\dot{V}(t) < 0$. The proof is completed.

### C. LEADER-FOLLOWING CONSENSUS OF MULTI-UUVS WITH UNCERTAIN FACTORS AND TIME-VARYING DELAYS

In the case of $\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \neq \mathbf{0}$. Consider the time-varying delay $\tau(t) \neq 0$, and $\tau(t)$ is continuously differentiable and satisfies Assumption 2, for the matrix form (13), we have:

*Theorem 3:* Suppose that Assumption 1,2 and 3 hold, the leader-following consensus of multi-UUVs system with (3) and (4) can be achieved under control protocol (10) if there are positive definite matrix $Q, \psi, Z \in R^{2n\times 2n}$, $\Phi \in R^{n\times n}$, and any matrices $X, Y \in R^{2n\times 2n}$ such that

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ \Xi_{31} & \Xi_{32} & \Xi_{33} \end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0, \tag{28}$$

where

$$\begin{aligned}
\Xi_{11} &= QA + A^T Q + hX + Y + Y^T + hA^T ZA + \psi + Q_1,\\
\Xi_{12} &= QB - Y + hA^T ZB, \quad \Xi_{13} = hA^T C_2,\\
\Xi_{21} &= B^T Q - Y^T + hB^T ZA,\\
\Xi_{22} &= -(1-\kappa)\psi + hB^T ZB,\\
\Xi_{23} &= hB^T C_2, \quad \Xi_{31} = hC_2^T A, \quad \Xi_{32} = hC_2^T B,\\
\Xi_{33} &= hZ_4 - (1-\kappa)\Phi,\\
Z &= \begin{bmatrix} Z_1 & Z_2 \\ Z_2 & Z_4 \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \end{bmatrix},\\
Z_i &\in R^{n\times n}, \quad i = 1,2,3,4, \quad C_j \in R^{2n\times n}, j = 1,2,\\
Q_1 &= \begin{bmatrix} 2\lambda_{\max}(G^2)\Phi & \mathbf{0} \\ \mathbf{0} & 2\lambda_{\max}(J^2)\Phi \end{bmatrix}.
\end{aligned}$$

*Proof:* Define Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{29}$$

with

$$V_1(t) = \varepsilon^T(t)(Q\otimes I_5)\varepsilon(t), \tag{30}$$

$$V_2(t) = \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{\varepsilon}^T(s)(Z\otimes I_5)\dot{\varepsilon}(s)ds d\theta, \tag{31}$$

$$V_3(t) = \int_{t-\tau(t)}^{t} \varepsilon^T(s)(\psi\otimes I_5)\varepsilon(s)ds, \tag{32}$$

$$\begin{aligned} V_4(t) = \int_{t-\tau(t)}^{t} &\tilde{F}^T(\tilde{x}(\theta), \tilde{v}(\theta), \theta)(\Phi\otimes I_5)\\ &\cdot\tilde{F}(\tilde{x}(\theta), \tilde{v}(\theta), \theta)d\theta. \end{aligned} \tag{33}$$

According to the Leibniz-Newton formula, it holds that

$$\begin{aligned}
&\varepsilon(t) - \varepsilon(t-\tau(t))\\
&\quad= \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)ds\\
&\quad= \int_{t-\tau(t)}^{t} (\varepsilon(s)(A\otimes I_5) + (B\otimes I_5)\varepsilon(s-\tau(s))\\
&\qquad + F(\tilde{x}(s-\tau(s)), \tilde{v}(s-\tau(s)), s))ds.
\end{aligned} \tag{34}$$

According to condition (9) in Lemma 3, the time derivative of $V_1(t)$ along system (13) is given by

$$\begin{aligned}
\dot{V}_1(t) \leq\ & \varepsilon^T(t)\left(\left(QA + A^T Q + Y + Y^T\right)\otimes I_5\right)\varepsilon(t)\\
& + h\varepsilon^T(t)(X\otimes I_5)\varepsilon(t)\\
& + 2\varepsilon^T(t)((QB - Y)\otimes I_5)\varepsilon(t-\tau(t))\\
& + \int_{t-\tau(t)}^{t} \dot{\varepsilon}^T(s)(Z\otimes I_5)\dot{\varepsilon}(s)ds.
\end{aligned} \tag{35}$$

$\dot{V}_2(t)$, $\dot{V}_3(t)$ and $\dot{V}_4(t)$ are given by

$$\begin{aligned}
&\dot{V}_2(t)\\
&\leq h((A\otimes I_5)\varepsilon(t) + (B\otimes I_5)\varepsilon(t-\tau(t))\\
&\quad + F(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t))^T\cdot(Z\otimes I_5)\\
&\quad \cdot((A\otimes I_5)\varepsilon(t) + (B\otimes I_5)\varepsilon(t-\tau(t))\\
&\quad + F(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t))\\
&\quad - \int_{t-\tau(t)}^{t} \dot{\varepsilon}^T(s)(Z\otimes I_5)\dot{\varepsilon}(s)ds\\
&= h\varepsilon^T(t)\left(A^T ZA\otimes I_5\right)\varepsilon(t)\\
&\quad + h\varepsilon^T(t-\tau(t))\left(B^T ZB\otimes I_5\right)\varepsilon(t-\tau(t))\\
&\quad + h\tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)(Z_4\otimes I_5)\\
&\quad \cdot\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)\\
&\quad + h\tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)\left(C_2^T A\otimes I_5\right)\varepsilon(t)\\
&\quad + h\varepsilon^T(t)\left(A^T C_2\otimes I_5\right)\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)\\
&\quad + h\tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)\left(C_2^T B\otimes I_5\right)\\
&\quad \times\varepsilon(t-\tau(t))\\
&\quad + h\varepsilon^T(t-\tau(t))\left(B^T C_2\otimes I_5\right)\tilde{F}(\tilde{x}(t-\tau(t)),\\
&\quad \tilde{v}(t-\tau(t)), t)\\
&\quad + h\varepsilon^T(t)\left(A^T ZB\otimes I_5\right)\varepsilon(t-\tau(t))\\
&\quad + h\varepsilon^T(t-\tau(t))\left(B^T ZA\otimes I_5\right)\varepsilon(t)\\
&\quad - \int_{t-\tau(t)}^{t} \dot{\varepsilon}^T(s)(Z\otimes I_5)\dot{\varepsilon}(s)ds
\end{aligned} \tag{36}$$

$$\begin{aligned}
&\dot{V}_3(t)\\
&= \int_{t-\tau(t)}^{t} \varepsilon^T(s)(\psi\otimes I_5)\varepsilon(s)ds\\
&\leq \varepsilon^T(t)(\psi\otimes I_5)\varepsilon(t)\\
&\quad - (1-\kappa)\varepsilon^T(t-\tau(t))(\psi\otimes I_5)\varepsilon(t-\tau(t)),
\end{aligned} \tag{37}$$

By Assumption 2 and Lemma 2, we have

$$\begin{aligned}\dot{V}_4(t) &= \tilde{F}^T(\tilde{x}(t), \tilde{v}(t), t)(\Phi \otimes I_5)\tilde{F}(\tilde{x}(t), \tilde{v}(t), t) \\ &\quad - (1-\dot{\tau}(t))\tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \\ &\quad \cdot (\Phi \otimes I_5)\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \\ &\leq 2\lambda_{\max}\left(G^2\right)\bar{x}^T(t)(\Phi \otimes I_5)\bar{x}(t) \\ &\quad + 2\lambda_{\max}\left(J^2\right)\bar{v}^T(t)(\Phi \otimes I_5)\bar{v}(t) \\ &\quad + (\kappa - 1)\tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \\ &\quad \cdot (\Phi \otimes I_5)\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t). \quad (38)\end{aligned}$$

Synthesizing (35)-(38), we have

$$\dot{V}(t) \leq \varsigma^T(t)(\Xi \otimes I_5)\varsigma(t) < 0, \quad (39)$$

where $\Xi$ is defined in (27) and

$$\varsigma(t) = \left(\varepsilon^T(t), \varepsilon^T(t-\tau(t)), \tilde{F}^T(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t)\right)^T.$$

thus, the proof of the Theorem is completed.

*Remark 3:* Motivated by the control protocol (10) with additional nonlinear function and the control protocol proposed in literature [28], the Lyapunov function (29) is proposed, and the part four of the Lyapunov function, that is, equation (33), refers to the literature [28]. The difference is that we take into account the fact that both the control protocol and additional nonlinear function exist time-varying delays simultaneously, and this design makes more sense in practice.

## IV. SIMULATION RESULTS

In this section, two numerical simulations are given to verify the effectiveness of the above results.

*Example 1:* Consider the multi-UUVs system composed of five followers labeled by UUV1-UUV5 and one leader labeled by UUVL with additional nonlinear function $\tilde{F}(\tilde{x}(t-\tau(t)), \tilde{v}(t-\tau(t)), t) \equiv \mathbf{0}$ and time-varying delay $\tau(t) = 0$. The desired path of the leader UUV is expressed as follows:

$$\begin{cases} \tilde{x}_0 = 60\cos(0.0022\pi t) + 25\sin(0.0011\pi t), \\ \tilde{y}_0 = 60\sin(0.0022\pi t) + 25\cos(0.0011\pi t), \\ \tilde{z}_0 = -0.12t. \end{cases}$$

The initial values of position and velocity $u(0)$ of each UUV is randomly distributed in the three-dimensional space and $[-2, 2]$, respectively. The initial value of pitch angles are set in the interval $[-\pi/18, \pi/18]$, and the heading angles are in the interval $[0, 2\pi]$.The initial states of follower UUV1-UUV5 and UUV L are shown in Table 1, and the communication topology is shown in Fig.1. For convenience, we use U1-U5 and UL to denote follower UUVs and leader UUV in the following table.

**TABLE 1.** Initial states of multiple UUVs.

| | UL | U1 | U2 | U3 | U4 | U5 | (unit) |
|---|---|---|---|---|---|---|---|
| $x(0)$ | 50 | 25.2 | 54.5 | 24.2 | 43.5 | 19 | $x/m$ |
| $y(0)$ | 0 | 48 | 35 | 50 | 27.8 | 22.5 | $y/m$ |
| $z(0)$ | 0 | -6.8 | -6.5 | -7.3 | -5.5 | -7 | $z/m$ |
| $\theta(0)$ | -0.17 | 0.13 | 0.1 | 0 | 0.02 | 0.12 | $\theta/rad$ |
| $\psi(0)$ | 1.6 | 0.8 | 1.8 | 1.6 | 2.7 | 0.35 | $\psi/rad$ |
| $u(0)$ | 0.8 | 0.1 | 0.1 | -0.3 | 0.25 | 0.5 | $u/(m\cdot s^{-1})$ |
| $v(0)$ | 0 | 0 | 0 | 0 | 0 | 0 | $v/(m\cdot s^{-1})$ |
| $\omega(0)$ | 0 | 0 | 0 | 0 | 0 | 0 | $\omega/(m\cdot s^{-1})$ |
| $q(0)$ | 0 | 0 | 0 | 0 | 0 | 0 | $q/(rad\cdot s^{-1})$ |
| $r(0)$ | 0 | 0 | 0 | 0 | 0 | 0 | $r/(rad\cdot s^{-1})$ |

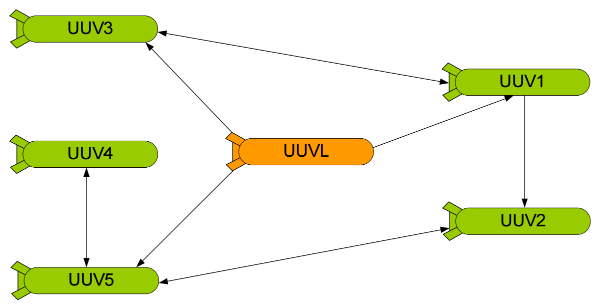



**FIGURE 1.** Communication topology.

The Laplacian matrix $L$ and $D$ are given as follows:

$$L = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & -1 & 2 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Clearly, Assumptions 1 is satisfied, and by the equation $WH + H^TW = I_5$, we can obtain that

$$W = \begin{bmatrix} 0.4052 & 0.1128 & 0.1976 & 0.0972 & 0.0818 \\ 0.1128 & 0.3333 & 0.0363 & 0.1667 & 0.1667 \\ 0.1976 & 0.0363 & 0.3488 & 0.0431 & 0.0322 \\ 0.0972 & 0.1667 & 0.0431 & 0.8333 & 0.3333 \\ 0.0818 & 0.1667 & 0.0322 & 0.3333 & 0.3333 \end{bmatrix},$$

with $\lambda_1(W) = 1.1167$, $\lambda_2(W) = 0.5506$, $\lambda_3(W) = 0.1650$, $\lambda_4(W) = 0.1278$, $\lambda_5(W) = 0.2940$. We choose $\alpha = 0.05$, $\beta = 6.4$, by some simple calculations, we can get:

$$\lambda_1(M^2) = 0.0168, \quad \lambda_2(M^2) = 0.2766, \quad \lambda_3(M^2) = 0.2372, \quad \lambda_4(M^2) = 0.2038, \quad \lambda_5(M^2) = 0.1258.$$

According to Theorem 1, the control protocol (10) is designed and the related control parameters are shown in Table 2:

**TABLE 2.** **The designed parameters in example 1.**

| $\alpha$ | $\beta$ | $k_1$ | $k_2$ | $l$ |
|---|---|---|---|---|
| 0.05 | 6.4 | 0.13 | 0.832 | 20 |

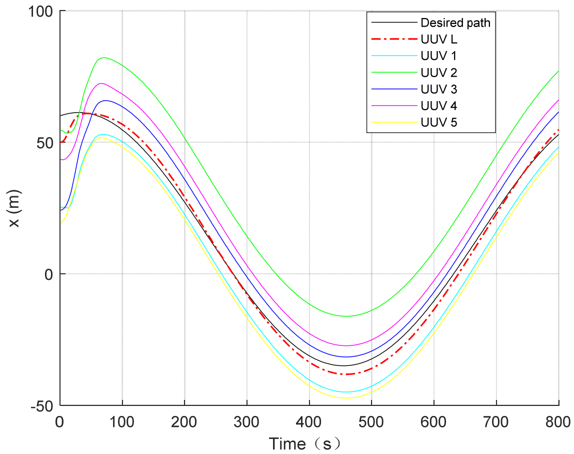



**FIGURE 2.** **Position state $x$ of multiple UUVs.**

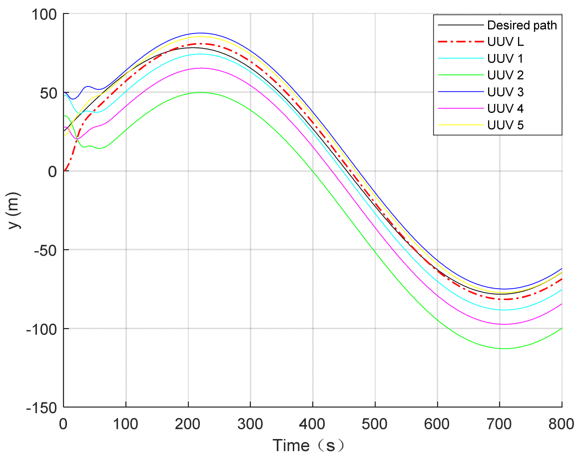



**FIGURE 3.** **Position state $y$ of multiple UUVs.**

Based on theorem 1 and the chosen parameters, the leader-following consensus of the multi-UUVs system with (3) and (4) can be achieved under control protocol (10). The simulation results are shown in Figs.2-12. The position and attitude states of leader UUV and follower UUVs are shown in Figs.2-6, and the velocity states shown in Figs.7-11. It is obvious that the follower UUVs could track the position states of the leader and converge to the leader UUV's desired path, and the velocity states of follower UUVs also finally converge to the leader's velocity states with high convergence accuracy. Fig.12 shows the three-dimensional trajectory of

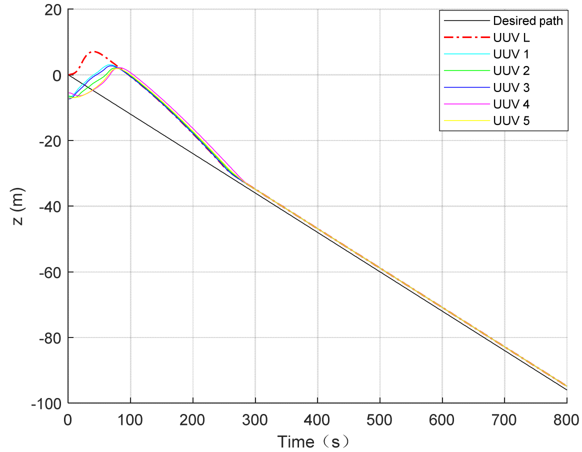



**FIGURE 4.** **Position state $z$ of multiple UUVs.**

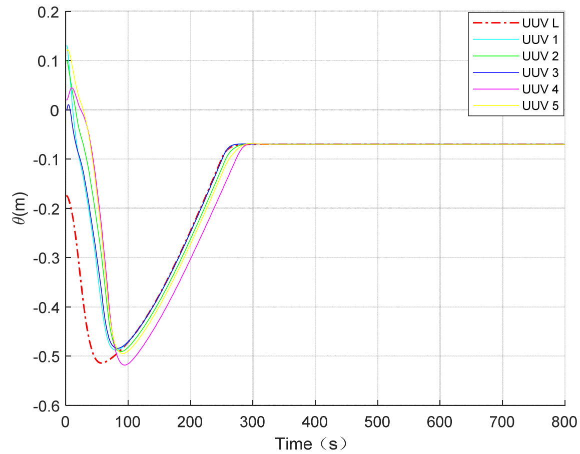



**FIGURE 5.** **Position state $\theta$ of multiple UUVs.**

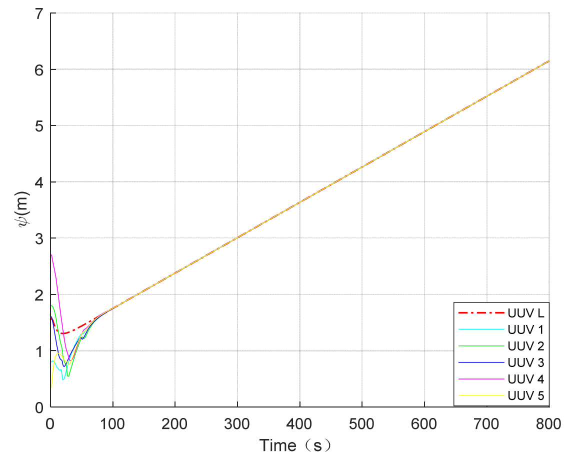



**FIGURE 6.** **Position state $\psi$ of multiple UUVs.**

leader and follower UUVs. It can be seen that the formation control of the multi-UUVs system can be achieved stably with any formation structure. The simulation results verify the Theorem 1 comprehensively.

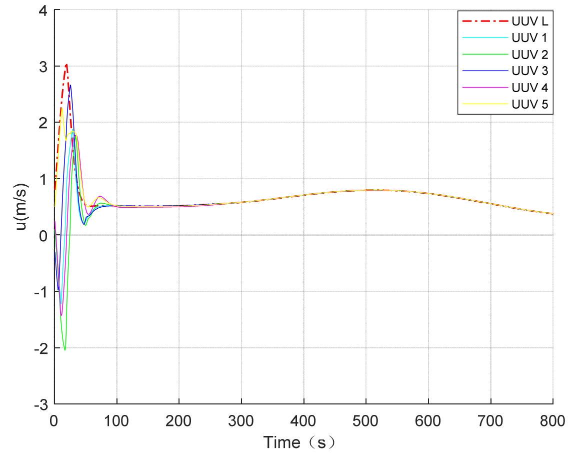



**FIGURE 7. Velocity state *u* of multiple UUVs.**

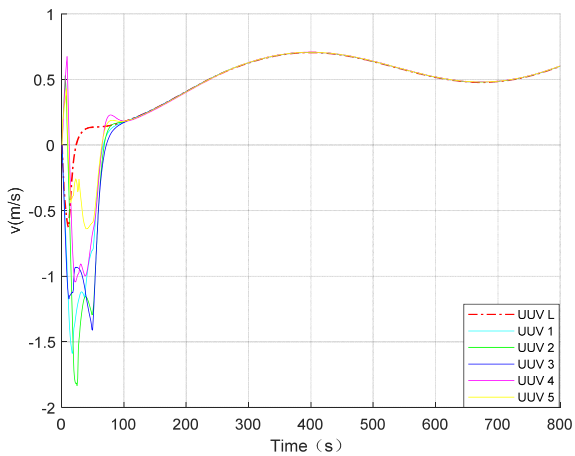



**FIGURE 8. Velocity state *v* of multiple UUVs.**

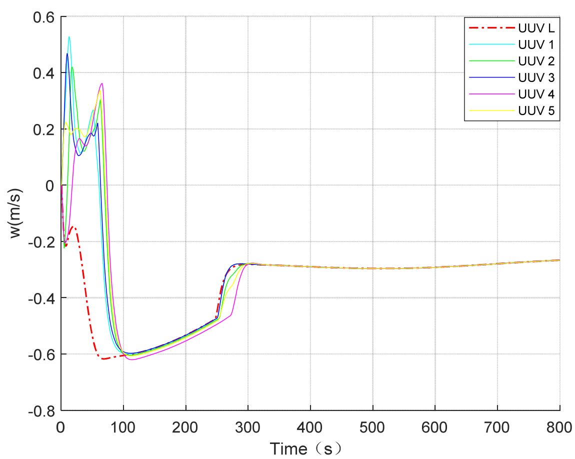



**FIGURE 9. Velocity state *w* of multiple UUVs.**

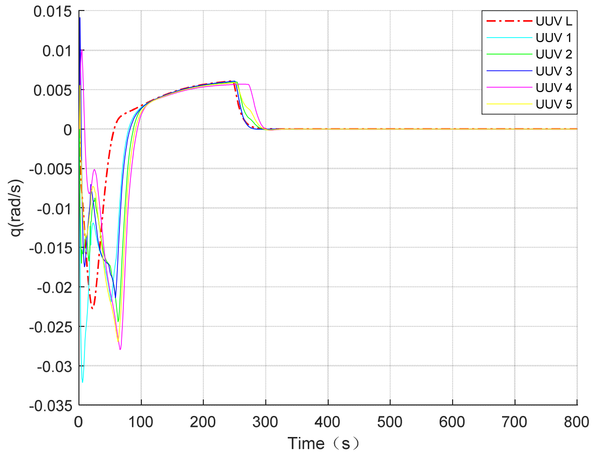



**FIGURE 10. Velocity state *q* of multiple UUVs.**

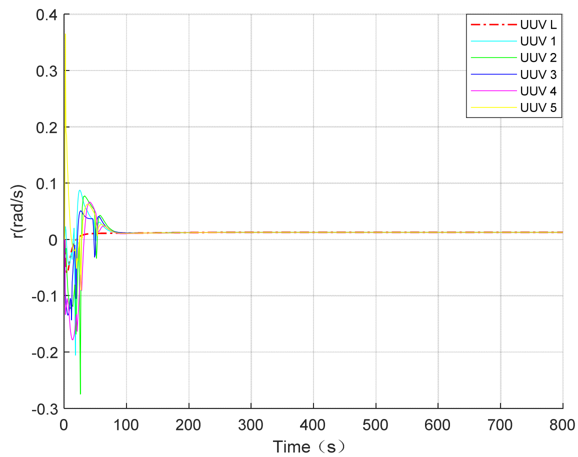



**FIGURE 11. Velocity state *r* of multiple UUVs.**

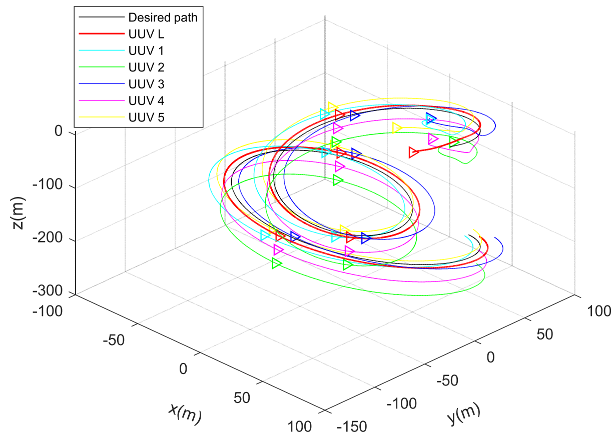



**FIGURE 12. 3D trajectory of leader and follower UUVs.**

*Example 2:* In this example, we illustrate the leader-following consensus of the multi-UUVs system with (3) and (4) can be achieved under control protocol (10) with uncertain factors and time-varying delays. As mentioned previously, consider the multi-UUVs system composed of five followers and one leader with additional nonlinear function:

$$\tilde{F}\left(\tilde{x}\left(t-\tau\left(t\right)\right), \quad \tilde{v}\left(t-\tau\left(t\right)\right), t\right) \neq \mathbf{0},$$

and the time-varying delay $\tau\left(t\right) \neq 0$.

**TABLE 3.** The designed parameters in example 2.

| $h$ | $\kappa$ | $k_1$ | $k_2$ | $l$ |
|---|---|---|---|---|
| 0.2 | 0.1 | 0.13 | 0.832 | 20 |

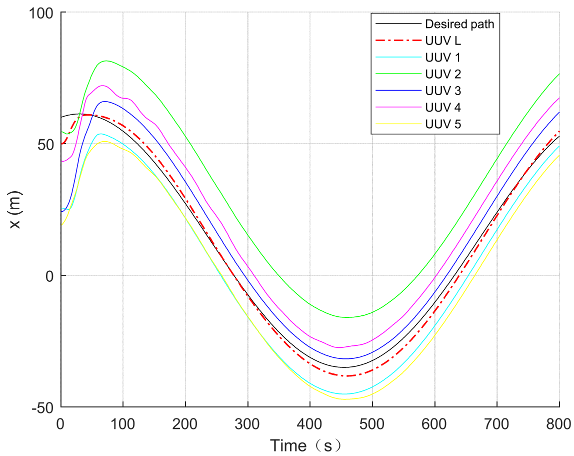



**FIGURE 13.** Position state $x$ of multiple UUVs with uncertain factors.

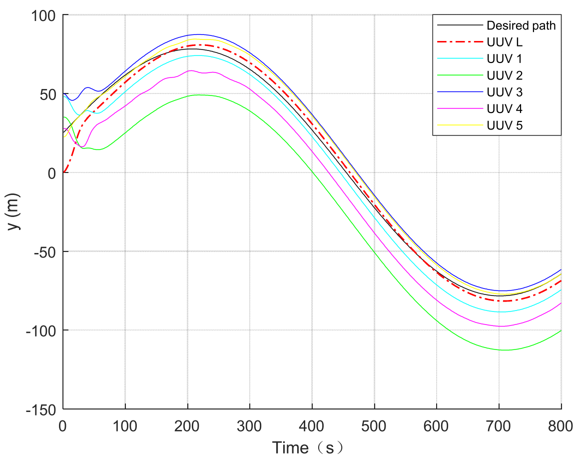



**FIGURE 14.** Position state $y$ of multiple UUVs with uncertain factors.

Let

$$\tilde{F}\left(\tilde{x}\left(t-\tau\left(t\right)\right),\tilde{v}\left(t-\tau\left(t\right)\right),t\right) = \begin{bmatrix} 0.4\tanh\left(\tilde{v}_1\left(t-\tau\left(t\right)\right)\right) \\ 0.01\sin\left(\tilde{x}_2\left(t-\tau\left(t\right)\right)\right)-0.45\tanh\left(\tilde{v}_2\left(t-\tau\left(t\right)\right)\right) \\ -0.01\cos\left(\tilde{x}_3\left(t-\tau\left(t\right)\right)\right) \\ 0.48\tanh\left(\tilde{v}_4\left(t-\tau\left(t\right)\right)\right) \\ 0.02\sin\left(\tilde{x}_5\left(t-\tau\left(t\right)\right)\right)-0.5\tanh\left(\tilde{v}_5\left(t-\tau\left(t\right)\right)\right) \end{bmatrix}$$

Clearly, the nonlinear function $\tilde{F}$ satisfies Assumption 2, and for this case, we have

$$\lambda_{\max}\left(G^2\right)=0.0004,\quad \lambda_{\max}\left(J^2\right)=0.25.$$

We consider the case of$\tau\left(t\right) = 0.1+0.05\cos(t)$ and the initial value of position and velocity, the communication topology, the Laplacian matrix $L$and matrix $D$, the desired path and the control parameters are same as the first case.

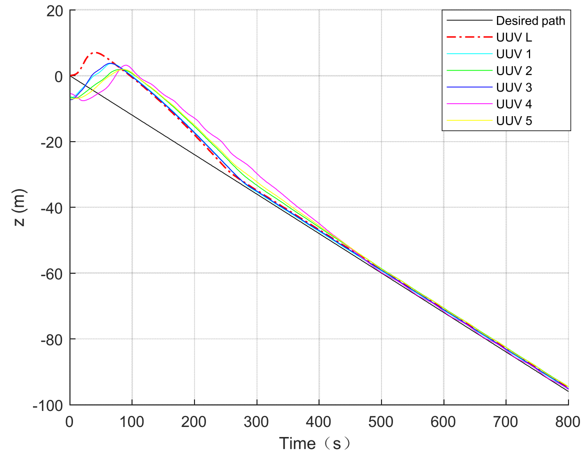



**FIGURE 15.** Position state $z$ of multiple UUVs with uncertain factors.

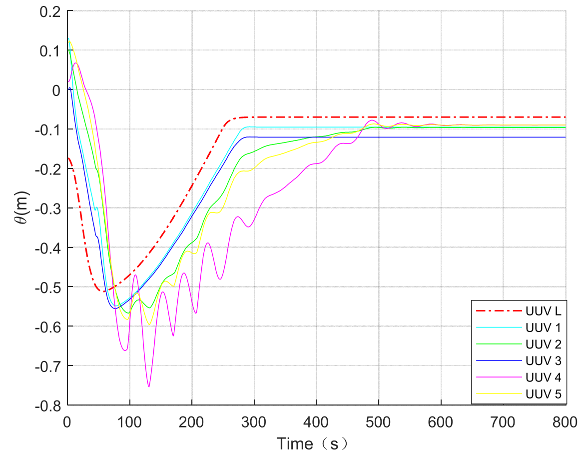



**FIGURE 16.** Position state $\theta$ of multiple UUVs with uncertain factors.

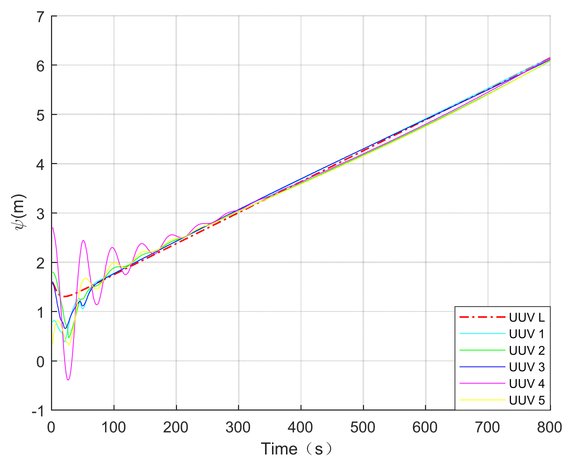



**FIGURE 17.** Position state $\psi$ of multiple UUVs with uncertain factors.

According to Theorem 3, the control protocol (10) is designed and the related parameters are shown in Table 3:

We choose the matrices $X=0.3I_{10\times10}$, $Y=\mathbf{0}_{10\times10}$, $Z=I_{10\times10}$, $\psi=0.1I_{10\times10}$, $\Phi=0.6I_{5\times5}$, it's easy to see that (28) hold, and $\psi$, $\Phi$ and $Z$are positive definite matrices.

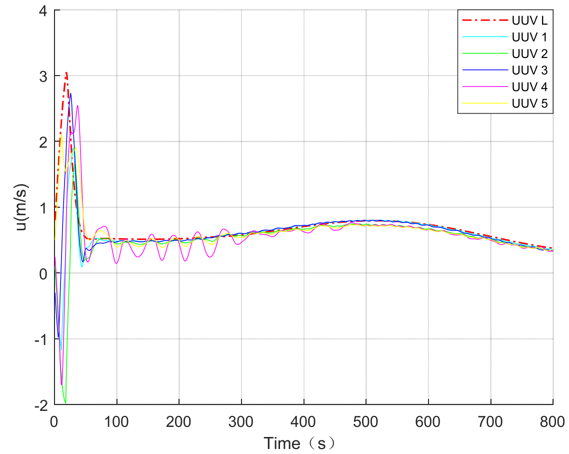



**FIGURE 18.** Velocity state *u* of multiple UUVs with uncertain factors.

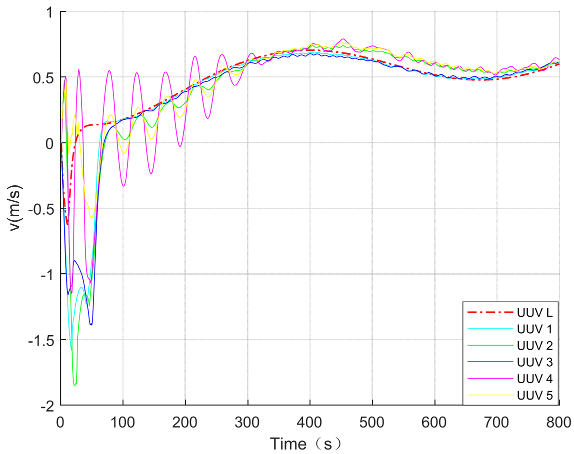



**FIGURE 19.** Velocity state *v* of multiple UUVs with uncertain factors.

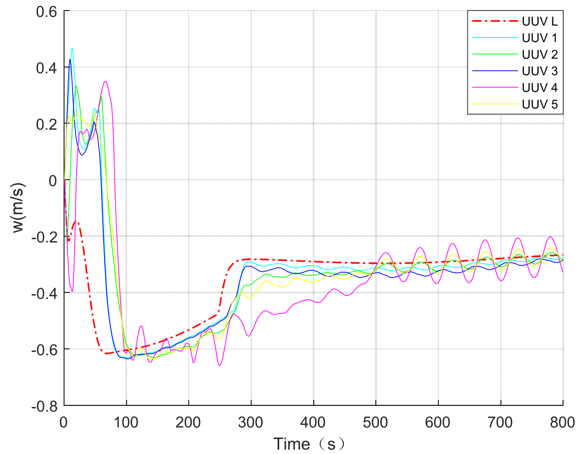



**FIGURE 20.** Velocity state *w* of multiple UUVs with uncertain factors.

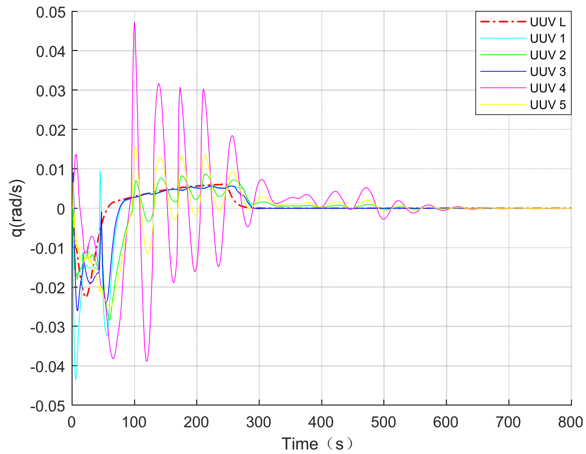



**FIGURE 21.** Velocity state *q* of multiple UUVs with uncertain factors.

Applying Theorem 3, the corresponding positive definite matrix $Q$ is solved as (40), as shown at the bottom of this page. The simulation images are demonstrated in Figs.13-23. The position and velocity states of leader UUV and follower UUVs are shown in Figs. 13-17 and Figs.18-22, respectively. From which we can see that the position and velocity states curves are shown to be more oscillatory than the first case, and all UUV's position and velocity states can also finally converge to the leader UUV's fixed states after making a large adjustment although the nonlinearity factor and time delay are existed. Fig.23 shows the three-dimensional trajectory of leader and follower UUVs. It can be seen that the formation control of the multi-UUVs system can also be achieved. The simulation results verify Theorem 3 comprehensively.

$$Q = \begin{bmatrix} 0.7417 & 0.1009 & 0.2145 & 0.0243 & 0.0431 & -0.6227 & -0.2095 & -0.4206 & -0.0926 & -0.1099 \\ 0.1009 & 0.6674 & 0.0061 & 0.0613 & 0.1587 & -0.2095 & -0.4653 & -0.0355 & -0.1917 & -0.3188 \\ 0.2145 & 0.0061 & 0.7135 & 0.0054 & 0.0064 & -0.4206 & -0.0355 & -0.5526 & -0.0312 & -0.0302 \\ 0.0243 & 0.0613 & 0.0054 & 0.9972 & 0.2334 & -0.0926 & -0.1917 & -0.0312 & -1.2544 & -0.5355 \\ 0.0431 & 0.1587 & 0.0064 & 0.2334 & 0.5884 & -0.1099 & -0.3188 & -0.0302 & -0.5355 & -0.3689 \\ -0.6227 & -0.2095 & -0.4206 & -0.0926 & -0.1099 & 0.8190 & 0.3632 & 0.6272 & 0.3378 & 0.2814 \\ -0.2095 & -0.4653 & -0.0355 & -0.1917 & -0.3188 & 0.3632 & 0.5615 & 0.1484 & 0.5986 & 0.5061 \\ -0.4206 & -0.0355 & -0.5526 & -0.0312 & -0.0302 & 0.6272 & 0.1484 & 0.6540 & 0.1446 & 0.1168 \\ -0.0926 & -0.1917 & -0.0312 & -1.2544 & -0.5355 & 0.3378 & 0.5986 & 0.1446 & 2.3534 & 1.1360 \\ -0.1099 & -0.3188 & -0.0302 & -0.5355 & -0.3689 & 0.2814 & 0.5061 & 0.1168 & 1.1360 & 0.6723 \end{bmatrix} \tag{40}$$

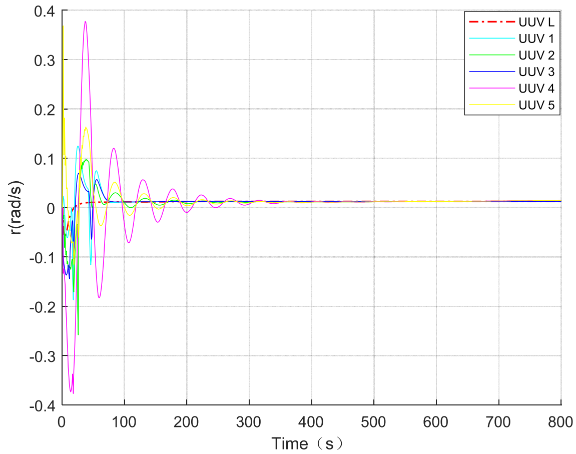



**FIGURE 22.** Velocity state *r* of multiple UUVs with uncertain factors.

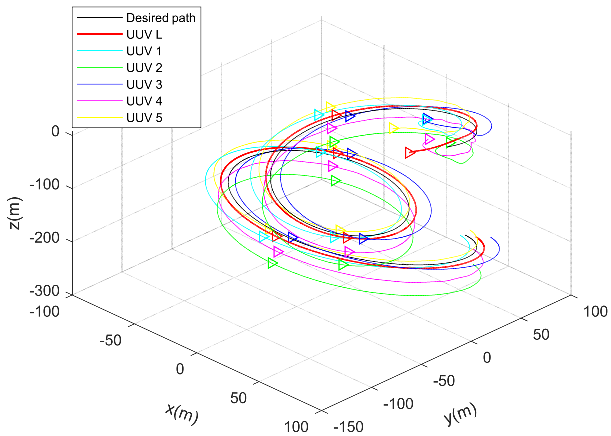



**FIGURE 23.** 3D trajectory of leader and follower UUVs.

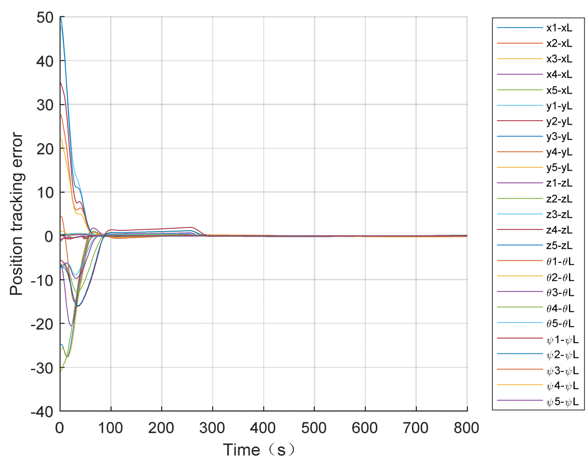



**FIGURE 24.** Position tracking error of the leader and follower UUVs in example 1.

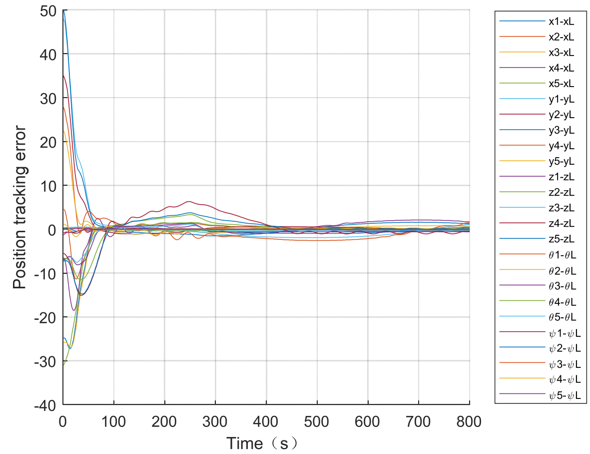



**FIGURE 25.** Position tracking error of the leader and follower UUVs in example 2.

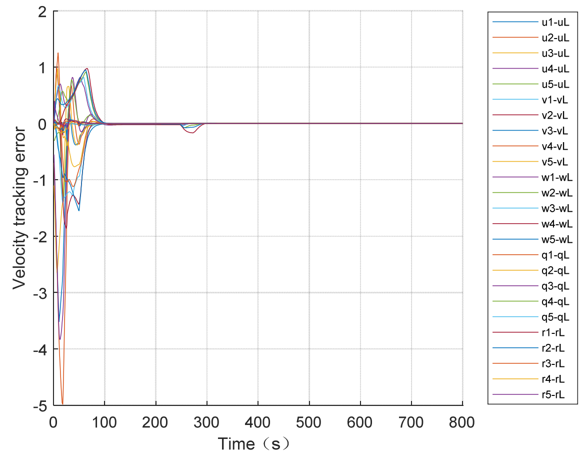



**FIGURE 26.** Velocity tracking error of the leader and follower UUVs in example 1.

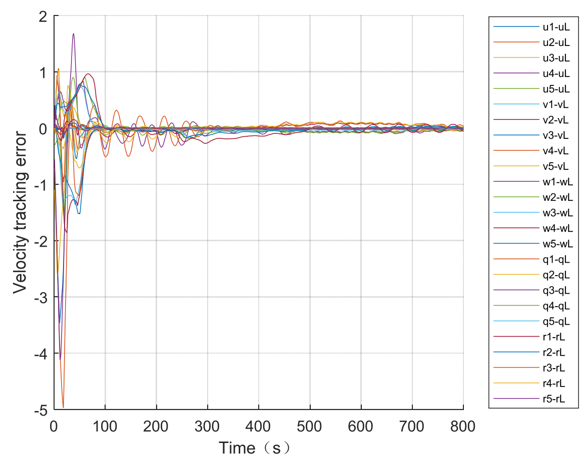



**FIGURE 27.** Velocity tracking error of the leader and follower UUVs in example 2.

The special case of $\tau(t) = 0$ in Theorem 3 is the result of Theorem 2 to be verified. So the verification process of Theorem 2 is omitted to prevent repetition.

In order to compare the two convergences above, the preset distance between each UUV is ignored. The position and velocity tracking errors of the leader and follower UUVs in the two examples are given in Figs.24-27. It can be seen that the position and velocity states in example 1 converge to the desired formation value at about 100s with smooth curves, and the consensus tracking is achieved in example 2 at about 400s with oscillation curves, which is obviously later than the first case. The reason for this imagination is the change of the additional nonlinear function with time-varying delay. It can be observed that the leader-following formation consensus problems of multi-UUVs system under control protocol (10) are indeed achieved and the simulations verify the results well.

## V. CONCLUSION

Formation control problems of leader-following multi-UUVs system were studied in this paper. We studied the consensus problem with no time delays firstly. Then consider the environmental disturbances in UUV's movement, it was proved that the leader-following consensus of the multi-UUVs system could still be achieved with or without time-varying delays when uncertain factors were concerned. Sufficient conditions are derived based on the Lyapunov-Krasovskii functional theory and matrix theory. Finally, numerical examples have been given to demonstrate the validity of the theoretical results. In the future work, the consensus algorithm of formation control for multi-UUVs with switching topologies and packet loss will be discussed.

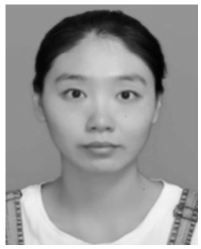
## REFERENCES

[1] W. Zhang, Y. Teng, S. Wei, H. Xiong, and H. Ren, "The robust H-infinity control of UUV with Riccati equation solution interpolation," *Ocean Eng.*, vol. 156, pp. 252–262, May 2018.
[2] C. D. Makavita, S. G. Jayasinghe, H. D. Nguyen, and D. Ranmuthugala, "Experimental study of command governor adaptive control for unmanned underwater vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 27, no. 1, pp. 332–345, Jan. 2019.
[3] S. Liu, Y. Liu, and N. Wang, "Robust adaptive self-organizing neuro-fuzzy tracking control of UUV with system uncertainties and unknown dead-zone nonlinearity," *Nonlinear Dyn.*, vol. 89, no. 2, pp. 1397–1414, Apr. 2017.
[4] Z.-P. Yan, Y.-B. Liu, J.-J. Zhou, W. Zhang, and L. Wang, "Consensus of multiple autonomous underwater vehicles with double independent Markovian switching topologies and timevarying delays," *Chin. Phys. B*, vol. 26, no. 4, Mar. 2017, Art. no. 40203.
[5] Z. Gao and G. Guo, "Fixed-time leader-follower formation control of autonomous underwater vehicles with event-triggered intermittent communications," *IEEE Access*, vol. 6, pp. 27902–27911, 2018.
[6] A. Khanzadeh and M. Pourgholi, "Fixed-time leader–follower consensus tracking of second-order multi-agent systems with bounded input uncertainties using non-singular terminal sliding mode technique," *IET Control Theory Appl.*, vol. 12, no. 5, pp. 679–686, Mar. 2018.
[7] F. King and G.-P. Liu, "Data-driven leader-follower output synchronization for networked non-Linear multi-agent systems with switching topology and time-varying delays," *J. Syst. Sci. Complex.*, vol. 31, no. 1, pp. 87–102, Feb. 2018.
[8] Y. Wang, M. Shan, Y. Yue, and D. Wang, "Vision-based flexible leader-follower formation tracking of multiple nonholonomic mobile robots in unknown obstacle environments," *IEEE Trans. Control Syst. Technol.*, to be published. doi: 10.1109/TCST.2019.2892031.
[9] X. Tan, J. Cao, and X. Li, "Consensus of leader-following multiagent systems: A distributed event-triggered impulsive control strategy," *IEEE Trans. Cybern.*, vol. 49, no. 3, pp. 792–801, Mar. 2019.
[10] M. H. Trinh, S. Zhao, Z. Sun, D. Zelazo, B. D. O. Anderson, and H.-S. Ahn, "Bearing-based formation control of a group of agents with leader-first follower structure," *IEEE Trans. Autom. Control*, vol. 64, no. 2, pp. 598–613, Feb. 2019.
[11] C.-E. Ren, Z. Shi, and T. Du, "Distributed observer-based leader-following consensus control for second-order stochastic multi-agent systems," *IEEE Access*, vol. 6, pp. 20077–20084, 2018.
[12] Y. Li, C. Hua, and X. Guan, "Distributed output feedback leader-following control for high-order nonlinear multiagent system using dynamic gain method," *IEEE Trans. Cybern.*, to be published. doi: 10.1109/TCYB.2018.2870543.
[13] Y. Xu, G. Ren, J. Chen, Y. Luo, L. Jia, X. Liu, Y. Yang, and Y. Xu, "A one-leader multi-follower Bayesian-stackelberg game for anti-jamming transmission in UAV communication networks," *IEEE Access*, vol. 6, pp. 21697–21709, 2018.
[14] Z.-P. Yan, Y.-B. Liu, C.-B. Yu, and J.-J. Zhou, "Leader-following coordination of multiple UUVs formation under two independent topologies and time-varying delays," *J. Central South Univ.*, vol. 24, no. 2, pp. 382–393, Feb. 2017.
[15] Z. Yan, X. Liu, J. Zhou, and D. Wu, "Coordinated target tracking strategy for multiple unmanned underwater vehicles with time delays," *IEEE Access*, vol. 6, pp. 10348–10357, 2018.
[16] G. V. Lakhekar and L. M. Waghmare, "Adaptive fuzzy exponential terminal sliding mode controller design for nonlinear trajectory tracking control of autonomous underwater vehicle," *Int. J. Dyn. Control*, vol. 6, no. 4, pp. 1690–1705, Jan. 2018.
[17] Z. Yan, L. Wang, T. Wang, Z. Yang, T. Chen, and J. Xu, "Polar cooperative navigation algorithm for multi-unmanned underwater vehicles considering communication delays," *Sensors*, vol. 18, no. 4, pp. 1044–1053, Mar. 2018.
[18] Z. Yan, G. Zhang, J. Xu, T. Chen, X. Du, and J. Li, "Clustering statistic Hough transform based estimation method for motion elements of multiple underwater targets," *IEEE Access*, vol. 6, pp. 23747–23766, 2018.
[19] Y. Jiang, Y. Zhang, and S. Wang, "Distributed adaptive consensus control for networked robotic manipulators with time-varying delays under directed switching topologies," *Peer-Peer Netw. Appl.*, to be published. doi: 10.1007/s12083-019-00751-1.
[20] Y. Guo, "Globally robust stability analysis for stochastic Cohen–Grossberg neural networks with impulse control and time-varying delays," *Ukrainian Math. J.*, vol. 69, no. 8, pp. 1220–1233, Jan. 2018.
[21] C.-J. Li and G.-P. Liu, "Data-driven consensus for non-linear networked multi-agent systems with switching topology and time-varying delays," *IET Control Theory Appl.*, vol. 12, no. 12, pp. 1773–1779, Aug. 2018.
[22] F. Xiao, Y. Shi, and W. Ren, "Robustness analysis of asynchronous sampled-data multiagent networks with time-varying delays," *IEEE Trans. Autom. Control*, vol. 63, no. 7, pp. 2145–2152, Jul. 2018.
[23] W. Zhang, C. Yu, Y. Teng, and S. Wei, "Research on UUV path tracking control based on model predictive control," *Chin. J. Sci. Instrum.*, vol. 38, no. 11, pp. 2659–2666, Nov. 2017.
[24] S. C. Martin and L. L. Whitcomb, "Nonlinear model-based tracking control of underwater vehicles with three degree-of-freedom fully coupled dynamical plant models: Theory and experimental evaluation," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 2, pp. 404–414, Mar. 2018.
[25] W. Gan, D. Zhu, and D. Ji, "QPSO-model predictive control-based approach to dynamic trajectory tracking control for unmanned underwater vehicles," *Ocean Eng.*, vol. 158, pp. 208–220, Jun. 2018.
[26] T. I. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control*. New York, NY, USA: Wiley, 2011.
[27] W. Zou, C. K. Ahn, and Z. Xiang, "Leader-following consensus of second-order nonlinear multi-agent systems with unmodeled dynamics," *Neurocomputing*, vol. 322, no. 17, pp. 120–129, Dec. 2018.
[28] D. Zhang, Q. Song, Y. Liu, and J. Cao, "Pinning consensus analysis for nonlinear second-order multi-agent systems with time-varying delays," *Asian J. Control*, vol. 20, no. 6, pp. 2343–2350, Nov. 2018.
[29] Y. Shang, "Couple-group consensus of continuous-time multi-agent systems under Markovian switching topologies," *J. Franklin Inst.*, vol. 352, no. 11, pp. 4826–4844, Nov. 2015.
[30] M.-C. Fan and W. Yue, "Global leader-following consensus of nonlinear multi-agent systems with unknown control directions and unknown external disturbances," *Appl. Math. Comput.*, vol. 331, pp. 274–286, Aug. 2018.

**ZHEPING YAN** was born in Hangzhou, China, in 1972. He received the B.S. degree in nuclear power plant, the M.S. degree in marine and marine engineering special auxiliary equipment and systems, and the Ph.D. degree in control theory and control engineering from Harbin Engineering University, Heilongjiang, China, in 1994, 1997, and 2001, respectively. He completed the Postdoctoral Research in mechanical and electrical engineering from the Harbin Institute of Technology, China, in 2004. In 2015, he was a Visiting Scholar with the National University of Singapore, Singapore. In 2016, he was a Visiting Scholar with Australian National University, Australia. In 2017, he was a Visiting Scholar with the University of Guelph, Canada. He is currently a Professor with the College of Automation and the Director of the Marine Assembly and Automatic Technology Institute, Harbin Engineering University. His main research interests include control systems of unmanned underwater vehicles and cooperative control of unmanned underwater vehicles.

**XIAOLI PAN** was born in Harbin, China, in 1982. She received the M.S. degree in mathematics from the Harbin Institute of Technology, China, in 2007. She is currently pursuing the Ph.D. degree in control science and engineering with Harbin Engineering University. Her research interests include multi-agents' consensus control and coordination control of multiple unmanned underwater vehicles.

**ZEWEN YANG** was born in Harbin, China, in 1991. He received the M.S. degree in control engineering from Northeast Forest University, China, in 2017. He is currently pursuing the Ph.D. degree in control science and engineering with Harbin Engineering University. His research interests include control system of unmanned underwater vehicles and coordination control of multiple unmanned underwater vehicles.



**LIDONG YUE** was born in Liaoning, China, in 1990. She received the B.S. degree in mathematics from Bohai University, China, in 2014, and the M.S. degree in mathematics from Northeastern University, China, in 2017. She is currently pursuing the Ph.D. degree in control science and engineering with Harbin Engineering University. Her research interests include coordination control of autonomous underwater vehicle (AUV) and consistency of multiple AUVs.