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A Novel Failure Mode and Effects Analysis Method Based on Fuzzy Evidential Reasoning Rules

WEN JIANG, ZHIPENG ZHANG, AND XINYANG DENG
School of Electronics and Information, Northwestern Polytechnical University, Xi'an 710072, China

Corresponding author: Xinyang Deng (xinyang.deng@nwpu.edu.cn)

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ABSTRACT Failure mode and effects analysis (FMEA) is an effective reliability analysis technique and has been used for safety and dependability analysis in a wide range of fields. In the traditional FMEA, a method called risk priority number (RPN) has been widely used to determine the risk levels of failure modes. However, the method is deficient in dealing with imprecise data. To overcome that shortcoming, we propose a novel method based on fuzzy evidential reasoning rules to study the risk evaluation of failure modes in an uncertainty evaluation environment. The main contributions of this work are twofold: First, by analyzing the classical risk priority number method, we extract the reasoning knowledge from RPN method to construct fuzzy evidential reasoning rules for risk evaluation based on virtue of Dempster-Shafer evidence theory and fuzzy set theory; Second, the initial risk assessment is modeled with fuzzy form based on basic probability assignment (BPA) and fuzzy number, which can perfectly reflect the uncertainties in practice. The approach establishes a new reasoning model for fuzzy risk evaluation in FMEA. Finally, an example for risk evaluation of failure modes during general anesthesia process is given to illustrate the effectiveness of the proposed method.

INDEX TERMS Dependability, failure mode and effect analysis, Dempster-Shafer evidence theory, fuzzy set theory, fuzzy evidential reasoning rule, risk priority number.

I. INTRODUCTION

Dependability is the most important property of safety critical systems that are used in many industries, including the aerospace, medical, and automotive and whose failure has the catastrophic effects on human life. The concept of dependability is defined as "the ability of the system to deliver services that can justifiably be trusted" [1]. In systems engineering field, dependability includes several analysis tools such as Fault Tree Analysis (FTA) [2], Preliminary hazard analysis (PHA) [3], Failure mode, effects and criticality analysis (FMECA), hazard and operability studies (HAZOPs) [4] and Failure mode and effect analysis (FMEA) [5]. As Liuz [6] mentioned, "The interviewed practitioners most frequently cited FMEA, FTA, and risk assessment. However, the most mentioned approach was FMEA (66 percent), not FTA (33 percent)". There are two reasons for this: First, it provides a documented method for assessing potential failure mechanisms, failure modes and their impact on system operation, resulting in a list of failure modes ranked and selecting a design with a high probability of successful operation and safety; Second, FMEA is a criteria for early planning of tests and an effective method for evaluating the effect of proposed changes to the design and operational procedures on mission success and safety. We take the requirements into account and assemble people with different experience to do the risk analysis. We make the FMEA on the requirements and try to find those that impact the safety of the system or product [6].

Failure mode and effect analysis (FMEA), as a systematic dependability analysis technique, is used to identify the components most likely to cause failures. Failure mode refers to the form of system failure, and effect analysis is to study the effect of component failure on the system.

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The theory of FMEA provides a basis for designers to develop improvements or upgrades by analyzing and identifying various potential failure modes, failure causes, and impacts in the system, thereby reducing post-loss losses and improving system reliability [7]–[9]. As an ex ante approach, FMEA aims to identify the preventive measures or improvement schemes and improve the reliability of system by risk assessment and ranking of possible failure modes of the system [10]–[13].

In the traditional FMEA, a method called risk priority number (RPN) is widely used to find high-risk failure modes. The method is simple in structure and easy to understand and implementation. Although RPN method is an attractive risk evaluation tool in FMEA, it still shows several weak points when applied in actual industrial situations, for example the possible missing of risk factors, without considering different importance between risk factors and so on, especially when criticality analysis is conducted using risk priority number. Hence, many new risk evaluation methods have been developed to overcome the weaknesses of conventional RPN, such as data envelopment analysis (DEA) [14]–[16], the technique for order of preference by similarity to ideal solution (TOPSIS) [17]–[19], VIKOR [20]–[22], multi-attribute failure mode analysis [23], decision making trial and evaluation laboratory (DMETEL) [24]–[26] and hybrid approaches [27]–[29]. Moreover, different failure analysis models based on uncertainty theories have been proposed in literature. Zammori and Gabbrielli [30] presented an advanced version of the failure mode, effects and criticality analysis (FMECA) [31], [32], which enhances the capabilities of the standard FMECA by taking into account possible interactions among the principal causes of failure in the criticality assessment. Wang *et al.* [33] proposed fuzzy risk priority numbers for prioritization of failure modes to deal with the problem that it may be inaccuracy in real applications to determine the risk priorities of failure modes using the RPN. Chin *et al.* [15] presented an FMEA approach based on the evidential reasoning approach that is useful to model the diversity and uncertainty of the assessment information and the evaluation of risk factors is the combination of risk levels of RPN with their corresponding percentages.

In the risk evaluation of FMEA, domain experts' knowledge and evaluation play a very important role. The evaluation is always described by crisp numbers in many methods of FMEA, especially RPN. RPN is a simple and effective approach. However, the crisp number is hardly given precisely by domain experts for risk evaluation in real and it loses much uncertainty information so that the evaluation result is less accurate. At the same time, the evaluation from human is always subjective and imprecise. Then how to reduce negative effectiveness of these uncertainties? Various theories have been proposed to solve the problem, such as evidence theory [34], belief entropy [35]–[37], belief function [38], [39], fuzzy set theory [40], D-number [41], Z-number [42] and so on [43]. In this study, we try to address the problem from a perspective of fuzzy evaluation. The risk factors are combined with occurrence, severity and detection like RPN. Linguistic item expressed by fuzzy set is used to describe the level of risk factor while the crisp number is used in RPN. In order to study the risk evaluation under uncertain circumstances, in this paper a novel failure mode and effects analysis method is proposed based on fuzzy evidential reasoning rules. The steps of the proposed method are listed as follows: First, we construct fuzzy evidential reasoning rules by analyzing the RPN method; Second, the risk evaluation expressed in basic probability assignment (BPA) for every failure mode is given by experts, then we can obtain reasoning evaluation results by fuzzy evidential reasoning based on the rules; Third, all failure modes are ranked in decreasing order according to the value that is obtained by defuzzifying the evaluation result. At last, the proposed method is validated through an illustrative example.

The rest of this paper is organized as follows. In Section II, the method's related concepts are briefly presented. In Section III, we introduce the main frame of proposed method and the main steps to construct a base of fuzzy evidential reasoning rules. In section IV, one example is provided to illustrate the proposed model. Furthermore, some comparisons and analysis with other methods are given to confirm the effectiveness of the proposed method. Finally, conclusions of this paper are presented in Section V.

II. PRELIMINARIES

A. RPN METHOD IN FMEA

FMEA is a structural and preventive reliability analysis approach that starts with known potential failure modes at one level, and investigates their effects on the next and higher level of system hierarchy [44]. In FMEA, the first is to identify all possible potential failure modes of the product or system. After that, analyze each failure mode with three risk factors: occurrence (O), severity (S) and detection (D), where O is the probability of the failure, S is the severity of the failure, and D is the probability of not detecting the failure. The analysis results can help analysts to identify and correct the failure modes that have a detrimental effect on the system and improve its performance during the stages of design and production [45].

The risk priority number (RPN) approach is usually used in FMEA to determine the prioritization of failure modes. Assuming V is the risk level of a failure mode, it is defined as

$$V = O \times S \times D. \tag{1}$$

For obtaining the risk number of a potential failure mode, three risk factors are evaluated using 10-point scales described in Table 1, Table 2 and Table 3 [46]–[48]. The higher the risk number of a failure mode, the greater the risk is for product system reliability. With respect to the risk numbers, failure modes can be ranked and then proper actions will be preferentially taken on the high-risk failure modes.

It can be found that the RPN method is a simple, intuitive and easy to operate and implement. However, there are some important defects or problems in this method that

TABLE 1. Suggested ratings for the occurrence (*O*) of a failure mode.

Probability of failure	Possible failure rates	Rank
Extremely high: failure almost inevitable	⩾ 1 in 2	10
Very high	1 in 3	9
Repeated failures	1 in 8	8
High	1 in 20	7
Moderately high	1 in 80	6
Moderate	1 in 400	5
Relatively low	1 in 2000	4
Low	1 in 15,000	3
Remote	1 in 150,000	2
Nearly impossible	⩽ 1 in 1,500,000	1

TABLE 2. Suggested ratings for the severity (*S*) of a failure mode.

Effect	Criteria: severity of effect	Rank
Hazardous	Failure is hazardous, and occurs without warning. It suspends operation of the system and/or involves noncompliance with government regulations	10
Serious	Failure involves hazardous outcomes and/or noncompliance with government regulations or standards	9
Extreme	Product is inoperable with loss of primary function. The system is inoperable	8
Major	Product performance is severely affected but functions. The system may not operate	7
Significant	Product performance is degraded. Comfort or convince functions may not operate	6
Moderate	Moderate effect on product performance. The product requires repair	5
Low	Small effect on product performance. The product does not require repair	4
Minor	Minor effect on product or system performance	3
Very minor	Very minor effect on product or system performance	2
None	No effect	1

TABLE 3. Suggested ratings for the detection (*D*) of a failure mode.

Detection	Criteria: likelihood of detection by design control	Rank
Absolute uncertainty	Design control does not detect a potential cause of failure or subsequent failure mode; or there is no design control	10
Very remote	Very remote chance the design control will detect a potential cause of failure or subsequent failure mode	9
Remote	Remote chance the design control will detect a potential cause of failure or subsequent failure mode	8
Very low	Very low chance the design control will detect a potential cause of failure or subsequent failure mode	7
Low	Low chance the design control will detect a potential cause of failure or subsequent failure mode	6
Moderate	Moderate chance the design control will detect a potential cause of failure or subsequent failure mode	5
Moderately high	Moderately high chance the design control will detect a potential cause of failure or subsequent failure mode	4
High	High chance the design control will detect a potential cause of failure or subsequent failure mode	3
Very high	Very high chance the design control will detect a potential cause of failure or subsequent failure mode	2
Almost certain	Design control will almost certainly detect a potential cause of failure or subsequent failure mode	1

need to be further solved [49]. For example, the impact of uncertainty on the evaluation process is not considered and the difference of their importance between three indicators, occurrence, severity and detection, is not considered. Many studies have provided improved methods for the above problems. For example, the three factors, occurrence, severity, and detection, are respectively given weights, the evaluation level, 1-10, is converted into a fuzzy linguistic term to reflect the uncertainty in the evaluation, and so on [49]–[53].

B. DEMPSTER-SHAFER EVIDENCE THEORY

Dempster-Shafer evidence theory (D-S evidence theory), introduced by Dempster [34] first and expanded by Shafer [54] later, is used to deal with the problem of uncertainty [37], [55] and widely used in decision making [56].

In the evidence theory, Θ called the frame of discernment (FOD) [57] is defined as a sample space [58]. It is composed of N exhaustive and exclusive hypotheses as follows

$$\Theta = \{H_1, H_2, \ldots, H_i, \ldots, H_N\}. \tag{2}$$

The power set of Θ is the set containing all the possible subsets of Θ, represented by 2^Θ. This set consists of 2^N elements: $2^\Theta = \{\emptyset, H_1, H_2, \ldots, H_N, \{H_1 \cup H_2\}, \ldots, \{H_1 \cup H_2 \cup \cdots \cup H_i\}, \ldots, \Theta\}$. For a FOD Θ, a basic probability assignment (BPA) is a function m from 2^Θ to [0,1], formally defines as

$$m : 2^\Theta \to [0, 1]. \tag{3}$$

The function m is also called a mass function. It must satisfy the following condition:

$$\sum_{A \in 2^\Theta} m(A) = 1, \quad m(\emptyset) = 0. \tag{4}$$

The mass $m(A)$ represents how strongly the evidence supports A [59]. For each subset $A \subseteq \Theta$, it is called a focal element of m if $m(A) > 0$. To deal with the uncertain data effectively, some aspects of D-S evidence theory have been developed well, including combination rule [60], [61] and conflict management [62].

In order to make decision in terms of BPA, an approach, called pignistic probability transformation (PPT), is proposed by Smets and Kennes [63] to derive a probability distribution from BPA. Let m be a mass function or BPA on FOD Θ, a PPT function $BetP_m : \Theta \to [0, 1]$ associated to m is defined by

$$BetP_m(x) = \sum_{x \in A, A \subseteq \Theta} \frac{1}{|A|} \frac{m(A)}{1 - m(\emptyset)}, \tag{5}$$

where $m(\emptyset) \neq 1$ and $|A|$ is the cardinality of proposition A.

Considering two pieces of evidence indicated by m_1 and m_2, Dempster's combination rule can be used to combine them and is defined as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B) m_2(C), & A \neq \emptyset; \\ 0, & A = \emptyset. \end{cases} \quad (6)$$

$$\text{where} \quad K = \sum_{B \bigcap C = \varnothing} m_1(B) m_2(C) \quad (7)$$

Assuming there are two BPAs m_1 and m_2 defined on $\Theta = \{a, b\}$:

$$m_1 : m_1(\{a\}) = 0.6, \quad m_1(\{b\}) = 0, \; m_1(\{a,b\}) = 0.4$$
$$m_2 : m_2(\{a\}) = 0.7, \quad m_2(\{b\}) = 0.2, \; m_2(\{a,b\}) = 0.1$$

According to Dempster's combination rule, the combined BPA m could be obtained as follows:

$$\begin{aligned} K &= m_1(\{a\}) \times m_2(\{b\}) + m_1(\{b\}) \times m_2(\{a\}) \\ &= 0.6 \times 0.2 + 0 = 0.12 \\ m(\{a\}) &= \frac{1}{1-K} \times \{m_1(\{a\}) \times m_2(\{a\}) \\ &\quad + m_1(\{a\}) \times m_2(\{a,b\}) + m_1(\{a,b\}) \times m_2(\{a\})\} \\ &= \frac{0.6 \times 0.7 + 0.6 \times 0.1 + 0.4 \times 0.7}{1 - 0.12} = 0.86 \\ m(\{b\}) &= \frac{1}{1-K} \times \{m_1(\{b\}) \times m_2(\{b\}) \\ &\quad + m_1(\{b\}) \times m_2(\{a,b\}) + m_1(\{a,b\}) \times m_2(\{b\})\} \\ &= \frac{0 + 0 + 0.4 \times 0.2}{1 - 0.12} = 0.09 \\ m(\{a,b\}) &= \frac{m_1(\{a,b\}) \times m_2(\{a,b\})}{1-K} \\ &= \frac{0.4 \times 0.1}{1 - 0.12} = 0.05. \end{aligned}$$

C. FUZZY SET THEORY

Fuzzy set theory was first introduced by Zadeh [40] in 1965 to deal with the uncertainty information [64]. It provides an efficiently simple way to express the vagueness or imprecise information for the situation in which subjective concepts are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [65].

Definition 1: Let X be the universe of discourse, a fuzzy set A is characterized by a membership function μ_A satisfying

$$\mu_A : X \rightarrow [0, 1]$$

where $\mu_A(x)$ is called the membership degree of $x \in X$ belonging to fuzzy set A.

Fuzzy number is defined in different forms depending on the characteristics of the problem. Triangular and trapezoidal fuzzy numbers are two of most widely used fuzzy numbers.

While $x, a_1, a_2, a_3 \in \mathbb{R}$, a triangular fuzzy number is usually denoted as $A = (a_1, a_2, a_3)$ shown in Figure 1, which

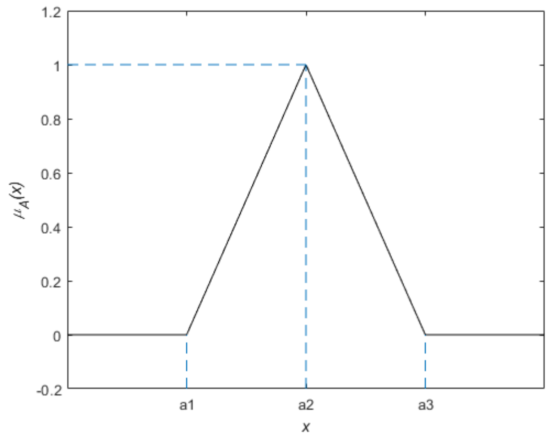


FIGURE 1. The membership function of triangular fuzzy number ***A***.

could be defined by a membership function μ_A as follows:

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 < x < a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x < a_3 \\ 0, & otherwise \end{cases} \quad (8)$$

In a triangular fuzzy number $A = (a_1, a_2, a_3)$, the element a_2 gives the maximal degree of membership, i.e. $\mu_A(a_2) = 1$, meaning that a_2 is the value with the highest degree of membership. At the same time, a_1 and a_3 are the lower and upper bound of the evaluation data, respectively.

III. PROPOSED METHOD

In the traditional RPN method, the risk factors were assessed with crisp numbers. However, because of the increasing complexity of system and the lack of knowledge, they may be not easy to be precisely evaluated in the real situation. Therefore in the paper, combining D-S evidence theory, fuzzy set theory and reasoning rules for the risk assessment of failure modes, a novel method is presented based on fuzzy evidential reasoning rules. The main steps are as follows shown in Figure 2 which contains four phases, including constructing rules, evaluation, process and rank.

A. TRANSFORM EACH RPN RECORD TO A FUZZY EVIDENTIAL REASONING RECORD

In this step, the knowledge suggested in RPN approach will be transformed to fuzzy evidential reasoning rules. A fuzzy evidential reasoning record is an IF-THEN rule whose components are BPAs defined on FOD consisting of fuzzy linguistic items. In the RPN, every risk factor is evaluated by a crisp number. The evaluation result, V, is obtained according to $V = O \times S \times D$. The four numbers construct a RPN record, which include three risk factor evaluation values and the corresponding result's value. Because of the uncertainty in the process of evaluation, fuzzy linguistic term, which is represented by triangular fuzzy number, is used to describe the damage degree of all factors in RPN to reduce uncertainty

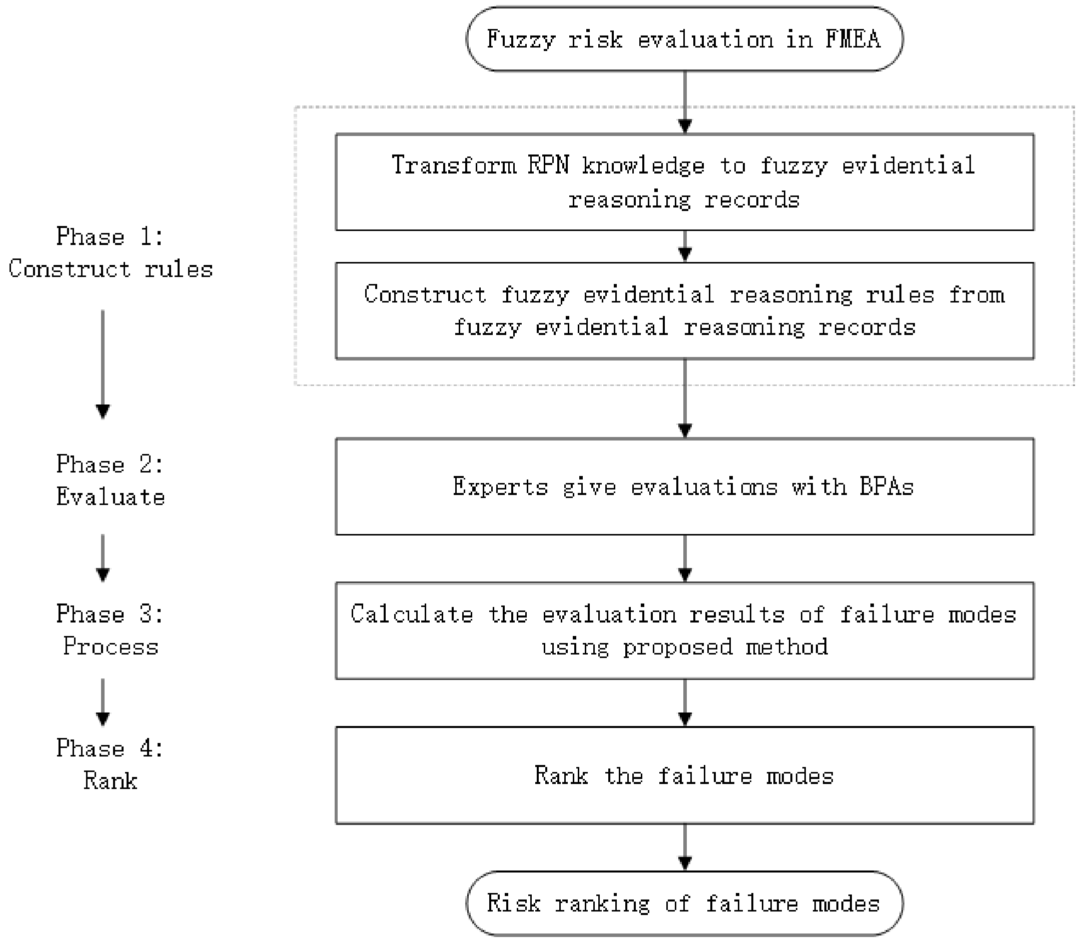


FIGURE 2. Flowchart of the proposed method for fuzzy risk evaluation.

effect. Then a RPN record is transformed to a fuzzy evidential reasoning record. The following are the main steps.

Step 1: define fuzzy linguistic terms to represent expert's evaluation on O, S, D and V

The three risk factors O, S and D are in the same range of values, so in this paper the same sample space is set for them and their three fuzzy linguistic terms are defined: Very Low (VL), Low (L), Middle Low (ML), Middle (M), Middle High (MH), High (H) and Very High (VH). For V, there are seven fuzzy linguistic terms: Very Good (VG), Good (G), Middle Good (MG), Fair (F), Middle Bad (MB), Bad (B) and Very Bad (VB). They can be expressed with FOD as follows:

$$\Theta_O = \Theta_S = \Theta_D = \{VL, L, ML, M, MH, H, VH\}$$
$$\Theta_V = \{VG, G, MG, F, MB, B, VB\}$$

In this paper, every fuzzy linguistic term is represented by a triangular fuzzy number given in Table 4 and Table 5. Then the membership function [66] of O, S, D is shown in Figure 3 and that of V is in Figure 4.

Step 2: list RPN records according to $V = O \times S \times D$. O, S and D range from 1 to 10. According to $V = O \times S \times D$, we get $V \in [1, 1000]$. So there are 1000 pieces of RPN record listed in Table 6.

TABLE 4. Linguistic item for evaluation of O, S, D.

Linguistic item	Fuzzy number
VL	(1.0,1.0,2.5)
L	(1.0, 2.5, 4.0)
ML	(2.5,4.0,5.5)
M	(4.0,5.5,7.0)
MH	(5.5,7.0,8.5)
H	(7.0,8.5,10.0)
VH	(8.5,10.0,10.0)

TABLE 5. Linguistic item for evaluation of V.

Linguistic item	Fuzzy number
VG	(1.0, 1.0, 167.5)
G	(1.0, 167.5, 334.0)
MG	(167.5, 334.0, 500.5)
F	(334.0, 500.5, 667.0)
MB	(500.5, 667.0, 833.5)
B	(667.0, 833.5, 1000.0)
VB	(833.5, 1000.0, 1000.0)

Step 3: transform the values of O, S, D and V to BPAs for each RPN record.

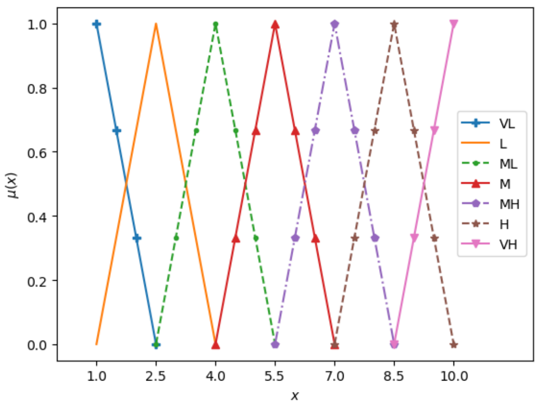


FIGURE 3. The membership function of risk factor (*O*, *S* or *D*).

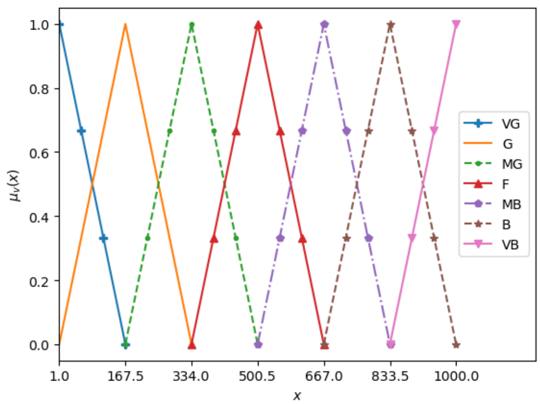


FIGURE 4. The membership function of *V*.

TABLE 6. RPN records.

No.	O	S	D	V
1	1	1	1	1
2	1	1	2	2
3	1	1	3	3
⋮	⋮	⋮	⋮	⋮
1000	10	10	10	1000

According to defined membership functions of fuzzy linguistic terms for O, S, D and V, one number x always corresponds to two levels $\widetilde{A}_1$ and $\widetilde{A}_2$. Assume $h_1 + h_2 = 1$, $\mu_{\widetilde{A}_1}(x) = h_1$ and $\mu_{\widetilde{A}_2}(x) = h_2$, then a mass function could be constructed as follows:

$$m(\widetilde{A}_1) = h_1, \quad m(\widetilde{A}_1, \widetilde{A}_2) = h_2, \; while \; h_1 \geq h_2 \tag{9}$$

$$m(\widetilde{A}_2) = h_2, \quad m(\widetilde{A}_1, \widetilde{A}_2) = h_1, \; while \; h_1 < h_2 \tag{10}$$

For example in Figure 5, $\mu_{VL}(2.0) = 0.333$, $\mu_L(2.0) = 0.667$ while $x = 2.0$. The BPA is obtained as $m(\{L\}) = 0.667, m(\{VL, L\}) = 0.333$.

In this way, the values of each RPN record are transformed to the corresponding record of BPAs(m_o, m_s, m_d, m_v). Through the above steps, for example, RPN record "IF $O = 1, S = 2, D = 10$ THEN $V = 20$" is transformed

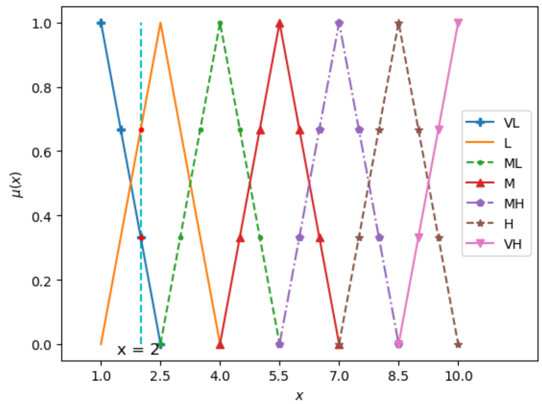


FIGURE 5. The example of transforming a crisp number to BPAs.

to the fuzzy evidential reasoning record:

$$\begin{aligned} IF \quad & m_o(\{VL\}) = 1.0 \\ & \wedge m_s(\{L\}) = 0.667, \quad m_s(\{VL, L\}) = 0.333 \\ & \wedge m_d(\{VH\}) = 1.0 \\ THEN \quad & m_v(\{VG\}) = 0.886, \quad m_v(\{VG, G\}) = 0.114. \end{aligned} \tag{11}$$

B. CONSTRUCT A BASE OF FUZZY EVIDENTIAL REASONING RULES

In the above steps, original knowledge in RPN method has been transformed to fuzzy evidential reasoning records. The record is still uncertain knowledge. In this section, we will transform uncertain fuzzy evidential reasoning records to certain fuzzy evidential reasoning rules, and for every record transform its uncertain property to corresponding rule's weight. A fuzzy evidential reasoning rule is an IF-THEN rule. Aggregate rules with the same antecedent, then construct a base of fuzzy evidential reasoning rules.

Step 1: decompose fuzzy evidential reasoning records to weighted fuzzy evidential reasoning rules

In a weighted fuzzy evidential reasoning rule, take a focal element from every BPA which is risk factor's evaluation result in the order of O, S, D. Then three focal elements are combined as antecedent of a rule and their product is taken as the rule's weight. For example, the fuzzy evidential reasoning record in (11) is transformed to the following weighted fuzzy evidential reasoning rules:

Rule 1:

$IF \quad (O = VL) \wedge (S = L) \wedge (D = VH),$

$THEN \quad m_{v_1}(\{VG\}) = 0.886, \; m_{v_1}(\{VG, G\}) = 0.114$

$\quad with\ a\ weight\ 1.0 \times 0.667 \times 1.0 = 0.667;$

Rule 2:

$IF \quad (O = VL) \wedge (S = \{VL, L\}) \wedge (D = VH),$

$THEN \quad m_{v_2}(\{VG\}) = 0.886, \; m_{v_2}(\{VG, G\}) = 0.114$

$\quad with\ a\ weight\ 1.0 \times 0.333 \times 1.0 = 0.333.$

Step 2: aggregate weighted fuzzy evidential reasoning rules

TABLE 7. A assumed weighted fuzzy evidential reasoning rules' base.

Rule	Antecedent	Consequent	Weight
Rule 1	$(O = VL) \wedge (S = L) \wedge (D = VH)$	$m_{v_1}(\{VG\}) = 0.886, m_{v_1}(\{VG, G\}) = 0.114$	$w_1 = 0.667$
Rule 2	$(O = VL) \wedge (S = \{VL, L\}) \wedge (D = VH)$	$m_{v_2}(\{VG\}) = 0.886, m_{v_2}(\{VG, G\}) = 0.114$	$w_2 = 0.333$
Rule 3	$(O = VL) \wedge (S = L) \wedge (D = VH)$	$m_{v_3}(\{VG\}) = 0.826, m_{v_3}(\{VG, G\}) = 0.174$	$w_3 = 0.667$
Rule 4	$(O = VL) \wedge (S = \{ML, L\}) \wedge (D = VH)$	$m_{v_4}(\{VG\}) = 0.826, m_{v_4}(\{VG, G\}) = 0.174$	$w_4 = 0.333$
Rule 5	$(O = VL) \wedge (S = ML) \wedge (D = VH)$	$m_{v_5}(\{VG\}) = 0.766, m_{v_5}(\{VG, G\}) = 0.234$	$w_5 = 1.0$

In this step, the weighted rules with the same antecedent will be aggregated to a fuzzy evidential reasoning rule. First, for a antecedent, gather all rules with that antecedent and normalize their weights to obtain new weights. Next weight every rule's consequent using its new weight. Then aggregate those weighted consequents with same focal element by summing them up to obtain the focal element's corresponding mass. Finally, we get a new BPA that also is the consequent of a rule. By the above steps, we aggregate original weighted rules and obtain new fuzzy evidential reasoning rules. These rules could be constructed a conditional mass table.

For example, assume the weighted rules listed in the Table 7 is constructed a complete weighted fuzzy evidential reasoning rules' base. The rules are combined with two rules of the above example and the following:

RPN record 2:

IF $O = 1,\ S = 3, D = 10$ THEN $V = 30$

Transformer:

corresponding fuzzy evidential reasoning record:

IF $m_o(\{VL\}) = 1.0$

$\wedge\, m_s(\{L\}) = 0.667,\ m_s(\{ML, L\}) = 0.333$

$\wedge\, m_d(\{VH\}) = 1.0$

THEN $m_v(\{VG\}) = 0.826,\ m_v(\{VG, G\}) = 0.174.$

Transformer:

corresponding fuzzy evidential reasoning rules:

Rule 3:

IF $(O = VL) \wedge (S = L) \wedge (D = VH),$

THEN $m_{V_3}(\{VG\}) = 0.826, m_{V_3}(\{VG, G\}) = 0.174$

with a weight 0.667;

Rule 4:

IF $(O = VL) \wedge (S = \{ML, L\}) \wedge (D = VH),$

THEN $m_{v_4}(\{VG\}) = 0.826, m_{v_4}(\{VG, G\}) = 0.174$

with a weight 0.333.

RPN record 3:

IF $O = 1,\ S = 4,\ D = 10$ THEN $V = 40$

Transformer:

corresponding fuzzy evidential reasoning record:

IF $m_o(\{VL\}) = 1.0$

$\wedge\, m_s(\{ML\}) = 1.0$

$\wedge\, m_d(\{VH\}) = 1.0$

THEN $m_v(\{VG\}) = 0.766,\ m_v(\{VG, G\}) = 0.234.$

Transformer:

corresponding fuzzy evidential reasoning rules:

Rule 5:

IF $(O = VL) \wedge (S = ML) \wedge (D = VH),$

THEN $m_{v_5}(\{VG\}) = 0.766,\ m_{v_5}(\{VG, G\}) = 0.234$

with a weight 1.0;

Assume N rules has the same antecedent and their consequents are made up of a weighted BPA set $\{m_{v_i}\ \textit{with a weight}\ w_i, i = 1, \ldots, N\}$. These rules would be aggregated a new rule. The new rule's antecedent is that antecedent. Its consequent m'_v in (13) can be calculated by weighted average method with the new weight w'_i] in (12).

$$w'_i = \frac{w_i}{\sum_{j=1}^{N} w_j} \tag{12}$$

$$m'_v(l) = \sum_{i=1}^{N} m_{v_i}(l) \times w'_i, \quad l \in 2^{\Theta_V} \tag{13}$$

Of them, Rule 1 and 3 with same antecedent $(O = VL) \wedge (S = L) \wedge (D = VH)$ would be aggregated to obtain a new fuzzy evidential reasoning rule. The antecedent of the new rule is $(O = VL) \wedge (S = L) \wedge (D = VH)$, and its consequent is expressed with a mass function m'_{v_1} according to (12) and (13) as follows:

$$\begin{aligned} w'_1 &= \frac{w_1}{w_1 + w_3} = 0.5 \\ w'_3 &= \frac{w_3}{w_1 + w_3} = 0.5 \\ m'_{v_1}(\{VG\}) &= m_{v_1}(\{VG\}) \times w'_1 + m_{v_3}(\{VG\}) \times w'_3 \\ &= 0.856 \\ m'_{v_1}(\{VG, G\}) &= m_{v_1}(\{VG, G\}) \times w'_1 \\ &\quad + m_{v_3}(\{VG, G\}) \times w'_3 = 0.144 \end{aligned}$$

So we get a fuzzy evidential reasoning rule expressed as follows:

IF $(O = VL) \wedge (S = L) \wedge (D = VH),$

THEN $m'_{v_1}(\{VG\}) = 0.856, m'_{v_1}(\{VG, G\}) = 0.144.$

like the above steps, all new rules are obtained as follows. They could be constructed a conditional mass table which is

TABLE 8. Crisp numbers of V's fuzzy linguistic terms.

linguistic item	VG	G	MG	M	MB	B	VB
defuzzified value(α)	56.5	167.5	334	500.5	667	833.5	944.5

based on the assumed rules' base.

$Rule1$:
$IF\ (O = VL) \wedge (S = L) \wedge (D = VH)$,
$THEN\ m'_{v_1}(\{VG\}) = 0.856, m'_{v_1}(\{VG, G\}) = 0.144$.
$Rule2$:
$IF\ (O = VL) \wedge (S = \{VL, L\}) \wedge (D = VH)$,
$THEN\ m'_{v_2}(\{VG\}) = 0.886, m'_{v_2}(\{VG, G\}) = 0.114$.
$Rule3$:
$IF\ (O = VL) \wedge (S = \{ML, L\}) \wedge (D = VH)$,
$THEN\ m'_{v_3}(\{VG\}) = 0.826, m'_{v_3}(\{VG, G\}) = 0.174$.
$Rule4$:
$IF\ (O = VL) \wedge (S = ML) \wedge (D = VH)$,
$THEN\ m'_{v_4}(\{VG\}) = 0.766, m'_{v_4}(\{VG, G\}) = 0.234$.

C. EVALUATE THE FAILURE MODE IN TERMS OF THE FUZZY EVIDENTIAL REASONING RULES

Through the above steps, the conditional mass table for risk evaluation in FMEA has been established. Therefore if we collected the evaluation of risk factors O, S, D for a failure mode, its risk can be calculated, indicated by a BPA, via standard network reasoning process. Assume a failure mode is donated by $\{m_o(i) = a_i, i \in 2^{\Theta_O}\}$, $\{m_s(j) = b_j, j \in 2^{\Theta_S}\}$ and $\{m_d(k) = c_k, k \in 2^{\Theta_D}\}$. For a combination $\{i, j, k\}$, it is taken as a antecedent $(O = i) \wedge (S = j) \wedge (D = k)$, then look up the conditional mass table to get its corresponding consequent $\{m_{v_{i,j,k}}(l) = e_l, l \in 2^{\Theta_V}\}$ and calculate its weight $w_{i,j,k}$ with (14) at the same time. Finally, aggregate the consequents and corresponding weights and get a mass function m_v according to (15).

$$w_{i,j,k} = m_o(i) \times m_s(j) \times m_d(k) \tag{14}$$

$$w'_{i,j,k} = \frac{w_{i,j,k}}{\sum_i \sum_j \sum_k w_{i,j,k}}$$

$$m_v(l) = \sum_i \sum_j \sum_k m_{v_{i,j,k}}(l) \times w'_{i,j,k} \tag{15}$$

D. RANK THE FAILURE RULES BY DEFUZZIFYING THE BPA OF EACH FAILURE MODE

In the above section, all failure modes' evaluations are obtained that are described with BPAs. To rank failure modes easily, a RPN is usually required. What we need to do is to reveal the values that linguistic terms take. At first, transform each evaluation result, which is represented by a BPA, to a probability distribution $Prob$ using PPT for each failure mode. Next, defuzzify each fuzzy linguistic term to crisp number in order to rank all failure modes. While A is a triangular fuzzy number indicated by (a_1, a_2, a_3), the defuzzified value α_A is obtained using the following formula:

$$\alpha_A = (a_1 + a_2 + a_3)/3. \tag{16}$$

For the triangular fuzzy numbers defined for V, their corresponding crisp numbers are shown in Table 8.

As a result, the new risk priority number (Y) can be obtained using weighted mean method:

$$Y = \sum_{A \in \Theta} Prob_A \times \alpha_A. \tag{17}$$

By decreasing order of their results Y, the priority of all failure modes can be determined. More attention is necessary for the bigger.

IV. ILLUSTRATIVE EXAMPLE

A. IMPLEMENTATION

In the section, the proposed method is used to illustrate its effectiveness for fuzzy risk evaluation in FMEA on a case of ranking the most serious failure modes during general anesthesia process [21]. Six potential failure modes are identified that are donated as FM 1, FM 2, FM 3, FM 4, FM 5, FM 6 and their evaluations from five decision makers are given in Table 9. High risky failure modes should be corrected with top priorities in the result. In this paper, we will solve the problem by using our proposed model and compare our result with that of literature [21].

Step 1: experts give the evaluation for each failure mode. By analyzing the data in Table 9, we can transform them to BPAs that are fuzzy evidential reasoning records presented in Table 10.

Step 2: According to (14) and (15), the reasoning evaluation results listed in Table 11 could be obtained. Next, transform the evaluation results expressed with BPAs to probabilities using PPT. For example, the value of $Prob_G$ in FM 1, it is $0.735 = 0.573 + 0.316 \div 2 + 0.008 \div 2$. The results of all failure modes are shown in Table 12.

Step 3: determine the ranking order of all failure modes according to the decreasing order of Y. Transform probability distribution of each failure mode to a crisp number. According to (17), the evaluation result is calculated and shown in Table 13. As we can see, failure mode 3 would be at the top of the priority list of attention, followed by failure modes 2, 6, 5, 1 and 4.

B. RESULT DISCUSSION

The above steps have clearly shown the process of using the proposed method to do the risk evaluation under fuzzy environment. Next we will compare the above result with another risk evaluation result shown in Table 14 which is from literature [21] with an extended VIKOR method.

TABLE 9. Judgments on six failure modes by FMEA team members under risk factors [21].

Risk factors			O					S					D		
Team members	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5	DM1	DM2	DM3	DM4	DM5
FM 1	M	M	M	MH	M	ML	ML	ML	M	M	M	ML	ML	ML	ML
FM 2	H	MH	H	MH	MH	H	MH	H	H	H	M	M	ML	M	M
FM 3	VH	MH	VH	VH	VH	MH	MH	MH	MH	MH	MH	M	MH	MH	M
FM 4	M	M	L	M	M	M	M	ML	M	M	VL	ML	VL	ML	VL
FM 5	M	ML	M	M	M	M	MH	MH	M	M	L	ML	L	L	L
FM 6	MH	H	M	MH	M	H	H	H	H	H	L	M	L	L	VL

TABLE 10. The evaluation for failure modes.

Failure modes	O	S	D
FM 1	$m_o(\{M\}) = 0.8$ $m_o(\{MH\}) = 0.2$	$m_s(\{ML\}) = 0.6$ $m_s(\{M\}) = 0.4$	$m_d(\{M\}) = 0.2$ $m_d(\{ML\}) = 0.8$
FM 2	$m_o(\{H\}) = 0.4$ $m_o(\{MH\}) = 0.6$	$m_s(\{H\}) = 0.8$ $m_s(\{MH\}) = 0.2$	$m_d(\{M\}) = 0.8$ $m_d(\{ML\}) = 0.2$
FM 3	$m_o(\{VH\}) = 0.8$ $m_o(\{MH\}) = 0.2$	$m_s(\{MH\}) = 1.0$	$m_d(\{MH\}) = 0.6$ $m_d(\{M\}) = 0.4$
FM 4	$m_o(\{M\}) = 0.8$ $m_o(\{L\}) = 0.2$	$m_s(\{M\}) = 0.8$ $m_s(\{ML\}) = 0.2$	$m_d(\{VL\}) = 0.6$ $m_d(\{ML\}) = 0.4$
FM 5	$m_o(\{M\}) = 0.8$ $m_o(\{ML\}) = 0.2$	$m_s(\{M\}) = 0.6$ $m_s(\{MH\}) = 0.4$	$m_d(\{L\}) = 0.8$ $m_d(\{ML\}) = 0.2$
FM 6	$m_o(\{M\}) = 0.4$ $m_o(\{MH\}) = 0.4$ $m_o(\{H\}) = 0.2$	$m_s(\{H\}) = 1.0$	$m_d(\{L\}) = 0.6$ $m_d(\{M\}) = 0.2$ $m_d(\{VL\}) = 0.2$

TABLE 11. Reasoning evaluation results of all failure modes.

FM	BPA
FM 1	$m_{v_1}(\{G\}) = 0.573$, $m_{v_1}(\{VG, G\}) = 0.316$, $m_{v_1}(\{VG\}) = 0.101$, $m_{v_1}(\{G, MG\}) = 0.008$, $m_{v_1}(\{MG\}) = 0.002$
FM 2	$m_{v_2}(\{MG\}) = 0.604$, $m_{v_2}(\{G, MG\}) = 0.154$, $m_{v_2}(\{MG, F\}) = 0.093$, $m_{v_2}(\{F\}) = 0.067$, $m_{v_2}(\{G\}) = 0.083$
FM 3	$m_{v_3}(\{F\}) = 0.532$, $m_{v_3}(\{MG, F\}) = 0.129$, $m_{v_3}(\{MG\}) = 0.289$, $m_{v_3}(\{G\}) = 0.021$, $m_{v_3}(\{G, MG\}) = 0.028$
FM 4	$m_{v_4}(\{VG\}) = 0.584$, $m_{v_4}(\{VG, G\}) = 0.213$, $m_{v_4}(\{G\}) = 0.203$
FM 5	$m_{v_5}(\{VG\}) = 0.106$, $m_{v_5}(\{VG, G\}) = 0.270$, $m_{v_5}(\{G\}) = 0.624$, $m_{v_5}(\{G, MG\}) = 0.000$
FM 6	$m_{v_6}(\{VG\}) = 0.149$, $m_{v_6}(\{VG, G\}) = 0.182$, $m_{v_6}(\{G\}) = 0.470$, $m_{v_6}(\{G, MG\}) = 0.060$, $m_{v_6}(\{MG\}) = 0.114$, $m_{v_6}(\{MG, F\}) = 0.015$, $m_{v_6}(\{F\}) = 0.010$

TABLE 12. Aggregation results of all failure modes.

FM	Probability
FM 1	$Prob_G = 0.735$, $Prob_{VG} = 0.259$, $Prob_{MG} = 0.006$
FM 2	$Prob_{MG} = 0.727$, $Prob_G = 0.159$, $Prob_F = 0.114$
FM 3	$Prob_F = 0.597$, $Prob_{MG} = 0.367$, $Prob_G = 0.035$
FM 4	$Prob_{VG} = 0.691$, $Prob_G = 0.309$
FM 5	$Prob_{VG} = 0.241$, $Prob_G = 0.759$, $Prob_{MG} = 0.000$
FM 6	$Prob_{VG} = 0.240$, $Prob_G = 0.591$, $Prob_{MG} = 0.152$, $Prob_F = 0.018$

TABLE 13. Risk ranking of failure modes by *Y* in decreasing order.

failure mode	FM 1	FM 2	FM 3	FM 4	FM 5	FM 6
Y	139.82	326.37	427.51	90.84	140.81	172.19
By Y	5	2	1	6	4	3

In literature [21], failure modes are ranked by the value of O, S and Q in decreasing order. By S index, the risk ranking of all failure modes from high to low is FM 3 > FM 2 > FM 6 > FM 1 > FM 5 > FM 4. Compared with the ranking obtained by the proposed approach, the result is same in failure modes 2, 3, 4 and 6. Although the rank of failure mode 1 and 5 is different, their evaluation values 139.82 and 140.81 of Y are close. By R index, the failure mode with the highest risk is failure mode 6, and others are followed by failure mode 3, 2, 5, 1 and 4. Comparing the result with that of this article, there are 3 failure modes with the same order: FM 1, FM 4 and FM 5. By Q index, the failure modes with the highest risk and the lowest risk is respectively FM 3 and FM 4. The ranking is basically same with that of the proposed value Y. In addition, failure mode 4 is the lowest risk in the four sorted methods and FM 3 is the highest risk in the rankings obtained by R index, Q index and the proposed value Y respectively. In the view of the group, the failure modes can be classified two groups: the higher risk group composed by failure mode 2, 3 and 6 and the lower risk group composed by failure mode 1, 4 and 5. We can also obtain the same classification result by evaluating with the proposed model. Through the above analysis and comparison, it shows the proposed method is an effective way for risk evaluation in FMEA.

TABLE 14. Risk ranking of failure modes by using the extended VIKOR method [21].

failure mode	FM 1	FM 2	FM 3	FM 4	FM 5	FM 6
By S index	4	2	1	6	5	3
By R index	5	3	2	6	4	1
By Q index	5	3	1	6	4	2

V. CONCLUSION

In this paper, a novel method based on fuzzy evidential reasoning rules is proposed for the fuzzy risk evaluation in FMEA. This approach can overcome the shortcoming of the traditional FMEA, whose evaluation is too simple to reflect the uncertainty property perfectly. This study provides a new solution for the fuzzy risk evaluation in an uncertainty evaluation environment. In the proposed method, the initial assessment of failure mode would be described with BPAs to retain more uncertain information and obtain more precise result in the risk evaluation. In addition, by analyzing the classical risk priority number method, the fuzzy evidential reasoning rules are constructed for risk evaluation based on virtue of Dempster-Shafer evidence theory and fuzzy set theory. The effectiveness of the proposed method is verified by an example of ranking the most serious failure modes during general anesthesia process. It is especially useful in situations where it is almost impossible to make a crisp evaluation. Additionally, to verify its effectiveness further, applying the proposed FMEA should be considered in further research for risk management decision making in other fields of quality and reliability engineering.

REFERENCES

[1] A. Avizienis, J. C. Laprie, B. Randell, and C. Landwehr, "Basic concepts and taxonomy of dependable and secure computing," *IEEE Trans. Depend. Sec. Comput.*, vol. 1, no. 1, pp. 11–33, Jan. 2004.

[2] H. Tanaka, L. T. Fan, F. S. Lai, and K. Toguchi, "Fault-tree analysis by fuzzy probability," *IEEE Trans. Rel.*, vol. 32, no. 5, pp. 453–457, Dec. 2009.

[3] C. A. Ericson, II, *Hazard Analysis Techniques for System Safety*. Hoboken, NJ, USA: Wiley, 2015.

[4] J. Dunjó, V. Fthenakis, J. A. VÃílchez, and J. Arnaldos, "Hazard and operability (HAZOP) analysis. A literature review," *J. Hazardous Mater.*, vol. 173, nos. 1–3, pp. 19–32, 2010.

[5] D. H. Stamatis, *Failure Mode and Effect Analysis: FMEA from Theory to Execution*. Milwaukee, WI, USA: ASQ, 2003.

[6] L. E. G. Martins and T. Gorschek, "Requirements engineering for safety-critical systems: Overview and challenges," *IEEE Softw.*, vol. 34, no. 4, pp. 49–57, Jul. 2017.

[7] N. Chanamool and T. Naenna, "Fuzzy FMEA application to improve decision-making process in an emergency department," *Appl. Soft Comput.*, vol. 43, pp. 441–453, Jun. 2016.

[8] R. Fattahi and M. Khalilzadeh, "Risk evaluation using a novel hybrid method based on FMEA, extended MULTIMOORA, and AHP methods under fuzzy environment," *Saf. Sci.*, vol. 102, pp. 290–300, Feb. 2018.

[9] E. Adar, M. Ince, B. Karatop, and M. S. Bilgili, "The risk analysis by failure mode and effect analysis (FMEA) and fuzzy-FMEA of supercritical water gasification system used in the sewage sludge treatment," *J. Environ. Chem. Eng.*, vol. 5, no. 1, pp. 1261–1268, 2017.

[10] H. Safari, Z. Faraji, and S. Majidian, "Identifying and evaluating enterprise architecture risks using FMEA and fuzzy VIKOR," *J. Intell. Manuf.*, vol. 27, no. 2, pp. 475–486, Apr. 2016.

[11] J. F. W. Peeters, R. J. I. Basten, and T. Tinga, "Improving failure analysis efficiency by combining FTA and FMEA in a recursive manner," *Rel. Eng. Syst. Saf.*, vol. 172, pp. 36–44, Apr. 2018.

[12] J. Ahn, Y. Noh, S. H. Park, B. I. Choi, and D. Chang, "Fuzzy-based failure mode and effect analysis (FMEA) of a hybrid molten carbonate fuel cell (MCFC) and gas turbine system for marine propulsion," *J. Power Sources*, vol. 364, pp. 226–233, Oct. 2017.

[13] D. Panchal and D. Kumar, "Risk analysis of compressor house unit in thermal power plant using integrated fuzzy FMEA and GRA approach," *Int. J. Ind. Syst. Eng.*, vol. 25, no. 2, pp. 228–250, 2017.

[14] W. W. Cooper, L. M. Seiford, and J. Zhu, "Data envelopment analysis," in *Handbook on Data Envelopment Analysis*. Boston, MA, USA: Springer, 2004, pp. 1–39.

[15] K.-S. Chin, Y.-M. Wang, G. K. K. Poon, and J.-B. Yang, "Failure mode and effects analysis using a group-based evidential reasoning approach," *Comput. Oper. Res.*, vol. 36, no. 6, pp. 1768–1779, 2009.

[16] P. A. A. Garcia, R. Schirru, and P. F. Frutuoso, "A fuzzy data envelopment analysis approach for FMEA," *Progr. Nucl. Energy*, vol. 46, nos. 3–4, pp. 359–373, 2005.

[17] C.-T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets Syst.*, vol. 114, no. 1, pp. 1–9, Aug. 2000.

[18] M. Braglia, M. Frosolini, and R. Montanari, "Fuzzy TOPSIS approach for failure mode, effects and criticality analysis," *Qual. Rel. Eng. Int.*, vol. 19, no. 5, pp. 425–443, Sep. 2003.

[19] W. Song, X. Ming, Z. Wu, and B. Zhu, "A rough TOPSIS approach for failure mode and effects analysis in uncertain environments," *Qual. Rel. Eng. Int.*, vol. 30, no. 4, pp. 473–486, 2014.

[20] S. Opricovic and G.-H. Tzeng, "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS," *Eur. J. Oper. Res.*, vol. 156, pp. 445–455, Jul. 2004.

[21] H.-C. Liu, L. Liu, N. Liu, and L.-X. Mao, "Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment," *Expert Syst. Appl.*, vol. 39, no. 17, pp. 12926–12934, 2012.

[22] S. Opricovic and G.-H. Tzeng, "Extended VIKOR method in comparison with outranking methods," *Eur. J. Oper. Res.*, vol. 178, no. 2, pp. 514–529, 2007.

[23] M. Braglia, "MAFMA: Multi-attribute failure mode analysis," *Int. J. Qual. Rel. Manage.*, vol. 17, no. 9, pp. 1017–1033, 2000.

[24] S. M. Seyed-Hosseini, N. Safaei, and M. J. Asgharpour, "Reprioritization of failures in a system failure mode and effects analysis by decision making trial and evaluation laboratory technique," *Rel. Eng. Syst. Saf.*, vol. 91, no. 8, pp. 872–881, 2006.

[25] Y.-C. Lee, M.-L. Li, T.-H. Huang, and T.-M. Yen, "Analysis of adopting an integrated decision making trial and evaluation laboratory on a technology acceptance model," *Expert Syst. Appl.*, vol. 37, no. 2, pp. 1745–1754, Mar. 2010.

[26] W. Zhang and Y. Deng, " Combining conflicting evidence using the DEMATEL method," *Soft Comput.*, vol. 23, no. 17, pp. 8207–8216, 2019. doi: 10.1007/s00500-018-3455-8.

[27] A. C. Kutlu and M. Ekmekçioğlu, "Fuzzy failure modes and effects analysis by using fuzzy TOPSIS-based fuzzy AHP," *Expert Syst. Appl.*, vol. 39, no. 1, pp. 61–67, Jan. 2012.

[28] A. Pillay and J. Wang, "Modified failure mode and effects analysis using approximate reasoning," *Rel. Eng. Syst. Saf.*, vol. 79, no. 1, pp. 69–85, Jan. 2003.

[29] Z. Zhang and X. Chu, "Risk prioritization in failure mode and effects analysis under uncertainty," *Expert Syst. Appl.*, vol. 38, no. 1, pp. 206–214, Jan. 2011.

[30] F. Zammori and R. Gabbrielli, "ANP/RPN: A multi criteria evaluation of the risk priority number," *Qual. Rel. Eng. Int.*, vol. 28, no. 1, pp. 85–104, Feb. 2012.

[31] A. Bouti and D. A. Kadi, "A state-of-the-art review of FMEA/FMECA," *Int. J. Rel., Qual. Saf. Eng.*, vol. 1, no. 4, pp. 515–543, 1994.

[32] M. Bertolini, M. Bevilacqua, and R. Massini, "FMECA approach to product traceability in the food industry," *Food Control*, vol. 17, no. 2, pp. 137–145, 2006.

[33] Y.-M. Wang, K.-S. Chin, G. K. K. Poon, and J.-B. Yang, "Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean," *Expert Syst. Appl.*, vol. 36, no. 2, pp. 1195–1207, Mar. 2009.

[34] A. P. Dempster, "Upper and lower probabilities induced by a multivalued mapping," *Ann. Math. Statist.*, vol. 38, no. 2, pp. 325–339, 1967.

[35] Z. Huang, L. Yang, and W. Jiang, "Uncertainty measurement with belief entropy on the interference effect in the quantum-like Bayesian networks," *Appl. Math. Comput.*, vol. 347, pp. 417–428, Apr. 2019.

[36] F. Xiao, "Multi-sensor data fusion based on the belief divergence measure of evidences and the belief entropy," *Inf. Fusion*, vol. 46, pp. 23–32, Mar. 2019.

[37] H. Cui, Q. Liu, J. Zhang, and B. Kang, "An improved deng entropy and its application in pattern recognition," *IEEE Access*, vol. 7, pp. 18284–18292, 2019.

[38] X. Deng, "Analyzing the monotonicity of belief interval based uncertainty measures in belief function theory," *Int. J. Intell. Syst.*, vol. 33, pp. 1869–1879, Sep. 2018.

[39] W. Jiang, C. Huang, and X. Deng, "A new probability transformation method based on a correlation coefficient of belief functions," *Int. J. Intell. Syst.*, vol. 34, no. 6, pp. 1337–1347, 2019.

[40] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[41] X. Deng and W. Jiang, "Evaluating green supply chain management practices under fuzzy environment: A novel method based on D number theory," *Int. J. Fuzzy Syst.*, vol. 21, no. 5, pp. 1389–1402, 2019.

[42] W. Jiang, Y. Cao, and X. Deng, "A novel Z-network model based on Bayesian network and Z-number," *IEEE Trans. Fuzzy Syst.*, to be published.

[43] H. Wang, H. Mo, R. Sadiq, Y. Hu, and Y. Deng, "Ordered visibility graph weighted averaging aggregation operator: A methodology based on network analysis," *Comput. Ind. Eng.*, vol. 88, pp. 181–190, Oct. 2015.

[44] R. K. Sharma, D. Kumar, and P. Kumar, "Systematic failure mode effect analysis (FMEA) using fuzzy linguistic modelling," *Int. J. Qual. Rel. Manag.*, vol. 22, no. 9, pp. 986–1004, Dec. 2005.

[45] H.-C. Liu, J.-X. You, M.-M. Shan, and L.-N. Shao, "Failure mode and effects analysis using intuitionistic fuzzy hybrid TOPSIS approach," *Soft Comput.*, vol. 19, no. 4, pp. 1085–1098, 2015.

[46] D.-S. Chang and K.-L. P. Sun, "Applying DEA to enhance assessment capability of FMEA," *Int. J. Qual. Rel. Manage.*, vol. 26, no. 6, pp. 629–643, 2009.

[47] K.-H. Chang and C.-H. Cheng, "A risk assessment methodology using intuitionistic fuzzy set in FMEA," *Int. J. Syst. Sci.*, vol. 41, no. 12, pp. 1457–1471, 2010.

[48] N. R. Sankar and B. S. Prabhu, "Modified approach for prioritization of failures in a system failure mode and effects analysis," *Int. J. Qual. Rel. Manage.*, vol. 18, no. 3, pp. 324–336, 2001.

[49] H.-C. Liu, L. Liu, and N. Liu, "Risk evaluation approaches in failure mode and effects analysis: A literature review," *Expert Syst Appl.*, vol. 40, no. 2, pp. 828–838, Feb. 2013.

[50] J. Yang, H.-Z. Huang, L.-P. He, S.-P. Zhu, and D. Wen, "Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using Dempster–Shafer evidence theory under uncertainty," *Eng. Failure Anal.*, vol. 18, no. 8, pp. 2084–2092, Dec. 2011.

[51] Y. Geum, Y. Cho, and Y. Park, "A systematic approach for diagnosing service failure: Service-specific FMEA and grey relational analysis approach," *Math. Comput. Model.*, vol. 54, nos. 11–12, pp. 3126–3142, 2011.

[52] A. P. Sant'Anna, "Probabilistic priority numbers for failure modes and effects analysis," *Int. J. Qual. Rel. Manage.*, vol. 29, no. 3, pp. 349–362, 2012.

[53] B. P. Nepal, O. P. Yadav, L. Monplaisir, and A. Murat, "A framework for capturing and analyzing the failures due to system/component interactions," *Qual. Rel. Eng. Int.*, vol. 24, no. 3, pp. 265–289, 2008.

[54] G. Shafer, *A Mathematical Theory of Evidence*, vol. 42. Princeton, NJ, USA: Princeton Univ. Press, 1976.

[55] X. Deng, W. Jiang, and Z. Wang, "Zero-sum polymatrix games with link uncertainty: A Dempster–Shafer theory solution," *Appl. Math. Comput.*, vol. 340, pp. 101–112, Jan. 2019.

[56] Z. He and W. Jiang, "An evidential Markov decision making model," *Inf. Sci.*, vol. 467, pp. 357–372, Oct. 2018.

[57] L. A. Zadeh, "Review of a mathematical theory of evidence," *AI Mag.*, vol. 5, no. 3, p. 81, 1984.

[58] R. Sun and Y. Deng, "A new method to identify incomplete frame of discernment in evidence theory," *IEEE Access*, vol. 7, no. 1, pp. 15547–15555, 2019.

[59] H. Xu and Y. Deng, "Dependent evidence combination based on decision-making trial and evaluation laboratory method," *Int. J. Intell. Syst.*, vol. 34, no. 7, pp. 1555–1571, 2019.

[60] X. Deng, D. Han, J. Dezert, Y. Deng, and Y. Shyr, "Evidence combination from an evolutionary game theory perspective," *IEEE Trans. Syst. Man, Cybern. B, Cybern.*, vol. 46, no. 9, pp. 2070–2082, Sep. 2016.

[61] X. Su, L. Li, H. Qian, M. Sankaran, and Y. Deng, "A new rule to combine dependent bodies of evidence," *Soft Comput.*, pp. 1–7, Feb. 2019. doi: 10.1007/s00500-019-03804-y.

[62] W. Jiang, "A correlation coefficient for belief functions," *Int. J. Approx. Reasoning*, vol. 103, pp. 94–106, Dec. 2018.

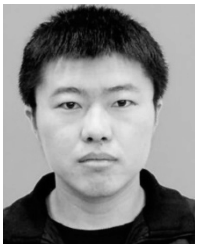
[63] P. Smets and R. Kennes, "The transferable belief model," *Artif. Intell.*, vol. 66, no. 2, pp. 191–234, Apr. 1994.

[64] X. Deng and W. Jiang, "D number theory based game-theoretic framework in adversarial decision making under a fuzzy environment," *Int. J. Approx. Reasoning*, vol. 106, pp. 194–213, Mar. 2019.

[65] F. Yang and H. Wei, "Fusion of infrared polarization and intensity images using support value transform and fuzzy combination rules," *Infr. Phys. Technol.*, vol. 60, pp. 235–243, Sep. 2013.

[66] B. Bede and S. G. Gal, "Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations," *Fuzzy Sets Syst.*, vol. 151, no. 3, pp. 581–599, 2005.

WEN JIANG received the bachelor's and master's degrees from Information Engineering University, Zhengzhou, China, in 1994 and 1997, respectively, and the Ph.D. degree from Northwestern Polytechnical University, Xi'an, China, in 2009, where she is currently a Professor and a Doctoral Advisor with the School of Electronics and Information. From September 2012 to October 2013, she was a Visiting Scholar with the University of Miami. She is also a Commentator of the National Natural Science Foundation of China. Her main research interests include information fusion and intelligent information processing. She is also a member of the Information Integration Branch, China Aviation Society, and a Reviewer for several SCI and EI journals.



ZHIPENG ZHANG received the bachelor's degree from Northwestern Polytechnical University, Xi'an, China, in 2018, where he is currently pursuing the master's degree in circuits and systems. His main research interests include uncertain information processing and fault diagnosis.

XINYANG DENG received the bachelor's, master's, and Ph.D. degrees from Southwest University, Chongqing, China, in 2010, 2013, and 2016, respectively. From August 2014 to August 2016, he studied at Vanderbilt University as a joint Ph.D. Student, which was supported by the China Scholarship Council. Since January 2017, he has been an Associate Professor with the School of Electronics and Information, Northwestern Polytechnical University. His main research interests include multi-source information fusion, Dempster–Shafer evidence theory, uncertain information modeling and processing, and evolutionary game theory. He is currently a Reviewer for several SCI journals.