IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received July 5, 2019, accepted July 30, 2019, date of publication August 6, 2019, date of current version August 30, 2019.

*Digital Object Identifier 10.1109/ACCESS.2019.2933456*

# A New Double-Wing Chaotic System With Coexisting Attractors and Line Equilibrium: Bifurcation Analysis and Electronic Circuit Simulation

**ACENG SAMBAS[1], SUNDARAPANDIAN VAIDYANATHAN[2], SEN ZHANG[3], YICHENG ZENG[3], MOHAMAD AFENDEE MOHAMED[4], AND MUSTAFA MAMAT[4]**

[1]Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Tasikmalaya 46196, Indonesia
[2]School of Electrical and Computing, Vel Tech University, Chennai 600062, India
[3]School of Physics and Opotoelectric Engineering, Xiangtan University, Xiangtan 411105, China
[4]Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Kuala Terengganu 21300, Malaysia

Corresponding author: Aceng Sambas (acengs@umtas.ac.id)

This work was supported in part by the Penelitian Kerja Sama Antar Perguruan Tinggi (PKPT) Research from the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia (KEMENRISTEKDIKTI) 2019 under Grant 2891/L4/PP/2019.

**ABSTRACT** This research work reports a double-wing chaotic system with a line of equilibrium points and constructs an electronic circuit via MultiSIM for practical implementation. Explicitly, the new chaotic system has a total of six terms with two quadratic nonlinearities and absolute function nonlinearity. Using the phase plots in MATLAB, we demonstrate that the new chaotic system has double-wing chaotic attractor. We describe the Lyapunov exponents and the Kaplan-Yorke fractal dimension of the new chaotic system. A novel feature of the new chaotic system is that the system has rest points located on the z-axis as well as two rest points not on the z-axis. Thus, the new system has infinite number of rest points and hidden attractor. We also exhibit that the new double-wing chaotic system has multi-stability and we illustrate the coexistence of attractors for two different sets of initial conditions. Some interesting dynamical properties such as offset boosting are also presented. Finally, we build an electronic circuit of the new chaotic system and show that the theoretical model has practical feasibility for implementation.

**INDEX TERMS** Chaos, chaotic systems, line equilibrium, circuit design.

## I. INTRODUCTION

Many fields of science and engineering feature applications of chaos theory and chaotic dynamical systems [1]. Nonlinear dynamical systems showing chaos are studied in several fields such as chemical reactors [2], steganography [3], encryption [4], secure communication [5], [6], etc. Recently, many dynamical systems with double-wing chaotic attractors have been studied by scientists such as Liu-Yang system [7], Lu-Xiao system [8], etc.

A new double-wing chaotic system (1) with double-wing attractor is introduced in this work. The new double-wing chaotic system (1) has a novel feature, *viz.* the rest points of the system (1) consist of the entire z-axis in $R^3$ and two points in the x-y plane symmetric about the z-axis.

The associate editor coordinating the review of this manuscript and approving it for publication was Hassan Ouakad.

In the recent years, significant studies have been done which focus on chaotic systems possessing infinitely many rest points on curves such as line [9], [10], square [11], cloud [12], circle [13], heart [14], axe [15], boomerang [16], etc.

Recent research has also focused upon finding chaotic systems with no equilibrium points [17]–[19] and chaotic systems with stable equilibrium points [20]. These special chaotic systems belong to the family of chaotic systems with hidden attractors [21], whose basins of attraction do not intersect with small neighborhoods of any equilibrium points. Chaotic systems with hidden attractors have received considerable attention in recent years [22]–[25].

Multi-stability in chaotic systems refers to the existence of multiple coexisting attractors for different initial conditions [1]. Chaotic systems with coexisting attractors have received significant attention in recent years [26]–[30].

**TABLE 1. Number of equilibrium points of multi-wing systems.**

| Chaotic Attractor | Multi-Wing Type | Number of Equilibrium Points |
|---|---|---|
| Rucklidge [31] | Two-wing system | 3 |
| Liu [32] | Two-wing system | 4 |
| Vaidyanathan [33] | Two-wing system | 2 |
| Lien *et al.* [34] | Two-wing system | 3 |
| Zhang *et al.* [35] | Three-wing system | 5 |
| Dadras *et al.* [36] | Three-wing system | 5 |
| Grassi et al. [37] | Four-wing system | 9 |
| Folifack Signing *et al.* [38] | Four-wing system | 5 |
| Volos *et al.* [39] | Four-wing system | 1 |
| This work | Two-wing system | $\infty$ |

In the chaos literature, there is significant interest in finding multi-wing chaotic systems such as two-wing systems [31]–[34], three-wing systems [35], [36], four-wing systems [37]–[39], etc. The multi-wing chaotic systems reported in [31]–[39] have a finite number of unstable equilibrium points and such chaotic systems belong to the family of chaotic systems with self-excited attractors [1]. Table 1 gives the number of equilibrium points of the multi-wing chaotic systems reported in [31]–[39].

In this research work, we introduce a new chaotic system with double-wring attractor and a line equilibrium. A novel feature of our chaotic system is that the system possesses rest points located on the z-axis as well as two rest points not on the z-axis. In comparison with the multi-wing chaotic systems having a finite number of equilibrium points (see Table 1), our new chaotic system has a line equilibrium with an infinite number of equilibrium points. Thus, it belongs to the family of chaotic systems with hidden attractors.

We also exhibit that the new chaotic system has multi-stability and we illustrate the coexistence of attractors for two different sets of initial conditions. Next, we build an electronic circuit of the new chaotic system via Multi-SIM. Finally, we build an experimental design of the new double-wing chaotic system with a real hardware circuit. Thus, we demonstrate that the theoretical chaotic model with an experimental hardware design has practical feasibility for implementation [39], [40].

## II. A NEW DOUBLE-WING DYNAMICAL SYSTEM EXHIBITING CHAOS AND A LINE OF REST POINTS

In this work, we propose a new 3-D dynamical system given by

$$\begin{cases} \dot{x} = yz \\ \dot{y} = x - y \\ \dot{z} = a|x| - bx^2 \end{cases} \tag{1}$$

In (1), $X = (x, y, z)$ is the state and $a, b$ are constant parameters. The dynamical system (1) has two quadratic nonlinear terms ($yz$ and $x^2$) and an absolute function nonlinearity ($|x|$). It is shown in this work that the system (1) is chaotic for $(a, b) = (5, 2)$.

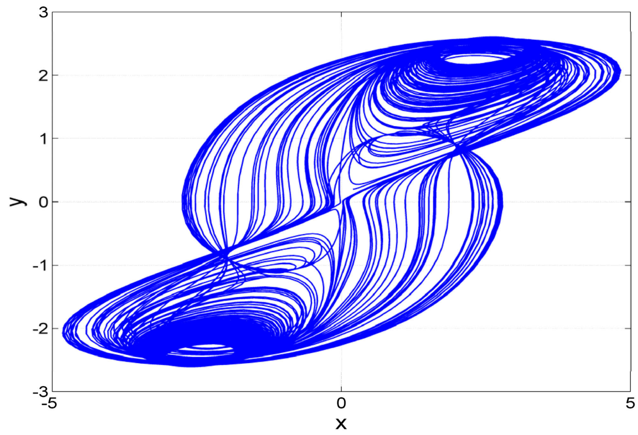



**FIGURE 1.** 2-D phase plot of the double-wing chaotic system (1) in the $(x, y)$-plane for $X_0 = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$.

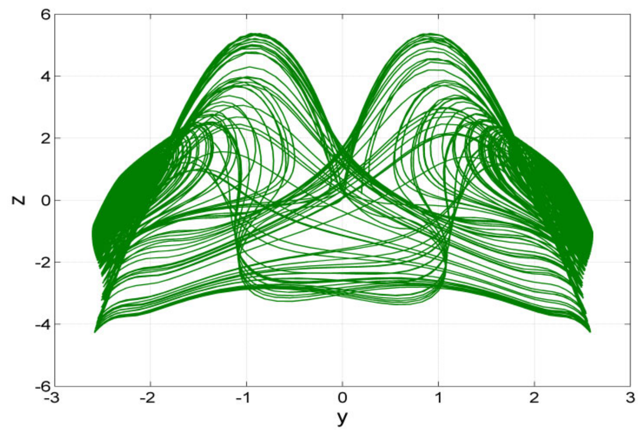



**FIGURE 2.** 2-D phase plot of the double-wing chaotic system (1) in the $(y, z)$-plane for $X_0 = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$.

For the choice of initial state $X_0 = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$. we estimate the Lyapunov exponents spectrum of the double-wing chaotic system (1) for $T = 1E5$ seconds in MATLAB as

$$LE_1 = 0.1425, \quad LE_2 = 0, \; LE_3 = -1.1425 \tag{2}$$

Since the Lyapunov exponents in (2) have the signs $(+, 0, -)$, it follows that the dynamical system (1) exhibits chaotic behaviour. Also, the Kaplan-Yorke dimension of the system (1) is found as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.1247 \tag{3}$$

In this work, we used the classical fourth-order Runge-Kutta method with step-size $h = 0.001$ in MATLAB to plot the numerical simulations of the dynamical system (1).

MATLAB planar plots of the double-wing chaotic system (1) are exhibited in Figures 1-3.

The dynamical system (1) is found to be invariant when coordinates are transformed as

$$S : (x, y, z) \mapsto (-x, -y, z) \tag{4}$$

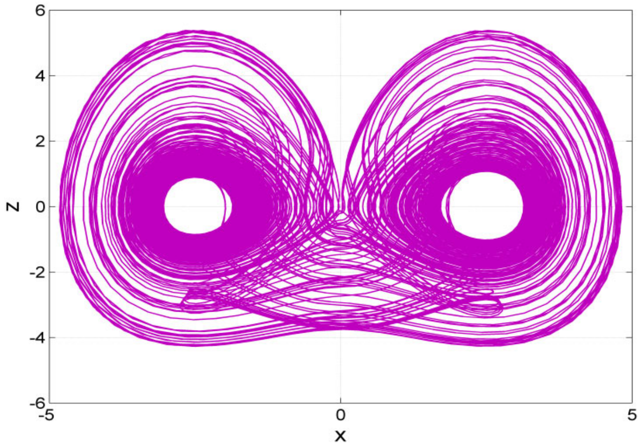



**FIGURE 3.** 2-D phase plot of the double-wing chaotic system (1) in the $(x, z)$-plane for $X_0 = (0.2, 0.2, 0.2)$ and $(a, b) = (5, 2)$.

This pinpoints that the dynamical system (1) has symmetry of rotation about the $z$-axis.

The rest points of the dynamical system (1) are found by determining roots of the following:

$$yz = 0 \tag{5a}$$

$$x - y = 0 \tag{5b}$$

$$5|x| - 2x^2 = 0 \tag{5c}$$

From (5b), $x = y$. Thus, the calculations amount to solving the following set of two equations in $x$ and $z$.

$$xz = 0 \tag{6a}$$

$$5|x| - 2x^2 = 0 \tag{6b}$$

There are two cases to consider:

*Case A:* When $x = 0$, $y = x = 0$. Also, both equations (6a) and (6b) are satisfied. Thus, the $z$-axis in $R^3$ consists of rest points of the system (1).

*Case B:* When $x \neq 0$, we get $z = 0$ from Eq. (6a). Solving the equation (6b), we get two solutions $x = 2.5$ and $x = -2.5$. We also note that $y = x$ from Eq. (5b).

Combining Cases A and B, the rest points of the system (1) consist of the entire $z$-axis in $R^3$ as well as the two points, $E_1 = (2.5, 2.5, 0)$ and $E_2 = (-2.5, -2.5, 0)$.

The origin $E_0 = (0, 0, 0)$ on the $z$-axis is also a rest point of the system (1).

Next, we undertake a stability analysis of the rest points $E_0, E_1, E_2$ and non-zero points on the $z$-axis. Let $J(X)$ denote the linearization matrix of the system (1) at $X = (x, y, z)$.

We take $(a, b) = (5, 2)$.

$J(E_0)$ is calculated using MATLAB as

$$J(E_0) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

The matrix $J(E_0)$ has the spectral values $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = -1$.

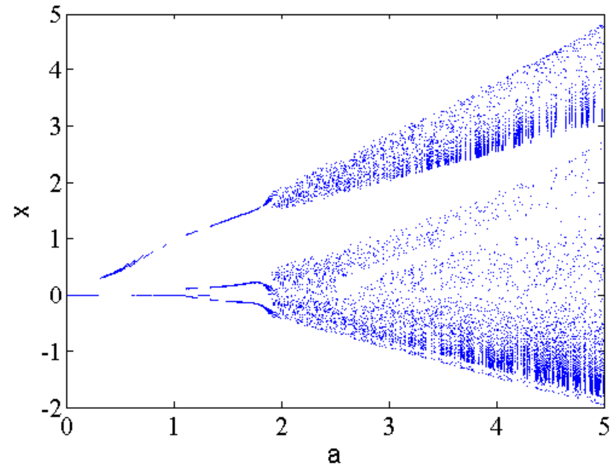



**FIGURE 4.** Bifurcation plot of the system (1) versus $a$ for $b = 2$ and $X_0 = (0.2, 0.2, 0.2)$.

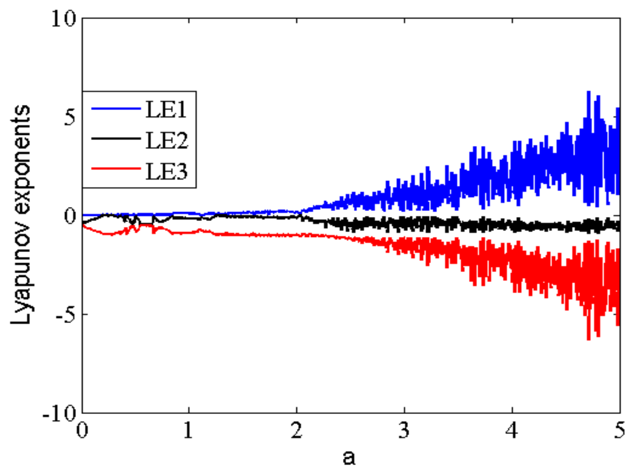



**FIGURE 5.** Lyapunov spectrum of the system (1) when varying $a$ for $b = 2$ and $X_0 = (0.2, 0.2, 0.2)$.

Thus, $E_0$ is a non-hyperbolic rest point of (1) and its stability is at a critical state. We cannot conclude the stability type of $E_0$ from the spectral values of $J(E_0)$ by the first method of Lyapunov. Using phase plots, we see that $E_0$ is unstable.

Next, the matrix $J(E_1)$ was found using MATLAB as

$$J(E_1) = \begin{bmatrix} 0 & 0 & 2.5 \\ 1 & -1 & 0 \\ -5 & 0 & 0 \end{bmatrix} \tag{8}$$

which has the spectral values $\lambda_1 = 0$ and $\lambda_{2,3} = \pm 3.5355i$. Thus, $E_1$ is a non-hyperbolic rest point of (1) and its stability is at a critical state. We cannot conclude the stability type of $E_1$ from the spectral values of $J(E_1)$ by the direct method of Lyapunov. Using phase plots, we find that $E_1$ is locally asymptotically stable.

Also, the matrix $J(E_2)$ was found using MATLAB as

$$J(E_2) = \begin{bmatrix} 0 & 0 & -2.5 \\ 1 & -1 & 0 \\ 5 & 0 & 0 \end{bmatrix} \tag{9}$$

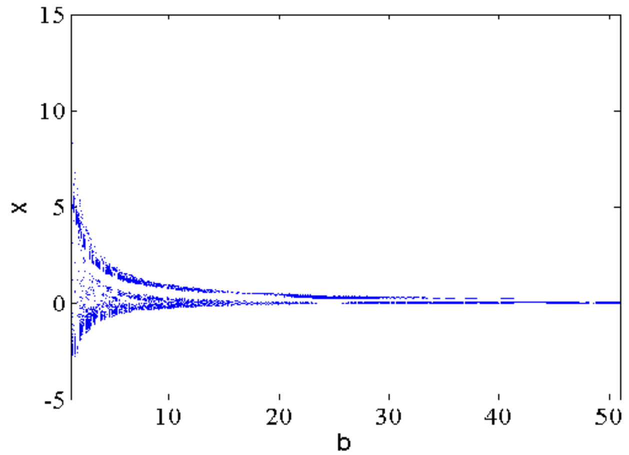



**FIGURE 6.** Bifurcation plot of the system (1) versus $b$ for $a = 5$ and $X_0 = (0.2, 0.2, 0.2)$.

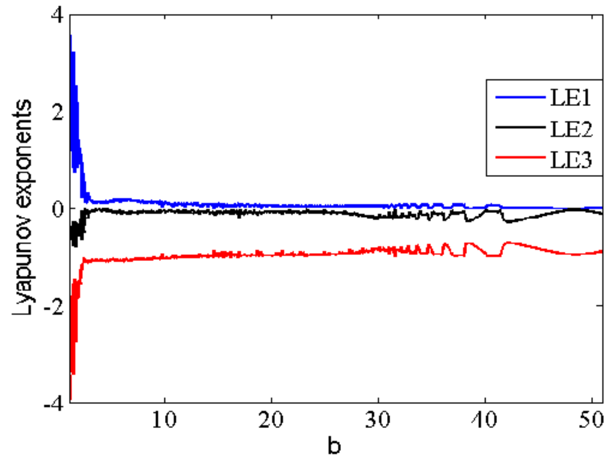



**FIGURE 7.** Lyapunov spectrum of the system (1) when varying $b$ for $a = 5$ and $X_0 = (0.2, 0.2, 0.2)$.

The matrix $J(E_2)$ has the spectral values $\lambda_1 = -1$ and $\lambda_{2,3} = \pm 3.1623i$. Thus, $E_2$ is a non-hyperbolic rest point of (1) and its stability is at a critical state. We cannot conclude the stability type of $E_2$ from the spectral values of $J(E_2)$ by the direct method of Lyapunov. Using phase plots, we find that $E_2$ is locally asymptotically stable.

For any non-zero rest point $X = (0, 0, k)$ on the z-axis, the matrix $J(X)$ was calculated as

$$J(X) = \begin{bmatrix} 0 & k & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The characteristic polynomial of $J(X)$ was found as

$$\varphi(\lambda) = \lambda \left(\lambda^2 + \lambda - k\right) = 0, \quad (k \neq 0) \tag{11}$$

Thus, $\lambda = 0$ is always a spectral value of $J(X)$.

When $k > 0$, $X = (0, 0, k)$ is on the positive side of the $z$-axis, and $\lambda^2 + \lambda - k$ is an unstable quadratic polynomial with real roots of opposite signs. In this case, $X = (0, 0, k)$ is a saddle point and unstable.

When $k < 0$, $X = (0, 0, k)$ is on the negative side of the z-axis, and $\lambda^2 + \lambda - k$ is a Hurwitz quadratic polynomial with stable roots. In this case, $X = (0, 0, k)$ is a critical rest point and the first method of Lyapunov does not enable us to conclude its stability. The rest points $X = (0, 0, k)$, $(k < 0)$ are non-isolated and hence they cannot be locally asymptotically stable. Using phase plot analysis, we find that $X = (0, 0, k)$, $(k < 0)$ are critically stable.

Finally, we check the dissipativity of (1) by finding $\nabla V$ along any volume flow of (1).

Indeed, it is seen that

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1 < 0 \tag{12}$$

Thus, the double-wing chaotic system (1) is dissipative.

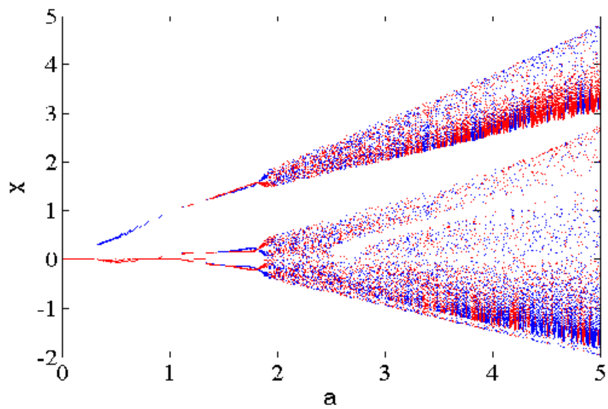



**FIGURE 8.** Coexisting bifurcation models of the double-wing chaotic system (1) when increasing the value of $a$ from 0 to 5 for $b = 2$, where the blue orbit starts with the I.C. and $X_0 = (0.2, 0.2, 0.2)$ and the red orbit starts with the I.C. and $Y_0 = (-0.2, -0.2, 0.2)$.

## III. DYNAMIC ANALYSIS OF THE NEW DOUBLE-WING CHAOTIC SYSTEM

### A. BIFURCATION DIAGRAM AND LYAPUNOV EXPONENTS

Bifurcation diagram is a miscellaneous tool to investigate the dynamics behavior of nonlinear systems [39], [40]. The bifurcation plot and the spectrum of Lyapunov exponents of the system (1) by changing $a$ are shown in Figures 4 and 5, respectively.

The system (1) exhibits periodic and chaotic behavior when changing the value of $a$ from 0 to 5. Explicitly, when $a < 2$, system (1) exhibits periodic behavior and when $a \geq 2$, system (1) has chaotic behavior.

In addition, it is observed from Figures 6 and 7 that system (1) displays chaos at the beginning and ultimately converts into periodic orbits with the increase of the control parameter $b$ in the range of [1], [51]. We note that the bifurcation diagram is consistent well with the corresponding Lyapunov exponents spectrum.

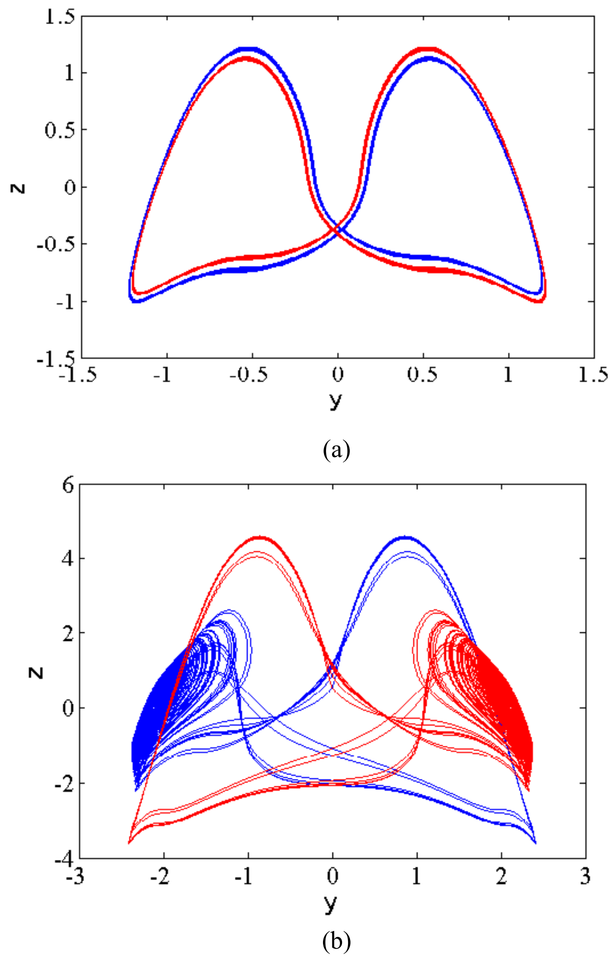



**FIGURE 9.** MATLAB planar plots of various coexisting attractors of the system (1) in the (y, z) plane: (a) the coexisting periodic attractors for $a = 1.75$ (b) the coexisting chaotic attractors for $a = 4.5$.

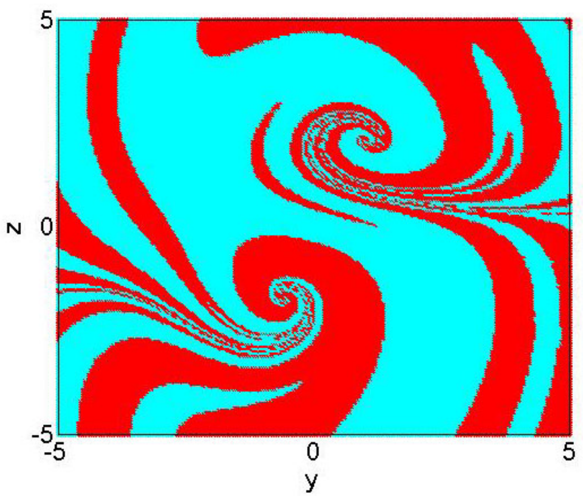



**FIGURE 10.** The basin of attraction of the coexisting chaotic attractors (cyan and red) of the system (1) in the $y - z$ plane for the cross section $x = 0.2$.

## B. COEXISTENCE OF ATTRACTORS

This section explores a study on finding various coexisting attractors for the system (1) in detail. The system (1)

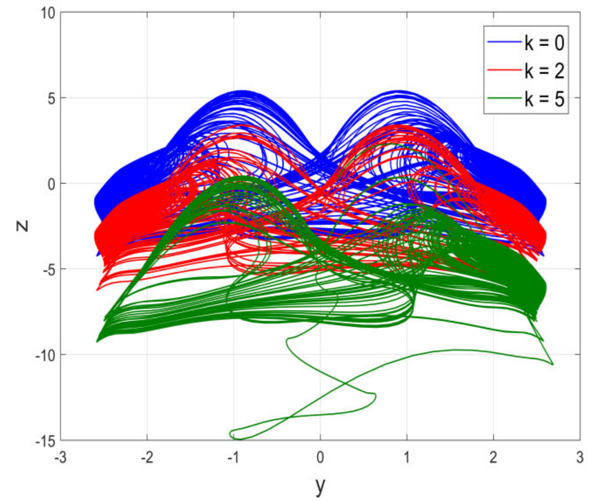



**FIGURE 11.** MATLAB planar plots in different planes and different values of the offset boosting controller $k$ in $y - z$ plane, $k = 0$ (blue color), $k = 2$ (red color), $k = 5$ (green color).

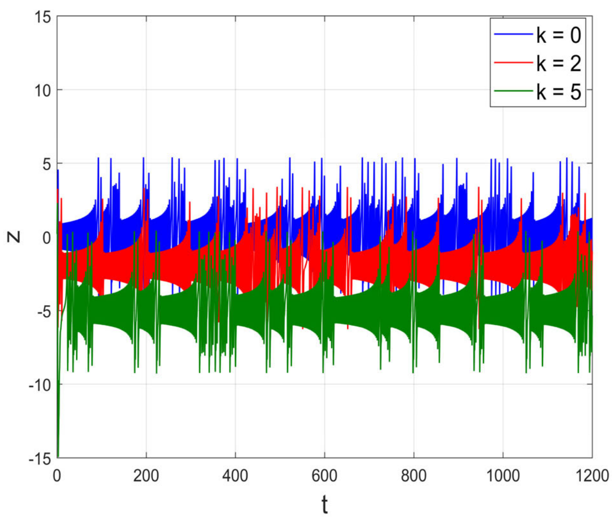



**FIGURE 12.** The state $z$ with different values of the offset boosting controller $k$: $k = 0$ (blue color); $k = 2$ (red color); $k = 5$ (green color).

remains invariant under the transformation $S : (x, y, z) \mapsto (-x, -y, z)$. Thus, any projection of the attractor has rotational symmetry in the $z-$ axis. Thus, (1) may exhibit coexisting attractors.

We set $b = 2$ and choose $a$ as control parameter in [0, 5], the coexisting bifurcation models of the state variable $x$ are plotted in Figure 8 in which the blue colored orbit begins with $X_0 = (0.2, 0.2, 0.2)$ and the red colored orbit begins with $Y_0 = (-0.2, -0.2, 0.2)$. As shown in Figure 8, there are coexisting attractors in system (1).

We set $a = 1.75$ and $b = 2$ the system (1) shows coexisting periodic attractors with respect to $X_0 = (0.2, 0.2, 0.2)$ (blue color) and the initial state (red color) as shown in Figure 9(a). When we set a $= 4.5$, the system (1) exhibits coexisting chaotic and periodic attractors corresponding to $X_0 = (0.2, 0.2, 0.2)$ (blue color) and $Y_0 = (-0.2, -0.2, 0.2)$. (red color) as shown in Figure 9(b).

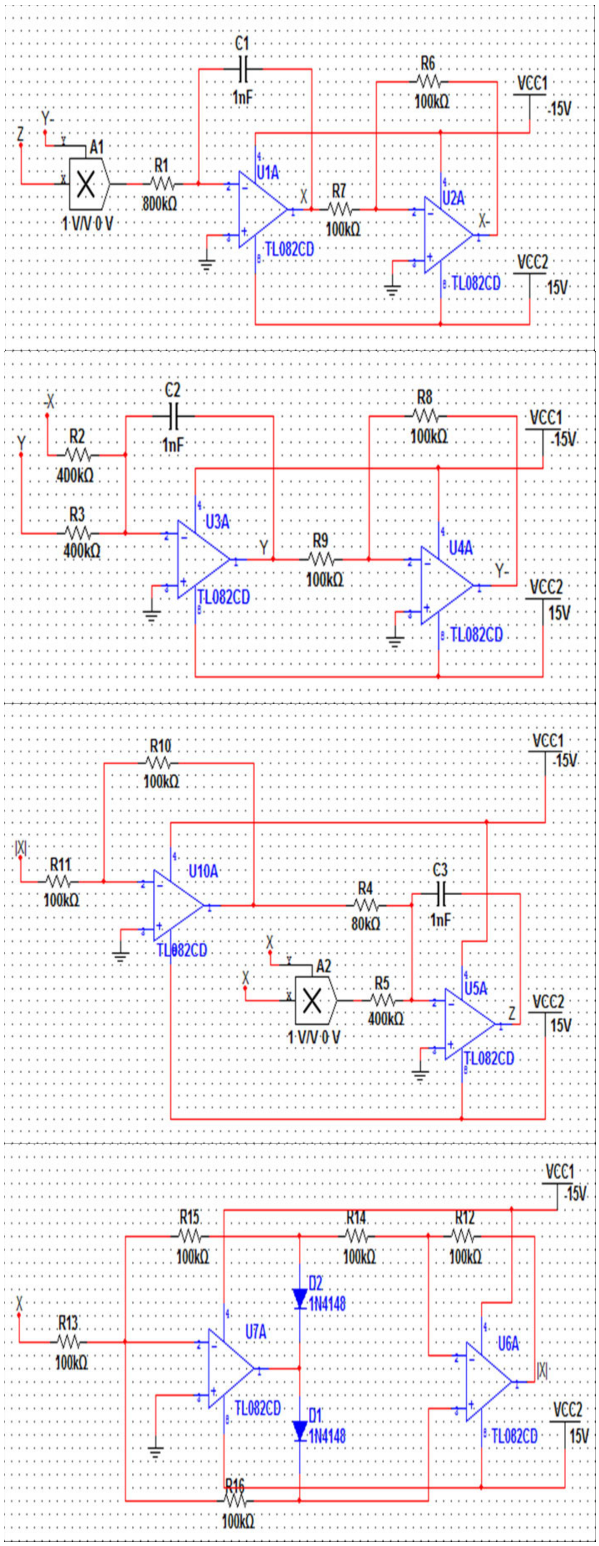



FIGURE 13. The circuit schematic of the double-wing chaotic system (15).

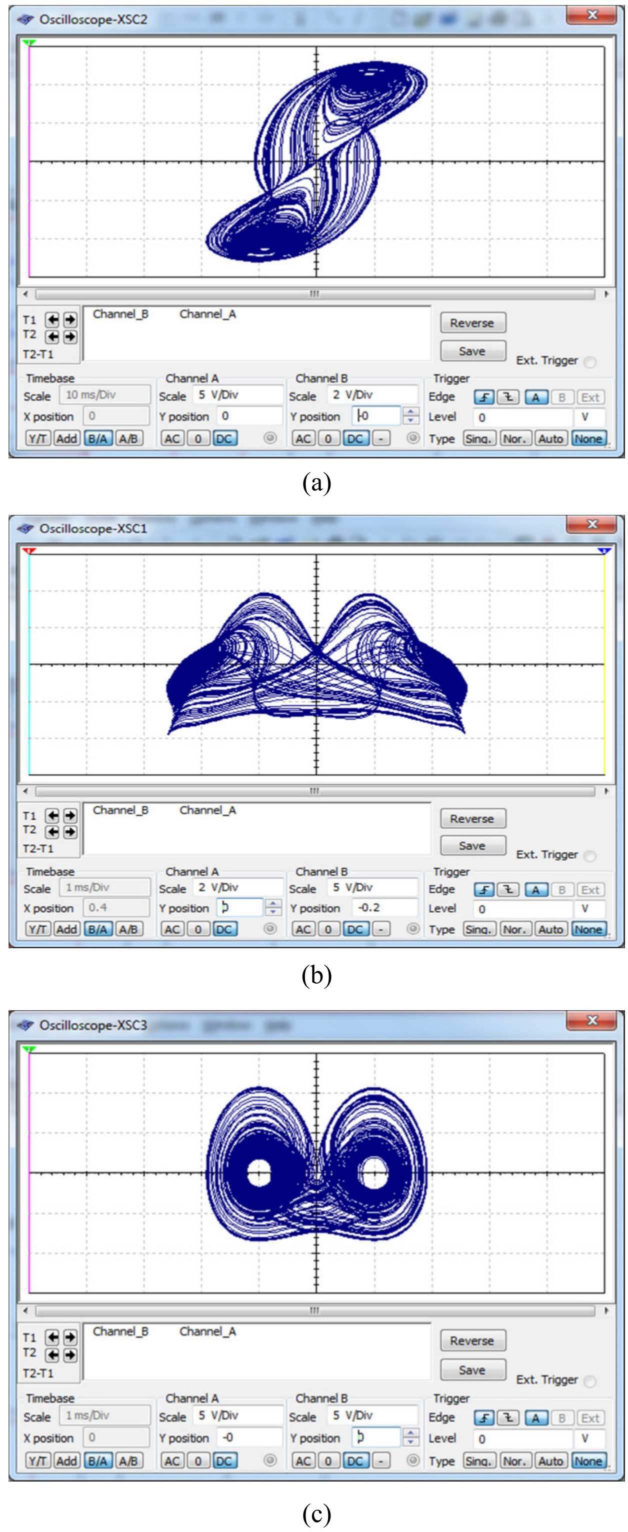



FIGURE 14. Multisim outputs of the double-wing chaotic system (15) in (a)$x-y$ plane, (b) $y-z$ plane, (c) $x-z$ plane.

As we know, basin of attraction is important for the study of hidden attractors and coexisting attractors and it is often defined as the set of initial conditions whose moving orbits converge to the specified attractor, from which much more information about the coexisting attractors can be gained [46]–[50]. Therefore, we plot the basin of attraction in the $y(0) - z(0)$ plane for $x(0) = 0.2$ of the coexisting chaotic attractors as shown in Figure 10 from which the cyan colored orbit starts from $X_0 = (0.2, 0.2, 0.2)$ and the red colored orbit starts from $Y_0 = (-0.2, -0.2, 0.2)$.

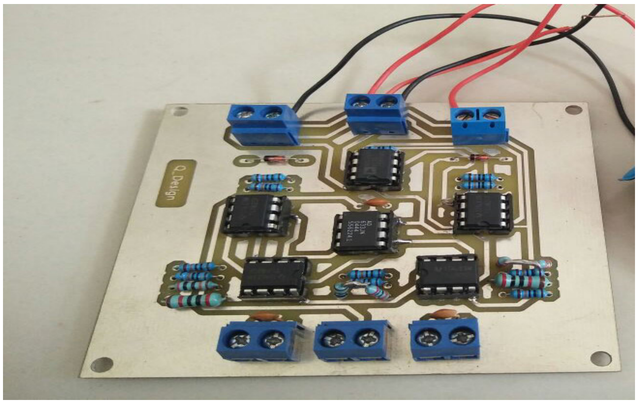


**FIGURE 15.** **The experimental circuit of the double-wing chaotic system (15).**

### C. OFFSET BOOSTING CONTROL

As the state $z$ appears only once in the second equation of (1), we can control the state $z$ conveniently. The state variable $z$ is offset-boosted by replacing $z$ with $z + k$ in which $k$ is a constant.

Thus, we consider the double-wing chaotic system (1) in the modified form as follows:

$$\begin{cases} \dot{x} = y(z+k) \\ \dot{y} = x - y \\ \dot{z} = a\,|x| - bx^2 \end{cases} \tag{13}$$

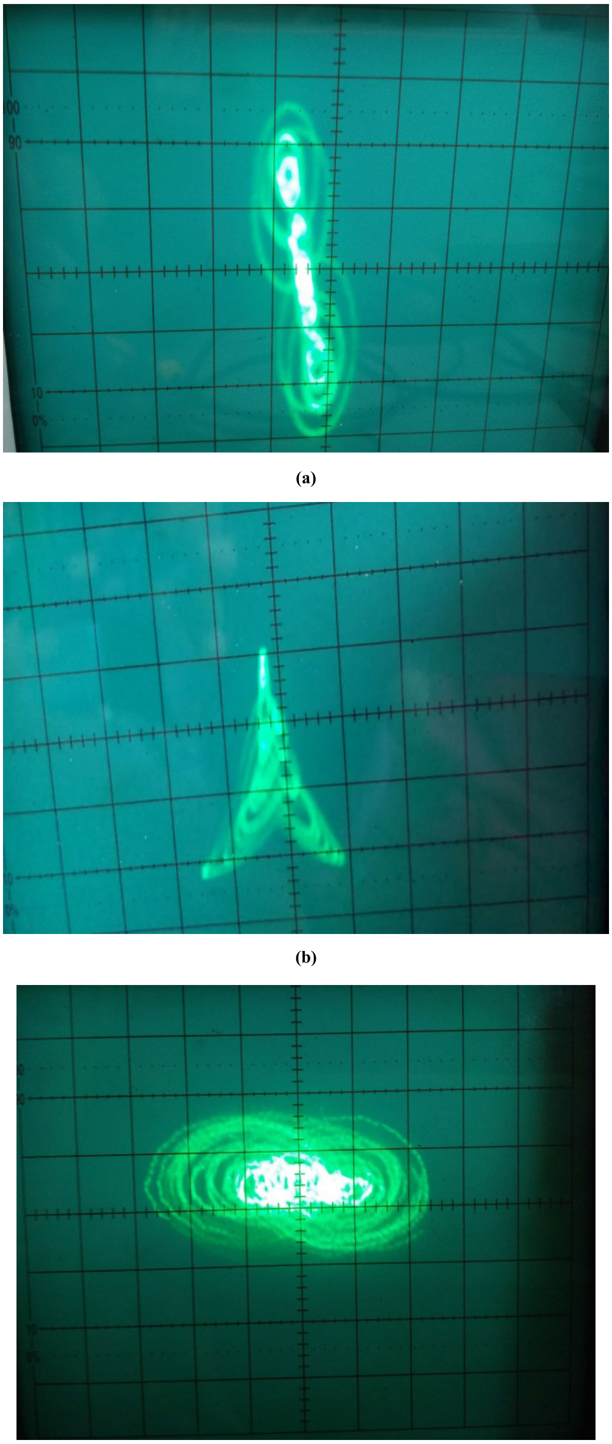
Consequently, the chaotic signal $z$ can be transformed from a bipolar signal to a unipolar signal when varying the control parameter $k$.

We fix $a = 5$, $b = 2$ and the initial state and $X_0 = (0.2, 0.2, 0.2)$. Various positions of the phase portraits of the chaotic attractors depicted in accordance with different values of the offset boosting controller $k$ for system (13) in the $(y, z)$ plane is shown in Figure 11. Figure 12 exhibits that as we change the value of $k$, the state $z$ is effectively boosted from a bipolar signal with chaos to a unipolar signal with chaos. From the above analysis, it is deduced that the double-wing nonlinear system (1) has potential chaos-based applications with the choice of the offset boosting control.

## IV. CIRCUIT DESIGN OF THE NEW DOUBLE-WING CHAOTIC SYSTEM

In this section, we present a circuit implementation of the theoretical model (1) by using electronic components. The electronic circuit of the new double-wing chaotic system (1) was executed in MultiSIM software. The circuit has basic electronic materials such as 16 resistors, 8 operational amplifiers (TL082CD), 2 diodes (1N4148), 2 multipliers (AD633JN) and 3 capacitors.

In this section, the three states $(x, y, z)$ of (1) have been rescaled as $X = 2x, Y = 2y, Z = 2z$. The rescaled



(a)

(b)

(c)

**FIGURE 16.** **Experimental phase portraits from the implemented circuit in (a) $x-y$ plane, (b) $y-z$ plane, (c) $x-z$ plane.**

double-wing chaotic system (1) is given below:

$$\begin{cases} \dot{X} = \dfrac{1}{2}yz \\ \dot{Y} = x - y \\ \dot{Z} = a|x| - \dfrac{b}{2}x^2 \end{cases} \tag{14}$$

By the use of Kirchhoff's circuit laws into the circuit in Figure 13, its circuital equations are derived as follows:

$$\begin{cases} \dot{X} = \dfrac{1}{C_1R_1}yz \\ \dot{Y} = \dfrac{1}{C_2R_2}x - \dfrac{1}{C_2R_3}y \\ \dot{Z} = \dfrac{1}{C_3R_4}|x| - \dfrac{1}{C_3R_5}x^2 \end{cases} \quad (15)$$

In Eq. (15), $X$, $Y$ and $Z$ correspond to the voltages on the integrators (U1A, U3A, U5A), respectively, while the power supply is $\pm 15$ *V.* We selected $R_1 = 800$ kΩ,$R_2 = R_3 = R_5 = 400$ kΩ, $R_4 = 80$ kΩ, $R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = 100$ kΩ, $C_1 = C_2 = C_3 = 1$nF.

The MultiSIM outputs of the double-wing chaotic system (15) are displayed in Figure 14 for *x*-*y*, *x*-*z* and *y*-*z* planes which agree with the MATLAB outputs of the double-wing chaotic system (1) shown in Figures 1-3.

Figure 15 shows the real circuit design of the double-wing chaotic system (15). Figure 16 illustrates the experimental results of the double-wing chaotic system (15) which match with the Multisim and MATLAB outputs of the same chaotic system.

## V. CONCLUSION

A new double-wing chaotic system with a line of rest points was proposed and investigated. Dynamic properties were studied such as rest points and stability, bifurcation diagram, multi-stability, coexistence of attractors and offset boosting control. As the double-wing chaotic system (1) has infinitely many equilibrium points, it was shown that it is a member of the family of hidden chaos attractors. Circuit implementation for the new double-wing chaotic system (1) was experimentally designed with a real circuit. The new double-wing chaotic system has potential applications in engineering areas such as voice encryption, image encryption, pseudo-random number generators (PNRG) and secure communication devices.

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**ACENG SAMBAS** received the M.Sc. degree in mathematics from Universiti Sultan Zainal Abidin (UniSZA), Malaysia, in 2015. He has been a Lecturer with the Muhammadiyah University of Tasikmalaya, Indonesia, since 2015. His current research interests include dynamical systems, chaotic signals, electrical engineering, computational science, signal processing, robotics, embedded systems, and artificial intelligence.

**SUNDARAPANDIAN VAIDYANATHAN** received the D.Sc. degree in electrical and systems engineering from Washington University at St. Louis, St. Louis, MO, USA, in 1996. He is currently a Professor with the Research and Development Centre, Vel Tech University, Chennai, India. He has published over 480 Scopus-indexed research publications. His current research interests include control systems, chaos theory, mathematical modeling, and scientific computing.

**SEN ZHANG** received the B.Sc. degree from the Zhengzhou University of Light Industry, in 2015, and the M.S. degree from the School of Physics and Opotoelectric Engineering, Xiangtan University, Xiangtan, China, in 2019. He is currently pursuing the Ph.D. degree with the School of Automation, Huazhong University of Science and Technology, Wuhan, China. His current research interests include memristive systems and circuits, chaos and fractional-order chaotic systems and circuits, and design and implementation of brain-like intelligent computing circuit based on memristor.

**YICHENG ZENG** graduated from Huaihua College, in 1982, the master's degree from Sichuan Normal University, in 1991, and the Ph.D. degree from Zhejiang University, in 2002. Since 2004, he has been a Professor of electronics with the Department of Photoelectric Engineering, Xiangtan University. His current research interests include nonlinear circuits and signal processing.

**MOHAMAD AFENDEE MOHAMED** received the Ph.D. degree in mathematical cryptography from Universiti Sultan Zainal Abidin, in 2011, where he is currently an Associate Professor. His research interests include both theoretical and application issues in the domain of data security, and mobile and wireless networking.

**MUSTAFA MAMAT** received the Ph.D. degree from UMT, in 2007, with specialization in optimization. He was first appointed as a Lecturer with Universiti Malaysia Terengganu (UMT), in 1999, where he was appointed as a Senior Lecturer, in 2008, and an Associate Professor, in 2010. He has been a Professor and the Dean of the Graduate School with Universiti Sultan Zainal Abidin (UniSZA), Malaysia, since 2013. He has successfully supervised more than 60 postgraduate students and published more than 150 research papers in various international journals and conferences.