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# Efficient Teleportation for High-Dimensional Quantum Computing

**WEN-LING XU, TIE-JUN WANG, AND CHUAN WANG, (Member, IEEE)**
School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China
State Key Laboratory of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, Beijing 100876, China

Corresponding author: Tie-Jun Wang (wangtiejun@bupt.edu.cn)

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**ABSTRACT** Performance of an entangled quantum channel is affected by classical feedback assisted in quantum communications. For example, in quantum-gate teleportation schemes, the capacity of an independent entangled quantum channel is reduced by utilizing two-way simultaneous classical communication (TWSCC$^{\leftrightarrow}$). However, by exploiting the superposition of high-dimensional quantum channels, the transmission efficiency of the quantum-gate teleportation can be dramatically improved with TWSCC$^{\leftrightarrow}$. In this study, we investigate the possibility of achieving an efficient scheme of nonlocal high-dimensional quantum computation by using hyperentangled photon pairs, atoms, and an optical micro-resonator coupled system. The feasibility and efficiency of the scheme are also discussed. Results prove that, for nonlocal quantum computing, high-dimensional quantum operation performs better than traditional methods that decompose the high-dimensional Hilbert space into two-dimensional quantum space under limited prior-shared maximally entangled resources.

**INDEX TERMS** Quantum computing, optical resonators, quantum mechanics.

## I. INTRODUCTION

Entanglement is an important resource that is widely used in quantum information processing [1], such as distributed quantum computation [2]–[4] and quantum cryptography [5]–[11]. For example, with nonlocal maximal entanglement and one-way classical communication, one can teleport an unknown quantum state without moving the particle itself [12]. According to the no-cloning theorem, the sender loses the quantum state while it is recovered on the receiver with the probability 100%. Similarly, with nonlocal maximal entanglement and classical communication, the gate-operation can be teleported without moving any control-particle itself [13]. Different from quantum-state teleportation, after quantum-gate teleportation, the unknown control-qubit remains at the sender while the target-qubit kept by the receiver is nonlocally controlled by control-qubit. Moreover, quantum-gate teleportation requires two-way (forward-back) classical communication (TWCC) instead of one-way classical communication in quantum-state teleportation.

Nonlocal quantum operation is an important component of nonlocal interactions and controlling. It not only plays a key role in realizing distributed quantum computation, but also exhibits potential applications in the domain of quantum communication and quantum information [2]–[4]. The quantum-gate teleportation, as an important realization method of nonlocal quantum operation, shows the nontrivial characteristics of quantum mechanics and attracts a lot of attention. In 2005, Zhou et al. proposed the first quantum-gate teleportation scheme using an atom-cavity coupled system [13]. The two-qubit controlled-Not (CNOT) gate [1] can be teleported from the controller (Alice) to the controllee (Bob) with a prior-shared two-dimensional maximally entangled Bell state and two-way simultaneous classical communication (TWSCC$^{\leftrightarrow}$). When the time of local operations is ignored, the communication time of TWSCC$^{\leftrightarrow}$ is equal to the transmission time of one-way classical communication. In 2015, Wang et al. illustrated that with a hyperentanglement (The entanglement of photons can simultaneously exist on multiple degrees of freedoms)-assisted quantum channel, the capacity for transmitting nonlocal CNOT gates can be doubled with TWSCC$^{\leftrightarrow}$ [14]. On the other hand, time consumption is an important issue in quantum communication, which is

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also related to the cost of entanglement in the context of distributed quantum computation [2], [4]. In 2012, Li et al. introduced the classes of nonlocal unitaries which leads to a fast protocol with TWSCC$^{\leftrightarrow}$ methods [15]. According to Ref. [15], two or more two-dimensional gates, which form a controlled abelian group, can be simultaneously teleported with only one prior two-dimensional maximally entangled Bell state assisted by TWSCC$^{\leftrightarrow}$ methods. It means that if two gates are noncommutative, although they have a common control-qubit, one needs two prior-shared two-dimensional maximally entangled Bell states to simultaneously teleport these two gates with a fast protocol. Otherwise, the transmission time is doubled by consuming only one prior-shared two-dimensional maximally entangled Bell state with other two-way classical communication (TWCC) methods. The time and entanglement resources are traded off in the gate-teleportation operations with TWSCC$^{\leftrightarrow}$. Thus, finding a better performance of nonlocal quantum computing with less entanglement and less time is an important task.

The manipulation of high-dimensional quantum information and quantum superposition enables an exponential computation speedup of the quantum computer than the classical machines [16]–[18]. In addition, high-dimensional quantum systems can also increase the channel capacity [19] and the security of quantum communication [20]. Moreover, the influence of noise can be efficiently eliminated [21] and the implementation of quantum logic gate can be simplified [22]. With these advantages, high-dimensional quantum information processing has attracted much more attention recently [1]. Motivated by the advantages of high-dimensional quantum systems, the present study investigates high-dimensional quantum control operation for nonlocal quantum computing. Specifically, by exploiting the superposition of high-dimensional entanglement-assisted channels, one can improve the quantum-gate teleportation capacity of the prior-shared entanglement with TWSCC$^{\leftrightarrow}$ method dramatically under limited prior-shared maximally entangled resources.

In this study, we investigate the possibility of achieving an efficient nonlocal high-dimensional quantum computation by using hyperentangled photon pairs, assisted with the atoms and optical-microresonator coupled system. The feasibility and efficiency of the scheme are discussed. Also, we present the upper bound of the gating transfer capacity of quantum-gate teleportation scheme with TWSCC$^{\leftrightarrow}$: with $n$ parallel two-dimensional maximally entangled quantum channels, one can deterministically teleport at most $n(n+1)/2$ two-dimensional two-body or multi-body nonlocal quantum gates, in which any two of these gates cannot be deterministically teleported in a two-dimensional maximally entangled quantum channel with TWSCC$^{\leftrightarrow}$. In our previous work [14], we proved that with hyperentanglement in two DOFs, one can teleport two CNOT gates in parallel. However, the capacity of the hyperentangled channel in Ref. [14] is not fully utilized. By exploiting the superposition between two independent Bell states channels, one can teleport another nonlocal controlled gate (three-qubit Toffoli [23] gate) simultaneously, which can be demonstrated by using four-dimensional quantum controlled gate teleportation. The efficiency of the quantum gate-teleportation can be improved by exploiting the superposition of high-dimensional quantum channels with TWSCC$^{\leftrightarrow}$.

## II. BASIC IDEAS FOR A FOUR-DIMENSIONAL NONLOCAL QUANTUM CONTROLLED-FLIP (CF) GATE TELEPORTATION

In this section, we show how to build a nonlocal four-dimensional two-qudit controlled-flip (CF) gate $U_{(\pm)}{}_B^A$ (the superscripts denote the control qudit and the subscripts represent the target qudit), in which if the state of control qudit is $|i\rangle$, after the gating operation, the state of the target qudit is changed as $|j\rangle \rightarrow |j \pm i\rangle$ $(i, j = 0, 1, 2, 3)$ where $i \pm j$ in the $|j \pm i\rangle$ means modding 4, with a hyperentangled Bell-state channel. The 4-dimensional CF gate $U_{(+)}{}_B^A$ can be decomposed into three two-dimensional quantum gates as $C_{B_1}^{A_1} C_{B_2}^{A_2} T_{B_1}^{A_2 B_2}$ or $C_{B_1}^{A_1} C_{B_2}^{A_2} T_{B_2}^{A_1 B_1}$. Here, the qubit $A_i$ belongs to Alice and the qubit $B_j$ belongs to Bob $(i, j, = 1, 2)$. TWSCC$^{\leftrightarrow}$ enables any one of the three gates $T_{B_2}^{A_1 B_1}$, $C_{B_2}^{A_2}$ and $C_{B_1}^{A_1}$ to be teleported with a prior-shared two-dimensional maximally entangled Bell state, but any two of these gates cannot be deterministically parallel in a two-dimensional maximally entangled quantum channel with TWSCC$^{\leftrightarrow}$. If parallel channels are not applicable, then three two-dimensional maximally entangled channels are needed to transmit these three gates in turn. However, by exploiting the superposition between channels, one can complete the same task with only two parallel two-dimensional maximally entangled quantum channels. Thus, high-dimension quantum computing teleportation can utilize two-dimensional maximally entangled resources with a higher capacity than the previous nonlocal quantum computing scheme [14].

We suppose that two four-dimensional network nodes A and B exist, and A is the control unit while B is the target unit. The initial states of these two nodes are prepared in $|\Psi\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A + \gamma|2\rangle_A + \xi|3\rangle_A$ and $|\Psi\rangle_B = \alpha'|0\rangle_B + \beta'|1\rangle_B + \gamma'|2\rangle_B + \xi'|3\rangle_B$ respectively, where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\xi|^2 = 1$ and $|\alpha'|^2 + |\beta'|^2 + |\gamma'|^2 + |\xi'|^2 = 1$. A nonlocal quantum CF gate is implemented when a four-dimensional maximally entangled Bell state is prior-shared by Alice and Bob. The state is $|\varphi_0\rangle_{CD} = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle)_{CD}$, and classical communication in each direction is necessary. Alice holds system AC and Bob holds system BD. The entire scheme can be understood through the following steps:

Step (I): A quantum CF operation $U_{(-)}{}_C^A$ occurs between A and C with A as the control qudit. The combined state of A, C and D becomes

$$\begin{aligned}
\frac{1}{2}[&|0\rangle_C(\alpha|00\rangle + \beta|11\rangle + \gamma|22\rangle + \xi|33\rangle)_{AD} \\
&+ |1\rangle_C(\alpha|01\rangle + \beta|12\rangle + \gamma|23\rangle + \xi|30\rangle)_{AD} \\
&+ |2\rangle_C(\alpha|02\rangle + \beta|13\rangle + \gamma|20\rangle + \xi|31\rangle)_{AD} \\
&+ |3\rangle_C(\alpha|03\rangle + \beta|10\rangle + \gamma|21\rangle + \xi|32\rangle)_{AD}]. \qquad (1)
\end{aligned}$$

Step (II): Simultaneously, Bob performs another four-dimensional CF operation $U_{(+)B}^{D}$ on B and D. Then Bob performs four-dimensional Fourier transform $\widetilde{F}$ on D, which can be described as

$$\begin{aligned}|0\rangle &\xrightarrow{\widetilde{F}} \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle + |3\rangle) = |\varphi_0\rangle,\\ |1\rangle &\xrightarrow{\widetilde{F}} \frac{1}{2}(|0\rangle + i|1\rangle - |2\rangle - i|3\rangle) = |\varphi_1\rangle,\\ |2\rangle &\xrightarrow{\widetilde{F}} \frac{1}{2}(|0\rangle - |1\rangle + |2\rangle - |3\rangle) = |\varphi_2\rangle,\\ |3\rangle &\xrightarrow{\widetilde{F}} \frac{1}{2}(|0\rangle - i|1\rangle - |2\rangle + i|3\rangle) = |\varphi_3\rangle. \end{aligned} \tag{2}$$

Alice and Bob can obtain the combined state of A, B, C, and D as

$$\begin{aligned}\frac{1}{2}[&|\varphi_0\rangle_D(\alpha|00\rangle + \xi|13\rangle + \gamma|22\rangle + \beta|31\rangle)_{CA}\\ &\otimes(\alpha'|0\rangle + \beta'|1\rangle + \gamma'|2\rangle + \xi'|3\rangle)_B\\ &+|\varphi_1\rangle_D(\beta|01\rangle + \alpha|10\rangle + \xi|23\rangle + \gamma|32\rangle)_{CA}\\ &\otimes(\alpha'|1\rangle + \beta'|2\rangle + \gamma'|3\rangle + \xi'|0\rangle)_B\\ &+|\varphi_2\rangle_D(\gamma|02\rangle + \beta|11\rangle + \alpha|20\rangle + \xi|33\rangle)_{CA}\\ &\otimes(\alpha'|2\rangle + \beta'|3\rangle + \gamma'|0\rangle + \xi'|1\rangle)_B\\ &+|\varphi_3\rangle_D(\xi|03\rangle + \gamma|12\rangle + \beta|21\rangle + \alpha|30\rangle)_{CA}\\ &\otimes(\alpha'|3\rangle + \beta'|0\rangle + \gamma'|1\rangle + \xi'|2\rangle)_B. \end{aligned} \tag{3}$$

Step(III): Alice and Bob measure the states of particles C and D on the basis of $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}_{CD}$ at the same time. Then, Alice and Bob exchange detection results with TWSCC$^{\leftrightarrow}$. If $C$ and $D$ are finally detected in the state $|00\rangle_{CD}$, then the state of $AB$ can be described as

$$\begin{aligned}&\alpha|0\rangle_A(\alpha'|0\rangle + \beta'|1\rangle + \gamma'|2\rangle + \xi'|3\rangle)_B\\ &+\beta|1\rangle_A(\alpha'|1\rangle + \beta'|2\rangle + \gamma'|3\rangle + \xi'|0\rangle)_B\\ &+\gamma|2\rangle_A(\alpha'|2\rangle + \beta'|3\rangle + \gamma'|0\rangle + \xi'|1\rangle)_B\\ &+\xi|3\rangle_A(\alpha'|3\rangle + \beta'|0\rangle + \gamma'|1\rangle + \xi'|2\rangle)_B. \end{aligned} \tag{4}$$

In this case, a four-dimensional 2-qudit CF gate $U_{(+)B}^{A}$ is performed nonlocally. The matrix form of the gate $U_{(+)B}^{A}$ can be described as

$$U_{(+)B}^{A} = \begin{bmatrix} I_4 & 0 & 0 & 0\\ 0 & X & 0 & 0\\ 0 & 0 & X^2 & 0\\ 0 & 0 & 0 & X^3 \end{bmatrix} \tag{5}$$

Here, $I_4 = \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}$ and $X = \begin{bmatrix} 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 1&0&0&0 \end{bmatrix}$, under the basis expanded by $\{|00\rangle, |01\rangle, |02\rangle, |03\rangle, |10\rangle, |11\rangle, |12\rangle, |13\rangle, |20\rangle, |21\rangle, |22\rangle, |23\rangle, |30\rangle, |31\rangle, |32\rangle, |33\rangle\}_{AB}$. If we encode a four-dimensional qudit by two two-dimensional qubits as $|0\rangle_K = |00\rangle_{K_2K_1}$, $|1\rangle_K = |01\rangle_{K_2K_1}$, $|2\rangle_K = |10\rangle_{K_2K_1}$, and $|3\rangle_K = |11\rangle_{K_2K_1}$ ($K = A, B, C$, and $D$). The four-dimensional CF gate $U_{(+)B}^{A}$ can be decomposed into three two-dimensional quantum gates as $C_{B_1}^{A_1}C_{B_2}^{A_2}T_{B_2}^{A_1B_1}$.

**TABLE 1.** Relation between final states of photons and corresponding single-spin operation.

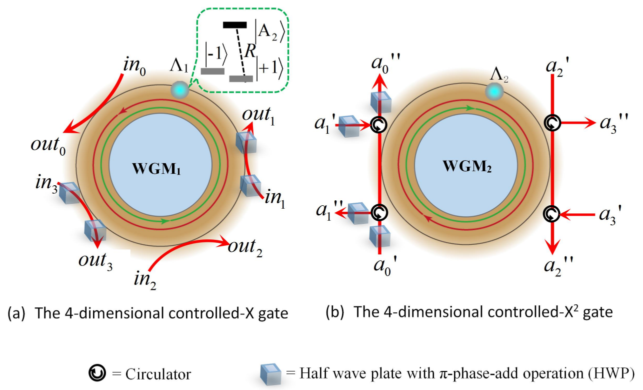
| Photons $D$ | Operations on A |
|---|---|
| $\|0\rangle_D$ | $I_A$ |
| $\|1\rangle_D$ | $F_A$ |
| $\|2\rangle_D$ | $F_A^2$ |
| $\|3\rangle_D$ | $F_A^3$ |
| Photons $C$ | Operations on B |
| $\|0\rangle_C$ | $I_B$ |
| $\|1\rangle_C$ | $X_B^3$ |
| $\|2\rangle_C$ | $X_B^2$ |
| $\|3\rangle_C$ | $X_B$ |

The four-dimensional maximally entangled Bell states can be decomposed into two two-dimensional Bell states $|\varphi_0\rangle_{CD} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{C_1D_1} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{C_2D_2}$,

If the state of the $CD$ is detected in the other state, then the CF gate performed on the AB could succeed after an additional single-qubit operation on $B$ or $A$. As a result, the correspondence between the measurement results and appropriate local single-qudit gate rotation operations on $A$ or $B$ is shown in Table 1. The deterministic CF gate is achieved with a success probability of 100% in principle using prior-shared entanglement, local operations, and classical communication. It also shows that one can teleport three gates ($T_{B_1}^{A_2B_2}$, $C_{B_2}^{A_2}$ and $C_{B_1}^{A_1}$) in parallel, with two prior-shared two-dimensional maximally entangled Bell states and TWSCC$^{\leftrightarrow}$. Here, $F = \begin{bmatrix} 1&0&0&0\\ 0&-i&0&0\\ 0&0&-1&0\\ 0&0&0&i \end{bmatrix}$ under the basis expanded by $\{|00\rangle, |01\rangle, |02\rangle, |03\rangle, |10\rangle, |11\rangle, |12\rangle, |13\rangle, |20\rangle, |21\rangle, |22\rangle, |23\rangle, |30\rangle, |31\rangle, |32\rangle, |33\rangle\}_{AB}$.

## III. DETERMINISTIC FOUR-DIMENSIONAL QUANTUM CF GATE USING WHISPERING GALLERY MODE MICRORESONATORS

In this section, we focus on the physical implementation of the local four-dimensional quantum CF gate. Here, the four-dimensional Bell state is encoded on the spatial-mode DOF of two photons because this DOF is more stable against the noise and can easily manipulate the states of the quantum systems. In particular, performing the high-dimensional quantum fast Fourier transform on the single photon is easy with only linear optics [24]. The two-photon hyperentanglement in the polarization and spatial-mode DOFs $|\varphi_0\rangle = \frac{1}{2}(|HH\rangle + |VV\rangle) \otimes (|a_1b_1\rangle + |a_2b_2\rangle)_{ab} = \frac{1}{2}(|00\rangle + |11\rangle + |22\rangle + |33\rangle)_{P_AP_B}$ can be generated by parametric down conversion techniques on nonlinear crystals [25], and with linear optics, its form can be changed into the following four-dimensional spatial-mode two-photon entangled state: $|\varphi_0\rangle = \frac{1}{2}(|a_0b_0\rangle + |a_1b_1\rangle + |a_2b_2\rangle + |a_3b_3\rangle)_{P_AP_B}$. Here, $a_i$ and $b_j$ are spatial-modes of photons $P_A$ and $P_B$, respectively ($i, j = 0, 1, 2, 3$). The basic building block of our high-dimensional CF gate is the four-dimensional controlled-X gate (if the state of control qubit is $|1\rangle$, after the gating operation, the state of the target




(a) The 4-dimensional controlled-X gate (b) The 4-dimensional controlled-$X^2$ gate

**FIGURE 1. (a) Schematic of four-dimensional controlled-X gate comprising a WGM microresonator and a $\Lambda$-type atom $\Lambda_1$. Two degenerate ground states $|\pm 1\rangle$ and an excited state $|A_2\rangle$ exist in an atom. The qubits in the atom system are usually encoded on the $|\pm 1\rangle$ state, while the $|A_2\rangle$ works as an auxiliary state. The transition $|+1\rangle \rightarrow |A_2\rangle$ is only induced by the $R$−polarized photon. An atom is fixed on the resonator, and four tapered fibers are coupled with a WGM resonator. The ports $in_0 - in_3$ and $out_0 - out_3$ represent input ports and output ports, respectively. (b) Schematic of the 4-dimensional controlled-$X^2$ gate comprising a WGM microresonator and a $\Lambda$-type atom $\Lambda_2$. The Half-wave plate adds a $\pi$ phase on the $R$ photon. $c_1$, $c_2$, $c_3$, and $c_4$ are four circulators rotating counterclockwise.**

four-dimensional qudit is changed as $|j\rangle \rightarrow |j+1\rangle$ where $j+1$ in the $|j+1\rangle$ means modding 4) and the four-dimensional controlled-$X^2$ gate (if the state of control qubit is $|1\rangle$, after the gating operation, the state of the target four-dimensional qudit is changed as $|j\rangle \rightarrow |j+2\rangle$ where $j+2$ in the $|j+2\rangle$ means modding 4) between a four-dimensional photon in spatial-mode DOF and an atomic qubit of a whispering-gallery-mode (WGM) microresonator coupled unit. The core of our architecture are the reflection and transmission rules of circularly polarized lights interacting with a WGM-waveguide system.

### A. INTERACTION BETWEEN A POLARIZED PHOTON AND A $\Lambda$-TYPE ATOM-WGM RESONATOR COUPLED SYSTEM

In our scheme, we use the three-level $\Lambda$-type atom-WGM-waveguide unit as the basic building block. As shown in Fig.1 (a), two degenerate ground states $|\pm 1\rangle$ exist in an atom and an excited state $|A_2\rangle$. The qubits in the atom system are usually encoded on the $|\pm 1\rangle$ state, while the $|A_2\rangle$ works as an auxiliary state. The transition $|+1\rangle \rightarrow |A_2\rangle$ is only induced by the $R$-polarized photon with the transition frequency $\omega_0$, while the transition $|-1\rangle \rightarrow |A_2\rangle$ is induced by the $L$-polarized photon. We couple four tapered fibers with a WGM resonator, and exploit the laser cooling to fix the atom on the resonator. In Fig.1(a), the ports $in_0 - in_3$ and $out_0 - out_3$ represent input ports and output ports, respectively. The frequency of the input photon is the same as that of the bare resonator. When the atom is coupled with the resonator strongly, the resonance frequency of the resonator is modified so that the buildup of the resonator field is prevented. In this case, the input photon is off-resonant with the resonator and is transmitted via the bus waveguide with the spatial mode unchanged. That is to say, the single-photon input from port $in_j$ is output from port $out_j$. When the atom is not coupled with the resonator, the photon input from port $in_j$ is coupled into the bare resonator and be dropped to the field $out_{j+1}$ ($j = 0, 1, 2, 3$) finally.

The interaction between a single-mode quantum electromagnetic field and a single two-level atom is the simplest model of the interaction between a quantum electromagnetic field and a substance; this interaction is the Rabi model, which is a J-C model after the rotating wave approximation. Compared with the J-C model, an $\Lambda$-type atom system has similar characteristics. We assume that we input a single photon pulse with a frequency of $\omega_p$ from port $in_0$ and obtain the input field $a_{in}$ that drives the Heisenberg equations of motion for the cavity field. The equations can be expressed as:

$$\begin{aligned}\frac{da}{dt} &= -[i(\omega_c - \omega_p) + \frac{\kappa_1}{2} + \frac{\kappa_2}{2}]a(t) - g\sigma_-(t) + \sqrt{\kappa_1}a_{in} \\ \frac{d\sigma_-}{dt} &= -[i(\omega_0 - \omega_p) + \gamma]\sigma_-(t) - g\sigma_z(t)a(t)\end{aligned} \tag{6}$$

where $a$ is the cavity-field annihilation operator driven by the input field, and $\omega_0$ and $\omega_c$ denote the frequency of the atom and the resonator field, respectively. $\sigma_z$ and $\sigma_-$ represent the inversion operator and lowering operator of the atom. $\kappa_1$ and $\kappa_2$ represent the coupling losses of the fiber and resonator, respectively. $\gamma$ stands for the atomic spontaneous emission rate, and $g$ is the coupling strength between the resonator and the atom.

To maximize the drop and add efficiencies in our scheme, we make $\kappa_1 = \kappa_2 = \kappa$. The atom is prepared in the ground state. We have $\langle\sigma_z\rangle = -1$ through the entire process to ensure the weak excitation through the single-photon pulse on the $\Lambda$-type atom. The input and output relation can be described as $a_{out} = a_{in} - \sqrt{\kappa}a$. According to this relation, the reflection and transmission coefficients for the coupled system can be expressed as

$$\begin{aligned}t_{(\omega_p)} &= \frac{i(\omega_c - \omega_p)[i(\omega_0 - \omega_p) + \gamma] + g^2}{[i(\omega_c - \omega_p) + \kappa][i(\omega_0 - \omega_p) + \gamma] + g^2} \\ r_{(\omega_p)} &= -\frac{\kappa[i(\omega_0 - \omega_p) + \gamma]}{[i(\omega_c - \omega_p) + \kappa][i(\omega_0 - \omega_p) + \gamma] + g^2}\end{aligned} \tag{7}$$

Through the complex dynamics, we can determine that if the initial state of the $\Lambda$-type atom is $|+1\rangle$, when the $R$ circularly polarized single-photon pulse interacts with the atom-WGM resonator system, the transition is $|+1\rangle \rightarrow |A_2\rangle$. In this case, the $R$ photon transports through the input path with a phase shift $\phi$. On the contrary, if the initial state of the $\Lambda$-type atom is $|-1\rangle$, then $R$ photon is coupled with the resonator and the photon is transported from port $|in_j\rangle$ to $|out_{j+1}\rangle$ with the phase shift $\phi_0$.

When the system is in resonance, namely, $\omega_0 = \omega_c = \omega_p$, the reflection and transmission coefficients can be simplified. The rule of the spin-selective optical transition for the four-dimensional hybrid quantum gate unit shown in Fig.1 (a) can be expressed as

$$|in_j, +1\rangle \rightarrow |out_j, +1\rangle, |in_j, -1\rangle \rightarrow -|out_{j+1}, -1\rangle \quad (8)$$

where, $j = 0, 1, 2, 3$. According to this rule, we can know that if the initial state of the atom is $|-1\rangle$, then the $R$-polarized photon flips with an acquired phase shift $\pi$. Otherwise, the $R$-polarized photon maintains the original state in the initial state of the atom which is $|+1\rangle$.

### B. IMPLEMENTATION OF A FOUR-DIMENSIONAL LOCAL QUANTUM CF GATE BETWEEN TWO QUBITS AND A FOUR-DIMENSIONAL PHOTON

In this section, we investigate the possibility of building a four-dimensional CF gate as shown in Fig.1. The control qubits are stationary in the $\Lambda$-type atom 1 ($\Lambda_1$) and $\Lambda$-type atom 2 ($\Lambda_2$) whose initial state is $|\phi\rangle_{\Lambda_2\Lambda_1} = (\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\xi|11\rangle)_{\Lambda_2\Lambda_1}$, here $|\alpha|^2+|\beta|^2+|\gamma|^2+|\xi|^2 = 1$, $|+1\rangle_\Lambda = |0\rangle_\Lambda$ and $|-1\rangle_\Lambda = |1\rangle_\Lambda$, while the four-dimensional spatial-mode state of the single-photon $P_A$ serves as the target qudit whose initial state is $|\phi\rangle_{P_A} = \frac{1}{2}(a|a_0\rangle+b|a_1\rangle+c|a_2\rangle+d|a_3\rangle)_{P_A}$ with $|a|^2+|b|^2+|c|^2+|d|^2 = 1$. Here, the $|a_j\rangle (j = 0, 1, 2, 3)$ denotes the four-dimensional spatial-mode state of the photon $P_A$, which is correspondingly given by

$$\begin{aligned}|a_0\rangle &= |in_0\rangle_{P_A}, |a_1\rangle = |in_1\rangle_{P_A},\\ |a_2\rangle &= |in_2\rangle_{P_A}, |a_3\rangle = |in_3\rangle_{P_A}.\end{aligned} \quad (9)$$

As shown in Fig.1(a), the incident photon $P_A$ may pass through the half-waves plates (HWPs), which can add a $\pi$ phase on the photon. After the photon flies through the four-dimensional controlled-X gate as shown in Fig.1(a), the state of the composite system ($P_A - \Lambda_1 - \Lambda_2$) is changed as

$$\begin{aligned}&|\phi\rangle_{\Lambda_2\Lambda_1}|\phi\rangle_{P_A}\\ &= \frac{1}{2}\alpha|00\rangle_{\Lambda_2\Lambda_1}(a|out_0\rangle+b|out_1\rangle+c|out_2\rangle+d|out_3\rangle)_{P_A}\\ &\quad+\frac{1}{2}\beta|01\rangle_{\Lambda_2\Lambda_1}(a|out_3\rangle+b|out_0\rangle+c|out_1\rangle+d|out_2\rangle)_{P_A}\\ &\quad+\frac{1}{2}\gamma|10\rangle_{\Lambda_2\Lambda_1}(a|out_0\rangle+b|out_1\rangle+c|out_2\rangle+d|out_3\rangle)_{P_A}\\ &\quad+\frac{1}{2}\xi|11\rangle_{\Lambda_2\Lambda_1}(a|out_3\rangle+b|out_0\rangle+c|out_1\rangle+d|out_2\rangle)_{P_A}.\end{aligned} \quad (10)$$

Then, the photon $P_A$ passes through the four-dimensional controlled-$X^2$ gate as shown in Fig.1 (b), where the $c_1$-$c_4$ are circulators (counterclockwise). Based on the assumption that the state of $\Lambda_2$ is $|-1\rangle$, the photons will couple into the brae resonator; otherwise, when the state of $\Lambda_2$ is $|+1\rangle$, the atom couples with the resonator strongly and the photon remains in the initial mode. When the spatial-mode of photon is encoded as follows

$$\begin{aligned}|out_0\rangle &= |a'_0\rangle_{P_A}, |out_1\rangle = |a'_1\rangle_{P_A},\\ |out_2\rangle &= |a'_2\rangle_{P_A}, |out_3\rangle = |a'_3\rangle_{P_A},\end{aligned} \quad (11)$$

after the four-dimensional controlled-$X^2$ gate, the state of the composite system ($P_A - \Lambda_1 - \Lambda_2$) is changed as

$$\begin{aligned}&|\phi\rangle_{\Lambda_2\Lambda_1}|\phi\rangle_{P_A}\\ &= \frac{1}{2}\alpha|00\rangle_{\Lambda_2\Lambda_1}(a|a''_0\rangle+b|a''_1\rangle+c|a''_2\rangle+d|a''_3\rangle)_{P_A}\\ &\quad+\frac{1}{2}\beta|01\rangle_{\Lambda_2\Lambda_1}(a|a''_3\rangle+b|a''_0\rangle+c|a''_1\rangle+d|a''_2\rangle)_{P_A}\\ &\quad+\frac{1}{2}\gamma|10\rangle_{\Lambda_2\Lambda_1}(a|a''_2\rangle+b|a''_3\rangle+c|a''_0\rangle+d|a''_1\rangle)_{P_A}\\ &\quad+\frac{1}{2}\xi|11\rangle_{\Lambda_2\Lambda_1}(a|a''_1\rangle+b|a''_2\rangle+c|a''_3\rangle+d|a''_0\rangle)_{P_A}.\end{aligned} \quad (12)$$

According to Eq. (12), if we encode a four-dimensional qudit by 2 two-dimensional atom qubits as $|0\rangle_\Lambda = |00\rangle_{\Lambda_2\Lambda_1}$, $|1\rangle_\Lambda = |01\rangle_{\Lambda_2\Lambda_1}$, $|2\rangle_\Lambda = |10\rangle_{\Lambda_2\Lambda_1}$, and $|3\rangle_\Lambda = |11\rangle_{\Lambda_2\Lambda_1}$, then we can successfully perform the four-dimensional CF gating $U^\Lambda_{(-),P_A}$ successively.

If one encodes the spatial-mode of the photon as

$$\begin{aligned}|a_0\rangle &= |in_0\rangle_{P_A}, |a_1\rangle = |in_3\rangle_{P_A},\\ |a_2\rangle &= |in_2\rangle_{P_A}, |a_3\rangle = |in_1\rangle_{P_A},\end{aligned} \quad (13)$$

then, the four-dimensional CF gating $U^\Lambda_{(+),P_A}$ can be achieved. Thereafter, with $U^\Lambda_{(\pm),P_A}$ and nonlocal entanglement, one can perform the four-dimensional quantum gate teleportation by following the three steps described in Section 2.

### C. FEASIBILITY AND EFFICIENCY OF FOUR-DIMENSIONAL CF OPERATION

The atom and microresonator coupled system is the basic element of our scheme. In this section, we provide a brief discussion of the experimental implementation of the key element of our scheme. The key element is the four-dimensional CF operation, which is based on the Jaynes-Cummings model. As described in Ref. [26], under the resonance condition with the help of HWPs, the reflection and transmission coefficients $t(\omega_p)$ and $r(\omega_p)$ of the coupled atom-WGM cavity system can be written as

$$t(\omega_p) = \frac{2G^2}{1+2G^2}, r(\omega_p) = \frac{1}{1+2G^2}, \quad (14)$$

where $G = g/\sqrt{\kappa\gamma}$. The nitrogen-vacancy (NV) centers in diamonds [26], [27] are promising $\Lambda$-type three-level candidates for our scheme given its long coherence time at room temperature [28], [29] and its application in quantum error correction [30]. For simplification, we consider the cases in which the initial photon state is $|a_0\rangle$. After a ideal $U^\Lambda_{(+),P_A}$, the hybrid system becomes $|E(0)\rangle = (\xi_0|0\rangle|a_0\rangle+\xi_1|1\rangle|a_1\rangle+\xi_2|2\rangle|a_2\rangle+\xi_3|3\rangle|a_3\rangle)_{\Lambda_2\Lambda_1\ P_A}$ with the initial atomic state is $(\xi_0|0\rangle+\xi_1|1\rangle+\xi_2|2\rangle+\xi_3|3\rangle)_{\Lambda_1\Lambda_2}$ ($\Sigma^3_{i=0}|\xi_i|^2 = 1$). In the imperfect resonance case, the state of the hybrid system becomes $|E(0)'\rangle = (t^2\xi_0|0\rangle|a_0\rangle + rt\xi_0|0\rangle|a_1\rangle + rt\xi_0|0\rangle|a_2\rangle + r^2\xi_0|0\rangle|a_3\rangle + t\xi_1|1\rangle|a_1\rangle + r\xi_1|1\rangle|a_3\rangle+t\xi_2|2\rangle|a_2\rangle+r\xi_2|2\rangle|a_3\rangle+\xi_3|3\rangle|a_3\rangle)_{\Lambda_2\Lambda_1\ P_A}$. The fidelity is $F = |\langle E(0)'|E(0)\rangle|^2$. To obtain the average fidelity by averaging over all input states, we apply the idea of the

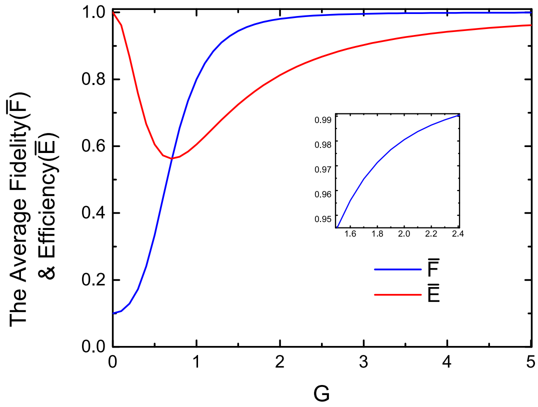



**FIGURE 2.** Average fidelity ($\bar{F}$, the blue line) and efficiency ($\bar{E}$, the red line) of four-dimensional CF operation as a function of the parameter $G$. Here, $F = \frac{[|\xi_0|^2t^2+(|\xi_1|^2+|\xi_2|^2)t+|\xi_3|^2]^2}{[|\xi_0|^2(r^2+t^2)^2+(|\xi_1|^2+|\xi_2|^2)(t^2+r^2)+|\xi_3|^2]}$ and $E = |\xi_0|^2(r^2+t^2)^2 + (|\xi_1|^2+|\xi_2|^2)(t^2+r^2) + |\xi_3|^2$.

first kind of curved surface integral in four-dimensional case to obtain an average value. The average fidelity of our CF operation for all input states on the hybrid system is

$$
\begin{aligned}
F &= \frac{[|\xi_0|^2t^2 + (|\xi_1|^2 + |\xi_2|^2)t + |\xi_3|^2]^2}{[|\xi_0|^2(r^2+t^2)^2 + (|\xi_1|^2 + |\xi_2|^2)(t^2+r^2) + |\xi_3|^2]} \\
\bar{F} &= \frac{\int_S F dS}{\int dS} \\
&= \frac{\int_0^1 dx_0 \int_0^{1-x_0} dx_1 \int_0^{1-x_0-x_1} \sqrt{4}F(x_j)dx_2}{\int dS}
\end{aligned} \tag{15}
$$

where $x_j = |\xi_j|^2$ $(j = 0, 1, 2)$, and $|\xi_3|^2 = 1 - x_0 - x_1 - x_2$. $S$ is the generalized area (volume) of parallel three-dimensional surface composed of 6 vectors in the four-dimensional space in which 4 vertices are $x_0 = 1, x_1 = 1, x_2 = 1, x_3 = 1$. The length of each vector is $\sqrt{2}$. Here, $x_i$ is the axis of four-dimensional space. Thus, we can easily determine that $\int dS$ is $\frac{\sqrt{4}}{3!}$

To evaluate the performance of the gate operation, we numerically simulated the results of the fidelities as a function of $G$ in the resonant case, as shown in Fig.2. When $G$ is larger than 1.6, the fidelity of the CF operation is higher than 0.95. As shown by the results of Ref. [26], $G \geq 3$ in the resonance situation is not difficult to achieve in experiments. The experimental cavity-QED parameters $[g, \kappa, \gamma_{total}, \gamma]/2\pi = [0.3, 26, 0.013,$ and $0.0004]$ GHz of an NV center coupled to a microdisk with $Q \sim 10^4$ has been demonstrated [30]. The coherent coupling rate of the NV center in Ref. [30] is almost one-third of the total coupling rate within the narrow-band zero phonon line (ZPL), but the ZPL emission rate has been enhanced to 70% [31]–[34] in similar experimental systems. Based on these experimental parameters, the result of our simulation is in good qualitative consistency with the theoretical speculations.

## IV. DISCUSSION

Through TWSCC$^{\leftrightarrow}$, any one of three gates $T_{B_2}^{A_1B_1}$, $C_{B_2}^{A_2}$, and $C_{B_1}^{A_1}$ can be teleported with a prior shared two-dimensional maximally entangled Bell state, but any two of these gates cannot be deterministically paralleled in a two-dimensional maximally entangled quantum channel with TWCC$^{\leftrightarrow}$. Although $C_{B_1}^{A_1}$ and $C_{B_2}^{A_2}$ are commutative, the control party (the system $A_1A_2$) and controllee party (the system $B_1B_2$) are both four-dimensional quantum systems for the combination of these two nonlocal gates. Only one prior-shared two-dimensional maximally entangled Bell state is not enough for teleporting the two gates $C_{B_1}^{A_1}$ and $C_{B_2}^{A_2}$. We notice that the control party of $T_{B_2}^{A_1B_1}$ is nonlocal, so the controlled-Abelian-group-rule described in Ref. [15] is not applicable to this case. To accomplish the task of teleporting two gates $T_{B_2}^{A_1B_1}$ and $C_{B_1}^{A_1}$ or $T_{B_2}^{A_1B_1}$ and $C_{B_2}^{A_2}$ simultaneously with TWSCC$^{\leftrightarrow}$ using only one prior nonlocal two-dimensional Bell state, we can try to construct a fast protocol similar to that in Ref. [13], [15]. However, extra nonlocal CNOT gates, which require another prior-shared maximally entangled resources are needed to accomplish this task deterministically (with 100% probability). If parallel channels are not applicable, then three two-dimensional maximally entangled channels are needed to teleport three nonlocal gates $T_{B_2}^{A_1B_1}$, $C_{B_2}^{A_2}$, and $C_{B_1}^{A_1}$ in turn.

In the four-qubit $A_1A_2B_1B_2$ system, besides $T_{B_2}^{A_1B_1}$, $C_{B_2}^{A_2}$, and $C_{B_1}^{A_1}$, one and only one nonlocal controlled-flip gate $T_{B_1}^{A_2B_2}$ which can be deterministically teleported with a two-dimensional maximally entangled Bell-state channel but can not be parallel teleported with any one of the other three gates using a two-dimensional maximally entangled quantum channel and TWSCC$^{\leftrightarrow}$. Thus, the results can proved that, with only two prior-shared two-dimensional maximally entangled Bell state resources and TWSCC$^{\leftrightarrow}$, one can not teleport four nonlocal gates $T_{B_2}^{A_1B_1}$, $T_{B_1}^{A_2B_2}$, $C_{B_2}^{A_2}$ and $C_{B_1}^{A_1}$, deterministically and simultaneously. The upper bound of the gating transfer capacity of quantum-gate teleportation with TWSCC$^{\leftrightarrow}$ and 2 two-dimensional maximally entangled quantum channels is that: one can deterministically teleport at most three nonlocal two-body or multi-body unitary controlled gates in which any two of these three gates cannot be parallel within only one two-dimensional maximally entangled quantum channel. Thus, with our scheme, the nonlocal four-dimensional quantum computing not only improves the quantum-gate teleportation efficiency of paralleled two-dimensional maximally entangled quantum resources with TWSCC$^{\leftrightarrow}$ but also makes the best use of parallel maximally entangled quantum resources with the shortest communication time.

With a good knowledge of four-dimensional quantum computing teleportation in the hybrid system, we are able to extend our scheme to a $2^n$-dimensional case. In a nonlocal $2^n$-dimensional quantum system $\tilde{A}\tilde{B}$ in which a logic qudit $\tilde{K} = K_n \otimes \cdots \otimes K_2 \otimes K_1$ is constructed with n two-dimensional physical qubits $K_1$,$K_2$,... and $K_n$ ($K$ = $A$ or $B$), a $2^n$-dimensional $CF$ gate operation can be identified by means of the nonlocal two-dimensional quantum two-qubit or multi-qubit gates-operation sequence as

$$\begin{aligned} CF_{\tilde{B}}^{\tilde{A}} &= C_{B_1}^{A_1} C_{B_2}^{A_2} \dots C_{B_n}^{A_n} \cdot T_{B_1}^{A_2B_2} T_{B_2}^{A_3B_3} \dots T_{B_{n-1}}^{A_nB_n} \\ &\cdot T_{B_1}^{A_3B_3B_2} T_{B_2}^{A_4B_4B_3} \dots T_{B_{n-2}}^{A_nB_nB_{n-1}} \dots \\ &\cdot T_{B_1}^{A_{n-1}B_{n-1}B_{n-2}\dots B_2} T_{B_2}^{A_nB_nB_{n-1}\dots B_3} \\ &\cdot T_{B_1}^{A_nB_nB_{n-1}\dots B_2}, \end{aligned} \tag{16}$$

where one needs n two-qubit CNOT gates $C_{B_i}^{A_i}$(i = 1,2,...n), (n-1) two-dimensional three-qubit Toffoli gates $T_{B_{j-1}}^{A_jB_j}$($j$ = 2, 3, ... $n$), ($n$ − 2) four-qubit Toffoli gates $T_{B_{k-2}}^{A_kB_kB_{k-1}}$ ($k$ = 3, ... $n$),..., one $n$-control-bit Toffoli gate $T_{B_1}^{A_nB_nB_{n-1}\dots B_2}$ and so on. According to this conclusion, one can easily prove that any two of these gates cannot be paralleled within only one two-dimensional maximally entangled quantum channel with a fast proposal. However, with non-simultaneous TWCC, one prior-shared two-dimensional maximally entangled Bell state is enough to teleport some pairs of these gates. For example, in the four-dimensional case, $T_{B_1}^{A_2B_2}$ and $C_{B_2}^{A_2}$ can be teleported simultaneously with twice the time of the fast proposal using non-simultaneous TWCC. Therefore, a trade off occurs between the time and entanglement resources for nonlocal quantum computing with TWSCC$^{\leftrightarrow}$.

A $2^n$-dimensional CF gate operation also can be identified by means of the gate operation sequence as

$$CF_{2^n} = CX_{2^n} \otimes CX_{2^n}^2 \otimes CX_{2^n}^4 \otimes \cdots \otimes CX_{2^n}^{2^{n-1}} \tag{17}$$

where $CX_{2^n}^k$ (k = 1,2,4, ...$2^{n-1}$) represents the controlled flip operation between a two-dimensional qubit state of the $i$-th atom $\Lambda_i$ ($i$ = $log_2^k + 1$) and a $2^n$-dimensional photon in the spatial-mode DOF. Any $2^n$-dimensional nonlocal $CX_{2^n}^k$ operation can be constructed by the 2-dimensional maximally entangled quantum channel and local $CX_{2^n}^k$ operation on the hybrid system. Thus, $n$ two-dimensional maximally entangled quantum channels are enough to teleport a $2^n$-dimensional CF gate with $2^n$-dimensional local quantum operation in the shortest time.

Although the TWSCC$^{\leftrightarrow}$ limits the capacities of entanglement-assisted quantum channels, the high-dimensional local operations of the quantum system, which exploit the superposition between independent channels, can overcome this deficiency to avoid wasting additional nonlocal entanglement resources. Therefore, with $n$ two-dimensional maximally entangled quantum channels, one can deterministically teleport at most $n(n+1)/2$ two-body or multi-body nonlocal quantum gates with a two-dimensional nonlocal controller, in which any two of these gates can not be paralleled within only one two-dimensional maximally entangled quantum channel. If parallel channels are not applicable, only $n$ two-body or multi-body nonlocal quantum gates of Eq.(16) with a fast protocol can be transferred, and then gating transfer capacity of $n$ independent two-dimensional maximally

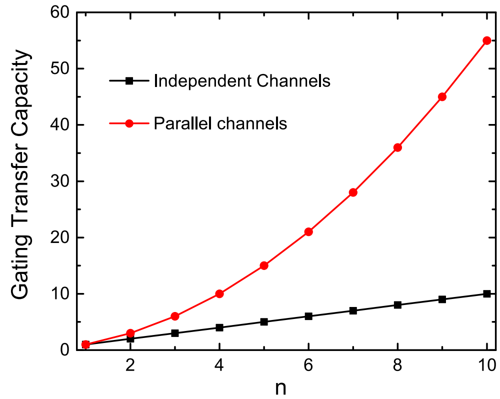



**FIGURE 3.** The gating transfer capacity of quantum-gate teleportation scheme with n independent or n parallel two-dimensional maximally entangled quantum channels using TWSCC$^{\leftrightarrow}$.

entangled channels is only proportional to $n$. Thus, high-dimension quantum computing teleportation can utilize two-dimensional maximally entangled resources with a higher capacity than the previous parallel two-dimensional quantum computing scheme [14]. To vividly demonstrate the merits of our scheme, we calculate the gating transfer capacity of the present scheme as pictured in Fig. 3. The black curve "independent channels" represents the number of nonlocal gates that can be teleported by $n$ independent two-dimensional maximally entangled quantum resources, whereas the red line "parallel channels" indicates that of our scheme. Apparently, in our scheme, the gating transfer capacity of $n$ parallel two-dimensional maximally entangled quantum channels can be improved remarkably. Furthermore, less time is required in a nonlocal computation manner, and it is on a shorter temporal scale for the experimental realization. According to the preceding discussion, under limited prior-shared maximally entangled resources, nonlocal high-dimensional quantum control-operation outperforms the traditional method, which decomposes the high-dimensional Hilbert space into two-dimensional quantum space, with high efficiency and short communication time.

## V. SUMMARY

In summary, the capacity of the high-dimensional quantum channel for the operation has been discussed. We have proposed a scheme for the nonlocal quantum computation. By using the atom-microresonator coupling local system and hyperentanglement, we have conducted the high-dimensional nonlocal quantum computation between two different quantum network nodes, thereby greatly improving the efficiency of nonlocal and long-distance quantum computing. Compared with the previously proposed schemes that often use photon as a single flying qubit to transmit interaction between two separate nodes, our method consists of an entangled qubit pair as a quantum channel. Thus, environmental noise can be suppressed by entanglement purification operation. Moreover, the present scheme can be effectively combined with the long-distance quantum communication proposals, such as hyperentanglement purification and hyper-entanglement repeater schemes.

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**WEN-LING XU** received the B.S. degree in physics from Xidian University, Xi'an, China, in 2016. She is currently pursuing the Ph.D. degree with the Beijing University of Posts and Telecommunications, Beijing.

**TIE-JUN WANG** received the B.S. degree in physics from the Beijing Institute of Technology, Beijing, China, in 2006, and the Ph.D. degree in physics from Beijing Normal University, Beijing, in 2011. She is currently an Associate Professor with the Beijing University of Posts and Telecommunications, Beijing.

**CHUAN WANG** received the B.S. degree in physics from Shandong University, Jinan, China, in 2003, and the Ph.D. degree in physics from Tsinghua University, Beijing, in 2008. He is currently a Professor with the Beijing University of Posts and Telecommunications, Beijing.