IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received July 20, 2019, accepted August 5, 2019, date of publication August 9, 2019, date of current version September 30, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2934323

Multiple Due Time Surgical Scheduling With Truncated Learning and Deteriorating Effect

MINSHAN SONG[1,2], YULIN ZHANG[1], AND YUE ZHU[1]
[1]School of Economics and Management, Southeast University, Nanjing 211189, China
[2]School of Science, Jiangsu University of Science and Technology, Zhenjiang 212003, China

Corresponding author: Yulin Zhang (zhangyl@seu.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 71671036, in part by the Major Project of Philosophy and Social Science Research in Colleges and Universities in Jiangsu Province under Grant 2018SJZDA005, and in part by the Scientific Innovation Research of Graduate Students in Jiangsu Province under Grant KYLX_0211.

ABSTRACT The surgical scheduling problem that incorporates the surgical duration, the setup time, the turnover time and the due time is studied in this paper. The actual surgical duration is affected by the normal duration, the surgical sequence, the accumulated experience of surgical teams and a controlling parameter. Besides, the surgical duration, the setup time and the turnover time are affected by the deteriorating effect, which means when the start time, the setup time or the turnover time of a surgery is postponed, the actual duration, the actual setup time or the actual turnover time will be prolonged. A schedule problem is formulated to minimize the maximum surgical tardiness. By building and analyzing the surgical scheduling model, each surgical team should operate surgeries according to a non-decreasing order of patients' normal surgical duration. Furthermore, a branch-and-bound algorithm is provided to solve the surgical teams scheduling problem. Our experimental results show the effectiveness and stability of our proposed algorithm.

INDEX TERMS Deteriorating effect, surgical scheduling, truncated learning effect, various due times.

I. INTRODUCTION

In surgical scheduling, when the actual completion time of a surgery exceeds a given due time, a surgical tardiness occurs and brings harm to the postoperative recovery of the corresponding patient. Besides, it also downgrades the patient's evaluation to the corresponding hospital. Therefore, surgical tardiness is an important criterion for a surgical schedule. An optimal schedule should ensure surgeries be completed on time to limit the tardiness as short as possible. In general, some departments, such as urology, orthopaedics or phthalmology, only own one fixed operating room for elective patients [18]. The department head needs to determine surgical or surgical groups sequence of the operating room each day according to the operation time. In this case, a single operating room sequencing problem is studied. The objective is to find an optimal surgical schedule minimizing the maximum surgical tardiness.

The associate editor coordinating the review of this article and approving it for publication was Shih-Wei Lin.

There are many factors affecting the surgical tardiness, such as the surgical duration, the setup time, the turnover time and the due time. The surgical duration is not fixed, which depends on the normal duration, the surgical sequence, the accumulated experience of surgical teams, a controlling parameter and its start time. In real schedules, the actual surgical duration decreases with the accumulation of the experience of a surgical team. This is called the learning effect. Ziaee *et al.* [1] found that as the completed number of surgeries increases, the average surgical duration reduces gradually. It is 95.4 minutes for the first 15 surgeries, 84 minutes for the next 15 ones and 78.3 minutes for the last 15 ones. Specifically, for Laparoscopic Roux-en-Y Gastric Bypass (LRYGBP), among 393 surgeries operated by a same surgeon, the surgical duration decreases significantly as the number of surgeries increases [2].

However, the actual surgical duration won't keep decreasing. There exists a critical state. As enough experience is accumulated, the actual surgical duration becomes stable. This is called the truncated learning effect. The critical state of different types of surgeries is different. In the Robotic-Assisted Laparoscopic Colorectal Surgery, after 15 to 25 surgeries, the actual duration would not decrease any more [3], while the number of LRYGBP surgeries is 400 for reaching its critical state [2].

Besides, surgeons' accumulated experience also affects the duration of the following surgeries. Grantcharov *et al.* [4] categories the surgeons of the Laparoscopic Surgery into three groups, i.e., masters, intermediates and beginners respectively. They pointed out that the order of the surgical duration stopping decreasing is masters, intermediates and beginners.

On the other hand, the deterioration of patients prolongs the actual surgical duration. This is called the deteriorating effect. When investigating the computed tomography of hepatic portal venous gas, Chan *et al.* [5] concluded that the deferral of surgeries would make patients' condition worse and bring difficulties to surgeons. In order to depict this phenomenon, Wu *et al.* [6] and Zhang *et al.* [7] regarded the actual surgical duration as a linear combination of the normal surgical duration and the start time, where the normal surgical duration refers to the duration when patients are operated without deferral. When a surgery is deferred, the actual surgical duration increases.

Similarly, the setup time and the turnover time are also affected by the deteriorating effect. The setup time is the time interval between two consecutive surgeries, which is the time of cleaning operating tables and anaesthetizing patients. The turnover time is the time interval between two surgical teams, which is the time of switching medical equipment. The actual setup time or the actual turnover time can be prolonged when the start time of the setup or the turnover is deferred. In order to describe this phenomenon, Wang and Liu [8] regarded the two kinds of time as a linear combination of their normal duration and their start time.

Different from the surgical duration, the setup time and the turnover time, the due time is usually fixed. Because of the constraint of the due time, surgical teams should try to complete surgeries earlier. Otherwise, patients have to continue to receive surgeries without food and water, and to endure both the physical and mental pressure [9].

Therefore, it is more realistic to take the surgical duration, the setup time, the turnover time and the due time into account when formulating a surgical scheduling. However, similar attempts are limited. If regarding surgeries, patients and operating rooms as workers, jobs and machines respectively, the surgical scheduling problem can be formulated as a single machine flow shop scheduling model with variable task durations. In recent years, this type of scheduling problem has been widely studied and provide theoretical basis to our study [10]–[17].

However, the truncated learning effect and the deteriorating effect are usually considered separately in the above flow shop scheduling literature. Besides, the setup time or the turnover time is not taken into consideration. Therefore, this paper is the first to consider the surgical duration, the setup time, the turnover time and the due time simultaneously in the flow shop problem. Specifically, the surgical duration is affected by both the truncated learning effect and the deteriorating effect.

The remainder of this paper is organized as follows. A brief review of surgical scheduling is provided in Section 2. In Section 3, an optimal surgical schedule problem that minimizes the maximum surgical tardiness is formulated. In Section 4, an optimal scheduling model is built, and the sequence for surgeries and surgical teams are given respectively in Section 5. After that, computational experiments are presented to show the process of formulating a surgical schedule in Section 6. Finally, the conclusions of the paper and the topics for future research are provided in Section 7.

II. LITERATURE REVIEW

There is a large body of literature on the management problem of operating rooms. Cardoen *et al.* (2010) [19] and Demeulemeester *et al.* (2013) [20] provided a comprehensive review on the planning and scheduling problem. As pointed out by Cardoen *et al.* [19], surgical scheduling requires the execution of two main steps: advanced scheduling and allocation scheduling. Advanced scheduling assigns patients to different operating rooms, and then allocation scheduling determines the surgical sequence in each operating room. This work is motivated by the allocation scheduling problem to determine the sequence of operations on daily basies.

For a fixed surgical duration, Abdeljaouad *et al.* [21] used a two-dimensional strip packing model to order different groups of operations with aim to minimize the completion time. A simulation model was proposed by Liang *et al.* [22] to sequence operations with multi-objectives. Several studies integrated advance and allocation scheduling problems. In order to maximize the scheduled surgical cases, Castro and Marques [23] proposed a new two-level decomposition algorithm for the mixed-integer linear programming model considering surgery priorities. Roshanaei *et al.* [24] extended the problem from a unique hospital to hospitals' network, and developed a novel logic-based Benders' decomposition approach to solve the proposed mixed-integer programming model.

For the surgical duration uncertainty, some researches considered stochastic surgical durations. Lee and Yih [25] incorporated fuzzy time duration in the flow shop model with restraint of beds in the unit Post-Anesthesia Care Unit. A genetic algorithm was proposed to solve the model. Van Essen *et al.* [26] considered emergency surgeries which will be performed immediately after ongoing surgery was completed and surgery duration was assumed to be stochastic. Latorre *et al.* [27] further integrated emergency surgeries and restraint of beds together. They developed a metaheuristic based on a genetic algorithm to solve the proposed integer linear programming model. Kroer *et al.* [28] further considered emergency patients with stochastic arrivals and surgical durations to minimize overtime and proposed heuristics to solve the stochastic mixed-integer programming model. Several researchers considered the uncertainty of surgical duration due to some facts. Molina *et al.* [29] believed that surgical duration is affected by surgeon's experiences and modeled surgical durations based on experience levels. An approximate algorithm was proposed to the parallel flow shop model. Wang *et al.* [18] investigated surgical scheduling with patients' priorities and linear deteriorating effect on surgical duration. A meta-heuristic algorithm was proposed to solve the flow shop model. Similar to Wang *et al.* [18], this paper examined the single day surgical scheduling for single operating room. Moreover, surgeons' experiences were taken into account and the sequencing problem for surgical groups aims to minimize the maximum tardiness.

III. PROBLEM FORMULATION

A. PROBLEM DESCRIPTION

In one working day, a single operating room scheduling problem is studied. The operating room can be used to operate various types of surgeries. The setup time exists between consecutive surgeries, while the turnover time is between different surgical teams. They both have the deteriorating effect. The actual surgical duration is affected by the truncated learning effect and the deteriorating effect. For each surgical team, there is a due time. The surgical tardiness occurs when the completion time of a surgical team exceeds the given due time. For hospitals, it is necessary to limit the surgical tardiness as short as possible. The objective is to find an optimal surgical schedule that minimizes the maximum surgical tardiness. This schedule consists of the sequence for surgeries and the sequence for surgical teams entering the operating room.

B. ASSUMPTIONS AND NOTATIONS

1) ASSUMPTIONS

There are three basic hypotheses in this paper.

Assumption 1: Patients and surgery types are known in advance. No cancellation is allowed.

Assumption 2: The surgical teams enter the operating room taking turns. That means only when a surgical team completes all the surgeries, the next team can enter the operating room.

Assumption 3: A certain environment. Stochastic factors resulting from complex surgical processes are not taken into account.

Assumption 1 assures the certain number of patients and their surgical types. Therefore, surgical teams involving in surgical schedules are also known. The patients dispatched to each surgical team are determined in advance. Assumption 2 is provided because Luo *et al.* [31] indicated that it can improve efficiency when scheduling the same type of surgeries consecutively.

2) NOTATIONS

The following notations will be used throughout the whole paper.

N: Number of patients.

M: Number of surgical teams.

G_i: Surgical team i, $i = 1, 2, \ldots, M$.

n_i: Number of patients operated by surgical team G_i, $i = 1, 2, \ldots, M$, $n_1 + n_2 + \cdots + n_M = N$.

$n_{[i]}$: Number of patients operated by the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$.

J_{ij}: The patient whose serial number is j and operated by surgical team G_i, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_i$.

α_{ij}: The normal surgical duration of patient J_{ij}. It can be acquired in advance from historical data, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_i$.

$\alpha_{i[q]}$: The normal surgical duration of the qth patient operated by surgical team G_i, $i = 1, 2, \ldots, M$, $q = 1, 2, \ldots, n_i$.

E_i: The accumulation of the normal surgical duration of surgical team G_i, reflecting surgical team G_i's experience, $i = 1, 2, \ldots, M$.

k_i: The learning effect parameter of surgical team G_i, $k_i \geq 0$, $i = 1, 2, \ldots, M$.

β_i: The controlling parameter of surgical team G_i, $0 < \beta_i < 1$, $i = 1, 2, \ldots, M$.

a_i: The deteriorating effect parameter of surgical team G_i, $a_i \geq 0$, $i = 1, 2, \ldots, M$.

p_{ij}: The actual surgical duration of patient J_{ij} satisfies $p_{ij} = \alpha_{ij} \max\{(1+E_i+\sum_{a=1}^{r-1} \alpha_{i[q]})^{-k_i}, \beta_i\}+a_i t$, where r means that J_{ij} is the rth patient operated by surgical team G_i and t is the start time of patient J_{ij}'s surgery, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_i$

$p_{[i][j]}$: The actual surgical duration of the patient who is the jth to be operated by the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_{[i]}$.

m_i: The normal setup time of the surgeries operated by surgical team G_i, $i = 1, 2, \ldots, M$.

b_i: The deteriorating effect parameter of the setup time of the surgeries operated by surgical team G_i, $b_i \geq 0$, $i = 1, 2, \ldots, M$.

s_{ij}: The actual setup time of patient J_{ij} satisfies $s_{ij} = m_i + b_i t$, where t is the start time of patient J_{ij}'s setup, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_i$.

$s_{[i][j]}$: The actual setup time of the patient who is the jth to be operated by the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, n_{[i]}$.

γ_i: The normal turnover time of surgical team G_i, $i = 1, 2, \ldots, M$.

c_i: The deteriorating effect parameter of the turnover time of surgical team G_i, $c_i \geq 0$, $i = 1, 2, \ldots, M$.

δ_i: The actual turnover time of surgical team G_i satisfies $\delta_i = \gamma_i + c_i t$, where t represents the start time of surgical team G_i's turnover, $i = 1, 2, \ldots, M$.

$\delta_{[i]}$: The turnover time of the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$.

d_i: The due time of surgical team G_i, $i = 1, 2, \ldots, M$.

$d_{[i]}$: The due time of the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$.

T_i: The surgical tardiness of surgical team G_i, $i = 1, 2, \ldots, M$.

$T_{[i]}$: The surgical tardiness of the ith surgical team entering the operating room, $i = 1, 2, \ldots, M$.

π: A given surgical sequence.

$C_{ij}(\pi)$: The surgical completion time of patient J_{ij} in sequence π, $i = 1, 2, \ldots, M, j = 1, 2, \ldots, n_i$.

$C_{[i][j]}(\pi)$: The surgical completion time of the jth patient operated by the ith surgical team entering the operating room, $i = 1, 2, \ldots, M, j = 1, 2, \ldots, n_{[i]}$.

$$x_{ijr} = \begin{cases} 1, & \text{patient } \mathrm{J_{ij}} \text{ is the rht to be operated} \\ 0, & \text{otherwise,} \end{cases}$$

$i = 1, 2, \ldots, M, j, r = 1, 2, \ldots, n_i$

$$y_{ih} = \begin{cases} 1, & G_i \text{ is the } h\text{th to entering operating room} \\ 0, & \text{otherwise,} \end{cases}$$

for $i, h = 1, 2, \ldots, M$

The binary variables x_{ijr} and y_{ih} determine the sequence for surgeries and the sequence for surgical teams entering the operating room. They are the decision variables in this paper.

IV. SCHEDULING MODEL AND ANALYSIS

A. SCHEDULING MODEL

When taking various surgical due times into account, it is likely to encounter the surgical tardiness. Gocgun and Puterman [32] pointed out that it is necessary to provide a schedule so that the possible surgical tardiness as short as possible; otherwise it may generate extra cost. Therefore, the minimization of the maximum surgical tardiness is set as the objective of the schedule model as follows.

$$\min\ T_{\max} = \max_{1\le h\le M}\{T_{[h]}\} \tag{1}$$

$$\text{s.t.}\ \sum_{i=1}^{M} y_{ih} = 1 \quad \forall h = 1, 2, \cdots, M \tag{1-1}$$

$$\sum_{h=1}^{M} y_{ih} = 1 \quad \forall h = 1, 2, \cdots, M \tag{1-2}$$

$$\sum_{j=1}^{n_i} x_{ijr} = 1 \quad \forall i = 1, 2, \cdots, M; \quad \forall r = 1, 2, \cdots, n_i \tag{1-3}$$

$$\sum_{r=1}^{n_i} x_{ijr} = 1 \quad \forall i = 1, 2, \cdots, M; \quad \forall j = 1, 2, \cdots, n_i \tag{1-4}$$

$$\alpha_{i[q]} = \sum_{j=1}^{n_i} \alpha_{ij} x_{ijq} \quad \forall i = 1, 2, \cdots, M \tag{1-5}$$

$$\delta_{[1]} = \sum_{i=1}^{M} y_{i1}\gamma_i \tag{1-6}$$

$$s_{[1][1]} = \sum_{i=1}^{M} y_{i1}(m_i + b_i\gamma_i) \tag{1-7}$$

$$p_{[1][1]} = \sum_{i=1}^{M} y_{i1}[\sum_{j=1}^{n_i} x_{ij1}\alpha_{ij} \max\{(1+E_i)^{-k_i}, \beta_i\} + a_i(\delta_{[1]} + s_{[1][1]})] \tag{1-8}$$

$$C_{[1][1]} = \delta_{[1]} + s_{[1][1]} + p_{[1][1]} \tag{1-9}$$

$$s_{[1][l]} = \sum_{i=1}^{M} y_{i1}(m_i + b_i C_{[1][l-1]}) \quad \forall l = 2, 3, \cdots, n_i \tag{1-10}$$

$$p_{[1][l]} = \sum_{i=1}^{M} y_{i1}[\sum_{j=1}^{n_i} x_{ijl}\alpha_{ij} \max\{(1+E_i + \sum_{q=1}^{l-1}\alpha_{i[q]})^{-k_i}, \beta_i\} + a_i(C_{[1][l-1]} + s_{[1][l]})] \quad \forall l = 2, 3, \cdots, n_i \tag{1-11}$$

$$C_{[1][l]} = C_{[1][l-1]} + s_{[1][l]} + p_{[1][l]} \quad \forall l = 2, 3, \cdots, n_i \tag{1-12}$$

$$\delta_{[h]} = \sum_{i=1}^{M} y_{i1}(\gamma_i + c_i C_{[h-1][n_{[h-1]}]}) \quad \forall h = 1, 2, \cdots, M \tag{1-13}$$

$$s_{[h][1]} = \sum_{i=1}^{M} y_{ih}[m_i + b_i(C_{[h-1][n_{[h-1]}]} + \delta_{[h]})] \quad \forall h = 1, 2, \cdots, M \tag{1-14}$$

$$p_{[h][1]} = \sum_{i=1}^{M} y_{ih}[\sum_{j=1}^{n_i} x_{ij1}\alpha_{ij} \max\{(1+E_i)^{-k_i}, \beta_i\} + a_i(\delta_{[1]} + s_{[1][1]} + C_{[h-1][n_{[h-1]}]})] \quad \forall h = 1, 2, \cdots, M \tag{1-15}$$

$$C_{[h][1]} = C_{[h-1][n_{[h-1]}]} + \delta_{[h]} + s_{[h][1]} + p_{[h][1]} \quad \forall h = 1, 2, \cdots, M \tag{1-16}$$

$$s_{[h][l]} = \sum_{i=1}^{M} y_{ih}(m_i + b_i C_{[h][l-1]}) \quad \forall l = 2, 3, \cdots, n_i \quad \forall h = 1, 2, \cdots, M \tag{1-17}$$

$$p_{[h][l]} = \sum_{i=1}^{M} y_{ih}[\sum_{j=1}^{n_i} x_{ijl}\alpha_{ij} \max\{(1+E_i + \sum_{q=1}^{l-1}\alpha_{i[q]})^{-k_i}, \beta_i\} + a_i(C_{[h][l-1]} + s_{[h][l]})] \quad \forall l = 2, 3, \cdots, n_i \quad \forall h = 1, 2, \cdots, M \tag{1-18}$$

$$C_{[h][l]} = C_{[h][l-1]} + s_{[h][l]} + p_{[h][l]} \quad \forall l = 2, 3, \cdots, n_i \quad \forall h = 1, 2, \cdots, M \tag{1-19}$$

$$d_{[h]} = \sum_{i=1}^{M} y_{ih} d_i \quad \forall h = 1, 2, \cdots, M \tag{1-20}$$

$$T_{[h]} = \max\{C_{[h][n_{[h]}]} - d_{[h]}, 0\} \quad \forall h = 1, 2, \cdots, M \tag{1-21}$$

Expression (1) indicates that the objective is to minimize the maximum surgical tardiness. Expression (1-1) and (1-2) restrict that each surgical team can only enter the operating room once and the operating room can only allow one surgical team for surgery at a time. Expression (1-3) and (1-4) show that each patient can only be operated once and a surgical team can only operate one surgery at a time. Expressions (1-5) to (1-19) calculate the completion time of each surgery. Expression (1-20) is the surgical due time of the hth surgical team entering the operating room. Expression (1-21) represents the surgical tardiness of the hth surgical team entering the operating room. Referring to Graham *et al.* [33], the surgical scheduling model can be simplified as follows.

$$1\left|\begin{array}{l}\delta_i = \gamma_i + c_i t,\ s_{ij} = m_i + b_i t,\\ p_{ij} = \alpha_{ij}\max\{(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i},\ \beta_i\} + a_i t,\ d_i\end{array}\right| T_{\max} \tag{2}$$

In expression (2), "1" denotes a single operating room; δ_i, s_{ij}, p_{ij} calculate the turnover time, the setup time and the surgical duration surgical respectively. d_i represents due time of surgical team G_i. $T_{\max}$ indicates that the objective is to minimize the maximum surgical tardiness.

B. SEQUENCE FOR EACH SURGERY TEAM

To solve the model (1), a theorem is proposed to determine the sequence for surgeries. Before the theorem, a lemma is presented.

Lemma 1: If $a, b, k, x \geq 0, y, \lambda \geq 1$, then:

$$\lambda[(1+a)(1+b)y^{-k} - (y+x)^{-k}] - [(1+a)(1+b)y^{-k} - (y+\lambda x)^{-k}] \geq 0$$

Through derivation, Lemma 1 can easily be proved.

For a given surgical team G_i, its surgical tardiness can be expressed as follows.

$$T_i = \max\{C_{i,n_i} - d_i, 0\} \tag{3}$$

C_{i,n_i} is the time when G_i completes all the surgeries and setups. From expression (3), it is apparent that for each surgical team, the optimal sequence minimizing the maximum surgical tardiness equals to the one minimizing makespan.

Theorem 1: In each surgical team, it is optimal to schedule surgeries according to the non-decreasing order of patients' normal surgical duration.

Proof:

Case (1): surgical team G_i only needs to operate one surgery. In this case, Theorem 1 holds.

Case (2): surgical team G_i needs to operate at least two surgeries. Assume that there exists an optimal sequence $\pi_1 = (S_1, J_{iu}, J_{iv}, S_2)$ satisfying $\alpha_{iu} \leq \alpha_{iv}$. S_1 and S_2 may either contain some patients or not. Patient J_{iu} is the rth patient to be operated and arrival time is t. Another sequence $\pi_1' = (S_1, J_{iv}, J_{iu}, S_2)$ can be generated by exchanging the position of patient J_{iu} and J_{iv}. The proof of Theorem 1 equals to the proof of $C_{iv}(\pi_1) \leq C_{iu}(\pi_1')$.

From expression (1-5) to (1-19), the difference between the completion time of patient J_{iu} in sequence π_1' and the one of patient J_{iv} in sequence π_1 can be acquired as follows.

$$\begin{aligned}C_{iu}(\pi_1') - C_{iv}(\pi_1) &= \alpha_{iv}[(1+a_i)(1+b_i)\max\{(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}, \beta_i\}\\ &\quad - \max\{(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i}, \beta_i\}]\\ &\quad - \alpha_{iu}[(1+a_i)(1+b_i)\max\{(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}, \beta_i\}\\ &\quad - \max\{(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iv})^{-k_i}, \beta_i\}]\end{aligned} \tag{4}$$

When $0 < \beta_i < (1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iv})^{-k_i}$, expression (4) equals to

$$\begin{aligned}C_{iu}(\pi_1') - C_{iv}(\pi_1) &= \alpha_{iv}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}\\ &\quad -(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i}]\\ &\quad -\alpha_{iu}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}\\ &\quad -(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iv})^{-k_i}]\end{aligned} \tag{5}$$

From Lemma 1, it is obvious that $C_{iv}(\pi_1) \leq C_{iu}(\pi_1')$. When

$$(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iv})^{-k_i} < \beta_i < (1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i},$$

expression (4) equals to

$$\begin{aligned}C_{iu}(\pi_1') - C_{iv}(\pi_1) &= \alpha_{iv}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}\\ &\quad -(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i}]\\ &\quad -\alpha_{iu}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i} - \beta_i]\end{aligned}$$

$$\geq \alpha_{iv}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i} - (1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i}] - \alpha_{iu}[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i} - (1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iv})^{-k_i}] \quad (6)$$

From Lemma 1, it is apparent that $C_{iv}(\pi_1) \leq C_{iu}(\pi_1')$.

When $(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]}+\alpha_{iu})^{-k_i} < \beta_i < (1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i}$, expression (4) equals to

$$C_{iu}(\pi_1') - C_{iv}(\pi_1) = (\alpha_{iv}-\alpha_{iu})[(1+a_i)(1+b_i)(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i} - \beta_i] \geq 0 \quad (7)$$

When $(1+E_i+\sum_{q=1}^{r-1}\alpha_{i[q]})^{-k_i} < \beta_i < 1$, expression (4) equals to

$$C_{iu}(\pi_1')-C_{iv}(\pi_1)=(\alpha_{iv}-\alpha_{iu})[(1+a_i)(1+b_i)\beta_i-\beta_i]\geq 0 \quad (8)$$

As β_i changes, it always holds that $C_{iv}(\pi_1) \leq C_{iu}(\pi_1')$. From the analysis of case (1) and case (2), the proof of Theorem 1 is completed.

C. SEQUENCE FOR SURGICAL TEAMS

In the previous subsection, Theorem 1 determines the sequence for surgeries. In this subsection, Theorem 2 provides a lower bound of sequence with maximum surgical tardiness to speed up the sequencing procedure.

Lemma 2: If $a_2 \geq a_1 > 0, b_2 \geq b_1 > 0$, then the following inequality always holds:

$$\max\{a_2-b_2, a_1-b_1, 0\} \leq \max\{a_1-b_2, a_2-b_1, 0\}$$

Lemma 2 can be proved easily.

Assuming that at time t, G_i completes its turnover, then the time surgical team G_i completing surgeries and setups can be denoted by p_{G_i}. The expression of p_{G_i} is

$$p_{G_i} = A_i + B_i t \quad (9)$$

Relating parameters are as follows.

$$A_i = \sum_{l=1}^{n_i}\{m_i[(1+a_i)^l(1+b_i)^{l-1}] + [(1+a_i)(1+b_i)]^{n_i-l}\alpha_{i[l]} \times \max\{(1+E_i+\sum_{q=1}^{l-1}\alpha_{i[q]})^{-k_i}, \beta_i\}\}$$
$$B_i = [(1+a_i)(1+b_i)]^{n_i} - 1.$$

Afterwards, the problem determining the sequence for surgical teams entering the operating room can be converted into the following single machine scheduling problem.

$$1\,|p_i = A_i + B_i t, \delta_i = \gamma_i + c_i t, d_i|\,T_{\max} \quad (10)$$

A branch-and-bound algorithm is designed to determine the sequence for surgical teams entering the operating room.

For surgical teams, the non-decreasing order of their parameters A_i, B_i, c_i, γ_i, d_i are $A_{(1)} \leq \ldots \leq A_{(M)}$, $B_{(1)} \leq \ldots \leq B_{(M)}$, $c_{(1)} \leq \ldots \leq c_{(M)}$, $\gamma_{(1)} \leq \ldots \leq \gamma_{(M)}$ and $d_{(1)} \leq \ldots \leq d_{(M)}$ respectively. Define such a sequence PS, in which the first k $(1 \leq k \leq M)$ surgical teams are unknown while the last $(M-k)$ surgical teams are determined.

A lower bound of the sequence 's maximum surgical tardiness is provided to improves the sequencing procedure in theorem 2.

Theorem 2: For each sequence PS, a lower bound of its maximum surgical tardiness is

$$LB(PS)=\max\{\max_{1\leq i\leq k}\{\underline{C_{[i]}}-d_{(i)},0\}, \max_{k+1\leq i\leq M}\{\underline{C_{[i]}}-d_{[i]},0\}\}.$$

for $1 \leq i \leq k$,

$$\underline{C_{[i]}} = \sum_{l=1}^{i}\{[\gamma_{(l)}(1+B_{(l)})+A_{(l)}]\prod_{h=1}^{i-l}[(1+B_{(h)})(1+c_{(h)})]\},$$

and for $k+1 \leq i \leq M$,

$$\underline{C_{[i]}} = \underline{C_{[k]}}\prod_{l=k+1}^{i}[(1+C_{[l]})(1+B_{[l]})] + \sum_{l=k+1}^{i}\{A_{[l]}\prod_{h=l+1}^{i}[(1+B_{[h]})(1+C_{[h]})]\} + \sum_{l=k+1}^{i}\gamma_{[l]}[\prod_{h=l}^{i}(1+B_{[h]})\prod_{h=l+1}^{i}(1+C_{[h]})]$$

Proof: Let $C_{[i]}$ denote the completion time of the ith surgical team entering the operating room.

When $1 \leq i \leq k$, $C_{[i]}$ satisfies

$$C_{[i]} = \sum_{l=1}^{i}\{[\gamma_{[l]}(1+B_{[l]})+A_{[l]}]\prod_{h=l+1}^{i}[(1+B_{[h]})(1+c_{[h]})]\} \geq \sum_{l=1}^{i}\{[\gamma_{(l)}(1+B_{(l)})+A_{(l)}]\prod_{h=1}^{i-l}[(1+B_{(h)})(1+c_{(h)})]\} \quad (11)$$

The right part of inequality (11) denotes the lower bound of $C_{[i]}$. It can be marked as $\underline{C_{[i]}}$.

When $k+1 \leq i \leq \overline{M}$, the sequence for surgical teams from $k+1$ to M is determined. In this case, the lower bound of $C_{[i]}$ satisfies

$$\begin{aligned}\underline{C_{[i]}} = \underline{C_{[k]}} \prod_{l=k+1}^{i} [(1 + C_{[l]})(1 + B_{[l]})] \\ + \sum_{l=k+1}^{i} \{A_{[l]} \prod_{h=l+1}^{i} [(1 + B_{[h]})(1 + C_{[h]})]\} \\ + \sum_{l=k+1}^{i} \gamma_{[l]} [\prod_{h=l}^{i} (1 + B_{[h]}) \prod_{h=l+1}^{i} (1 + C_{[h]})] \end{aligned} \quad (12)$$

The maximum tardiness of the sequence PS satisfies

$$\begin{aligned} T_{\max}(PS) \\ = \max\{\max_{1\leq j\leq k} \{C_{[j]} - d_{[j]}, 0\}, \max_{k+1\leq j\leq M} \{C_{[j]} - d_{[j]}, 0\}\} \\ \geq \max\{\max_{1\leq j\leq k} \{\underline{C_{[j]}} - d_{[j]}, 0\}, \max_{k+1\leq j\leq M} \{\underline{C_{[j]}} - d_{[j]}, 0\}\} \end{aligned} \quad (13)$$

From Lemma 2, the lower bound of sequence PS's maximum surgical tardiness is

$$LB(PS) = \max\{\max_{1\leq j\leq k} \{\underline{C_{[j]}} - d_{(j)}, 0\}, \max_{k+1\leq j\leq M} \{\underline{C_{[j]}} - d_{[j]}, 0\}\} \quad (14)$$

Therefore, the proof of Theorem 2 is completed.

V. SEQUENCING ALGORITHM

Based on the analysis of the scheduling model, a branch-and-bound algorithm is provided to determine the order for all surgeries in the operating room. Detail steps of the algorithm is as follows.

Step 1: Use theorem 1 to determine the order for surgeries in each surgery team and calculate the corresponding parameters A_i and B_i;

Step 2: Set an initial sequence $PS = (G_{[1]}, G_{[2]}, \ldots, G_{[M]})$. Where $G_{[1]}, G_{[2]}, \ldots, G_{[M]}$ are unknown. Let $S = \{G_1, G_2, \ldots, G_M\}$ is the set of unscheduled surgical teams. The dimension of it is K. Set K equals to M initially;

Step 3: Calculate the lower bound of PS's maximum surgical tardiness when $G_{[K]}$ equals to each surgical team in set S respectively. Among all the surgical teams, choose the one with the minimum lower bound to be G_l;

Step 4: Let $G_{[K]} = G_l, K = K - 1, S = S\backslash\{G_l\}$;

Step 5: If $K = 0$, then turn to step 6; otherwise, turn to step 2;

Step 6: Output the sequence $PS = (G_{[1]}, G_{[2]}, \ldots, G_{[M]})$.

The sequence PS derived from the algorithm is the optimal sequence.

VI. COMPUTATIONAL EXPERIMENTS

In this section, computational experiments are conducted to verify the performance of our sequencing algorithm. The algorithm is coded in matlab using version R2015b and performed the experiments on a personal computer with a 2.67 GHz Intel Core i5 CPU and 4 GB RAM under Windows XP.

Most parameters are set from a survey in the orthopaedics department at a large Class-3 Level-A hospital in Jiangsu Province, China. Eight operations are performed in one operation room per day. The number of surgery groups is less than 4, the number of operations for each group is randomly set and the total number of operations N is less than 8. The normal processing time α_{ij} is set according to the operation time of fracture patients, and generated from a uniform distribution U[0.5,3], and the time unit is hour. Normal set up and turnover time are both generated from a uniform distribution U[0.05,0.2]. The deteriorating rate is different from surgeries and varies from zero to 0.003. That is, deteriorating rates are set randomly from U[0,0.03]. Each member in a surgeon team has a different schedule. For example, some surgeons' schedule is half-day operation and half-day outpatient. However, anesthesiologists need to participate in another team's surgery. The workhour of staff in a hospital is eight hours per day. In this case, the due date for each surgical group are uniform generated using a uniform distribution U[2,8]. The parameters in the learning effect are obtained from the data in Bjorgul *et al.* [30].

First, we consider 8 operations with 2-4 surgical groups which is close to the reality in the orthopaedics department. The running time is compared between full enumeration for the programming model and our algorithm in Table 1.

TABLE 1. Comparison between full enumeration and proposed algorithm.

N =8	full enumeration		proposed algorithm	
	Mean	Max	Mean	Max
M =2	0.0156	0.0185	0.00001	0.00002
M =3	0.027	0.0322	0.00002	0.00003
M =4	0.031	0.042	0.00002	0.00003

TABLE 2. Comparison for A large scale environment.

N =100	full enumeration		proposed algorithm	
	Mean	Max	Mean	Max
M =6	1.2761	1.5259	0.0034	0.0055
M =8	17.656	18.2935	0.0768	0.09106
M =10	639.709	680.0867	0.3635	0.5827

Table 1 shows that our algorithm performs well in the real orthopaedics department. However, in other departments, the number of operations will be greater than in the orthopaedics department. And it is necessary to check whether the proposed algorithm is efficiency in a large scale environment. Therefore, a test on the performance between full enumeration and our algorithm with 100 operations is conducted in Table 2.

The results in Table 2 indicate the stability and efficiency of the proposed algorithm. Especially for more surgical groups, the proposed algorithm is better.

VII. CONCLUSIONS AND DISCUSSIONS

This paper mainly studied the surgical scheduling problem which incorporates the surgical duration, the setup time, the turnover time and the due time. The surgical duration is affected by both the truncated learning effect and the deteriorating effect. Its actual value depends on the normal duration, the surgical sequence, the accumulated experience of a surgical team, a controlling parameter and the start time. Besides, both the setup time and the turnover time are affected by the deteriorating effect. As the start time postpones, the actual setup time or the turnover time prolongs. Furthermore, there are various due time constraints, corresponding to each surgical team respectively. When the actual completion time of a surgical team exceeds the given due time, the surgical tardiness occurs, bringing negative evaluation to the hospital and harm to patients' postoperative recovery. For this reason, the maximum surgical tardiness minimization is selected as the objective.

By building and analyzing the surgical scheduling model, an optimal schedule is presented to achieve the objective. The study indicates that in each surgical team, it is optimal to schedule surgeries according to the non-decreasing order of patients' normal surgical duration. Further, a branch-and-bound algorithm is provided to determine the sequence of all surgeries. Finally, a set of numerical simulations are presented to show the process of formulating a surgical schedule in reality.

The defect of this paper is that the medical system only consists of a single operating room without any other resources in upstream and downstream. However, real-world surgical scheduling is complicated, due to multiple factors such as surgical nurses, post-surgery activity scheduling and multiple operating room scheduling. Thus, further work will focus on these problems. More complex medical systems will be the direction in the future.

REFERENCES

[1] S. A. Ziaee, M. M. Sichani, A. H. Kashi, and M. Samzadeh, "Evaluation of the learning curve for percutaneous nephrolithotomy," *Urol. J.*, vol. 7, no. 4, pp. 226–231, 2010.

[2] G. H. Ballantyne, D. Ewing, R. F. Capella, J. F. Capella, D. Davis, H. J. Schmidt, A. Wasielewski, and R. J. Davies, "The learning curve measured by operating times for laparoscopic and open gastric bypass: Roles of surgeon's experience, institutional experience, body mass index and fellowship training," *Obesity Surg.*, vol. 15, no. 2, pp. 172–182, Feb. 2005.

[3] M. B. Bokhari, C. B. Patel, D. I. Ramos-Valadez, M. Ragupathi, and E. M. Haas, "Learning curve for robotic-assisted laparoscopic colorectal surgery," *Surg. Endosc.*, vol. 25, no. 3, pp. 855–860, Mar. 2011.

[4] T. P. Grantcharov, L. Bardram, P. Funch-Jensen, and J. Rosenberg, "Learning curves and impact of previous operative experience on performance on a virtual reality simulator to test laparoscopic surgical skills," *Amer. J. Surg.*, vol. 185, no. 2, pp. 146–149, Feb. 2003.

[5] S.-C. Chan, Y.-L. Wan, Y.-C. Cheung, S.-H. Ng, A. M.-C. Wong, and K.-K. Ng, "Computed tomography findings in fatal cases of enormous hepatic portal venous gas," *World J. Gastroenterol.*, vol. 11, no. 19, pp. 2953–2955, May 2005.

[6] C.-C. Wu, W.-H. Wu, W.-H. Wu, P.-H. Hsu, Y. Yin, and J. Xu, "A single-machine scheduling with a truncated linear deterioration and ready times," *Inf. Sci.*, vol. 256, pp. 109–125, Jan. 2014.

[7] X. Zhang, H. Wang, and X. Wang, "Patients scheduling problems with deferred deteriorated functions," *J. Combinat. Optim.*, vol. 30, no. 4, pp. 1027–1041, Nov. 2015.

[8] J.-J. Wang and Y.-J. Liu, "Single-machine bicriterion group scheduling with deteriorating setup times and job processing times," *Appl. Math. Comput.*, vol. 242, pp. 309–314, Sep. 2014.

[9] R. E. Wachtel and F. Dexter, "Influence of the operating room schedule on tardiness from scheduled start times," *Anesthesia Analgesia*, vol. 108, no. 6, pp. 1889–1901, Jun. 2009.

[10] T. C. E. Cheng, C.-J. Hsu, Y.-C. Huang, and W.-C. Lee, "Single-machine scheduling with deteriorating jobs and setup times to minimize the maximum tardiness," *Comput. Oper. Res.*, vol. 38, no. 12, pp. 1760–1765, Dec. 2011.

[11] C.-C. Wu, Y. Yin, and S.-R. Cheng, "Some single-machine scheduling problems with a truncation learning effect," *Comput. Ind. Eng.*, vol. 60, no. 4, pp. 790–795, May 2011.

[12] C.-C. Wu, S.-R. Cheng, W.-H. Wu, Y. Yin, and W.-H. Wu, "The single-machine total tardiness problem with unequal release times and a linear deterioration," *Appl. Math. Comput.*, vol. 219, no. 20, pp. 10401–10415, Jun. 2013.

[13] X.-R. Wang, J.-B. Wang, J. Jin, and P. Ji, "Single machine scheduling with truncated job-dependent learning effect," *Optim. Lett.*, vol. 8, no. 2, pp. 669–677, Feb. 2014.

[14] Y.-P. Niu, J. Wang, and N. Yin, "Scheduling problems with effects of deterioration and truncated job-dependent learning," *J. Appl. Math. Comput.*, vol. 47, nos. 1–2, pp. 315–325, Feb. 2015.

[15] C.-C. Wu and W.-C. Lee, "Scheduling linear deteriorating jobs to minimize makespan with an availability constraint on a single machine," *Inf. Process. Lett.*, vol. 87, no. 2, pp. 89–93, Jul. 2003.

[16] G. Peng, W. Cheng, and W. Yi, "A general variable neighborhood search for single-machine total tardiness scheduling problem with step-deteriorating jobs," *J. Ind. Manag. Optim.*, vol. 10, no. 2, pp. 1071–1090, Oct. 2017.

[17] F. Liu, Y. Yang, and Y. Y. Lu, "Solution algorithms for single-machine group scheduling with ready times and deteriorating jobs," *Eng. Optim.*, vol. 51, no. 5, pp. 862–874, May 2019.

[18] D. Wang, F. Liu, Y. Yin, J. Wang, and Y. Wang, "Prioritized surgery scheduling in face of surgeon tiredness and fixed off-duty period," *J. Combinat. Optim.*, vol. 30, no. 4, pp. 967–981, Nov. 2015.

[19] B. Cardoen, E. Demeulemeester, and J. Beliën, "Operating room planning and scheduling: A literature review," *Eur. J. Oper. Res.*, vol. 201, no. 3, pp. 921–932, Mar. 2010.

[20] E. Demeulemeester, J. Beliën, B. Cardoen, and M. Samudra, "Operating room planning and scheduling," in *Handbook of Healthcare Operations Management*. New York, NY, USA: Springer, 2013, pp. 121–152.

[21] M. A. Abdeljaouad, N. El Houda Saadani, and Z. Bahroun, "A dichotomic algorithm for an operating room scheduling problem," in *Proc. Int. Conf. Control, Decis. Inf. Technol. (CoDIT)*, Nov. 2014, pp. 134–139.

[22] F. Liang, Y. Guo, and R. Y. Fung, "Simulation-based optimization for surgery scheduling in operation theatre management using response surface method," *J. Med. Syst.*, vol. 39, no. 11, p. 159, Nov. 2015.

[23] P. M. Castro and I. Marques, "Operating room scheduling with generalized disjunctive programming," *Comput. Oper. Res.*, vol. 64, no. 1, pp. 262–273, Dec. 2015.

[24] V. Roshanaei, C. Luong, D. M. Aleman, and D. R. Urbach, "Collaborative operating room planning and scheduling," *Informs J. Comput.*, vol. 29, no. 3, pp. 558–580, Jul. 2017.

[25] S. Lee and Y. Yih, "Reducing patient-flow delays in surgical suites through determining start-times of surgical cases," *Eur. J. Oper. Res.*, vol. 238, no. 2, pp. 620–629, Oct. 2014.

[26] J. T. van Essen, E. W. Hans, J. L. Hurink, and A. Oversber, "Minimizing the waiting time for emergency surgery," *Oper. Res. Health Care*, vol. 1, nos. 2–3, pp. 34–44, 2012.

[27] G. Latorre-Núñez, A. Lüer-Villagra, V. Marianov, C. Obreque, F. Ramis, and L. Neriz, "Scheduling operating rooms with consideration of all resources, post anesthesia beds and emergency surgeries," *Comput. Ind. Eng.*, vol. 97, no. 1, pp. 248–257, Jul. 2016.

[28] L. R. Kroer, K. Foverskov, C. Vilhelmsen, A. S. Hansen, and J. Larsen, "Planning and scheduling operating rooms for elective and emergency surgeries with uncertain duration," *Oper. Res. Health Care*, vol. 19, no. 1, pp. 107–119, Dec. 2018.

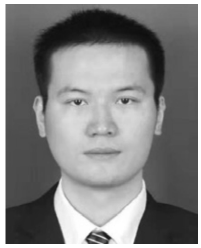
[29] J. M. Molina-Pariente, V. Fernandez-Viagas, and J. M. Framinan, "Integrated operating room planning and scheduling problem with assistant surgeon dependent surgery durations," *Comput. Ind. Eng.*, vol. 82, no. 1, pp. 8–20, Apr. 2015.
[30] K. Bjorgul, W. M. Novicoff, and K. J. Saleh, "Learning curves in hip fracture surgery," *Int. Orthop.*, vol. 35, no. 1, pp. 113–119, Jan. 2011.
[31] L. Luo, D. D. Yao, X. Huang, Y. You, Y. Cheng, Y. Shi, J. Liu, and R. Gong, "Sequence-dependent anesthesia-controlled times: A retrospective study in an ophthalmology department of a single-site hospital," *Anesth. Analg.*, vol. 119, no. 1, pp. 62–151, Jul. 2014.
[32] Y. Gocgun and M. L. Puterman, "Dynamic scheduling with due dates and time windows: An application to chemotherapy patient appointment booking," *Health Care Manag. Sci.*, vol. 17, no. 1, pp. 60–76, Mar. 2014.
[33] R. L. Graham, E. L. Lawler, J. K. Lenstra, and A. H. G. R. Kan, "Optimization and approximation in deterministic sequencing and scheduling: A survey," *Ann. Math.*, vol. 5, pp. 287–326, 1979.

MINSHAN SONG received the B.S. degree in mathematics from Soochow University, in 2008, and the M.S. degree in mathematics from the National University of Singapore, in 2009. She is currently pursuing the Ph.D. degree in management science and engineering with Southeast University, Nanjing, China.

Her research interests include healthcare management and scheduling, queuing theory, and operational research.

YULIN ZHANG received the B.S. degree in mathematics from Nanjing University, the M.S. degree in systems engineering, in 1991, and the Ph.D. degree in management science and engineering from Southeast University, in 2002.

He is currently a Professor with the School of Management Science and Engineering, Southeast University. His research interests include service operation and revenue management, supply chain management, and business intelligence and decision support systems.



YUE ZHU received the M.S. degree in management science and engineering from Southeast University, in 2016.

His research interests include healthcare management and scheduling, and scheduling.