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Space Harmonics Elimination for Fractional-Slot Windings With Two-Slot Coil Pitch

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ABSTRACT Recently, fractional-slot windings with two-slot coil pitch are receiving more and more attention due to their lower space harmonic contents in stator magnetomotive force (MMF) compared with the widely-used fractional-slot nonoverlapping counterparts. To further reduce the detrimental space harmonics for this type of winding, various solutions featured by varied coil turn ratios are investigated in this paper. Firstly, all the feasible slot and pole number combinations for fractional-slot windings with two-slot coil pitch are discussed. Secondly, two types of Slot Harmonic Only Windings (SHOW), i.e., SHOW I and SHOW II, are illustrated on the 18-slot/10-pole (18S10P) and 24S10P windings, through which only the main harmonic and its slot harmonics will be present in stator MMF harmonics. Then, to overcome the drawback of reduced main harmonic magnitude owing to both the decreased winding factor and poor slot area utilization associated with the SHOW, two solutions, i.e., 6-0 solution and 5-1 solution for 18S10P and four alternatives, i.e., 7-1 solution, 6-2 solution, 5-3 solution and 4-4 solution for 24S10P, are proposed based on the SHOW I or SHOW II. It is found that the 5-1 solution for 18S10P and 5-3 solution for 24S10P winding are the most desirable ones considering the tradeoff between larger main harmonic magnitude and lower harmonic contents. Furthermore, electromagnetic performance comparisons are carried out between 24S10P permanent magnet (PM) machines with 5-3 solution and 12S10 two layer and 24S10P two layer machines. Although the average torque of the 24S10P machine with 5-3 solution has a slight decrease, its torque ripple and radial magnetic force density rotor eddy-current losses are greatly suppressed compared with the two layer counterparts.

INDEX TERMS Fractional-slot winding, slot harmonic only, space harmonic, two-slot coil pitch.

I. INTRODUCTION

The objectives of designing AC windings are always the same, i.e., amplitude of main harmonic (generating torque) is as large as possible, while for the rest harmonics, the smaller the better. Harmonics other than the main harmonic can cause serious problems such as detrimental parasitic torques, which can be harmful especially at low speeds, causing acoustic noise and vibrations, and inducing rotor eddy current losses [1]–[2].

Plenty of approaches by modifying the winding layouts have been proposed to reduce harmonic contents of AC windings. The mostly employed method for the conventional integral slot windings is by distributing and chording the winding. A coil pitch to pole pitch around 5/6 is preferably adopted in double-layer windings to counteract the 5th and 7th harmonics. In addition, to overcome the limited ability of harmonic elimination for the distributing and chording method (only one of the major low-order harmonics can be eliminated if designed properly), double-layer graded distributed windings characteristic by varied coil turn ratios and slot distributions are put forward in [3]–[6], through which all harmonics except the main harmonic and its slot harmonics are present in the stator magnetomotive force (MMF). While there is a continuous interest in finding integral slot windings with even lower harmonic content, the emerging fractional-slot nonoverlapping windings have been received considerable attention owing to advantages of short end winding, high slot filling factor, low manufacturing cost and fault tolerance suitability [7], [8]. However, fractional-slot nonoverlapping windings are more susceptible to the aforementioned problems due to their relative rich MMF harmonic contents,

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especially the subharmonics [9]. Although the multiphase winding [10] or the star and delta winding [11] has been proposed to eliminate the sub, super harmonics, they will either increase the driving cost or have a potential of harmful circulating currents. Besides, fractional-slot nonoverlapping multilayer windings with various coil turn ratios are put forward to improve the quality of the MMFs [12]–[17]. The number of varied coil turn ratios required for elimination of all space harmonics except the slot harmonics is closely related to the slot per pole per phase q [14].

In addition, fractional-slot windings with two-slot coil pitch are claimed to be competitive in low stator space harmonic contents [18]–[21], which can be treated as a compromise between fractional-slot nonoverlapping windings and integral slot windings. The splitting and shifting windings, e.g., the 24-slot/10-pole (24S10P) winding investigated in [22]–[25], are in essence fractional-slot windings with coil pitch of two slot. To further reduce the harmonic contents of fractional-slot windings with two-slot coil pitches, methods of employing varied coil turn ratios are studied in [26]–[28]. However, only a certain method is proposed in these papers and they fail to investigate the relationship among all these harmonic elimination methods with varied coil turn ratios. Besides, the associated problem of unequal conductors per slot alongside these methods is hardly mentioned.

The purpose of this paper is to investigate different harmonic elimination solutions for the two-slot coil pitches fractional-slot windings. The relationship among these solutions will also be revealed. In addition, those methods with unequal conductors per slot should be avoided and alternative solutions with equal conductors and the least number of varied turn ratio should be welcomed.

This paper will be organized as follows. In section II, all the feasible slot and pole number combinations for fractional-slot windings with two-slot coil pitch are discussed. In section III, two types of SHOWs are studied in terms of their pros and cons. Alternative harmonic elimination solutions, i.e., 6-0 solution and 5-1 solution for the 18S10P winding and 6-2 solution, 5-3 solution and 4-4 solution for 24S10P winding are proposed based on the Slot Harmonic Only Winding (SHOW) and optimal coil turn ratios are calculated in section IV, whilst the electromagnetic performance comparisons are carried out between 24S10P permanent magnet (PM) machines with 5-3 solution and 12S10P and 24S10P two layer machines in section V. Section VI is the conclusion.

II. FRACTIONAL-SLOT WINDINGS WITH TWO-SLOT COIL PITCH

A. FEASIBLE SLOT AND POLE NUMBER COMBINATIONS

The periodicity of the two layer windings is given by

$$t = GCD\{Z, p\} \tag{1}$$

where Z is stator slot number, p is winding pole pair number, and GCD is short for greatest common divisor. Owing to this periodicity, analysis of windings can be performed on base windings whose slot number Z_0 and pole pair number p_0 equal to Z/t and p/t, respectively.

TABLE 1. Slot and pole number combinations.

$Z_0\backslash p_0$	2	4	8	10	14	16	20	22
3	*yes*	*yes*	-	-	-	-	-	-
6	*yes*	-	-	*yes*	-	-	-	-
9	*yes*	*yes*	*yes*	*yes*	-	-	-	-
12	*yes*	-	-	-	*yes*	-	-	-
15	*yes*	*yes*	*yes*	-	*yes*	*yes*	-	-
18	*yes*	-	-	*yes*	*yes*	-	-	*yes*
21	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*
24	*yes*	-	-	*yes*	*yes*	-	-	*yes*
27	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*	*yes*

The feasible slot/pole number combinations for symmetrical three phase windings is given as:

$$Z_0 = 3j \quad j = 1, 2, 3\ldots \tag{2}$$

The commonly-used slot and pole number combinations for three phase base windings are listed in TABLE 1.

For two layer windings with equal positive and negative phase-belt (60° phase-belt for three phase ones), their space harmonics will contain orders of $v = 3n\pm1$ ($n = 0, 1, 2$) when j (or Z_0) is odd, and orders of $v = 6n\pm1$ ($n = 0, 1, 2$) order when j (or Z_0) is even [27]. The space harmonic can be categorized into the sub-harmonics ($v < p_0$), the super-harmonics ($v > p_0$) and the slot harmonics ($v = Z_0k \pm p_0$) whilst the $v = p_0$ harmonic is named the main harmonic. For windings with equal slot pitches, the winding factors of the slot harmonics are the same as that the main harmonic, which cannot be reduced to zero by varying the coil distribution. However, the sub-harmonic and super-harmonics can be eliminated through varied coil turns and distributions, as well be explained later.

B. WINDINGS WITH TWO-SLOT COIL PITCH

The average coil pitch τ can be defined as

$$\tau = \frac{Z_0}{2p_0}(\tau \in R) \tag{3}$$

To obtain higher winding factor of the main harmonic p_0, the coil pitch y_1 should be as close to the pole pitch as possible.

$$y_1 = int(\tau) \pm i \begin{cases} i \in N \\ y_1 \geq 1 \end{cases} \tag{4}$$

From (3) and (4), it can be observed that typical slot and pole number combinations for base windings with two-slot coil pitches are 9S4P, 15S8P, 18S10P, 21S10P, 24S10P, 24S14P, 27S14P and 27S16P. As stated, windings with odd slot number have richer harmonic contents compared with ones with even slot number. Thus, the 18S10P, 24S10P and 24S14P windings seem to be more preferable than the others. Only the 18S10P and 24S10P windings are chosen for investigation, whose winding layout and star of phasors are shown

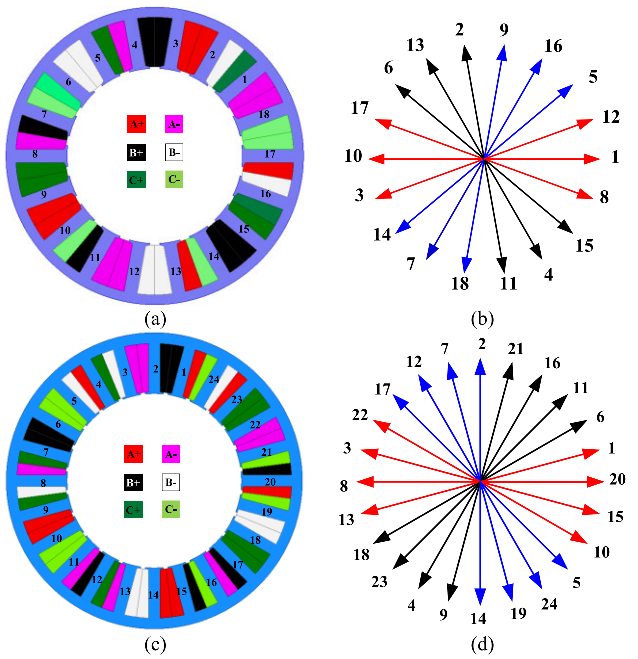

FIGURE 1. 18S10 and 24S10P two layer winding. (a) Winding layouts of 18S10P. (b) Star of phasors of 18S10P. (c) Winding layouts of 24S10P. (d) Star of phasors of 24S10P.

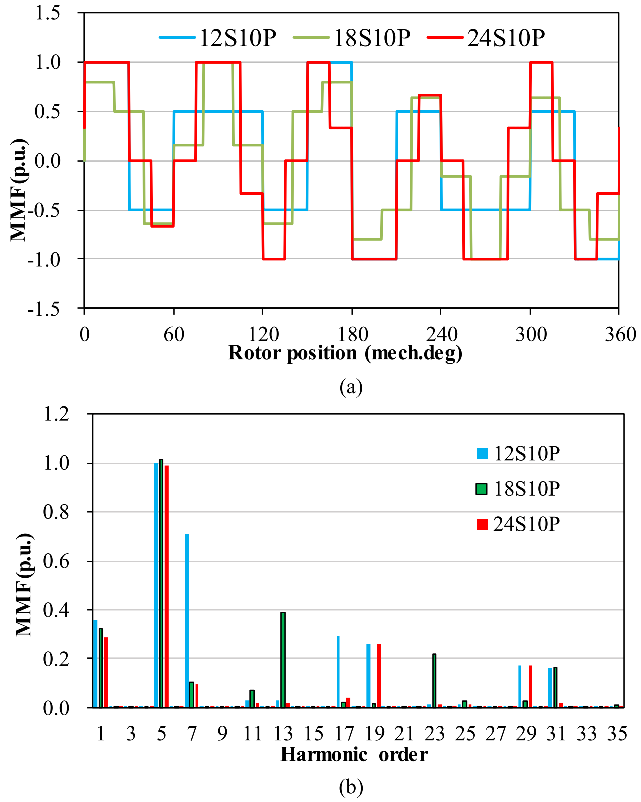

FIGURE 2. Stator MMF distributions and harmonic contents for 12S10P, 18S10P and 24S10P double layer windings. (a) Stator MMF distributions. (b) Harmonics.

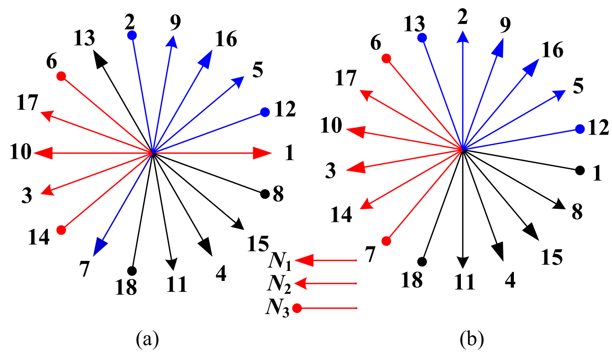

FIGURE 3. Star of phasors of 18S10P winding with two SHOW. (a) SHOW I. (b) SHOW II.

in Fig. 1. Since the 24S14P and 24S10P windings have the same distribution factor and all the analysis for the 24S10P machine in this paper is applicable to the 24S14P one as well. Each phasor in the star of phasor represents one coil with coil pitch being two-slot.

Figure 2 gives a comparison of stator MMF distributions and harmonic contents between the popular 12S10P fractional-slot nonoverlapping winding and the 18S10P, 24S10P windings. As can be seen, the annoying 7th super-harmonic, which is the most responsible one for producing the 2nd mode vibration in the 12S10P machine [28], has been reduced by 85.2% for the 18S10P and 87.3% for 24S10P windings, respectively. However, the 1st sub-harmonic in the two windings, which can induce large rotor eddy current losses especially at high speed, is only slightly decreased compared with the 12S10P winding. Various solutions characteristic by varied coil turn ratios and distributions will be put forward to remove both the sub-harmonic (1st) and super-harmonics (especially the 7th) in the stator MMF of the 18S10P and 24S10P winding hereinafter.

III. SLOT HARMONIC ONLY WINDINGS

The concept of Slot Harmonic Only Winding was proposed in [17], where only the main harmonic and its slot harmonics are present in the stator MMF. By varying coil turn ratios and their distributions, all the winding space harmonics except the main harmonic and its slot harmonics can be eliminated. Two key points are mentioned as follows:

1) Owing to the periodicity of the distribution factors, harmonics having the identical distribution factor will viewed as the same kind. For the 18S10P winding, stator MMF harmonics can be categorized into three kinds, i.e., $v = 18k\pm1$, $v = 18k\pm5$, $v = 18k\pm7$ order harmonics, whilst for the 24S10P winding, four kinds, i.e., $v = 24k\pm1$, $v = 24k\pm5$, $v = 24k\pm7$, $v= 24k\pm11$ order harmonics are classified.

2) As mentioned in [14], the available number of varied turn ratios for a certain winding equals to the numerator of q (q is slots per pole per phase) minus one. For the 18S10P and 24S10P windings, the value is 2 and 3 respectively.

Fig. 3 shows two types of two-turn ratio-applied star of phasors for the 18S10P winding, i.e. N_2/N_1, and N_3/N_1 whilst Fig. 4 draws the three-turn ratio-applied star of phasors for the 24S10P winding, i.e. N_2/N_1, N_3/N_1 and N_4/N_1. It should be noted that each phasor represents a coil pair (the two coils of each coil pair has a phase difference by 180 degree in

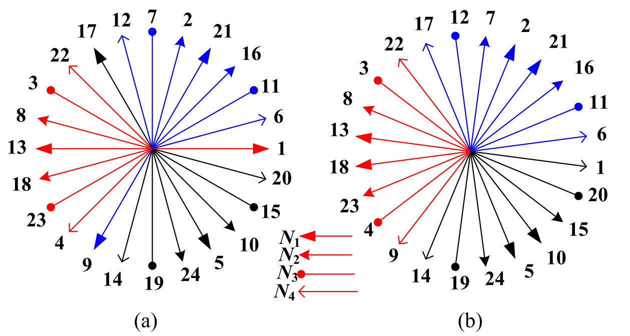

FIGURE 4. Star of phasors of 24S10P winding with two SHOWs. (a) SHOW I. (b) SHOW II.

TABLE 2. Values of optimal turns ratios with SHOWs.

SHOW I	N_2/N_1	N_3/N_1	N_4/N_1
18S10P	1.484	0.790	/
24S10P	1.633	1.155	0.598
SHOW II	N_2/N_1	N_3/N_1	N_4/N_1
18S10P	0.653	0.227	/
24S10P	0.767	0.482	0.164

stator circumference) [15]. By choosing the optimal values of N_2/N_1, N_3/N_1 and N_4/N_1, the $v = 18k\pm1$, $v = 18k\pm5$, $v = 18k\pm7$ order harmonics in 18S10P and the $v = 24k\pm1$, $v = 24k\pm7$, $v= 24k\pm11$ order harmonics in 24S10P will be eliminated. Since only the main harmonic ($v = 5$) and its slot harmonic ($v = 24k\pm5$ for 18S10P and $v= 24k\pm5$ for 24S10P) will be present in stator MMF with the two solutions, they are named Slot Harmonic Only Winding I (SHOW I) and Slot Harmonic Only Winding II (SHOW II).

Using star of phasors of the to-be removed harmonics for 18S10P and 24S10P windings with both SHOW I and SHOW II, which are shown in Fig. 23 and Fig. 24 in Appendix A, optimal turn ratios can be calculated. The optimal values for the 18S10P and 24S10P with SHOW are tabulated in TABLE 2.

Figure 5 shows the FE-predicted normalized stator MMF harmonic spectrums for 18S10P and 24S10P windings with two layer, SHOW I and SHOW II. It should be mentioned that throughout this paper the based current sheet model is employed in FE analysis to obtain the MMF harmonics excluding the effect of stator effects [29]. As can be observed, for windings with either SHOW I or SHOW II, stator MMF harmonics except the $v = 18k\pm5$ order harmonics for 18S10P and $v = 24k\pm5$ for 24S10P will be removed, which confirms the effectiveness of the SHOW. It should be mentioned that during the FE simulations, the maximum conductors per slot for windings both SHOW is kept the same as that of the two layer counterpart to make sure the identical maximum current density. It can also be observed that the amplitudes of the 5th order harmonic for SHOW I and SHOW II are reduced by 7.51% and 6.26% for 18S10P and by 6.38% and 5.55% for 24S10P compared with that of the two layer ones, which will unavoidably do harm to the machine torque density. This not only owns to the decreased equivalent distribution factors associated with the SHOW, but also largely comes down to

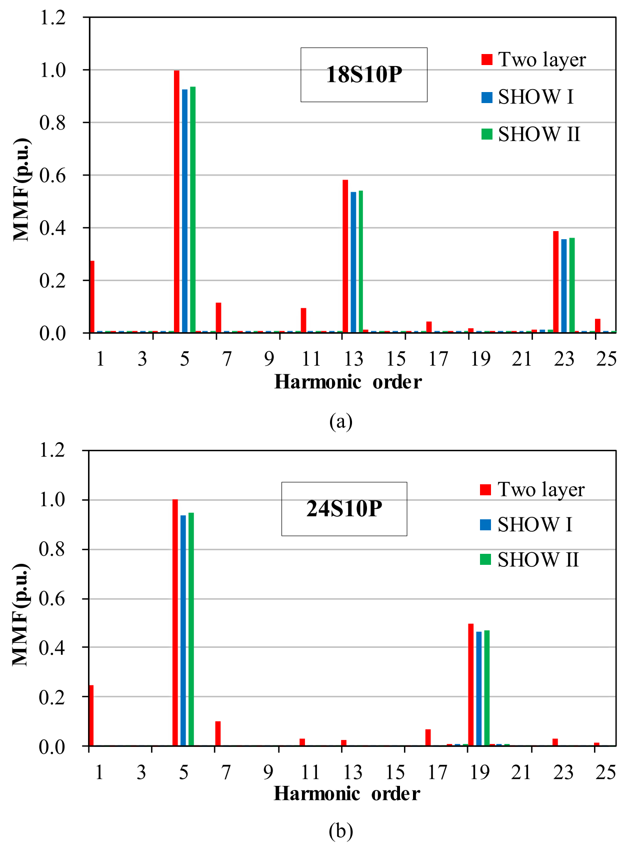

FIGURE 5. FE-predicted normalized stator MMF harmonic spectrums of windings with SHOW. (a) 18S10P. (b) 24S10P.

the issue of unequal number of conductors per slot N_{per_slot}, lowering the space utilization of the slot areas. For example, the slot area utilization ratio of the 18S10P and 24S10P with SHOW I is reduced by 1.2%, and 1.7%, respectively.

In the following section, alternative harmonic elimination solutions for the 18S10P and 24S10P windings with reduced available varied coil turn ratios but equal number of conductors per slot will be proposed and the relationship among the SHOW and alternative solutions shall be established. What should be pointed out is that these alternative solutions are proposed at the cost of less prefect harmonic elimination ability since the available varied coil turn ratios have been reduced.

IV. ALTERNATIVE HARMONIC ELIMINATION SOLUTIONS

Alternative harmonic elimination solutions for 18S10P and 24S10P windings with equal conductors each slot are divided into SHOW I-based solutions and SHOW II-based solutions, their relationship with SHOW will be revealed and the optimal value of varied coil turn ratios will be calculated as well.

A. SHOW I-BASED SOLUTIONS

For the 18S10P winding, equal N_{per_slot} can be satisfied by setting $N_2 + N_3 = N_1^{'} + N_1$ in the star of phasors of SHOW I, as shown in Fig. 6. The 5-1 solution is achieved by setting $N_2 = N_1^{'}$ and $N_3 = N_1$ with only one available turn

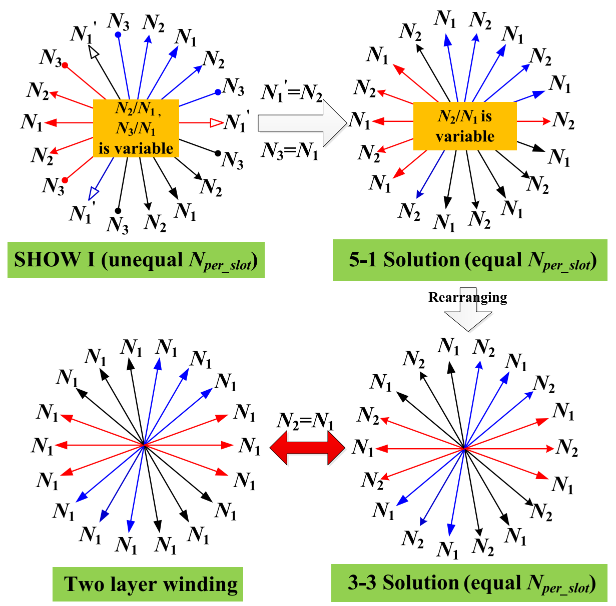

FIGURE 6. Derivation of SHOW I-based solutions for 18S10P winding.

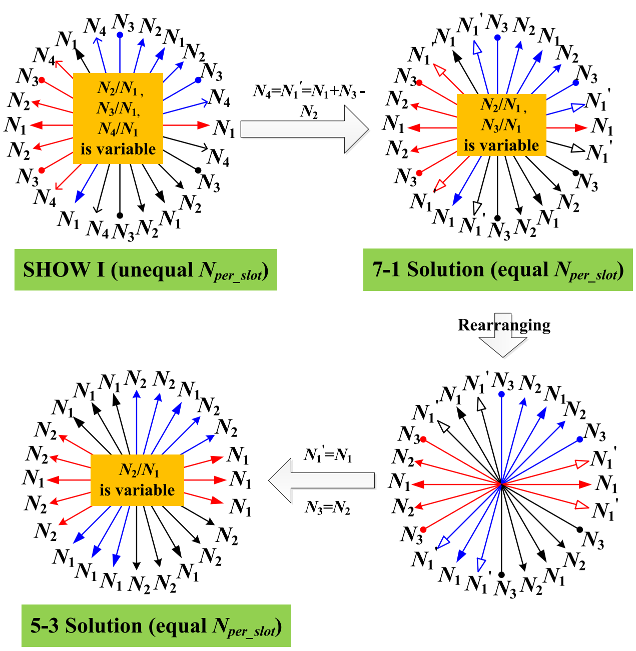

FIGURE 7. Derivation of SHOW I-based solutions for 24S10P winding.

ratio, N_2/N_1. Rearranging the star of phasors of 5-1 solution and maintaining the characterastic of equal N_{per_slot}, the 3-3 solution can be obtained. It is actually a two layer winding and no variable turn ratio is available. The 5-1 solution and 3-3 solution are named since there are 5 red phasors opposite 1 red phasor and 3 opposite to 3 red phasors for them, respectively. The naming rule is obeyed for other alternative solutions. The $v = 18k\pm1$ order harmonics is selected to be removed from the winding for 5-1 solution with optimal values of N_2/N_1.

As shown in Fig. 7, the 7-1 solution and 5-3 solution for the 24S10P winding are derived from the SHOW I with

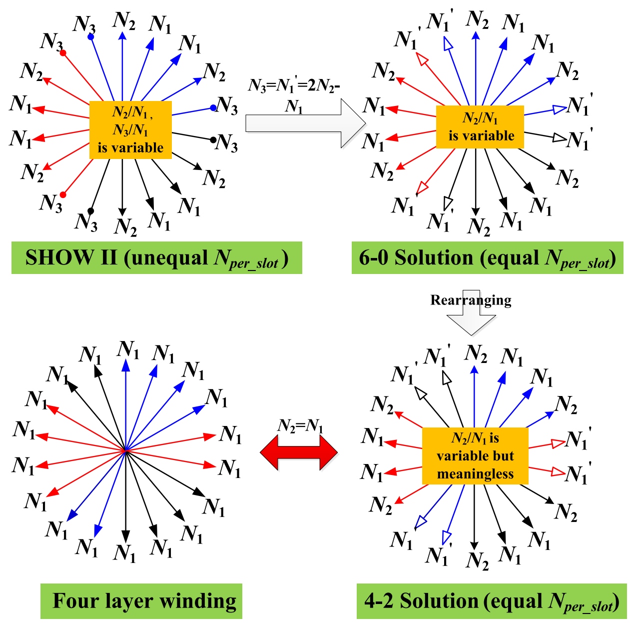

FIGURE 8. Derivation of SHOW II-based solutions for 18S10P winding.

the constraint of equal N_{per_slot}. For the 7-1 solution, two variable turn ratios, i.e., N_2/N_1, N_3/N_1 are available. The $v = 24k\pm1$ and $v = 24k\pm7$ order harmonics are selected to be removed from the winding with this solution with optimal values of N_2/N_1, N_3/N_1. For the 5-3 solution, only one variable turn ratio N_2/N_1 is available to eliminate the $v = 24k\pm1$ order harmonic.

B. SHOW II-DERIVED SOLUTIONS

The 6-0 solution and 4-2 solution for 18S10P are derived from the SHOW II, as shown in Fig. 8. The 4-2 solution with equal N_{per_slot}, is actually the four layer winding with no variable available. The $v = 18k\pm1$ order harmonics is selected to be removed from the winding for 6-0 solution with optimal values of N_2/N_1.

Derivation process of SHOW II-based solutions for 24S10P winding with equal N_{per_slot}, i.e., 6-2 solution and 4-4 solution is shown in Fig. 9. Although equal N_{per_slot} is satisfied with SHOW II for 24S10P windings, its distribution factor of the main harmonic is low, to obtain higher distribution factor and maintain the characteristic of equal N_{per_slot}, the 6-2 solution is derived and the 4-4 solution can also be obtained to further increase the distribution factor for the main harmonic. However, for the 6-2 solution, only two variable turn ratios are available, the $v = 24k\pm1$ and $v = 24k\pm7$ order harmonics can be removed from the winding with this solution. For the 5-3 solution and 4-4 solution, only one variable turn ratio can be chosen to eliminate the $v = 24k\pm1$ order harmonics.

C. C OPTIMAL COIL TURN RATIOS

The alternative solutions, i.e., 6-0 solution and 5-1 solution for 18S10P and 7-1 solution, 6-2 solution, 5-3 solution

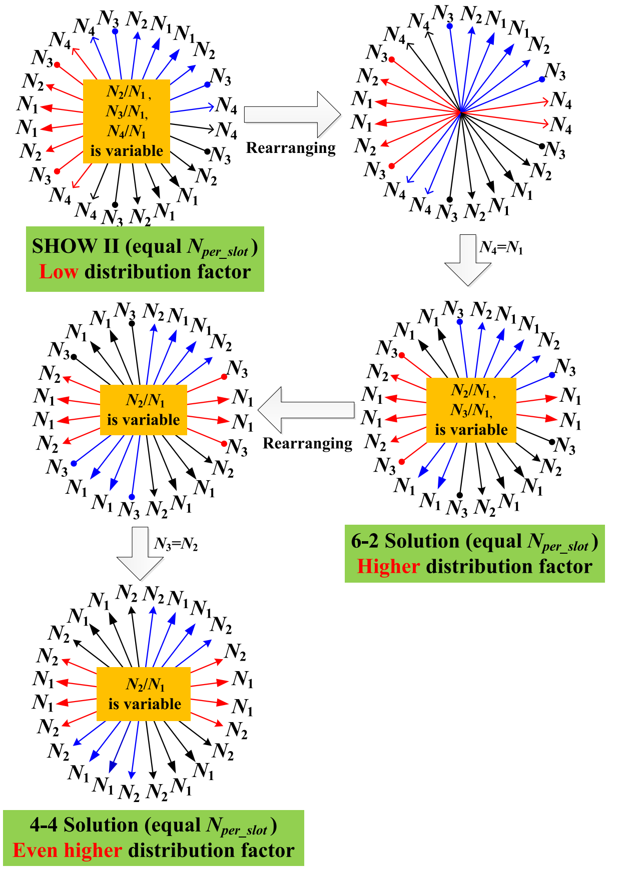

FIGURE 9. Derivation of SHOW II-based solutions for the 24S10P winding.

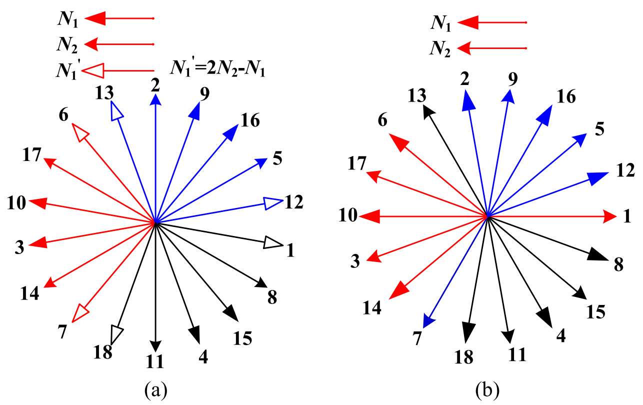

FIGURE 10. Star of phasors of 5-1 solution for the 18S10P winding.(a) 6-0 solution. (b) 5-1 solution.

and 4-4 solution for 24S10P are shown together in Fig. 10 and Fig. 11, respectively. As stated before, for the 7-1 solution and 6-2 solution, two varied coil turn ratios, i.e., N_2/N_1, N_3/N_1 are obtainable whilst for the 6-0 solution, 5-1 solution, 5-3 solution and 4-4 solution, only one is available. By using the star of phasors of the to-be removed harmonics for the these solutions, as shown in Fig. 25 and Fig. 26 in Appendix B, optimal turn ratios can be calculated. Optimal N_2/N_1 of the 6-0 solution, 5-1 solution for eliminating v =18k±1 order harmonics and optimal N_2/N_1 and N_3/N_1 of the 7-1 solution and 6-2 solution for eliminating

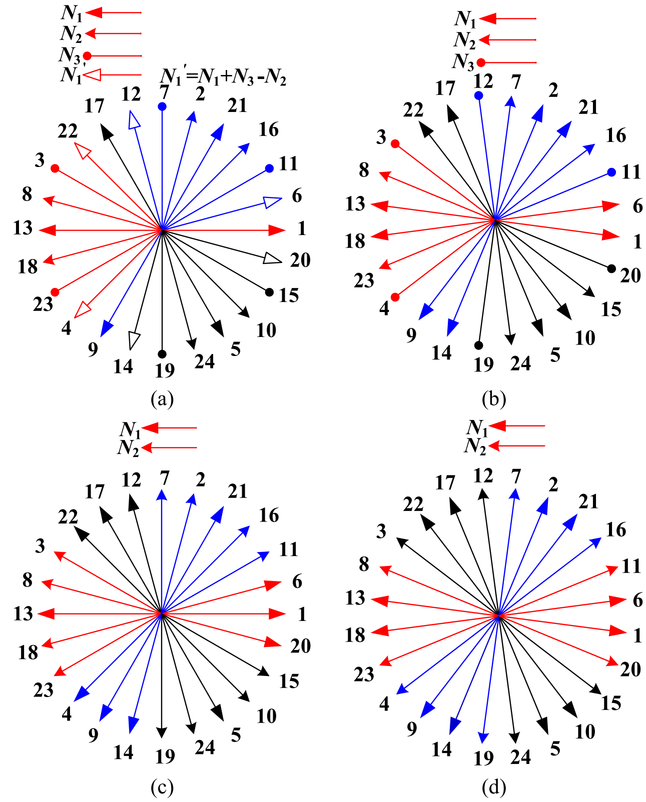

FIGURE 11. Star of phasors of the v = 24k ±5 order harmonics. (a) 7-1 solution. (b) 6-2 solution. (c) 5-3 solution. (d) 4-4 solution.

TABLE 3. Optimal values of N_2 N_1 and N_3 N_1 of the alternative solutions.

18S10P	N_2/N_1	24S10P	N_2/N_1	N_3/N_1
6-0 solution	0.582	7-1 solution	2.034	1.468
5-1 solution	2.532	6-2 solution	2	0.828
		5-3 solution	2.073	/
		4-4 solution	2.073	/

the v =24k±1 and v =24k±7 order harmonics and optimal N_2/N_1 of the 5-3 solution and 4-4 solution for eliminating the v =24k±1 order harmonics are listed in TABLE 3.

The FE predicted stator MMF harmonic spectrums of the 6-0 solution, 5-1 solution for 18S10P and 7-1 solution, 6-2 solution, 5-3 solution and 4-4 solution for 24S10P are shown in Fig. 12, with a comparison with that of the two layer counterparts. As can be seen, for the 18S10P winding, the $v = 18k$±1 order harmonics can be removed with 6-0 solution and 5-1 solution with optimal turn ratios. The amplitude of the v = 18k±7 order harmonics of 5-1 solution is slightly high than the of the 6-0 solution. For 24S10P winding, the $v = 24k$±1 and $v = 24k$±7 order harmonics can be removed with 7-1 solution and 6-2 solution and both 5-3 solution and 4-4 solution can eliminate the $v = 24k$±1 order harmonics in stator MMF. The amplitude of the $v = 24k$±7 order harmonic can be reduce to nearly to zero with the 5-3 solution whilst it has an increase by 71.8% for the 4-4 solution compared with the two layer winding. Regarding to the amplitude of the 5th harmonic, it has a decrease by 4.12% 2.65%, 6.36%, 2.96%, 2.1% and 1.26% respectively for the 6-0 solution,

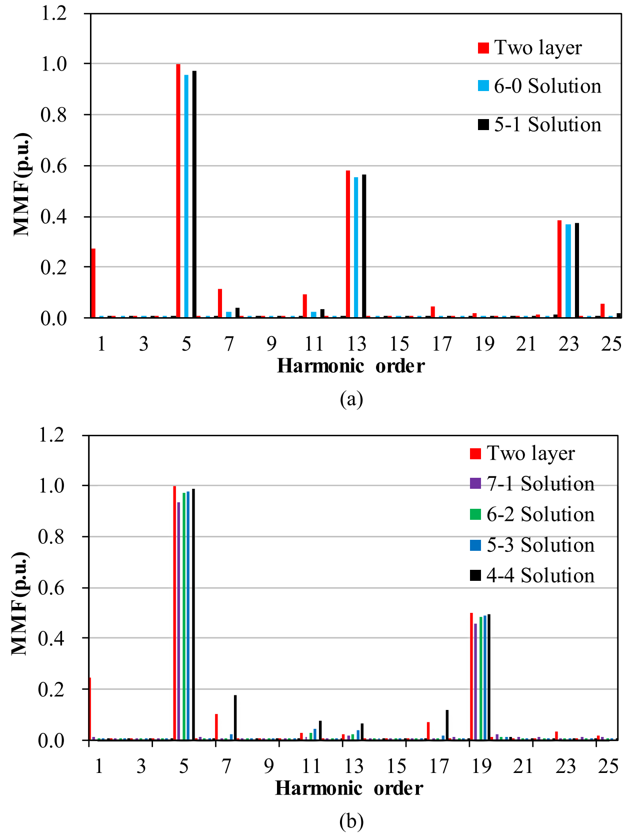

FIGURE 12. FE predicted normalized stator MMF harmonic spectrums with different solutions. (a) 18S10P. (b) 24S10P.

5-1 solution, 7-1 solution, 6-2 solution, 5-3 solution and 4-4 solution. Generally, solutions with stronger harmonic reduction ability but with less varied turn ratio numbers are preferred. The 5-1 solution for 18S10P and 5-3 solution for the 24S10P winding seems to be the most desirable ones among all the alternative solutions.

In the following section, a 24S10P permanent magnet (PM) machine with 5-3 solution will be investigated in term of average torque, torque ripple and rotor losses. Similar study can also be carried out the 18S10P PM machine with the 5-1 solution, which will not be given in this paper.

V. 24S10P PM MACHINE WITH THE 5-3 SOLUTION

Figure 13 shows the winding layouts of the 24S10P PM machine with 5-3 solution and the 10-pole PM rotor with sleeve. To enrich the comparative study, the 24S10P machine is also compared with the 12S10P machine. For the sake of comparison, the same rotor and magnet volumes are used. In addition, the same stator outer and inner diameters are assumed. All the stator, rotor and PM parameters are given in Table 4.

Since it is more preferable to adopt an integral turn ratio in practical for the 24S10P PM machine with 5-3 solution. The influence of turn ratio on the machine performance in terms of armature airgap flux density, torque characteristic and rotor eddy-current loss are first investigated.

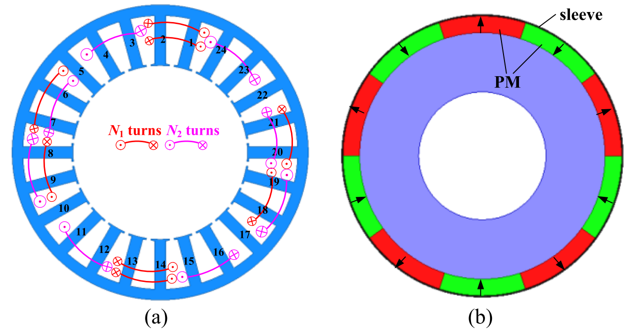

FIGURE 13. 24S10P PM machine with 5-3 solution. (a) Winding arrangement of phase A. (b) 10-pole rotor with sleeve.

TABLE 4. Machine parameters.

	24S10P	12S10P
Stator outer diameter D_{so} (mm)	90	90
Stator inner diameter D_{io} (mm)	53	53
Tooth width w_t (mm)	3.5	6.8
Yoke thichness t_y (mm)	3.25	3.75
Airgap length l_g (mm)	0.7	0.7
Sleeve length l_s (mm)	0.3	0.3
Rotor outer diameter D_{ro} (mm)	51	51
PM thichness l_m (mm)	3	3
PM remanence B_r (T)	1.2	1.2
PM relative permeability μ_r	1.05	1.05
Stack length l_a (mm)	50	50
Slot packing factor k_s	0.5	0.5
Current density J_c (A/mm^2)	5.6	5.6
Rated speed ω_r (rpm)	3000	3000

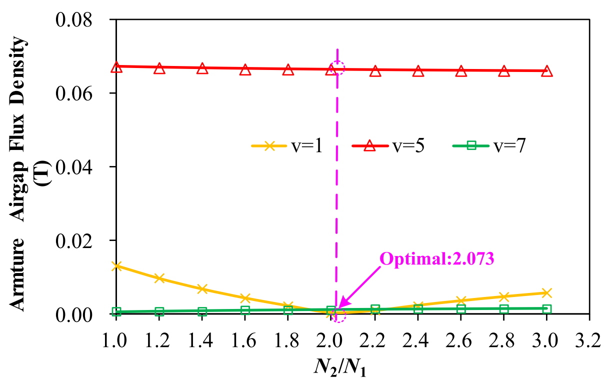

FIGURE 14. Variations of armature airgap flux density harmonics with turn ratio N_2/N_1.

A. INFLUENCE OF COIL TURN RATIO ON MACHINE PERFORMANCE

The influence of turn ratio N_2/N_1 on armature airgap flux density harmonics is illustrated in Fig. 14. As can be seen, magnitude of the 5th order harmonic of armature airgap flux density reduces with the increase of N_2/N_1. Magnitude of the 1st harmonic first decreases and then increases with N_2/N_1, verse for the 7th harmonic. Although the optimal value is 2.073 to eliminate the first harmonic, adoption of the turn ratio being 2 will maintain the almost the same amplitude of the $v = 5$ and $v = 7$ harmonics whilst the $v = 1$ harmonic is still kept to be near zero.

Variations of average torque and rotor eddy-current losses with N_2/N_1 is shown in Fig. 15. The average torque decreases

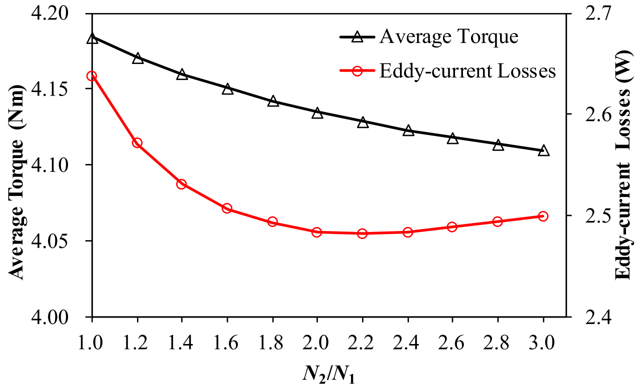

FIGURE 15. Variations of average torque and eddy-current losses with N_2/N_1, current density = 5.6A/mm2, rotor speed = 6000 rpm.

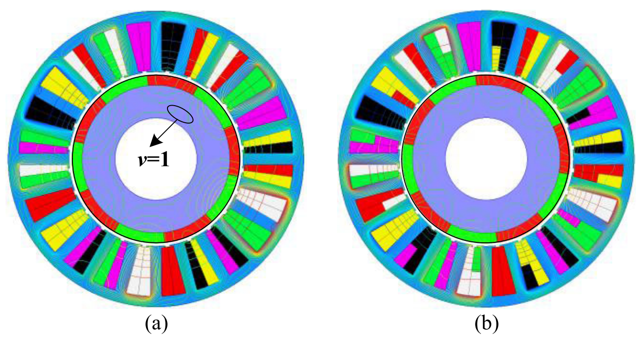

FIGURE 16. Armature flux distributions when the flux linkage of phase A reaches maximum. (a) Two layer. (b) Practical 5-3 solution.

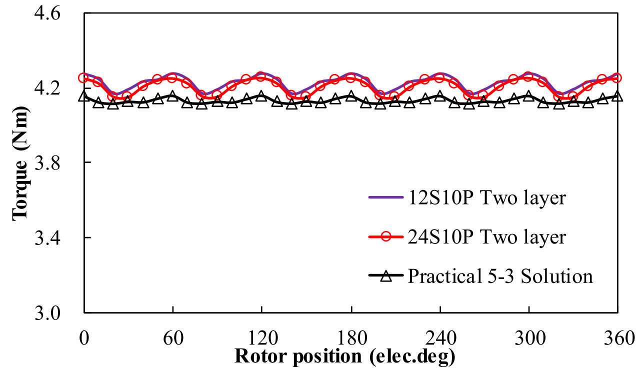

FIGURE 17. Variations of torque with rotor position, current density = 5.6A/mm2.

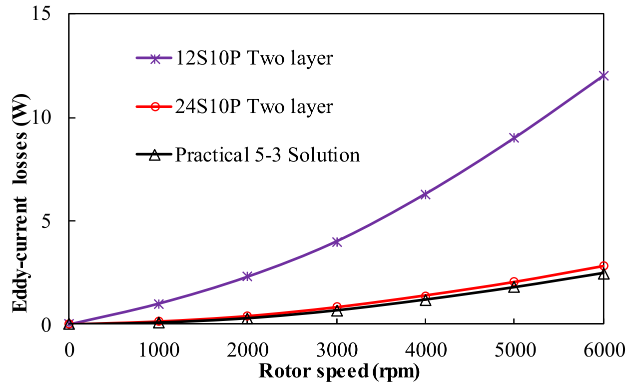

FIGURE 18. Variations of the eddy current losses with the rotor speed, current density = 5.6A/mm^2.

gradually as N_2/N_1 decreases from 1 to 3. Rotor eddy-current losses first reduces with the increase of N_2/N_1 and then increases, reaching a minimum value around N_2/N_1 equal 2.

Since the 24S10P PM machine with 5-3 solution and N_2/N_1 being 2 can have almost the same electromagnetic performance with one with 5-3 solution and optimal turn ratio (2.073), 5-3 solution with N_2/N_1 being 2 will be called the practical 5-3 solution. Electromagnetic performance of 24S20P PM machines with practical 5-3 solution will be compared with that of the two layer ones.

B. MACHINE PERFORMANCE COMPARISONS

The flux distributions of the armature field of two layer and practical 5-3 solution 24S10P PM machines when the flux linkage of phase A reaches maximum is shown in Fig. 16. It is clear to see that the subharmonic with pole pair equal to 1 is obvious in the two layer winding machine whilst it disappears in the machine with practical 5-3 solution.

Variations of torque with rotor position for the 24S10P two layer, 12S10P two layer, and 24S10P machine with practical 5-3 solution is shown in Fig. 17, whilst the torque components are listed in TABLE 5, where the supplied three-phase currents are in phase with its phase back-EMFs and the current density is 5.6A/mm^2. It shows that practical 5-3 solution machine has a reduction of average torque by 2.3% and 1.67% compared with the 12S10 two layer and 24S10P two layer ones, respectively. However, the torque ripple is much lower for the practical 5.3 solution machine, mainly due to the reduction of the 6th harmonic torque, as observed from TABLE 5.

Rotor eddy-current losses consists of the PM eddy-current loss, sleeve eddy-current loss and rotor iron eddy-current loss. Fig. 18 shows the variations of rotor eddy-current losses with rotor speed. Due to the reduction of sub-harmonic (v =1) and 7th harmonic with the practical 5-3 solution, the rotor eddy-current losses are largely reduced for the second machine compared with the 12S10P and 24S10P two layer counterparts, especially at high speed. At rotor speed of 6000rpm, the rotor eddy-current losses can be reduced by 79.3% and 16.7% respectively, compared with the 12S10P and 24S10P two layer counterparts with the employment of the practical 5-3 solution, which is beneficial to the irreversible demagnetization withstand capability.

Fig. 19 shows the radial force density distributions and their harmonic contents under rated load condition. Only harmonic order less and equal 6 are given because lower order harmonic of radial magnetic force density tends to play an important role in noise and vibration. It can be seen that amplitude of the most harmful 2nd harmonic for the practical 5-3 solution machine has a reduction by 98% and 72.9% compared with that of the 12S10P two layer and 24S10P two layer machines respectively. The 4th and 6th harmonic also be reduced at the same time, which indicates a lower noise

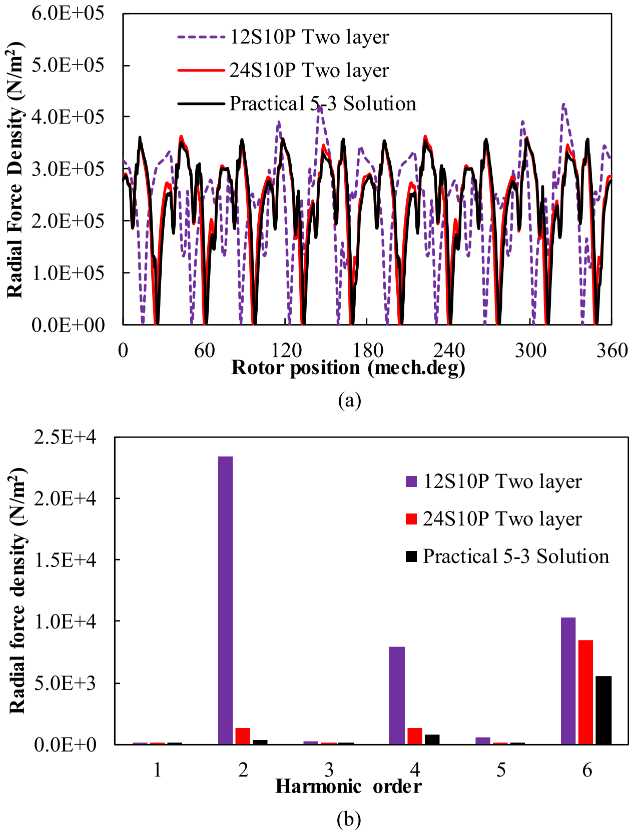

FIGURE 19. Variations of radial force density distributions with rotor position under rated load, current density = 5.6A/mm². (a) Waveforms. (b) Harmonics.

TABLE 5. Torque components.

	12S10P Two layer	24S10P Two layer	Practical 5-3 Solution
Average Torque (Nm)	4.21	4.24	4.14
Torque ripple (%)	2.41	2.5	1.02
6th harmonic	0.05	0.04	0.016
12th harmonic	0.010	0.015	0.009

and vibration level for 24S10P machine with the practical 5-3 solution.

VI. EXPERIMENT

To verify the aforementioned analysis, the 24S10P PM machine with the two layer and practical 5-3 solution have been prototyped and tested for validation. Fig. 20 shows the pictures of the 24-slot stator and winding distribution with two layer winding and practical 5-3 solution. The coil turn of the two layer winding is 6, whilst coil turn N_1 and N_2 for the practical 5-3 solution is 4 and 8 respectively. Fig. 21 shows the test platform.

The variations of FEA predicted and measured phase back-EMFs with rotor position are compared in Fig. 22, whilst TABLE 6 shows its harmonic contents. It can be seen from

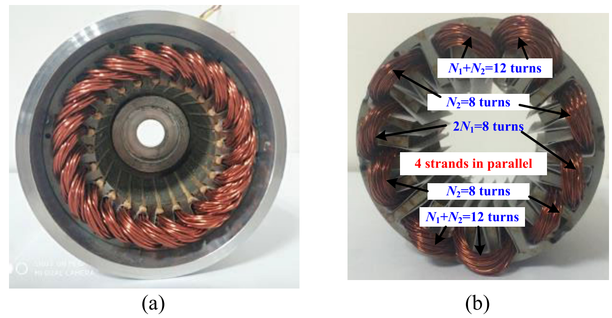

FIGURE 20. 24S10P stator windings of prototype machines. (a) Two layer winding. (b) Coil distributions of one phase for the practical 5-3 solution.

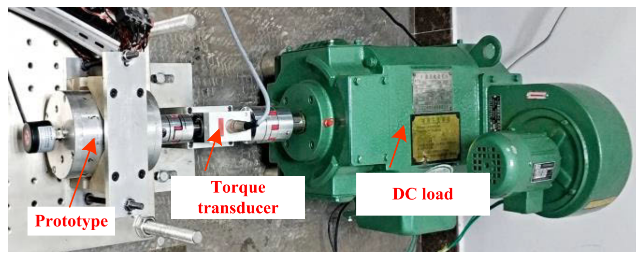

FIGURE 21. Test platforms.

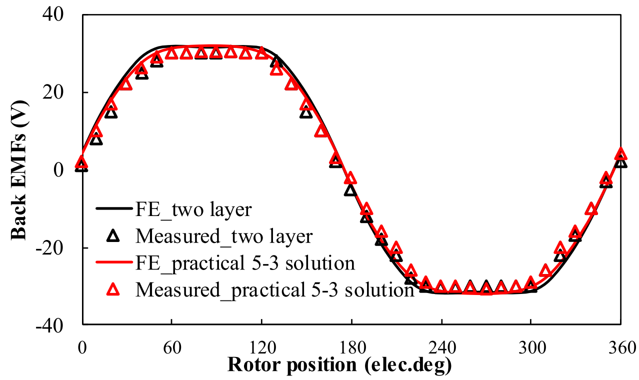

FIGURE 22. Variation of FE predicted and measured phase back-EMFs (3000 rpm).

TABLE 6. Harmonics in back-EMFs.

Two layer	FE (V)	Measured (V)	Practical 5-3 solution	FE (V)	Measured (V)
1	36.2	56.5	1	35.5	56.5
3	4.51	1.81	3	1.92	3.56
5	0.28	0.10	5	0.15	0.01
7	0.07	0.05	7	0.06	0.11
9	0.18	0.22	9	0.12	0.14
11	0.04	0.14	11	0.18	0.005
13	0.04	0.18	13	0.09	0.04

TABLE 6 that compared with the two layer counterpart, the employment of practical 5-3 solution can have a very low content of the 5th and 11th back-EMFs harmonics whilst the 7th back-EMF harmonic have a slight increase. This is due to that the 5th and 11th harmonics of phase back-EMFs have the same winding factor as that of the $v = 24k\pm1$ and

$v = 24k \pm 7$ order harmonics in stator MMFs, which is almost reduced to zero for the practical 5-3 solution whilst the 7th back-EMFs harmonic share one winding factor with the $v = 24k \pm 11$ order harmonic in stator MMFs. It can confirm that the amplitude of the $v = 24k \pm 1$ and $v = 24k \pm 7$ order harmonics of the 24S10P can be reduced to near zero with the practical 5-3 solution.

VII. CONCLUSION

Various solutions for elimination of space harmonics in stator MMF of fractional-slot winding with two-slot coil pitches are investigated in this paper. Two kinds of slot harmonic only windings (SHOW) are first proposed to remove all the harmonics except the main harmonic and its slot harmonics. In order to overcome the drawback of poor slot area utilization of the SHOW, two alternative solutions, i.e., 6-0 solution, 5-1 solution for 18S10P and four solutions, i.e., 7-1 solution, 6-2 solution, 5-3 solution and 4-4 solution for 24S10P are put forward. Several conclusions can be drawn:

1) All the proposed harmonic elimination solutions will inevitably compromise the winding factor of the main harmonic. The 5-1 solution for 18S10P and 5-3 solution for 24S10P are proved be the most favorable ones, considering the tradeoff made between the less harmonic contents and larger magnitude of main harmonic.

2) For 18S10P PM machines with 5-1 solution and 24S10P ones with 5-3 solution, the torque ripple, rotor eddy-current loss and noise and vibration level can be much lower than that of their corresponding two layer machines.

Appendix A

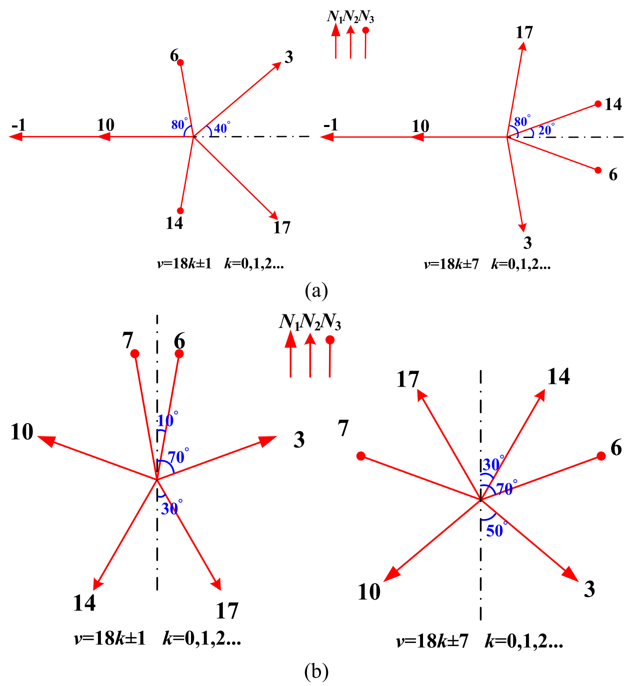

FIGURE 23. Star of phasors of to-be removed harmonics for the 18S10P winding with SHOW. (a) SHOW I. (b) SHOW II.

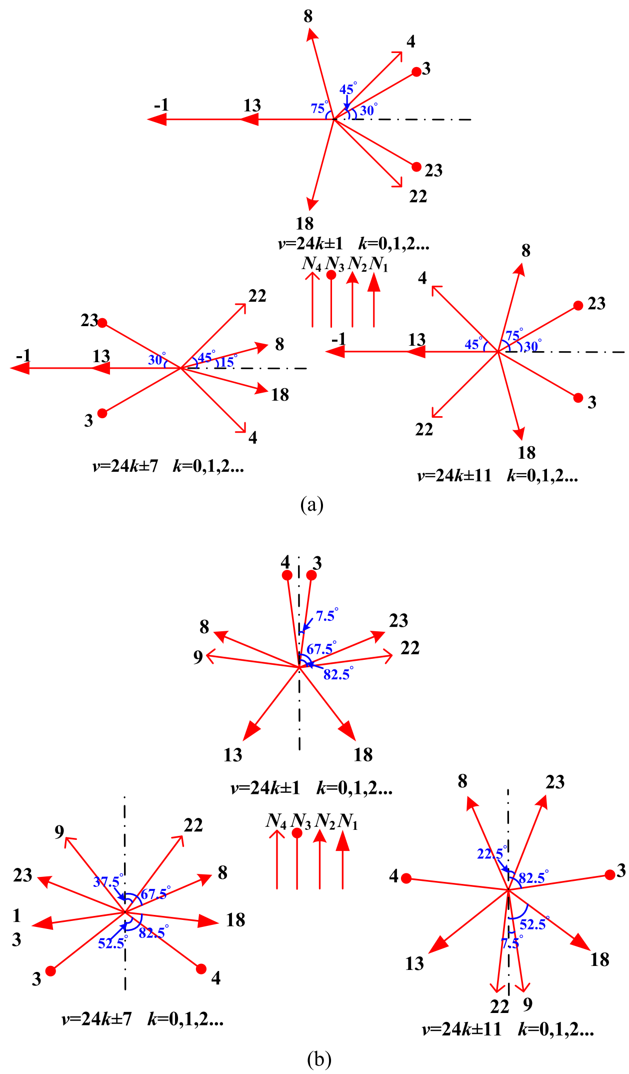

FIGURE 24. Star of phasors of to-be removed harmonics for the 24S10P winding with SHOW. (a) SHOW I. (b) SHOW II.

Appendix B

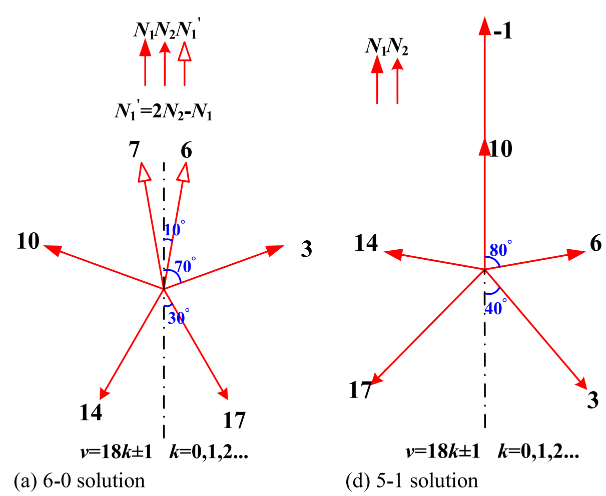

FIGURE 25. Star of phasors of the to-be removed harmonics for the 18S10P winding.

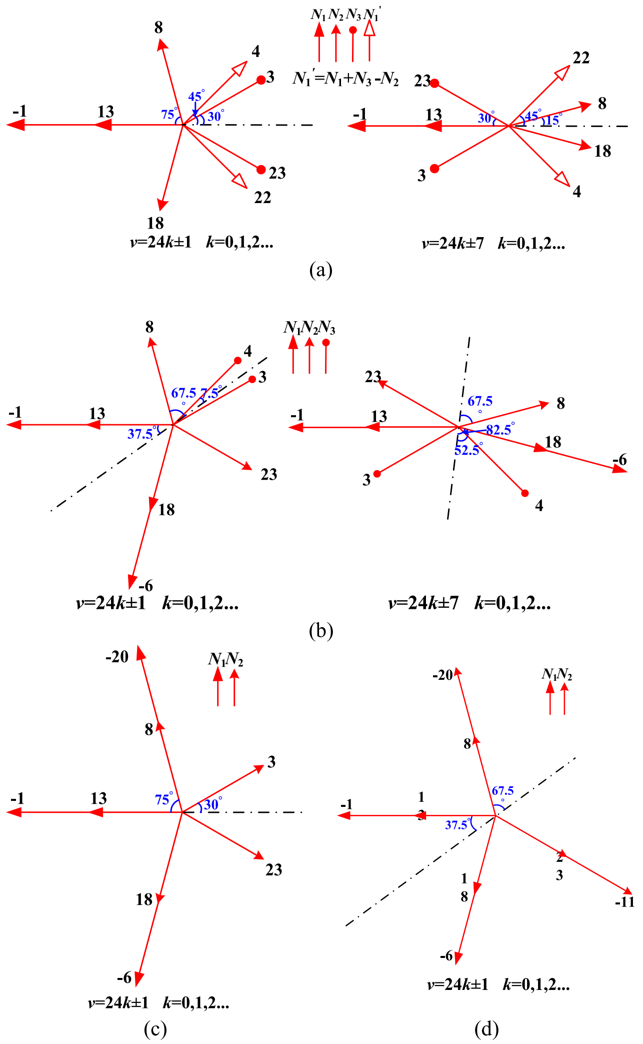

FIGURE 26. Star of phasors of the to-be removed harmonics for the 24S10P winding. (a) 7-1 solution. (b) 6-2 solution. (c) 5-3 solution. (d) 4-4 solution.

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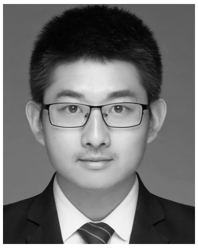

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