IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received July 17, 2019, accepted August 1, 2019, date of publication August 6, 2019, date of current version September 12, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2933529

Two-Phase Fluctuating Flow of Dusty Viscoelastic Fluid Between Non-Conducting Rigid Plates With Heat Transfer

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The authors extend their appreciation to the Deanship of Scientific Research at Majmaah University for funding this work under project number No (RGP-2019-3).

ABSTRACT The objective of this article is to investigate the combined effects of the magnetic field and heat transfer on electrically conducting viscoelastic incompressible dusty fluid, flowing between two non-conducting rigid plates. The flow generation in the aforementioned fluid is caused by the oscillating pressure gradient and heat transfer. It is also assumed that all the dust particles having spherical shapes are homogeneously distributed in the fluid. In order to investigate analytical solutions, Light Hill method has been used. Furthermore, the effects of different parameters like an elastic parameter, radiation parameter, Reynolds number and Grashof number on velocity and applied shear stress have been discussed. A noteworthy relation of applied magnetic field with velocity and applied shear stresses is noticed, from the present study it is depicted that in boundary layer flow velocity increases and shear stress decreases with increase in applied magnetic field, while in ordinary course of nature, by increasing applied magnetic field decrease occurs in fluid velocity and shear stress increases.

INDEX TERMS Two phase fluctuating flow, viscoelastic dusty fluid, heat transfer, magnetohydrodynamic.

I. INTRODUCTION

From several decades, the researchers are working on multiphase flows due to wide applications in blood flow, enhancement of heat transfer in gas cooling systems, the motion of inert particles in atmosphere, ordinary flame, rocket exhaust, dust precipitators (used to remove pollution from smokestacks), dusty plasma devices (DPDs) and many more. As for the idea of multiphase flow is concerned, in multiphase flows, the effect of different physical parameters on both the velocities of a dust particle (dusty phase) and base fluid (fluid phase) is scrutinized.

For the very first time in 1967, Soo [1] represented the basic theory of multiphase flow. Michael and Miller [2] in 1966 investigated the flow of dusty gas produced by the motion of the infinite plate. The stability of the laminar flow of gas is studied by Saffman [3]. In cylindrical coordinates and over the flat plate the flows of dusty fluid are discussed by Healy [4]. Vimala [5] also investigated the flow of dusty fluid in a channel. Furthermore, in 1976 Gupta and Gupta [6], analyzed the dusty gas flow in a closed channel with an arbitrary time-varying pressure gradient. Venkateshappa *et al.* [7] used the hodograph method and also took the magnitude of velocity constant in each individual streamline. Venkatesh and Kumara [8] also investigated the unsteady flow of conducting dusty fluid between oscillating plates along the wavy wall. From there findings they concluded, that velocity profiles for fluid and dust particles having parabolic nature. The velocities approach zero for large values of time, while at the center of the channel the velocities are minimum. Further, Ghosh and Sana [9] also carried out the investigation of the transversely applied magnetic field to the fluid having dust particles of spherical shape. In another paper, Gosh and Gosh [10] shared their efforts about dusty fluid by finding that in presence of pulsation when the concentration of the dust particles is increased,

The associate editor coordinating the review of this article and approving it for publication was Yunjie Yang.

it increases the velocity of the dusty fluid near the plate in the case when time is large while rotation is small. The study of the two-dimensional flow of dusty fluid over a stretching sheet in the presence of radiation is carried out by Gireesha *et al*. [11], for the analysis of this phenomenon they have transformed the highly non-linear momentum and heat equation into coupled ordinary differential equations by similarity transformation. From their major findings, they have claimed that both the fluid and particles velocities retard by increasing the applied magnetic field. Gosh and Debnath [12] explored the flow of the hydromagnetic stock of dusty fluid. They have employed Operational method for finding the solutions of fluid velocity, particles velocity, and shear stress. The influence of large and small time on solutions is also discussed in detail. In 1988 Debnath and Gosh [13], extended this work about dusty fluid. They discussed the unsteady flow of hydromagnetic dusty fluid between a closed channel. During this inquisition, they found the influence of dust particles on fluid velocity related to the time period of oscillation of the plates. For the small time period oscillations frequency of the plates increases which retards the velocity of the fluid with an increase of dust particles, while with the large time period of oscillations, the frequency of the plates decreases and velocity increases with increase in a number of dust particles. The flow of electrically conducting dusty fluid at steady state is investigated by Attia and Abdeen [14]. They have performed a numerical investigation of steady hydromagnetic electrically conducting non-Newtonian Oldroyd 8-constant dusty fluid which is moving in a circular pipe through finite differences. The investigation of steady and transient solutions of MHD second grade fluid is carried out by Ali *et al*. [15]. From their findings, they concluded that by increasing second grade parameter velocity decreases, while in case of the influence of MHD term and radiation parameter both the velocity and skin friction have reverse behavior i.e.by increasing MHD velocity is decreased and shear stress increased. While in case of increasing radiation parameter velocity increased and shear stress decreased. Furthermore, the closed form solutions of unsteady flow of second grade fluid with free conviction are analyzed by Ali *et al.* [16]. By analyzing the effects of different parameters they have found that by increasing second grade parameter velocity decreases. When both phase angle and time are increased skin friction also increases while temperature decreases with an increase in Pr.

The aforementioned entire discussion was to facilitate the readers about under investigation phenomenon, in which the fluctuating flow of viscoelastic dusty fluid with heat transfer is considered. Ali and Sheikh [17] overviewed viscoelasticity of fluids, additionally, they explained comprehensively MHD and its applications in real life. The unsteady MHD flow of second grade fluid with heat and mass transfer in a porous medium is investigated by Ali *et al*. [18] through Laplace transformation. They have briefly discussed the parametric influence on velocity. Through the porous medium, the MHD flow of second grade fluid is examined by Khan *et al*. [19].

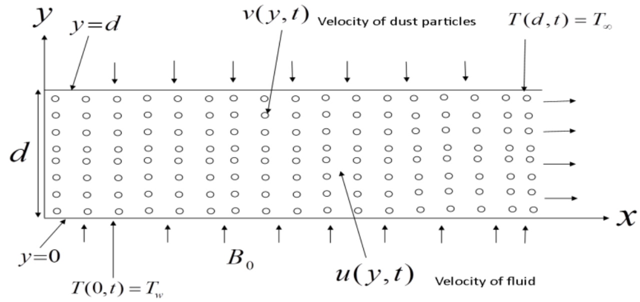

FIGURE 1. Physical model of the problem.

They have discussed the influence of the magnetic field and porous medium on the flow of second grade fluid. Recently, Ali *et al*. [20] worked on two-phase flow. For the first time, they have investigated the flow of magnetic particles in blood with isothermal heat transfer through Caputo-Fabrizio frictional model. Here in this article, we are also investigating the two-phase flow of dusty fluid in a closed channel with heat transfer. Dey [21], worked out on the flow of Oldroyd hydromagnetic dusty fluid in a horizontal channel with energy dissipation and volume fraction. From their work, they have cited some findings, which are: velocity of the fluid is maximum in the neighborhoods of upper plates. The decrease occurs in fluid velocity by increasing values of relaxation and retardation time parameters. Roach *et al.* [22] developed equations for the flow of viscous dusty gas. They used the method of intrinsic volume averaging and take the viscosities of the flowing phase as the function of pressure. In present work, we have considered viscoelastic fluid flow, in which the dust particles are uniformly distributed in the fluid with MHD and heat transfer analysis.

II. MATHEMATICAL FORMULATION

The fluid flow is only in x-direction, and we have considered the oscillatory flow of viscoelastic dusty fluid in the closed channel. It's also assumed that due to the influence of the transversely applied magnetic field B_0, the dusty fluid is electrically conducting. Because of the small magnitude of the external magnetic field, the polarization is negligible so the internal electric field is zero. The temperature of the lower boundary of the channel is T_w, while at the upper plate the ambient temperature T_∞ is considered. The influence of radiation in the energy equation is also taken into account. By using the assumption of Boussinesq approximation and in order to avoid likeness it is preferred to refer [8] and [9] for the basic equations of momentum and energy for dusty fluid in two-phase flow.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (\upsilon + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} + \frac{K_0 N_0}{\rho}(v-u) - \frac{\sigma B_0^2 u}{\rho} + g\beta_T(T-T_\infty), \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

where $-\frac{\partial q_r}{\partial y} = 4\alpha_0^2(T-T_\infty)$.

In equation (1), $u(y,t)$ representing velocity of fluid while $v(y,t)$ represents velocity of dust particles which are uniformly distributed in the viscoelastic fluid, α_0 is mean radiation absorption coefficient, q_r is radiation heat flux, c_p is specific heat capacity, k is thermal conductivity, g is gravitational acceleration, ρ is fluid density, μ viscosity, α_1 is viscoelastic parameter, σ electrical conductivity, while B_0 is induced magnetic field.

This can be represented by Newton's law of motion as:

$$m\frac{\partial v}{\partial t} = K_0(u - v), \tag{3}$$

subject to the boundary conditions:

$$u(0,t) = 0, \quad u(d,t) = U(t),$$

$T(d,t) = T_\infty, T(0,t) = T_w$, where $U(t) = u_0\left(1 + \frac{\varepsilon}{2}(e^{i\omega t} + e^{-i\omega t})\right)$ is free stream velocity.

In order to calculate the velocity of the dust particles we assume the velocity of the form [23]:

$$v(y,t) = v_0(y)e^{i\omega t}, \tag{4}$$

from equation (3), we get. (for detail calculi see Appendix A)

$$v(y,t) = (\frac{K_0}{mi\omega + K_0})u(y,t), \tag{5}$$

by incorporating velocity of dust particles, equation (1) becomes:

$$\begin{aligned}\frac{\partial u}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \left(\upsilon + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} \\ &\quad + \frac{K_0 N_0}{\rho}\left\{\left(\frac{K_0}{mi\omega + K_0}\right)u - u\right\} \\ &\quad - \frac{\sigma B_0^2 u}{\rho} + g\beta_T(T - T_\infty),\end{aligned} \tag{6}$$

above boundary layer for free stream velocity equation (6) will take the following form:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{dU}{dt} + \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial t \partial y^2} - \frac{\sigma B_0^2}{\rho}(u - U) \\ &\quad + \frac{K_0 N_0}{\rho}\left(\frac{K_0}{(mi\omega + K_0)} - 1\right)(u - U) \\ &\quad + g\beta_T(T - T_\infty),\end{aligned} \tag{7}$$

To make equation (2) and equation (7) dimensionless we introduce the following dimensionless variables.

$$\begin{aligned}u^* &= \frac{u}{u_0}, \quad U^* = \frac{U}{u_0}, \; y^* = \frac{y}{d}, \; t^* = \frac{u_0 t}{d}, \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty}.\end{aligned} \tag{8}$$

For the sake of simplicity $(*)$ sign has been emitted but understood. Dimensionless momentum and energy equations are:

$$\begin{aligned}\mathrm{Re}\frac{\partial u}{\partial t} &= \mathrm{Re}\frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t \partial y^2} \\ &\quad + (K_2 - K_1)(u - U) - M(u - U) + Gr\theta,\end{aligned} \tag{9}$$

$$Pe\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta. \tag{10}$$

with dimensionless boundary condition:

$$\begin{aligned}&u(0,t) = 0, \quad u(1,t) = U(t), \\ &\because U(t) = 1 + \frac{\varepsilon}{2}(e^{i\omega t} + e^{-i\omega t}), \\ &\theta(0,t) = 1, \quad \theta(1,t) = 0.\end{aligned} \tag{11}$$

where:

$$\begin{aligned}\mathrm{Re} &= \frac{u_0 d}{\upsilon}, \quad \alpha = \frac{\alpha_1 u_0}{\rho \upsilon d}, \; K_1 = \frac{K_0 N_0 d^2}{\rho \upsilon}, \\ K_2 &= \frac{K_0^2 N_0 d^2}{\rho \upsilon (mi\omega + K_0)}, \quad M = \frac{\sigma B_0^2 d^2}{\rho \upsilon}, \\ Gr &= \frac{g\beta_T d^2(T_w - T_\infty)}{\upsilon u_0}, \quad Pe = \frac{\rho c_p u_0 d}{k}, \\ N^2 &= \frac{4\alpha_0^2 d^2}{k}\end{aligned}$$

The above dimensionless parameters Re, α, K_1, K_2, M, Gr, Pe, N are representing Reynolds number, second grade fluid parameter, dusty fluid parameters (which represent the concentration of dust particles in the fluid), magnetic parameter, Grashofnumber, Peclet number, and radiation parameter respectively.

To solve equation (10), we assume the solution of the form:

$$\theta(y,t) = \theta_0(y) + \theta_1(y)e^{i\omega t}, \tag{12}$$

using equation (12) the solution of equation (10) is given as:

$$\theta(y,t) = \frac{\sin(N - Ny)}{\sin N}. \tag{13}$$

Using equation (13) in equation (9) we have:

$$\begin{aligned}\mathrm{Re}\frac{\partial u}{\partial t} &= \mathrm{Re}\frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t \partial y^2} \\ &\quad + (K_2 - K_1)(U - u) - M(u - U) \\ &\quad + Gr\left\{\frac{\sin(N - Ny)}{\sin N}\right\},\end{aligned} \tag{14}$$

III. SOLUTION OF THE MOMENTUM EQUATION

To solve equation (14) the Light Hill method [23] is used which is given as:

$$u(y,t) = F_0(y) + \frac{\varepsilon}{2}(F_1(y)e^{i\omega t} + F_2(y)e^{-i\omega t}), \tag{15}$$

by incorporating equation (15) in equation (14), we get the following values for $F_0(y)$, $F_1(y)$ and $F_2(y)$.

$$\begin{aligned}F_0(y) &= -(1 + A)\cosh\sqrt{m_1} \\ &\quad + (1 + A)\frac{\cosh\sqrt{m_1}}{\sinh\sqrt{m_1}}\sinh y\sqrt{m_1} \\ &\quad + 1 + A\frac{\sin(N - Ny)}{\sin N},\end{aligned} \tag{16}$$

where $A = \frac{Gr}{N^2 + M + (K_2 - K_1)}$ and $m_1 = M + K_2 - K_1$,

$$\begin{aligned}F_1(y) &= -\cosh y\sqrt{m_3} \\ &\quad + \cosh\sqrt{m_3}\frac{\sinh y\sqrt{m_3}}{\sinh\sqrt{m_3}} + 1,\end{aligned} \tag{17}$$

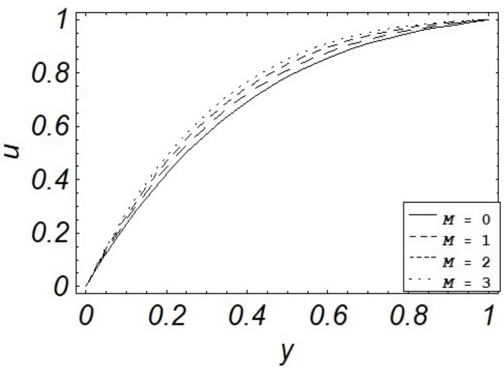

FIGURE 2. Velocity graph for different values of M at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $\alpha = 5$, $R = 1$, $Gr = 2$, $\epsilon = 0.001$.

with $m_3 = \frac{M+K_2-K_1+\mathrm{Re}i\omega}{1+\alpha i\omega}$

$$F_2(y) = \frac{\sinh(y\sqrt{m_4} - \sqrt{m_4})}{\sinh\sqrt{m_4}}, \tag{18}$$

where $m_4 = \frac{M+K_2-K_1-\mathrm{Re}i\omega}{1-\alpha i\omega}$.

Finally incorporating equations (16), (17) and (18) in equation (15) we get the following form of

$$\begin{aligned} u(y,t) = &-(1+A)\cosh\sqrt{m_1}+(1+A)\frac{\cosh\sqrt{m_1}}{\sinh\sqrt{m_1}}\sinh y\sqrt{m_1} \\ &+1+A\frac{\sin(N-Ny)}{\sin N}+\frac{\sinh(y\sqrt{m_4}-\sqrt{m_4})}{\sinh\sqrt{m_4}}e^{-i\omega t} \\ &+\frac{\varepsilon}{2}\left\{1-\cosh y\sqrt{m_3}+\cosh\sqrt{m_3}\frac{\sinh y\sqrt{m_3}}{\sinh\sqrt{m_3}}\right\}e^{i\omega t}. \end{aligned} \tag{19}$$

Equation (19) satisfies the boundary conditions which shows the validity of our calculations.

IV. SKIN FRICTION

In order to find skin friction for the viscoelastic dusty fluid dimensional form is given as:

$$\tau = (\mu + \alpha_1\frac{\partial}{\partial t})\frac{\partial u}{\partial y}, \tag{20}$$

To make equation (20) dimensionless let consider $\tau^* = \frac{\tau d^2}{\mu\upsilon}$ and incorporating the remaining dimensionless variables from equation (8) also emitting the $(*)$ sign for the sake of simplification, we have:

$$\tau = \mathrm{Re}\frac{\partial u}{\partial y} + \alpha\frac{\partial^2 u}{\partial t\partial y}. \tag{21}$$

V. GRAPHICAL RESULTS AND DISCUSSION

In this section, the effects of different parameters on velocity, shear stress, and temperature profile have been discussed. During the investigation, it has been observed that in boundary layer flow by increasing M velocity increases while in free stream area, velocity decreases by increasing M as shown in Fig. 2. As viscoelastic parameter α having a direct relation

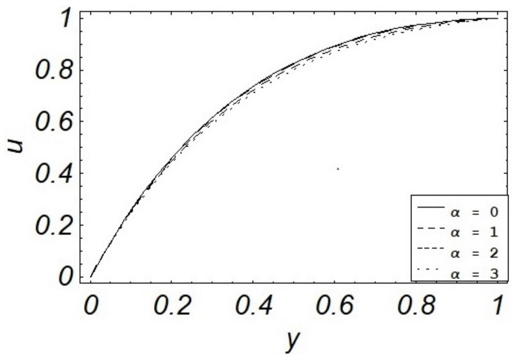

FIGURE 3. Velocity graph for different values of α at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $M = 1$, $R = 1$, $Gr = 2$, $\epsilon = 0.001$.

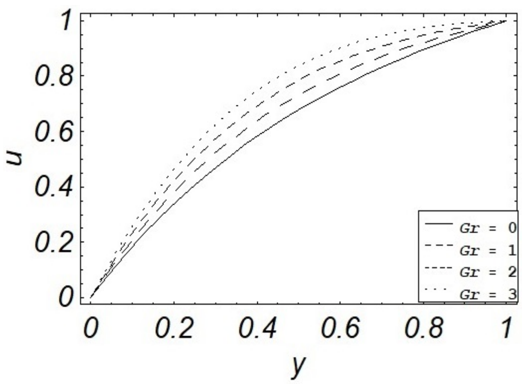

FIGURE 4. Velocity graph for different values of Gr at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

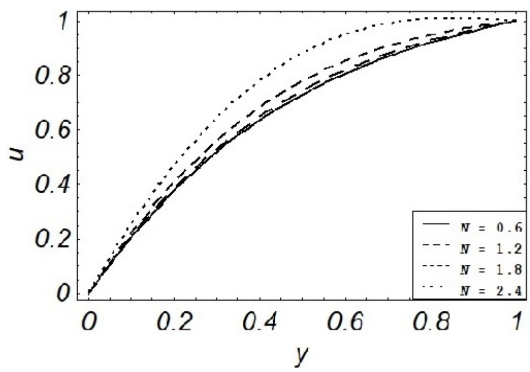

FIGURE 5. Velocity graph for different values of N at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $M = 1$, $K1 = 2$, $\alpha = 5$, $R = 1$, $Gr = 2$, $\epsilon 0.001$.

with inertial forces, so by increasing α the inertial forces increases, and as a result, these inertial forces decrease the velocity of the fluid as shown in Fig. 3. About the relation of Gr with velocity, it has observed that increase in Gr bringing a decrease in viscosity, this decrease in viscosity occurs due to the increase of bouncy forces and as a result velocity of the fluid increases, so increase in Gr brings an increase in velocity, labeled in Fig. 4. By scrutinizing the effect of radiation on the velocity it has noticed that by increasing radiation parameter N, the temperature of the fluid increases, this

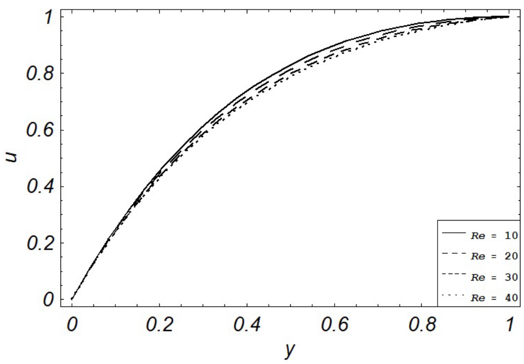

FIGURE 6. Velocity graph for different values of Re at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $M = 1$, $\alpha = 1$, $Gr = 2$, $\epsilon = 0.001$.

increase of temperature brings an increase in kinetic energy which increases the velocity of the fluid, shown in Fig. 5. Inside boundary layer velocity of the viscoelastic dusty fluid decreases with increase in Re. During this phenomenon, it depicted that even in viscoelastic dusty fluid Re can be used for boundary layer control because inside boundary layer increase of Re brings retardation in fluid velocity as interpreted in Fig. 6.

Relation of the dust particle's density is also scrutinized in this article. It has been considered that the shape of the dust particle is spherical, so by Stocks drag formula ($k_1 = 6\pi r\mu$) as clear from this relation by increasing k_1 the viscous forces of viscoelastic dusty fluid decreases as a result velocity increases, so by increasing the number of dust particles velocity increases clear from Fig. 7.

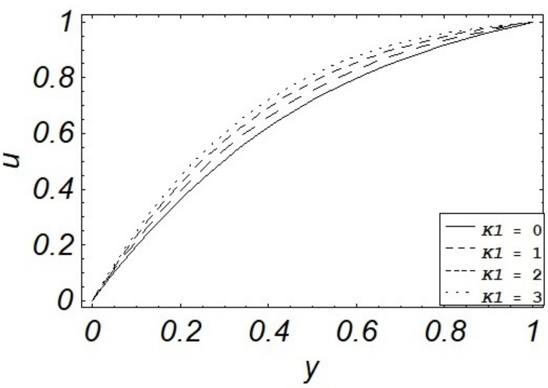

FIGURE 7. Velocity graph for different values of $K1$ at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $Gr = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

In this section, the effects of different parameters on velocity, shear stress, and temperature profile have been discussed. During the investigation, it has been observed that in boundary layer flow by increasing M velocity increases while in free stream area, velocity decreases by increasing M as shown in Fig. 2. As viscoelastic parameter α having a direct relation with inertial forces, so by increasing α the inertial forces increases, and as a result, these inertial forces decrease the velocity of the fluid as shown in Fig. 3. About the relation of Gr with velocity, it has observed that increase in Gr bringing a decrease in viscosity, this decrease in viscosity occurs due to the increase of bouncy forces and as a result velocity of the fluid increases, so increase in Gr brings an increase in velocity, labeled in Fig. 4. By scrutinizing the effect of radiation on the velocity it has noticed that by increasing radiation parameter N, the temperature of the fluid increases, this increase of temperature brings an increase in kinetic energy which increases the velocity of the fluid, shown in Fig. 5.

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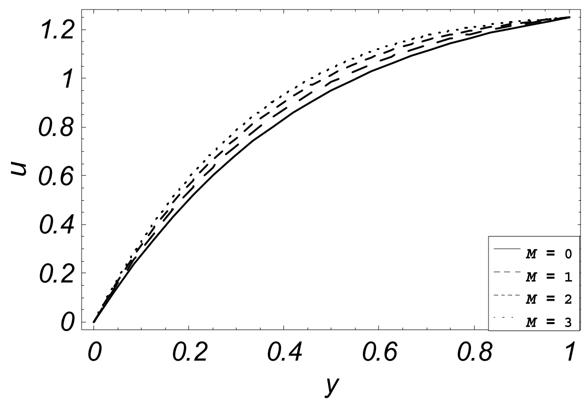

FIGURE 8. Velocity graph for different values of $K1$ at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $Gr = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.5$.

Fig. 8 is depicted for the influence of M on the velocity which has discussed before, but here, in this case, the larger value of $\boldsymbol{\varepsilon}$ has been considered which can show the oscillating part of velocity as well, when $\boldsymbol{\varepsilon}$ is a very small number, then oscillating part is negligible which shows constant velocity at the upper plate. When $\boldsymbol{\varepsilon} = 0.001$ $u = 1$ while in the case when $\boldsymbol{\varepsilon} = 0.5$ $u = 1.2$.

The relation of different parameters with applied shear stress is discussed here. As already mentioned that by increasing M boundary layer velocity increase this increase in velocity is an agreement that less amount of applied shear stresses will be required, due to this reason we are able to claim that by increasing M applied shear stress decrease as shown in Fig. 9. The intermolecular forces are decreases with increase in Gr so by increasing Gr applied shear stress will decrease. So Gr and applied shear stress are inversely related to each other, as shown in Fig. 10. In this investigation, we observe the dual

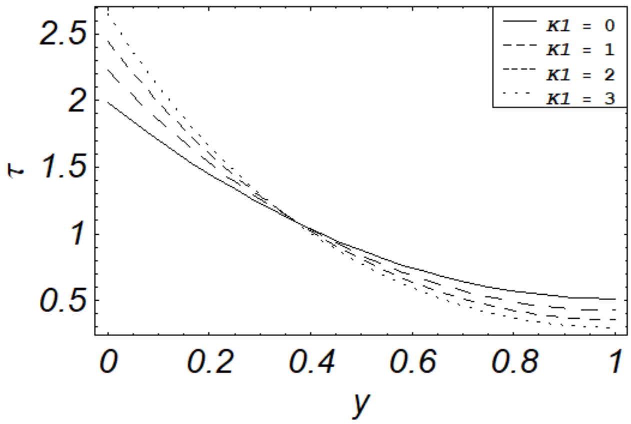

FIGURE 9. Relation of shear stress with different values of M at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $Gr = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

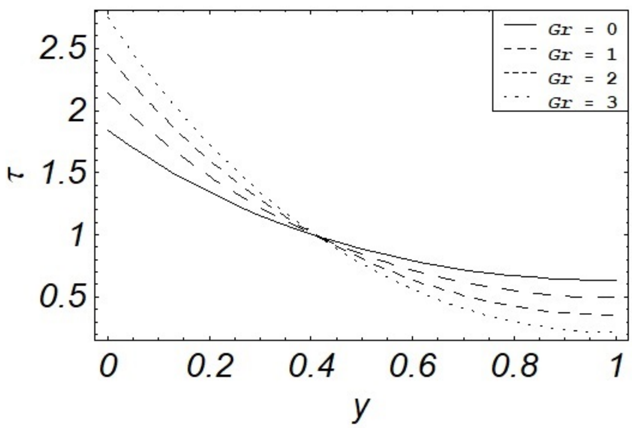

FIGURE 10. Relation of shear stress with different values of Gr at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $K1 = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

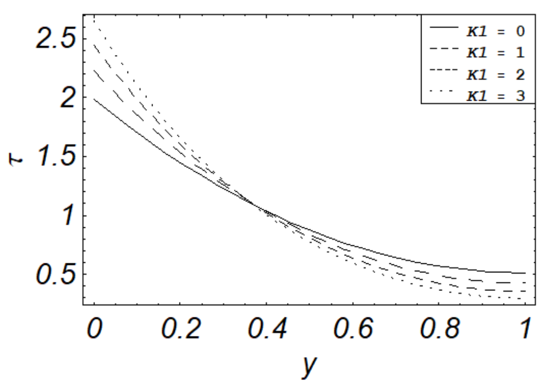

FIGURE 11. Relation of shear stress with different values of $K1$ at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $N = 1$, $Gr = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

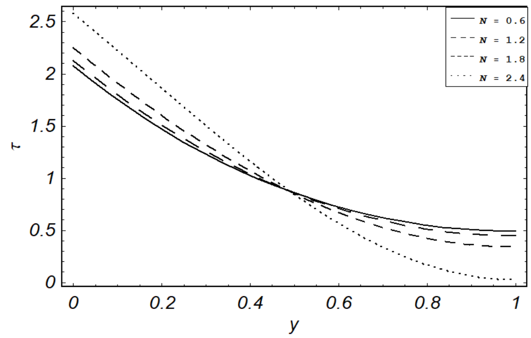

FIGURE 12. Relation of shear stress with different values of N at $t = 1$ and $Pe = 0.5$, $\omega = 0.5$, $K1 = 1$, $Gr = 2$, $M = 1$, $R = 1$, $\alpha = 2$, $\epsilon = 0.001$.

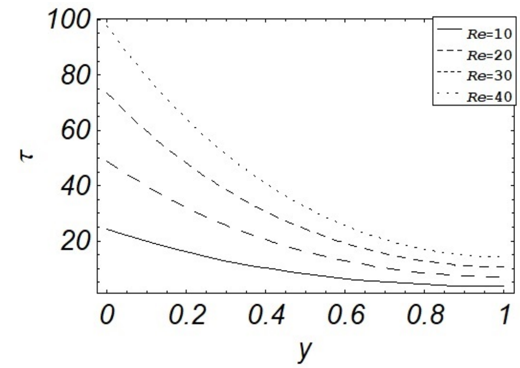

FIGURE 13. Relation of shear stress with different values of Re at $t = 1$ and $Pe = 0.5$, $\omega = b0.5$, $N = 1$, $Gr = 2$, $M = 1$, $K1 = 1$, $\alpha = 2$, $\epsilon = 0.001$.

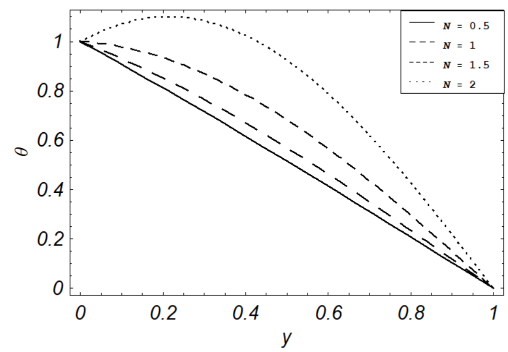

FIGURE 14. Relation of temperature with N.

behavior of shear stress with the concentration of dust particles because in boundary layer flow the amount of applied shear stress for flow generation decreases while above boundary layer applied shear stress increases by increasing the number of dust particles as shown in Fig. 11. Increase of radiation parameter increases $K.E$ of molecules, decrease intermolecular forces also decrease the viscous forces, so it shows that by increasing radiation parameter applied shear stress will decrease, clear from figure 12. From Fig. 13 the relation of applied shear stresses and Re has shown as in the previous section we already discuss that by increasing Re velocity is decreasing, so here we can claim that less amount of applied shear stress will be required, in other words, we can say that by increasing Re applied shear stresses decreases. Fig. 14 tells about the relation of temperature and radiation parameters, as clear from the fig by increasing N temperature of the dusty fluid will increase.

The influence of M, Gr, N, α and K_1 has shown in table 1. One can observe from the table that increases of M brings an increase in skin friction, while this phenomenon is quite

TABLE 1. Parametric influence of skin friction.

N	0.1	0.2	0.3	0.4	0.5	0.6
Nu	.9966	.9866	.9698	.9460	.9152	.8770

TABLE 2. Relation of nusselt number with radiation parameter.

t	Re	M	Pe	Gr	N	α	K_1	ω	ε	Cf
1	1	0.5	1	0.5	1	0.8	0.4	0.01	0.05	1.9813
1	1	**1**	1	0.5	1	0.8	0.4	0.01	0.05	2.1121
1	1	0.5	1	**1**	1	0.8	0.4	0.01	0.05	2.1507
1	1	0.5	1	0.5	**2**	0.8	0.4	0.01	0.05	2.0696
1	1	0.5	1	0.5	1	**1.6**	0.4	0.01	0.05	1.9818
1	1	0.5	1	0.5	1	0.8	**0.8**	0.01	0.05	2.0866

identical that increase of M will increase Cf. During observing the effect of Gr on Cf it has been pictured that an increase occurs in Cf with an increase in Gr. The same behavior we can notice in case of N and α, because by increasing radiation parameter and viscoelastic parameter Cf increases. While from table 2, it has shown that an increase of N brings a decrease in Nusselt number.

APPENDIX A

$$m\frac{\partial v}{\partial t} = K_0(u - v), \tag{A1}$$

$$v(y, t) = v_0 e^{i\omega t}, \tag{A2}$$

$$m\frac{\partial}{\partial t}(v_0 e^{i\omega t}) = K_0(u - v_0 e^{i\omega t}), \tag{A3}$$

$$i\omega v_0 e^{i\omega t} = \frac{K_0}{m}(u - v_0 e^{i\omega t}), \tag{A4}$$

$$v_0\left(\frac{mi\omega + K_0}{K_0}\right) e^{i\omega t} = u, \tag{A5}$$

$$v_0 = u\left(\frac{K_0}{mi\omega + K_0}\right) e^{-i\omega t}, \tag{A6}$$

$$v = u\left(\frac{K_0}{mi\omega + K_0}\right). \tag{A7}$$

REFERENCES

[1] S. L. Soo, *Fluid Dynamics of Multiphase Systems*. Waltham, MA, USA: Blaisdell, 1967, p. 524.

[2] D. H. Michael and D. A. Miller, ''Plane parallel flow of a dusty gas,'' *Mathematica*, vol. 13, no. 1, pp. 97–109, 1966.

[3] P. G. Saffman, ''On the stability of laminar flow of a dusty gas,'' *J. Fluid Mech.*, vol. 13, no. 1, pp. 120–128, 1962.

[4] J. V. Healy, ''Perturbed two-phase cylindrical type flows,'' *Phys. Fluids*, vol. 13, no. 3, pp. 551–557, 1970.

[5] C. S. Vimala, ''Flow of a dusty gas between two oscillating plates,'' *Defense Sci. J.*, vol. 22, no. 4, pp. 231–236, 1972.

[6] R. K. Gupta and S. C. Gupta, ''Flow of a dusty gas through a channel with arbitrary time varying pressure gradient,'' *Zeitschrift für angewandte Mathematik und Physik ZAMP*, vol. 27, no. 1, pp. 119–125, 1976.

[7] Y. Venkateshappa, R. Basavanna, B. J. Gireesha, and K. R. Gopinath, ''Viscous dusty fluid flow with constant velocity magnitude,'' *Electron. J. Theor. Phys.*, vol. 5, no. 17, pp. 237–252, 2008.

[8] P. Venkatesh and B. C. P. Kumara, ''Exact solution of an unsteady conducting dusty fluid flow between non-torsional oscillating plate and a long wavy wall,'' *J. Sci. Arts*, vol. 13, no. 1, pp. 97–106, 2013.

[9] A. K. Ghosh and P. Sana, ''On hydromagnetic channel flow of an Oldroyd-B fluid induced by rectified sine pulses,'' *Comput. Appl. Math.*, vol. 28, no. 3, pp. 365–395, 2009.

[10] S. Ghosh and A. K. Ghosh, ''On hydromagnetic rotating flow of a dusty fluid near a pulsating plate,'' *Comput. Appl. Math.*, vol. 27, no. 1, pp. 1–30, 2008.

[11] B. J. Gireesha, A. J. Chamkha, S. Manjunatha, and C. S. Bagewadi, ''Mixed convective flow of a dusty fluid over a vertical stretching sheet with non-uniform heat source/sink and radiation,'' *Int. J. Numer. Methods Heat Fluid Flow*, vol. 23, no. 4, pp. 598–612, 2013.

[12] A. K. Ghosh and L. Debnath, ''Hydromagnetic Stokes flow in a rotating fluid with suspended small particles,'' *Appl. Sci. Res.*, vol. 43, no. 3, pp. 165–192, 1986.

[13] L. Debnath and A. K. Ghosh, ''On unsteady hydromagnetic flows of a dusty fluid between two oscillating plates,'' *Appl. Sci. Res.*, vol. 45, no. 4, pp. 353–365, 1988.

[14] H. A. Attia and M. A. M. Abdeen, ''Steady MHD flow of a dusty incompressible non-Newtonian Oldroyd 8-constant fluid in a circular pipe,'' *Arabian J. Sci. Eng.*, vol. 38, no. 11, pp. 3153–3160, 2013.

[15] F. Ali, M. Norzieha, S. Sharidan, I. Khan, and T. Hayat, ''New exact solutions of Stokes' second problem for an MHD second grade fluid in a porous space,'' *Int. J. Non-Linear Mech.*, vol. 47, no. 5, pp. 521–525, Jun. 2012.

[16] F. Ali, I. Khan, and S. Shafie, ''Closed form solutions for unsteady free convection flow of a second grade fluid over an oscillating vertical plate,'' *PLoS ONE*, vol. 9, no. 2, 2014, Art. no. e85099.

[17] F. Ali and N. A. Sheikh, ''Introductory chapter: Fluid flow problems,'' in *Fluid Flow Problems*. London, U.K.: IntechOpen, 2018.

[18] F. Ali, N. A. Sheikh, M. Saqib, and I. Khan, ''Unsteady MHD flow of second-grade fluid over an oscillating vertical plate with isothermal temperature in a porous medium with heat and mass transfer by using the laplace transform technique,'' *J. Porous Media*, vol. 20, no. 8, pp. 671–690, 2017.

[19] T. Hayat, I. Khan, R. Ellahi, and C. Fetecau, ''Some MHD flows of a second grade fluid through the porous medium,'' *J. Porous Media*, vol. 11, no. 4, pp. 389–400, 2008.

[20] F. Ali, A. Imtiaz, I. Khan, and N. A. Sheikh, ''Flow of magnetic particles in blood with isothermal heating: A fractional model for two-phase flow,'' *J. Magn. Magn. Mater.*, vol. 456, pp. 413–422, Jun. 2018.

[21] D. Dey, ''Dusty hydromagnetic oldryod fluid flow in a horizontal channel with volume fraction and energy dissipation,'' *Int. J. Heat Technol.*, vol. 34, no. 3, pp. 415–422, Sep. 2016.

[22] D. Roach, M. A. Zaytoon, and M. H. Hamdan, ''On the flow of dusty gases with pressure—Dependent viscosities through porous structures,'' *Int. J. Enhanced Res. Sci., Technol. Eng.*, vol. 5, no. 9, Sep. 2016.

[23] C. Comstock, ''The poincare-lighthill perturbation technique and its generalizations,'' *SIAM Rev.*, vol. 14, no. 3, pp. 433–446, 1972.

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