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High-Efficiency Expectation Propagation Detector for High-Order Massive MIMO Systems

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ABSTRACT Multiple-inputs multiple-outputs (MIMO) technology, including massive MIMO, plays an important role in modern wireless communication systems. Massive MIMO systems with high-order modulation can promote spectrum efficiency. The algorithm complexity and detection performance are the main challenges for the massive MIMO system. Thus, the low complexity and high-performance MIMO detector is important for the practical massive MIMO application. The Expectation Propagation (EP) detector outperforms many conventional detectors in high-order massive MIMO scenarios. However, its complexity increases exponentially with the number of sending antennas and the modulation order. In this paper, we propose a novel information updating scheme for EP MIMO detection algorithm to achieve high performance with low complexity. The high-efficiency EP detector is based on the expectation propagation algorithm with the jointed scheme of successive updating, sorting updating and sphere search aided algorithm. Numerical results show the high-efficiency EP detector reduces over 85% complexity of the original EP detector for the scenario $N_t = N_r = 20$ with 64-QAM modulation, and the gain on complexity becomes more evident with the increase of antenna scale and the modulation order. The high-efficiency EP detector can outperform the original EP detector in different high-order massive scenarios. Compared with MMSE algorithm, the proposed scheme can get huge performance gain with 1.5 times complexity for high-order massive MIMO systems.

INDEX TERMS Expectation propagation, massive MIMO detector, low complexity, high performance, high-order modulation.

I. INTRODUCTION

MIMO technology is one of the key technologies in modern wireless communication systems. Through multiple antennas at sending and receiving sides, MIMO can improve the system capacity and throughput without increasing spectrum resources and the transmitting power of antennas. At present, MIMO is adopted in many communication systems, including long-term evolution system (LTE), WiFi and microwave transmission networks, etc. With more antennas at sending and receiving sides, massive MIMO can obtain more diversity and multiplexing gain, which makes it be one of the key candidates for the 5^{th} generation (5G) mobile communication system. Massive MIMO is necessary for future wireless communication systems with high-throughput and high-stability.

For massive MIMO systems, the increase of antenna scales takes huge capacity gain as well as some technical issues and many researchers have worked to solve them [1], [2]. MIMO symbols detection is one of them. Since MIMO system was proposed, many MIMO detectors have been widely researched and some of them have been adopted in practical systems [3]. Maximum likelihood (ML) detector [4] can reach the optimal performance while its complexity increases exponentially with modulation order and the number of antennas, which is unacceptable for practical massive MIMO systems. The achievable rate of practical massive MIMO for different channel fading have been analyzed in [5] and [6]. The practical MIMO detector can be generally divided into linear detectors and non-linear detectors. The linear massive MIMO detection algorithms include Neumann series approximation algorithm [7], Chebyshev iteration algorithm [8], conjugate gradient algorithm [9], Zero Forcing (ZF) and Linear Minimum Mean Square Error (LMMSE) [10]. The non-linear massive MIMO detection algorithms include Sphere Decoding (SD) [11], Tree Search (TS) [12], Lattice Reduction Aided (LRA) [13], Triangular approximate semidefinite relaxation (TASER) [14] and K-Best signal detection algorithm [15]. Generally, non-linear

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detectors have better performance and higher complexity than linear detectors, while the latter can improve their performance with some schemes including Successive Interference Cancellation (SIC) [16] and Parallel Interference Cancellation (PIC) [17]. For example, LMMSE-PIC detector [18] can evidently improve the performance of the LMMSE detector, while its complexity also increases accordingly. It is proved that the iterative LMMSE detector can achieve the Gaussian capacity of MIMO system for Gaussian signaling [19] and the AMP receiver can achieve the constrained capacity of MIMO systems for any fixed input distributions [20].

In recent years, iterative detectors have attracted extensive attention [21]. Belief Propagation (BP) algorithm [22] is based on Message Passing (MP), it can approximate the posterior probability distribution for the Factor Graph (FG) or Markov Random Fields (MRF). Especially, the approximation result is equal to the exact posterior probability distribution for the FG without loops. BP detector is also researched by lots of scholars [23]–[25]. However, the FG of MIMO (including massive MIMO) is fully connected, which means there are lots of short loops (the number of nodes is not more than 4) in the FG. As a result, the performance of BP MIMO detector is poor. Gaussian Tree Approximation (GTA) detector is based on an optimal tree approximation of the Gaussian density of the unconstrained linear system, which can improve the performance of BP in MIMO detection [26]. It's further improved by GTA-SIC detector [27]. However, their performances are still not satisfactory. The Gaussian message passing (GMP) is used to further reduce the complexity of iterative receiver in the fully connected FG and obtain a capacity-approaching performance [28], [29]. Thus the convergence and performance of the loopy message passing receiver have been analyzed and improved. Another iterative approximated algorithm is Expectation Propagation. It is worth mentioning that there are two kinds of EP. The first one is based on EP algorithm which approximates a distribution by approximating components of the distribution and updates them by moment matching. EP algorithm is widely adopted in machine learning and achieves remarkable accomplishment after being proposed by Minka [30], [31]. The second one is similar to BP and MP, it propagates messages (such as expectations, this is why the scheme is named) between nodes in the FG and updates the messages until they converge. Both two EP schemes are widely adopted in communication systems. Such as channel estimation (CE), MIMO detection and channel decoding [32]–[36].

EP MIMO detector based on EP algorithm (the first kind, we call it ''Original EP'') was first proposed in [32] with features of fast convergence and outstanding performance. The EP MIMO detector is cited by a lot of MIMO researches. For example, the EP detector extends to joint BP with variation MP (BP-EP-VMP) in [37] and EP based on the conjugate gradient (EP-CG) is used for signal detection in [38]. It is also adopted in joint channel estimation and symbols detection. Such as CE based on the orthogonal pilot in [39] and blind CE in [40]. EP detector is improved in [41] based on the expectation consistency (EC) framework.

However, there are three drawbacks of the original iterative EP MIMO detector. Firstly, the original EP detector has to calculate a matrix inversion whose size is equal to the antenna scale in each iteration, which causes the high complexity and restricts its practicability in massive MIMO systems. Secondly, the EP detector indistinguishably updates different posterior distributions of the transmitted symbols, which could decrease the updating efficiency. At last, the original EP detector must perform some heavy loaded mathematical operations, including *multiplication* and *exp*, for every transmitted symbol in each iteration and the whole constellation. It's unacceptable for high-order massive MIMO systems. We have proposed a low-complexity EP MIMO detection algorithm in [42], but its limitation is still obvious.

To improve the mentioned drawbacks of the original EP detector, we propose a novel information updating algorithm to achieve high-efficiency EP detector for high-order massive MIMO systems. The novel updating algorithms include successive updating, sorting updating and sphere search aided. The jointed application of these three novel information updating algorithms can achieve high-efficiency EP MIMO detector. Both of the original EP detector and the proposed high-efficiency EP detector approximate and update the posterior distributions of transmitted symbols iteratively. However, the proposed high-efficiency EP detector adopts different updating strategies as well as space compression. The contributions are listed as follows:

- **Successive updating.** The high-efficiency EP detector changes the batch updating of the original EP detector into successive updating, which means it does not update the approximated joint posterior distribution (AJPoD) simultaneously after all approximated marginal prior distributions (AMPrDs) have been updated. It updates the AJPoD immediately after a single AMPrD has been updated. Through successive updating scheme, the high-efficiency EP detector can avoid the matrix inversion of the original EP detector and improve the updating efficiency. Thus, the algorithm complexity is decreased, and the convergence is sped up.
- **Sorting updating.** Since the high-efficiency EP detector updates the posterior distributions of transmitted symbols in successive mode, we can sort the information according to the reliability of posterior distributions and update the posterior distributions with high reliability preferentially, which makes the updating process more reliable and converges faster than the original EP detector. what's more, the performance is also improved.
- **Sphere search aided.** To reduce the huge calculation load in the updating process for high order modulations, the high-efficiency EP detector adopts the sphere search aided (SSA), which can decrease the number of mathematical operations without performance loss. Consequently, it reduces the complexity of the original

EP detector. The SSA scheme is especially effective in high-order modulation.

Thus, the proposed high-efficiency EP detector can not only reduce the complexity of the original EP detector in a single iteration but also speed up the convergence, which improves the practicability of the EP algorithm in MIMO symbols detection. What's more, the high-efficiency EP detector acquires better performance than EP detector whose performance is verified in [32] to be better than GTA and GTA-SIC detector.

We focus on symbols detection for high-order MIMO systems in this paper. We assume that receiving side gets perfect channel state information (CSI) and accurate estimation of channel noise power in our analysis. Although there is an estimation error in the practical system, it can be transformed into channel noise. The simulation results show the proposed algorithms are robust for imperfect CSI scenario. The different scenarios are studied in our simulation works, including different MIMO sizes, different modulation orders, symmetric and asymmetric MIMO systems, and imperfect CSI. According to simulation results, the proposed high-efficiency EP detector outperforms the original EP MIMO detector at the reduced complexity.

This paper is organized as follows. The system model and basic knowledge about MIMO detection are introduced in section II. Section III gives the details of the EP algorithm and the corresponding EP detector. Section IV introduces the proposed high-efficiency EP detectors. And Simulation results and conclusion are presented in section V and section VI, respectively.

Notation: The lowercase and uppercase in boldface denote column vector and matrix, respectively. $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^{-1}$ denote the conjugate transpose, transpose and inversion of a matrix, respectively. $\mathbb{E}[\cdot]$ and $\mathbb{V}[\cdot]$ denote the expectation and variance of random variables. $\mathcal{CN}(\mathbf{x}:\mathbf{a},\mathbf{B})$ and $\mathcal{CN}(x:a,b)$ denote complex joint Gaussian and complex Gaussian random variables, where the three parameters are random variable(s), mean(s) and (co)variance (matrix), respectively. For a matrix $\mathbf{M}$, $\mathbf{M}(i,j)$ is the element at i-th row and j-th column, $\mathbf{M}[i]$ and $\mathbf{M}\langle j\rangle$ are the i-th column vector and j-th row vector, respectively. $\mathcal{R}(\cdot)$ and $\mathcal{I}(\cdot)$ are the real part and imaginary part of a complex number. The operation $diag(\cdot)$ acts on a vector will get a diagonal matrix. $Vec(\boldsymbol{.})$ represents the vectorization of a matrix.

II. MIMO SYMBOLS DETECTION

Fig.1 is a M-QAM modulation MIMO system with N_t and N_r antennas at sending and receiving sides, respectively. Θ is the set of its constellation points, the transmitted symbols vector $\mathbf{x}=[x_1,x_2,\ldots,x_{N_t}]^T\in\mathbb{C}^{N_t\times 1}$ and $x_i\in\Theta$. The received symbols vector $\mathbf{y}=[y_1,y_2,\ldots,y_{N_r}]^T\in\mathbb{C}^{N_r\times 1}$ is given by:

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n}, \tag{1}$$

where $\mathbf{n}\in\mathbb{C}^{N_r\times 1}$ is the additive white circular-symmetric complex Gaussian noise vector whose entries follow $\mathcal{CN}(0,\sigma_n^2)$. $\mathbf{H}\in\mathbb{C}^{N_r\times N_t}$ denotes the flat Rayleigh fading channel matrix whose entries are independent and identically distributed (i.i.d.) with $\mathcal{CN}(0,1)$.

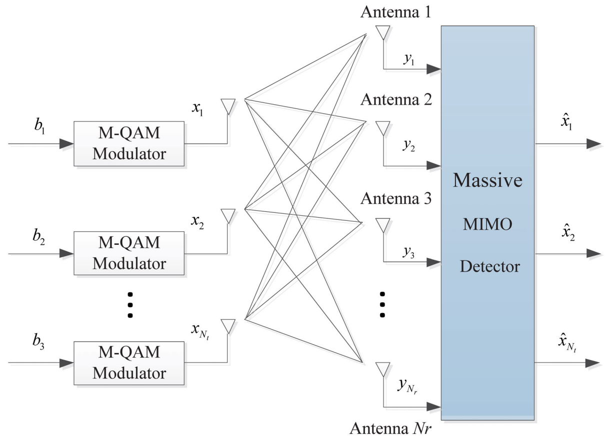

FIGURE 1. Conceptual illustration of MIMO detection.

The purpose of MIMO symbols detection is to obtain the optimal estimation $\hat{\mathbf{x}}$ of transmitted symbols $\mathbf{x}$. Generally, it needs to calculate the posterior probability $p(\mathbf{x}|\mathbf{y})$ and maximizes its value to obtain the optimal estimation.

$$\hat{\mathbf{x}}=\arg\max_{\mathbf{x}\in\Theta^{N_t}} p(\mathbf{x}|\mathbf{y}). \tag{2}$$

ML detector finishes this by searching the whole solution space.

$$\hat{\mathbf{x}}=\arg\min_{\mathbf{x}\in\Theta^{N_t}} \|\mathbf{H}\mathbf{x}-\mathbf{y}\|^2. \tag{3}$$

Linear detectors first equalize the receiving vector by an equilibrium matrix and then search for the partial optimal estimation for every transmitted symbol.

$$\hat{x}_i=\arg\min_{x_i\in\Theta}|x_i-z_i|. \tag{4}$$

where z_i is the equalized symbols. From Bayesian inference theory, there is:

$$p(\mathbf{x}|\mathbf{y})=\frac{p(\mathbf{x},\mathbf{y})}{p(\mathbf{y})}=\frac{p(\mathbf{y}|\mathbf{x})p(\mathbf{x})}{p(\mathbf{y})}. \tag{5}$$

Since the symbols y, the perfect CSI and channel noise power are known at the receiving sides, the $p(\mathbf{y}|\mathbf{x})$ follows $\mathcal{CN}(\mathbf{y}:\mathbf{H}\mathbf{x},\sigma_n^2\mathbf{I})$. And then the joint posterior distribution of transmitted symbols satisfies:

$$p(\mathbf{x}|\mathbf{y})\propto\mathcal{CN}(\mathbf{y}:\mathbf{H}\mathbf{x},\sigma_n^2\mathbf{I})\cdot p(\mathbf{x}). \tag{6}$$

In the MIMO communication system, the transmitted symbols are regarded as independent to each other (which is ensured by the randomness of transmitted symbols and interleaver in the system), that is $p(\mathbf{x})=\prod_{i=1}^{N_t}p(x_i)$. To improve the accuracy of MIMO symbols detection, a detector should calculate the following equation accurately.

$$\begin{aligned}p(\mathbf{x}|\mathbf{y})&\propto\mathcal{CN}(\mathbf{y}:\mathbf{H}\mathbf{x},\sigma_n^2\mathbf{I})\cdot\prod_{i=1}^{N_t}p(x_i)\\&=\mathcal{CN}\left(\mathbf{x}:(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y},\sigma_n^2(\mathbf{H}^H\mathbf{H})^{-1}\right)\cdot\prod_{i=1}^{N_t}p(x_i),\end{aligned} \tag{7}$$

Algorithm 1 Expectation Propagation

1: Initialize all approximation term $\tilde{t}_i(x)$.
2: Compute the posterior of x from the product of $\tilde{t}_i(x)$: $q(x) = f(x)\prod_i \tilde{t}_i(x) / \int f(x)\prod_i \tilde{t}_i(x)$
3: Repeat until all $\tilde{t}_i(x)$ converge:
4: Choose a factor term $\tilde{t}_i(x)$.
5: Remove $\tilde{t}_i(x)$ form $q(x)$ to get an 'old' posterior: $q^{\backslash i}(x) \propto q(x)/\tilde{t}_i(x)$.
6: Combine $q^{\backslash i}(x)$ with $t_i(x)$ and minimize the KL divergence to get a new posterior $q'(x)$ with normalization Z_i.
7: Update $\tilde{t}_i(x) = Z_i q'(x)/q^{\backslash i}(x)$.

where $p(x_i)$ are the marginal prior distributions of transmitted symbols.

III. EXPECTATION PROPAGATION MIMO DETECTOR

A. EXPECTATION PROPAGATION ALGORITHM

Expectation Propagation is one kind of approximated algorithms based on Bayesian inference [30], [31]. It is appropriate for the probability distributions which belong to exponential family.[1]

When an intractable distribution $p(x) = f(x)\prod_i t_i(x)$, where $f(x)$ belongs to exponential family, is difficult to calculate directly, EP can approximate it iteratively with a distribution $q(x) = f(x)\prod_i \hat{t}_i(x)$, where $\hat{t}_i(x)$ are approximations of $t_i(x)$ and belong to exponential family. And then $q(x)$ will be easier to calculate and it is updated as the following principle.

The similarity of two distributions can be described by Kullback Leibler (KL) divergence:

$$D_{KL}\left(p(x)||q(x)\right) = -\int p(x)\ln\frac{q(x)}{p(x)}dx. \tag{8}$$

From (8), we know that the smaller the value of KL divergence is, the more similar are the two distributions. In order to get the optimal approximation of $p(x)$, one feasible method is to compute the minimum of KL divergence by setting its gradient to zero. Then the optimal approximation $q_{opt}(x)$ satisfies the condition[2]:

$$\mathbb{E}_{q_{opt}(x)}[u(x)] = \mathbb{E}_{p(x)}[u(x)], \tag{9}$$

where $u(x)$ represents the sufficient statistics of $q_{opt}(x)$. (9) means that the expectations of sufficient statistics for approximated distribution are the same as the objective distribution. This is why the algorithm is called "Expectation Propagation". And because $q_{opt}(x)$ belongs to the exponential family, its expectations of sufficient statistics are the different order moments. Thus, this step is called "moment matching".

EP algorithm updates the $\tilde{t}_i(x)$ iteratively according to the criterion of minimum KL divergence. A general form of EP algorithm is shown as algorithm 1 in [30].

EP approximates the belief states by retaining the expectations of sufficient statistics, such as mean and variance, and iterates until these expectations are consistent throughout the Bayesian network. This makes it applicable to hybrid Bayesian networks with discrete and continuous nodes. Moreover, EP updates the approximated posterior distributions by moment matching, which is proved to be effective and can obtain excellent performance. These two features make EP draw extensive attention and successfully adopted in machine learning.

B. THE ORIGINAL EP MIMO DETECTOR

In the MIMO system, the distributions are Gaussian-like. Then EP algorithm can approximate them by Gaussian distributions which belong to exponential family. The sufficient statistics of Gaussian distributions include $\{x, x^2\}$ and their expectations are the first order and second order moments, which are the mean and variance of Gaussian distribution, respectively. When applied to MIMO symbols detection, EP algorithm approximates joint posterior distribution of the transmitted symbols. Just as mentioned in section II, the joint posterior distribution satisfies $p(\mathbf{x}|\mathbf{y}) \propto \mathcal{CN}(\mathbf{y}:\mathbf{Hx}, \sigma_n^2\mathbf{I}) \cdot \prod_{i=1}^{N_t} p(x_i)$. The original EP detector uses some unnormalized Gaussian distributions $\hat{p}(x_i) = exp(-1/2\lambda_i x_i^2 + \gamma_i x_i)$ to approximate the marginal prior distributions, then the AJPoD can be expressed as:

$$\begin{aligned}
\hat{p}(\mathbf{x}|\mathbf{y}) &\propto \mathcal{CN}(\mathbf{y}:\mathbf{Hx}, \sigma_n^2\mathbf{I}) \cdot \prod_{i=1}^{N_t}\hat{p}(x_i)\\
&\propto exp\left(-\frac{1}{2\sigma_n^2}(\mathbf{y}-\mathbf{Hx})^H\,\mathbf{I}(\mathbf{y}-\mathbf{Hx})\right)\prod_{i=1}^{N_t} exp\left(-\frac{1}{2}\lambda_i x_i^2 + \gamma_i x_i\right)\\
&= exp\left(-\frac{1}{2\sigma_n^2}(\mathbf{y}-\mathbf{Hx})^H\,\mathbf{I}(\mathbf{y}-\mathbf{Hx}) + \sum_{i=1}^{N_t}\left(-\frac{1}{2}\lambda_i x_i^2 + \gamma_i x_i\right)\right),
\end{aligned} \tag{10}$$

and it follows joint Gaussian distribution, that is:

$$\hat{p}(\mathbf{x}|\mathbf{y}) \sim \mathcal{CN}(\mathbf{x}:\mathbf{u},\mathbf{C}) \propto exp\left(-\frac{1}{2}(\mathbf{x}-\mathbf{u})^H\mathbf{C}^{-1}(\mathbf{x}-\mathbf{u})\right), \tag{11}$$

where $\mathbf{u}$ and $\mathbf{C}$ are the mean vector and covariance matrix of AJPoD, respectively. Comparing (10)) with (11), there are:

$$\mathbf{C} = \left(\sigma_n^{-2}\mathbf{H}^H\mathbf{H} + diag(\boldsymbol{\lambda})\right)^{-1}, \tag{12}$$

$$\mathbf{u} = \mathbf{C}\cdot\left(\sigma_n^{-2}\mathbf{H}^H\mathbf{y} + \boldsymbol{\gamma}\right), \tag{13}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_{N_t}]^T$, $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_{N_t}]^H$. For the transmitted symbols are independent to each other, the approximate marginal posterior distributions (AMPoDs) are the corresponding components of AJPoD. That is $\hat{p}(x_i|\mathbf{y}) \sim \mathcal{CN}\left(x_i : \mathbf{u}(i), \mathbf{C}(i,i)\right)$. Referring to the process of EP algorithm in section III, and the superscript (l) represents the l-th iteration, we summarize the details of original EP detector [32] as follow :

- *Step-1*: Initialize the parameters $\lambda_i^{(0)} = E_s^{-1}$, $\gamma_i^{(0)} = 0$, where E_s denotes the average symbol energy;

[1]More details about exponential family can be found in [43].
[2]The inference process can see [44].

- *Step-2*: Preprocessing: Compute the mean vector $\mathbf{u}^{(0)}$ and covariance matrix $\mathbf{C}^{(0)}$ of the AJPoD as (12) and (13).
- *Step-3*: Until the break condition or the maximum iterations, loop:
  1) Choose an AMPoD $\hat{p}(x_i|\mathbf{y})$, update the corresponding $\lambda_i^{(l)}$ and $\gamma_i^{(l)}$ as follows:
     a. Compute the marginal cavity distribution:

$$\hat{p}^{\setminus i(l)}(x_i|\mathbf{y}) = \frac{\hat{p}^{(l)}(x_i|\mathbf{y})}{\hat{p}^{(l)}(x_i)} \sim \mathcal{CN}(x_i : m_i^{(l)}, \varepsilon_i^{2(l)}), \tag{14}$$

where:

$$\varepsilon_i^{2(l)} = \frac{\mathbf{C}^{(l-1)}(i,i)}{1 - \mathbf{C}^{(l-1)}(i,i)\cdot\lambda_i^{(l-1)}}, \tag{15}$$

$$m_i^{(l)} = \varepsilon_i^{2(l)}\left(\frac{\mathbf{u}^{(l-1)}(i)}{\mathbf{C}^{(l-1)}(i,i)} - \gamma_i^{(l-1)}\right). \tag{16}$$

     b. Compute the mean $u_i^{*(l)}$ and variance $\sigma_i^{*2(l)}$ of replacement distribution $p_r^{(l)}(x_i|\mathbf{y}) = \hat{p}^{\setminus i(l)}(x_i|\mathbf{y})\cdot p(x_i)$, where $p(x_i)$ is a discrete uniform distribution about the constellation points of transmitted symbols when there is no prior knowledge.
     c. Match the moments of replacement distribution $p_r^{(l)}(x_i|\mathbf{y})$ and partly-updating distribution $p_{pu}^{(l)}(x_i|\mathbf{y}) = \hat{p}^{\setminus i(l)}(x_i|\mathbf{y})\cdot\hat{p}^{(l)}(x_i)$, then get the $\lambda_i^{(l)}$ and $\gamma_i^{(l)}$ by:

$$\lambda_i^{(l)} = \frac{1}{\sigma_i^{*2(l)}} - \frac{1}{\varepsilon_i^{2(l)}}, \tag{17}$$

$$\gamma_i^{(l)} = \frac{u_i^{*(l)}}{\sigma_i^{*2(l)}} - \frac{m_i^{(l)}}{\varepsilon_i^{2(l)}}. \tag{18}$$

  2) Check if all AMPoDs have been updated, if the answer is 'no', go back to 1).
  3) Update the mean vector $\mathbf{u}^{(l)}$ and covariance matrix $\mathbf{C}^{(l)}$ of the AJPoD as (12) and (13).
- *Step-4*: Output the estimation $\hat{\mathbf{x}}$ of transmitted symbols from the hard decision of AMPoDs $\hat{p}(x_i|\mathbf{y})$.

In the process of original EP detector, there may return a negative value in (17), which is illogical as a variance. It indicates that there is an unsuitable approximate distribution. In this situation, the detector gives up updating the approximation and uses the previous reserved results. Meanwhile, to improve the stability, updating (17) and (18) the detector can use a low-pass filter as follow:

$$\lambda_i^{(l)} = \beta\left(\frac{1}{\sigma_i^{*2(l)}} - \frac{1}{\varepsilon_i^{2(l)}}\right) + (1-\beta)\lambda_i^{(l-1)}, \tag{19}$$

$$\gamma_i^{(l)} = \beta\left(\frac{u_i^{*(l)}}{\sigma_i^{*2(l)}} - \frac{m_i^{(l)}}{\varepsilon_i^{2(l)}}\right) + (1-\beta)\gamma_i^{(l-1)}, \tag{20}$$

Besides, the detector controls the value of $\sigma_i^{*2(l)}$ by $\sigma_i^{*2(l)} = max\left(\mathbb{V}[p_r^{(l)}(x_i|\mathbf{y})], \tau\right)$. Both β and τ are experimental parameters.

EP detector is an application of the EP algorithm in the MIMO communication system, and it inherits the characteristic of good performance. EP detector also converges rapidly. Reference [32] shows that the EP detector with two iterations outperforms the Gaussian tree approximation (GTA) detector.

IV. HIGH-EFFICIENCY EP DETECTOR

A. MOTIVATION

Though the original EP detector has excellent performance, its complexity is very high. Reference [32] shows that the complexity of the original EP detector is $\mathcal{O}(LN_t^3 + LN_tM)$, where L denotes the iterations of the original EP detector, while the complexity of MMSE detector is $\mathcal{O}(N_t^3 + N_t^2)$.

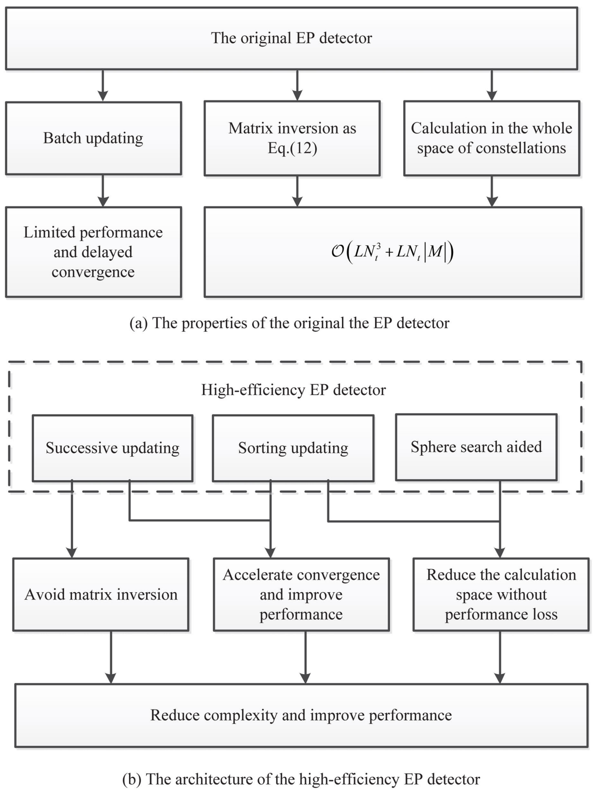

FIGURE 2. The illustrations of different EP detectors: (a) The properties of the original EP detector, (b) The architecture of the high-efficiency EP detector.

As shown in Fig.2(a), the main complexity of the original EP detector is caused by two parts. One is the computation of matrix inversion as (12). It is performed in each iteration, which makes the complexity become $\mathcal{O}(LN_t^3)$. The other is the computation in the updating process of *Step-3*-1)-b. To get the mean $u_i^{*(l)}$ and variance $\sigma_i^{*2(l)}$ of replacement distribution, the original EP detector must calculate the value of probability density of approximated Gaussian distribution for every transmitted symbol in the whole constellation space, that is where the item $\mathcal{O}(LN_tM)$ comes from. For a high-order massive MIMO system, both the values of N_t and M are big, and though the convergence speed of the original EP detector is faster than BP based algorithms, it still needs more than 6 or 7 iterations to achieve converge. These mean that to reach the excellent performance of the original EP detector needs a huge computation load.

Besides, the updating processes of the original EP detector for different transmitted symbols are indiscriminate, which restricts the updating accuracy. The gain in each iteration is limited. That is, the more iterative updating is required for the original EP detector. It also means there are enhanced spaces for the original EP detector in convergence and performance. Thus, to design a high-efficiency EP detector which can reduce the complexity of the original EP detector and further improve the performance of the original EP detector is our intention.

From the motivation, we proposed the high-efficiency EP MIMO detector which includes three main algorithms. They are successive updating, sorting updating and sphere search aided. Just shown as Fig.2(b), successive updating can avoid matrix inversion in the iteration, sorting updating and successive updating can achieve better updating performance and faster convergence than the original EP MIMO detector, SSA and sorting updating can reduce calculation loads. With better information updating performance, faster convergence speed and lower calculation loads, the proposed high-efficiency EP MIMO detector can achieve better detection performance than the original one with lower algorithm complexity.

B. SUCCESSIVE UPDATING

First of all, to solve the item $\mathcal{O}(LN_t^3)$ in complexity, which is from the matrix inversion as (12) in every iteration, we adopt the successive updating scheme in the EP detector (We call the EP detector with successive updating scheme as "EP-SU detector"). We notice that the matrix inversion is caused by batch updating scheme for AJPoD. That is, the AJPoD is updated after all AMPrDs have been updated, which brings two drawbacks. One is the AJPoD cannot be updated immediately; another is that all the AMPrDs take part in calculating when updating the AJPoD, which lead to high complexity.

In fact, the AJPoD of the transmitted symbols changes after any single AMPrD of a transmitted symbol has been updated. Based on this, we proposed EP-SU detector. Once a single AMPrD of a transmitted symbol has been updated, the EP-SU detector updates the AJPoD of transmitted symbols instantly, which can accelerate the updating speed. Besides, the successive updating scheme only needs the parameters of the latest updated transmitted symbol for AJPoD calculation, which can avoid the matrix inversion in the batch updating. So that the complexity can be reduced significantly. The details of EP-SU detector are realized as follows.

Firstly, EP-SU detector initializes and preprocesses as the EP detector does and it retains the updating process of a single transmitted symbol. After it obtains the AMPoDs of all transmitted symbols in the preprocessing, EP-SU detector updates the AMPrD of each transmitted symbol by iterative and successive mode.

The original EP MIMO detector computes $\mathbf{u}^{(l)}$ and $\mathbf{C}^{(l)}$ after obtaining all pairs of $(\lambda_i^{(l)}, \gamma_i^{(l)})$ while the proposed method updates the mean vector and covariance matrix of AJPoD once one pair of $(\lambda_i^{(l)}, \gamma_i^{(l)})$ is obtained. According to this update mode, assume $\mathcal{CN}(\mathbf{y} : \mathbf{Hx}, \sigma_n^2\mathbf{I})$ is a factor $\mathcal{F}$, and combine (10), the relation between the AJPoD before and after updating is[3]:

$$\begin{aligned}
\hat{p}_i^{(l)}(\mathbf{x}|\mathbf{y}) &= \mathcal{F}\prod_{j=1}^{i}\hat{p}^{(l)}(x_j)\cdot\prod_{k=i+1}^{N_t}\hat{p}^{(l-1)}(x_k) \\
&= \mathcal{F}\prod_{j=1}^{i-1}\hat{p}^{(l)}(x_j)\cdot\prod_{k=i}^{N_t}\hat{p}^{(l-1)}(x_k)\cdot\frac{\hat{p}^{(l)}(x_i)}{\hat{p}^{(l-1)}(x_i)} \\
&= \hat{p}_{i-1}^{(l)}(\mathbf{x}|\mathbf{y})\cdot\frac{\hat{p}^{(l)}(x_i)}{\hat{p}^{(l-1)}(x_i)} \\
&= \hat{p}_{i-1}^{(l)}(\mathbf{x}|\mathbf{y})exp\left(-\frac{1}{2}\Delta\lambda_i^{(l)}x_i^2+\Delta\gamma_i^{(l)}x_i\right),
\end{aligned} \quad (21)$$

where $\Delta\lambda_i^{(l)} = \lambda_i^{(l)} - \lambda_i^{(l-1)}$, $\Delta\gamma_i^{(l)} = \gamma_i^{(l)} - \gamma_i^{(l-1)}$. Assume that $\mathbf{C}_i^{(l)}$ is the covariance matrix of the AJPoD after i pairs of $(\lambda_i^{(l)}, \gamma_i^{(l)})$ have been updated in the l-th iteration. In this way there is $\mathbf{C}_N^{(l)} = \mathbf{C}_0^{(l+1)} = \mathbf{C}^{(l+1)}$. According to (21), we can get:

$$\mathbf{C}_i^{(l)} = \left(\left(\mathbf{C}_{i-1}^{(l)}\right)^{-1} + \Delta\lambda_i^{(l)}\mathbf{e}_i\mathbf{e}_i^T\right)^{-1}, \quad (22)$$

where $\mathbf{e}_i$ is the i-th column of identity matrix and there is a rank-1 theorem for N-order square matrix inversion:

$$(\mathbf{A}+\mathbf{u}\mathbf{v}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{u}\mathbf{v}^H\mathbf{A}^{-1}}{1+\mathbf{v}^H\mathbf{A}^{-1}\mathbf{u}}, \quad (23)$$

where $\mathbf{u} \in \mathbb{C}^{N\times1}$, $\mathbf{v} \in \mathbb{C}^{N\times1}$. Combining (22) and (23), then we have:

$$\mathbf{C}_i^{(l)} = \mathbf{C}_{i-1}^{(l)} - \frac{\Delta\lambda_i^{(l)}}{1+\Delta\lambda_i^{(l)}\mathbf{C}_{i-1}^{(l)}(i,i)}\mathbf{C}_{i-1}^{(l)}[i]\,\mathbf{C}_{i-1}^{(l)}\langle i\rangle. \quad (24)$$

In this way, we replace the complicated matrix inversion in each iteration with successive iterative vector multiplication and matrix addition, which decreases the complexity greatly. What's more, since $\mathbf{C}_i^{(l)}$ is a conjugate symmetric matrix, the complexity can be further decreased. After obtaining the covariance matrix, we only compute one mean of the next transmitted symbol, that is[4]:

$$\mathbf{u}_i^{(l)}(i+1) = \mathbf{C}_i^{(l)}[i+1]\cdot\left(\sigma_n^{-2}\mathbf{H}^H\mathbf{y}+\boldsymbol{\gamma}_i^{(l)}\right), \quad (25)$$

where $\boldsymbol{\gamma}_i^{(l)} = [\gamma_1^{(l)}, \gamma_2^{(l)}, \ldots, \gamma_i^{(l)}, \gamma_{i+1}^{(l-1)}, \ldots, \gamma_N^{(l-1)}]^T$ denotes the parameter whose former i components are updated.

[3]In order to mark simply, there we ignore the normalization factor and adopt '=' instead of '$\propto$'.

[4]When $i = N_t$, the index $i+1 = 1$, in a general form, $i = mod(i+1, N_t)$.

Besides reducing the complexity, the successive updating scheme can also speed up the convergence process of detector. To prove it, we compare the convergence of EP and EP-SU detectors. We present the changes of approximated means and variances of different detectors with the changes of iterations in Fig.3 and Fig.4. Both the 2 figures are got under the condition of $SNR = 10\log(N_t E_s \cdot \sigma_n^{-2}) = 23$ dB, 16-QAM modulation and $N_t = N_r = 32$.

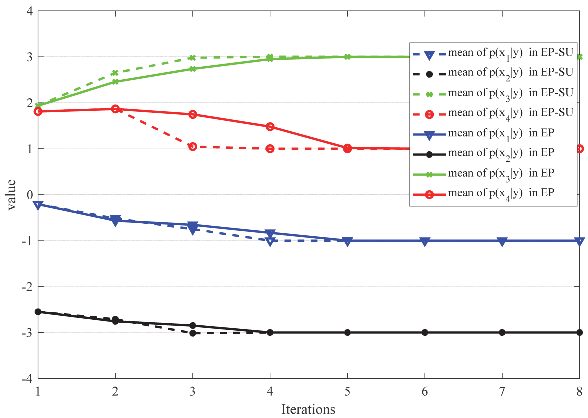

FIGURE 3. **The changes of approximated means of EP and EP-SU detectors when $N_t = N_r = 32$ and $SNR = 23$ dB.**

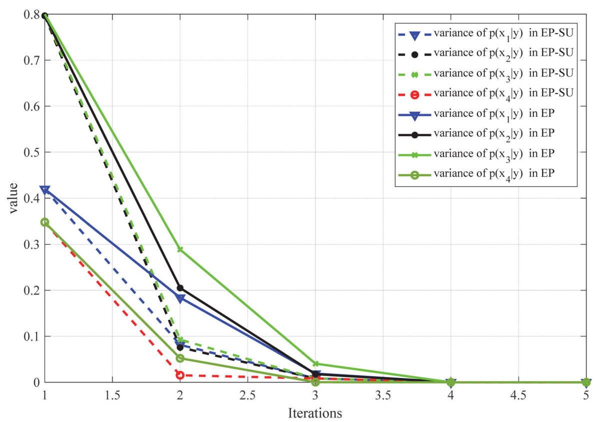

FIGURE 4. **The changes of approximated variances of EP and EP-SU detectors when $N_t = N_r = 32$ and $SNR = 23$ dB.**

In Fig.3 there are 4 values of the real and imaginary parts of exact means of transmitted symbols for 16-QAM, that is $\mathcal{R}(\Theta) = \mathcal{I}(\Theta) = \{+3, +1, -1, -3\}$. We choose 4 different posterior distributions to observe their convergence. They are $p(x_1|y), p(x_2|y), p(x_3|y), p(x_4|y)$ and they have different means. All of them converge to the exact values no matter the detector is EP or EP-SU. The changes of variances are presented in Fig.4, all the variances converge to zero. The convergence of means and variances indicates that both EP and EP-SU detectors are converged. Because of the same preprocessing of EP and EP-SU detectors, their initial approximated means and variances are the same. However, the following approximated values indicate that EP-SU detector

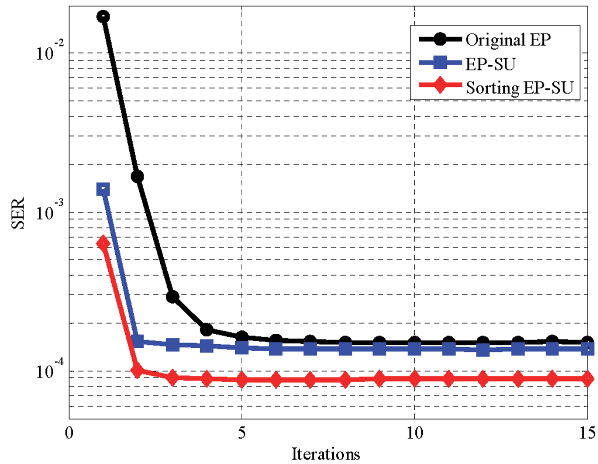

FIGURE 5. **The SER performance of different EP detectors with 16-QAM modulation when $N_t = N_r = 32$ and $SNR = 23$ dB.**

can converge to exact values faster, their Euclid distances from exact values and the approximated variances are both smaller.

C. SORTING UPDATING

In the update process of the original EP detector, the AMPrDs of transmitted symbols are batch updated and have no difference to each other, which means they are not updated in time sequence and have the same effect on AJPoD calculation. However, the reliability of different AMPoDs whose parameters (mean and variance) are used for updating AMPrDs are different because of different channel response coefficients and the different actual noise power. In general, the reliability of AMPoDs are determined by their variances: the smaller the variance, the more reliable the AMPoD is, and vice versa.

Because of the successive updating of EP-SU detector, the AJPoD updates after every AMPrD has been updated. Thus, the EP detector with successive updating can choose sorting updating. If we update the transmitted symbols in the sequence of serial numbers, such as in a sequence of $x_1, x_2, x_3, \ldots$, there will be a drawback. When the AMPoD of the present transmitted symbol is inaccurate or has low reliability, then updating this symbol preferentially leads to propagating the inaccuracy to other symbols. Although the subsequent updating possibly corrects this problem, it will slow down the convergence speed of the EP detector. Thus, we sort the variances of AMPoDs of transmitted symbols before every iteration and preferentially update the AMPrDs of symbols whose variance is smaller, which propagates the reliability of transmitted symbols optimally. The EP detector with sorting updating scheme (Because the sorting updating scheme must combine with successive updating, we call it "Sorting EP-SU detector") accelerates the convergence in some extent and improves the performance of the original EP detector.

We give the statistical results of convergence by SER performance for different scenarios in Fig.5 and Fig.6. We can

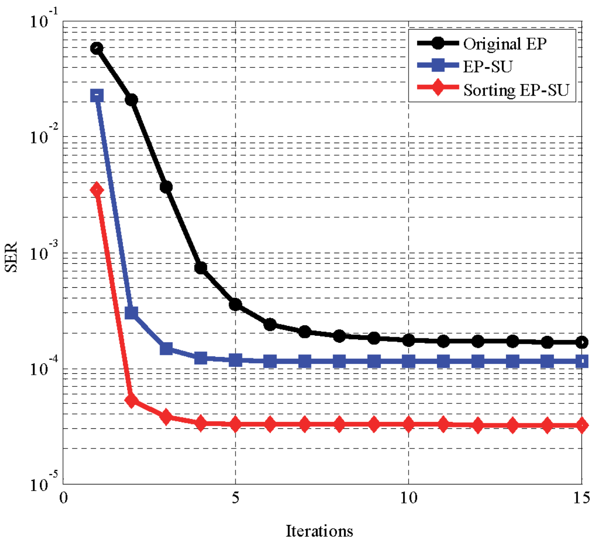

FIGURE 6. The SER performance of different EP detectors with 16-QAM modulation when $N_t = N_r = 80$ and $SNR = 21$ dB.

find that the EP-SU and the Sorting EP-SU can achieve better performance than the original EP with fewer iterations. Same as [32], the original EP detector almost converges after 6∼8 iterations. However, the proposed EP-SU detector and Sorting EP-SU detector converges with only 3∼4 iterations, which means the proposed method only needs about half iterations to reach convergence. With a half value of L, the complexity of EP detector is reduced a half. The performance of EP-SU detector in the former two iterations outperforms the original EP detector evidently. This is because the AJPoD of transmitted symbols updates LN_t times in EP-SU detector while it only updates L times in the original EP detector. Moreover, the batch updating of AJPoD causes the performance loss for the original EP detector. The sorting EP-SU detector improves the performance after all detectors converging. It verifies that the sorting updating scheme is as effective as expected.

Another conclusion is that the sorting updating scheme improves the robustness of EP-SU detector. The performance of EP-SU detector is unstable. This is because that the more reliable symbols are updated preferentially in a better condition or the less reliable symbols are updated preferentially in a worse condition. In both conditions, their reliability (low or high) is spread to other symbols when the AJPoD is updated, which causes a corresponding (worse or better) performance. In the worst case, the SER performance of the EP-SU detector may be worse than the original EP detector (This can be found in the next section). However, whatever the condition is, the SER performance of sorting EP-SU detector is always better than the other two (the original EP detector and EP-SU detector).

D. SPHERE SEARCH AIDED

As mentioned before, to get the mean $u_i^{*(l)}$ and variance $\sigma_i^{*2(l)}$ of replacement distribution, the original EP detector must calculate the value of probability density of approximated Gaussian distribution for every sample of constellation point of every transmitted symbol. The marginal cavity distribution $\hat{p}^{\backslash i(l)}(x_i|\mathbf{y}) \sim \mathcal{CN}(x_i : m_i^{(l)}, \varepsilon_i^{2(l)})$, then the values of probability density of transmitted symbol x_i are:

$$p(x_i = \theta_k) = \frac{1}{\sqrt{2\pi\varepsilon_i^{2(l)}}} \exp\left(-\frac{1}{2}\frac{(\theta_k - m_i^{(l)})^2}{\varepsilon_i^{2(l)}}\right), \quad (26)$$

where $\theta_k \in \Theta, k = 1, 2, \ldots, M$ are constellation samples. And then it uses $p(x_i = \theta_k)$ to calculate $u_i^{*(l)}$ and $\sigma_i^{*2(l)}$:

$$u_i^{*(l)} = \sum_{k=1}^{M} p(x_i = \theta_k) \cdot \theta_k, \quad (27)$$

$$\sigma_i^{*2(l)} = \sum_{k=1}^{M} p(x_i = \theta_k) \cdot \left(\theta_k - u_i^{*(l)}\right)^2. \quad (28)$$

From (26) to (28), there includes *exp*, *subtraction* and *multiplication* operations LN_tM times. Especially the *exp* operations has higher complexity in numerical calculation than the others [45]. To decrease the number of operations, we utilize the feature of EP algorithm and adopt SSA scheme to reduce the search space of solution.

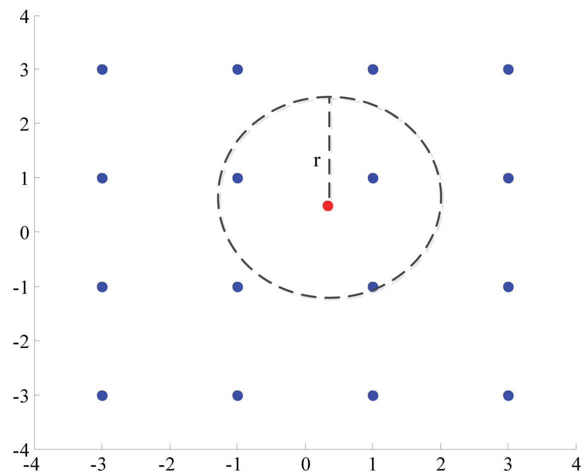

FIGURE 7. SSA scheme with 16-QAM for EP detector reduces the search space of solution.

EP algorithm converges fast, so in the preprocessing of EP detector, the AMPods of transmitted symbols can be regarded as reliable, especially when SNR is relatively high. Thus, these constellation points which are far from the mean of marginal cavity distribution $\hat{p}^{\backslash i(l)}(x_i|\mathbf{y})$ could hardly be the estimations of transmitted symbols. As a result, we only calculate the value of constellation samples whose Euclidean distance are smaller than the search radius r and set the other values to 0. Just shown as Fig.7, we only compute the probabilities $p(x_i = \theta_k)$ in the dotted circle and set the others to zeros.

Essentially, SSA scheme trades off between performance and complexity. In general, a bigger r brings better performance and higher complexity while a smaller one accompanies the worse performance and lower complexity. However, because the AMPoDs are reliable in updating processes, SSA scheme can keep a small value of r without performance loss. Thus, it can obviously reduce the mathematical operations without performance loss, especially in high-order modulation systems.

Concretely, we set the value of r as follows:

$$r = |\theta_k - m_i^{(l)}| = \rho\varepsilon_i^{(l)}, \tag{29}$$

where ρ is a parameter to balance the complexity and performance. Another worthy mentioned thing, when ρ in (29) is determined, the radius of search space depends on the value of variance $\varepsilon_i^{2(l)}$ of the first iteration, which is totally depended on the adopted scheme of MIMO detector. Obviously, the $\varepsilon_i^{2(l)}$ of the original EP detector is depended on the preprocessing. However, only the first $\varepsilon_1^{2(l)}$ of sorting EP-SU detector is depended on the preprocessing, the other variances $\varepsilon_{i,i\neq 1}^{2(l)}$ are depended on the covariance matrix after the AJPoD is updated $i-1$ times. This means sorting EP-SU detector has smaller radii r (lower complexity) than the original EP detector when they have the same ρ. Or, if both the two detectors have the same size of search space, the original EP detector will suffer more performance loss. Briefly, the SSA scheme is more appropriate for sorting EP-SU detector.

After we get this r in the first iteration, we use it to determine the search space, a subset of Θ which contains M_1 elements. Ignoring the normalized factor $(2\pi\varepsilon_i^{2(l)})^{-1/2}$, we put (26) into (27) and (28) to calculate $u_i^{*(l)}$ and $\sigma_i^{*2(l)}$:

$$u_i^{*(l)} = \sum_{k=1}^{M_1} \exp\left(-\frac{1}{2}\frac{(\theta_k - m_i^{(l)})^2}{\varepsilon_i^{2(l)}}\right) \cdot \theta_k, \tag{30}$$

$$\sigma_i^{*2(l)} = \sum_{k=1}^{M_1} \exp\left(-\frac{1}{2}\frac{(\theta_k - m_i^{(l)})^2}{\varepsilon_i^{2(l)}}\right) \cdot \left(\theta_k - u_i^{*(l)}\right)^2. \tag{31}$$

Then we test the SSA scheme. We compare the performance of sorting EP-SU detector with or without SSA under the condition of different ρ (Actually, these different adopted values of ρ let the exponential term in (26) is equal to 0.1, 0.01 and 0.001, respectively) and different SNR when $N_t = N_r = 20$ with 16-QAM, 64-QAM and 256-QAM modulation in Fig.9, and the average size of search space under the same condition in Fig.8. To ensure convergence, the detector iterates 4 times.

From Fig.9 we know that the SER performance of detector with SSA scheme is identical with the detector without SSA scheme when ρ is bigger than 3.0349. Actually, It is the 3δ principle for a Gaussian distribution which says the probability can reach 99.73% if its value is in the section $(\mu - 3\delta, \mu + 3\delta)$, where μ and δ are the mean and variance of the Gaussian distribution, respectively. Thus, we can set $\rho = 3$ to use SSA scheme in our proposed EP detector without performance loss.

In Fig.8 we regard the search space of detector without SSA scheme as 1. We transmit over 1 million symbols

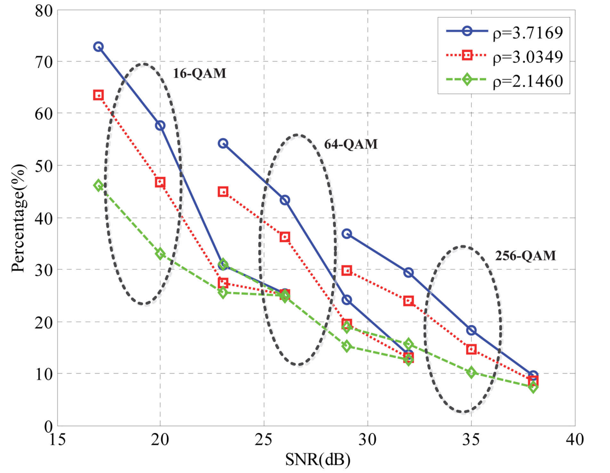

FIGURE 8. The percentage of the average size of search space under the condition of different SNR when $N_t = N_r = 20$ and 16-QAM, 64-QAM and 256-QAM modulation.

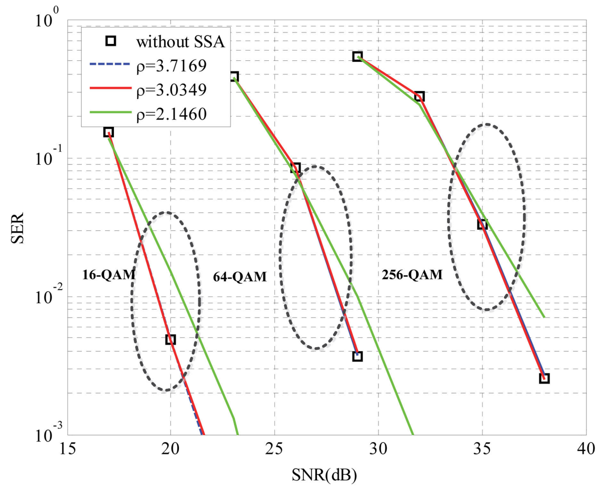

FIGURE 9. The performance of sorting EP-SU detector with or without SSA under the condition of different ρ and different SNR when $N_t = N_r = 20$ and 16-QAM, 64-QAM and 256-QAM modulation.

and calculate their average size M_1 of search space in one iteration for a single transmitted symbol. Then we get the $Percentage(\%) = M_1/M$. We also present these results in TABLE 1.

We can make three conclusions from TABLE 1:

1) When ρ is settled, the size of the search space becomes smaller with the increase of SNR. This is because when SNR increases, a lower $\varepsilon_i^{(l)}$ makes the search radius become smaller.
2) When SNR is settled, the size of search space changes with the parameter ρ. However, if SNR is big enough, the search radius in (29) is almost determined by $\varepsilon_i^{(l)}$ and the effect of ρ will be negligible.[5] This means we

[5]This means ρ will not change the number of search constellations when it varies in a range. In the extreme situations of $\rho = 0$ and $\rho = \infty$, the space will be null space and total space whatever the value of $\varepsilon_i^{(l)}$ is.

TABLE 1. The effect of SSA on performance and complexity in 20 × 20 MIMO with different modulation order.

modulation(ρ)	16QAM				64QAM				256QAM				SER/SNR
	2.1460	3.0349	3.7169	∞	2.1460	3.0349	3.7169	∞	2.1460	3.0349	3.7169	∞	
performance(dB)	18.4	18	18	18	26.6	26.5	26.5	26.5	34.6	34.4	34.4	34.4	0.05
	20.5	19.4	19.4	19.4	29	28.1	28.1	28.1	37.4	36.4	36.4	36.4	0.01
	21.4	20	20	20	29.8	28.8	28.7	28.7	\	37.3	37.2	37.2	0.005
complexity(%)	33.03	46.85	57.61	100	\	\	\	100	\	\	\	100	20 dB
	25.02	25.10	25.37	100	24.84	36.20	43.27	100	\	\	\	100	26 dB
	\	\	\	100	12.69	13.08	13.71	100	15.58	23.86	29.30	100	32 dB
	\	\	\	100	\	\	\	100	7.40	8.56	9.52	100	38 dB

1. $\rho = \infty$ means there is no SSA.
2. '\' means these data are missing and unimportant.

can set a very small value of ρ to decrease the size of search space if the concerned SNR is big enough (such as 26 dB for 16-QAM, 33 dB for 64-QAM, and 40 dB for 256-QAM).

3) SSA scheme is more useful in high-order modulation. When $\rho = 3.0349$ and a big enough SNR, the percentages of different modulation orders are 25.02%, 12.69%, and 7.40%, respectively. These percentages have corresponding search space with the calculation of constellation points about $M_1 = 4.00$ in 16-QAM, $M_1 = 8.12$ in 64-QAM and $M_1 = 18.94$ in 256-QAM.

The higher-efficiency EP detector improves the SER performance of the original EP detector with reduced complexity. In order to make the proposed high-efficiency detector more clear, we present it as Algorithm 2.

Algorithm 2 High-Efficiency EP Detector

Inputs: $\mathbf{y}$, $\mathbf{H}$, σ_n^2;
Initialization: $\lambda_i^{(0)} = E_s^{-1}$, $\gamma_i^{(0)} = 0$, $\beta = 0.2$, L, $\tau = 1 \times 10^{-7}$, ρ;
Preprocessing: Calculate covariance matrix and means vector as (12) and (13);
for l=1:L **do**
    Ascending sort the variances of AMPoDs; 0.5,0.5,0.5 *//sorting updating*
    for i=1:N_t **do**
        Choose the i-th transmitted symbol;
        Calculate the parameters of cavity marginal distribution as (15) and (16);
        Determine the search radius as (29) and find the M_1; 0.5,0.5,0.5 *//sphere search aided*
        Calculate the parameters of replacement distribution as (30) and (31);
        Update the parameters of approximated marginal prior distributions as (19) and (20);
        Update the covariance matrix of AJPoD as (24); 0.5,0.5,0.5 *//successive updating*
        Update the next element of mean vector as (25);
    end for
end for
Outputs: $\hat{x}_i = \arg\min_{x_i \in \Theta} \left| x_i - \mathbf{u}^{(l)}(i) \right|$

E. COMPLEXITY ANALYSIS

We analyze the proposed high-efficiency EP detector with the original EP detector in the aspect of complexity. Refer to the given complexity $\mathcal{O}(LN_t^3 + LN_tM)$ in [32], we now divide it into three items. The first one is $\mathcal{O}(N_t^3)$, the second one is $\mathcal{O}(M)$ and the last one is the product item $\mathcal{O}(L)$.

For the first item $\mathcal{O}(N_t^3)$ caused by matrix inversion, we handle it with successive updating scheme. We transform it into iterative vector multiplication. Actually its complexity is still $\mathcal{O}(N_t^3)$ (The complexity of a column vector times a row vector is N_t^2, and it executes N_t times in an iteration). However, the practical complexity of N_t times of vector multiplication is definitely less than matrix inversion. To differentiate them, we use a factor α_1. Then the complexity of EP-SU becomes $\mathcal{O}(N_t \times \alpha_1 N_t^2)$. If the matrix inversion is calculated with the Gaussian elimination method, we can get the value of $\alpha_1 = 0.25$ (When computing a N-order matrix invesion, the number of multipilication operations for Gaussian elimination method and iterative vector multiplication are $4N^3$ and N^3, respectively). Then we know that successive updating scheme reduces about 75% of the complexity of matrix inversion.

For the second item $\mathcal{O}(M)$ caused by the search space, the joint scheme of successive updating, sorting updating and sphere search aided solves it easily. Considering the SER performance is lower than 1×10^{-3} (This means a relatively high SNR), the complexity becomes $\mathcal{O}(\alpha_2 M)$, where α_2 is equal to 25.02% in 16-QAM, 12.69% in 64-QAM and 7.40% in 256-QAM.

For the third product item $\mathcal{O}(L)$ caused by iterations, the joint scheme of successive updating and sorting updating can solve it, too. The sorting EP-SU detectors can converge with only half iterations of the original EP detector. That is, the joint scheme changes the product item $\mathcal{O}(L)$ into $\mathcal{O}(\alpha_3 L)$, where $\alpha_3 = 0.5$. This change can reduce half of the whole complexity.

The proposed scheme will bring extra complexity when sorting for the variances and computing the sphere space. For sorting, the extra complexity is $\mathcal{O}(LN_t^2)$. The other one is $\mathcal{O}(N_tM)$ (The sphere space is settled in the first iteration).

What is worthy mentioned, the extra complexity $\mathcal{O}(N_tM)$ only contains the operation of comparison between the Euclid distance, while the abridged complexity contains all the operations in (30) and (31). Comparing to $\mathcal{O}(LN_t^3 + LN_tM)$, the extra complexity is negligible to the entirety.

It seems that the successive updating of the proposed scheme would cause higher delay and reduces the processing efficiency of the hardware systems. However, the processing unit for every received symbol in each iteration is the same, then in a hardware system, we can adopt pipeline architecture to guarantee the processing efficiency.

To sum up, if we take the complexity of preprocessing into account, the complexity of original EP should be $\mathcal{O}(N_t^3 + LN_t^3 + LN_tM)$, while the complexity of high-efficiency EP detector is $\mathcal{O}(N_t^3+\alpha_3\alpha_1LN_t^3+\alpha_3\alpha_2LN_tM)$. Thus, the higher-efficiency EP detector can significantly reduce the complexity of the original EP detector. Comparing to the complexity $\mathcal{O}(N_t^3+N_t^2)$, the complexity of the high-efficiency EP detector with L=4 is only about 1.5 times than MMSE detector.

V. PERFORMANCE ANALYSIS

In the previous section, we propose the high-efficiency EP detector with the joint scheme of successive updating, sorting updating and sphere search aided. We also discuss the complexity of the proposed EP detector and verify its high-efficiency. In this section, we are going to compare the performance of different detectors by simulation results. The performance of detectors is presented in terms of symbols error rate (SER) as a function of *SNR*. In the simulation system, we use the same models in other parts except for different MIMO detectors in MIMO detection. To focus on MIMO symbols detection and eliminate the impact of the channel code, we adopt the system without channel code.[6] Just like Fig.1, we transmit full streams in sending side, so we only consider the scenarios with $N_t \leq N_r$. We directly compare the outputs $\hat{\mathbf{x}}$ of MIMO detectors with the transmitted symbols $\mathbf{x}$ and then get the SER. For each antenna scale or modulation, we pay close attention to where SER is around 1×10^{-4} and we transmit at least 5 million symbols in all scenarios.

We compare SER performances of the following 4 kinds of MIMO detectors. The linear MMSE detector is a kind of linear detectors and it's adopted in many practical systems such as LTE system. The original EP detector is the object to optimize. The EP-SU detector is our former work in [42] and it is not ideal. The high-efficiency EP detector is proposed in this paper. The comparison of the original EP detector with some other MIMO detectors can be found in [32]. Combining with the conclusion of convergence in the previous section, we give the SER performance of the original EP detector with 1, 2, 4 and 6 iterations, while EP-SU and high-efficiency EP detectors with 1, 2 and 4 iterations. These iterations ensure all iterative detectors almost converge, which means their SER performance will nearly no longer change with the increase of iterations. For simplicity, we denote the performance curves for different detectors by "MMSE", "Original EP", "EP-SU" and "HE EP", respectively. As the mentioned 3δ principle, the parameter ρ of high-efficiency EP detector is set as $\rho = 3$.

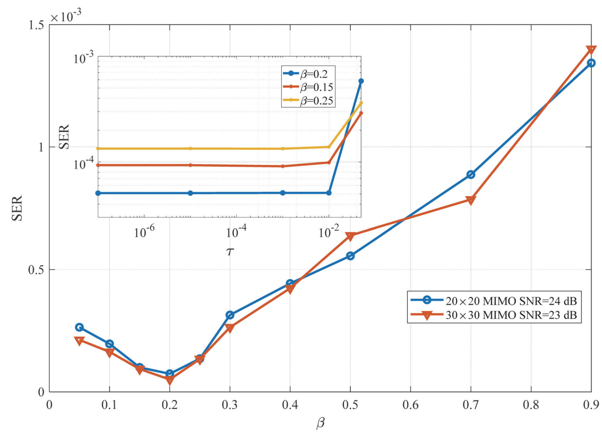

FIGURE 10. SER performance of different β and τ for the scenario of 16-QAM modulation.

To determine the parameters β and τ, we simulate some different conditions. The result is shown in Fig.10. Obviously, the criterion of minimum SER, which means relative optimal performance, can help us to determine that $\beta = 0.2$ and $\tau \in (10^{-7}, 10^{-3})$. To be consistent with [32] and [39], we adopt $\tau = 5 \times 10^{-7}$. The conditions of simulation are listed as TABLE 2.

TABLE 2. simulation conditions.

content	condition
channel	flat Rayleigh fading
antenna scales	$N_t \leq N_r$
CSI for receiver	perfect/imperfect
noise power for receiver	perfect known
modulation	QAM(16/64/256)
total number of transmitted symbols	over 5 millions
parameters	$L = 1/2/4/6$ $\rho = 3$ $\beta = 0.2$ $\tau = 5 \times 10^{-7}$

At first, we consider the scenarios where $N_t = N_r$ and the modulations are 16-QAM. The simulation results of different antenna scales are shown from Fig.11 to Fig.14.

We can make some similar conclusions from these 4 figures. The iterative EP detectors are much better than linear MMSE detector on SER performance for the case $N_t = N_r$. The SER performance of EP-SU detector could be unstable, which means its SER performance may be better or worse than the original EP detector for some cases. This is because the updating order of transmitted symbols is uncertain. When the symbols with higher reliability are updated preferentially (just like high-efficiency EP detector does),

[6]Our former work consider a system with convolution code, the details can be found in [42].

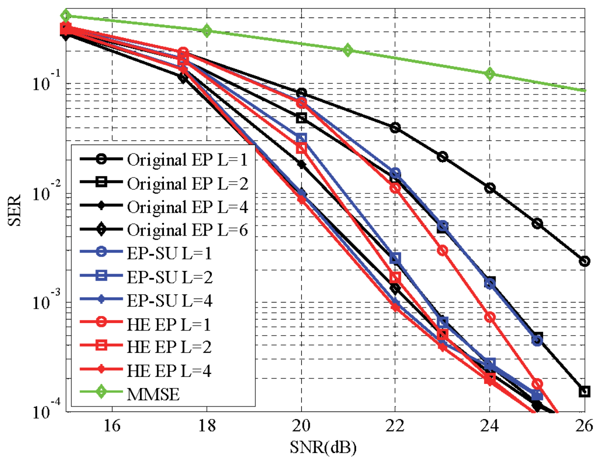

FIGURE 11. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 16$ and 16-QAM modulation.

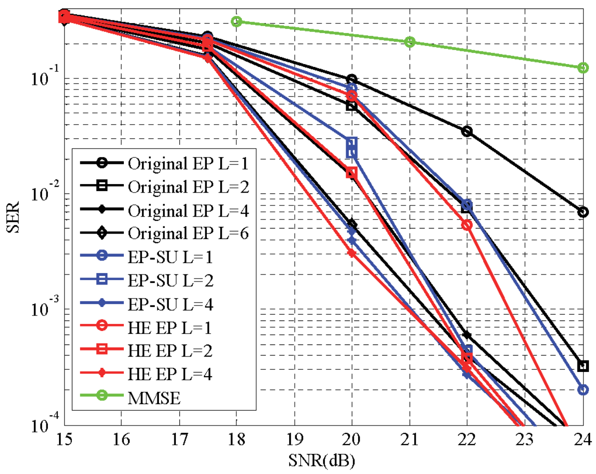

FIGURE 12. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 32$ and 16-QAM modulation.

it has better performance. Besides, if all the symbols are reliable, such as for the case of high SNR, EP-SU detector is also excellent. Thus, as high-efficiency EP detector is always better than EP-SU detector, it has strong robustness. Comparing to the original EP detector, high-efficiency EP detector needs fewer iterations to reach the same SER performance, or it has better SER performance with the same iterations. When both of them converge (original EP with L=6, and high-efficiency EP with L=4), the high-efficiency EP detector can improve the original EP detector about 0.5 dB in SER performance under different antennas scales. What is worthy mentioned is that the improvement of performance scales with the number of antennas.[7]

[7] In our title we use the word "massive", but for MIMO scale, both $N_t = N_r = 16$ and $N_t = N_r = 32$ cannot be regarded as massive. However, since we get similar results in different scales, and the proposed detector is better in bigger scales, it's reasonable to say the proposed detector is appropriate for massive MIMO system.

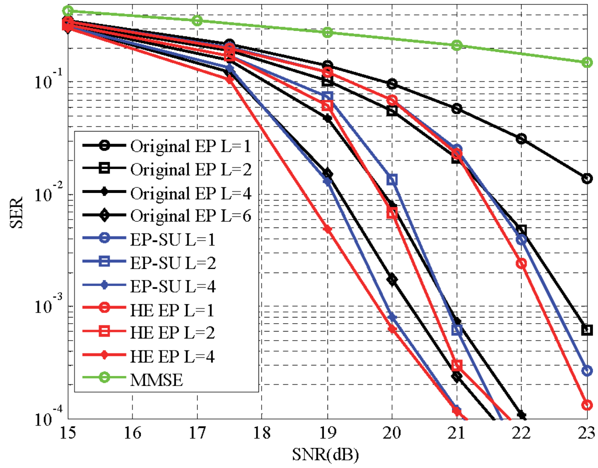

FIGURE 13. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 64$ and 16-QAM modulation.

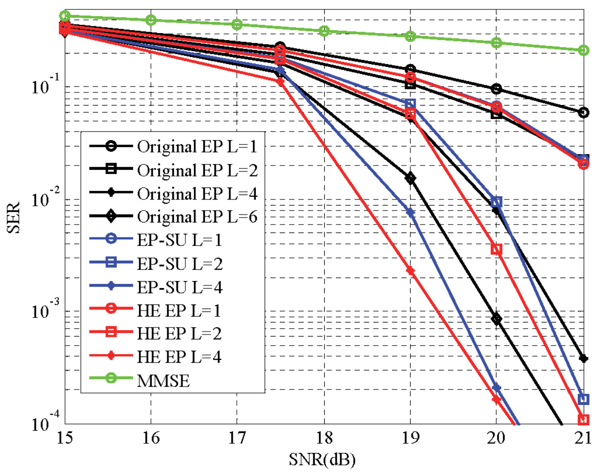

FIGURE 14. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 100$ and 16-QAM modulation.

We also consider the other two scenarios. The first one is the antenna scales with $N_t < N_r$, and the second one is the high-order modulation.

Fig.15 and Fig.16 present the SER performances of different MIMO detector with 16-QAM modulation for the scenario $N_t = 20, N_r = 40$ and $N_t = 20, N_r = 80$, respectively. As the number of receiving antennas is twice or four times the sending antennas, the diversity gain of the MIMO system will play an important role. In Fig.15, EP detectors outperform MMSE detector less than 2 dB. The high-efficiency EP detector only has a little advantage on convergence than the original EP detector, and they almost have the same SER performance after converging. It becomes clearer in Fig.16. The distinction among EP detectors is negligible. Iterative processes are unnecessary because the iterative EP detectors need only one iteration to converge. Since the diversity gain

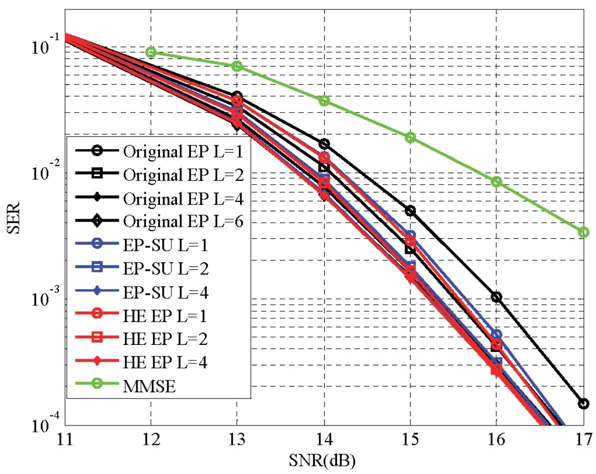

FIGURE 15. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = 20, N_r = 40$ and 16-QAM modulation.

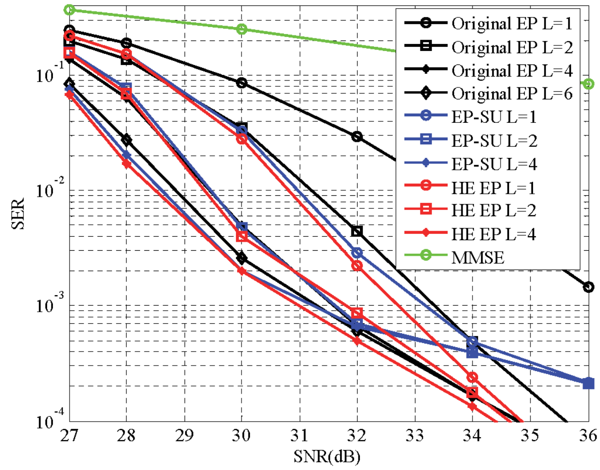

FIGURE 17. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 32$ and 64-QAM modulation.

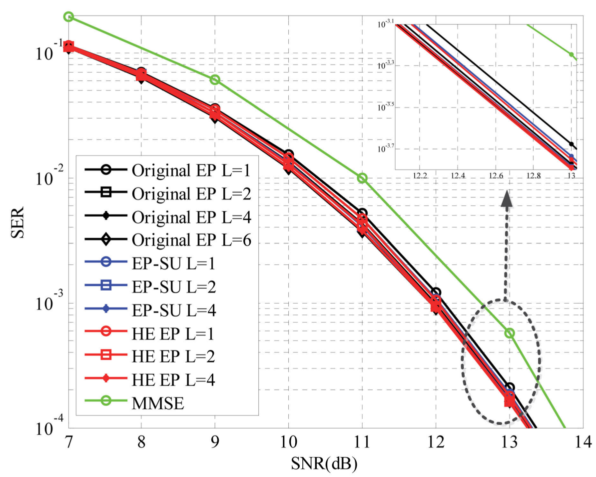

FIGURE 16. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = 20, N_r = 80$ and 16-QAM modulation.

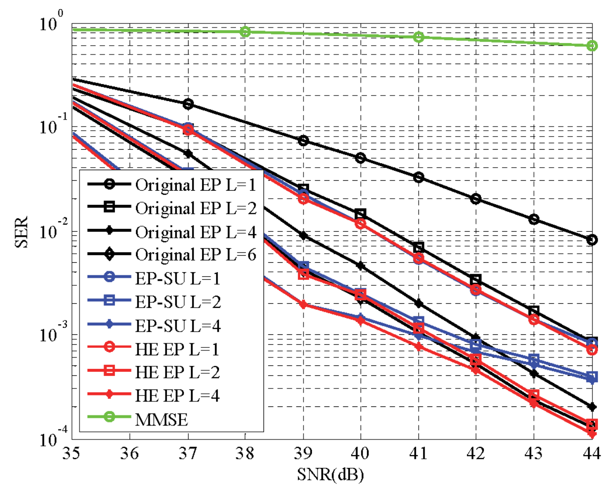

FIGURE 18. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 32$ and 256-QAM modulation.

makes a huge difference, the MMSE detector only loses 0.5 dB in SER performance than EP detectors.

Considering the multi-users MIMO system with perfect power control where each user has a single antenna (every user transmits an independent stream) and the base station has hundreds of antennas, the uplink transmission can be regarded as the scenarios $N_t \leq N_r$. Thus, when the number of active users is small or the system load is light ($N_t \ll N_r$), the diversity gain ensures the linear MMSE detector work well with low complexity and the EP detectors nearly have no advantage. However, when the MIMO system suffers from heavy load (a common scenario in modern wireless communication system), EP detectors show a great advantage over the MMSE detector and the proposed high-efficiency EP detector can further improve the SER performance of the original EP detector with below 12.5% ($\alpha_1 = 0.25, \alpha_3 = 0.5$) of its complexity.

The SER performance of different detectors for the other two high-order modulations scenarios are given by Fig.17 and Fig.18. These two figures indicate that the SER performance of the MMSE detector becomes worse with the increasing of modulation orders. The iterative detectors show a great advantage on high-order modulations. We can find that the EP-SU detector is worse than the original EP detector. This is because when modulation order becomes high, the gap of energy between different constellation points also becomes large. For example, the highest energy of 16-QAM is 18 ($\pm 3 \pm 3i$) while the highest energy of 256-QAM is 450 ($\pm 15 \pm 15i$), and both of their lowest energy is 2 ($\pm 1 \pm 1i$). Thus, when the noise works on different transmitted symbols, the reliability of symbols with low energy becomes very low and the EP-SU will propagate it to other symbols. So sorting updating scheme becomes indispensable when modulation order is high. After both the high-efficiency and original detectors converge, although the difference of SER performance between them still exists, it's not as evident as in 16-QAM modulation. Surely, when they have the same iterations, the high-efficiency EP detector evidently outperforms the original EP detector and we have verified the advantage of complexity become greater with modulation order increases in the previous section.

We have assumed the received side obtain the perfect CSI. Actually, in a practical system, the CSI will be imperfect for the receivers and they get CSI through channel estimation. We take the estimation error into consideration and give the SER performance for different detectors in the scenario with imperfect CSI in Fig.19. The Channel Estimation error in Fig.19 is obtained by MMSE estimation as [39], where $\delta_h = 10log_{10}\left(\|Vec(H) - Vec(\hat{H})\|^2/\|Vec(H)\|^2\right)$, where $\hat{H}$ represents the estimation value of real channel response. Results show HE EP detector is still the optimum for the imperfect CSI scenario.

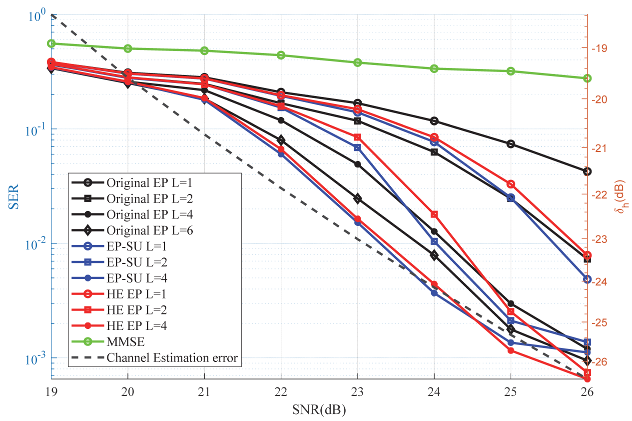

FIGURE 19. SER performance of MMSE, original EP, EP-SU, HE EP detectors with different iterations for the scenario of $N_t = N_r = 40$ and 16-QAM modulation with imperfect CSI.

In this section, we verified the SER performance of the proposed high-efficiency EP detector. Simulation results show it outperforms the original EP and MMSE detectors in different high-order massive MIMO scenarios.

VI. CONCLUSION

The low complexity and high-performance symbols detectors for high-order massive MIMO systems are enormous challenges. To solve this, we proposed a high-efficiency EP detector in this paper. The high-efficiency EP detector is based on the expectation propagation algorithm and adopts the joint scheme of successive updating, sorting updating and sphere search aided. Comparing to the original EP detector, the high-efficiency detector avoids the matrix inversion and reduces the size of the search space in each iteration, and it accelerates the convergence. Thus it reduces a lot of the complexity of the original EP detector, and the advantage of complexity scales with the number of antennas and modulation orders, which means our detector is more appropriate for high-order massive MIMO systems. Moreover, the proposed high-efficiency EP detector outperforms the original EP detector in different high-order massive MIMO scenarios.

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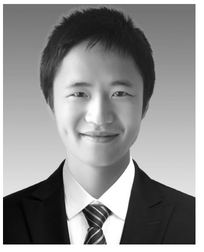

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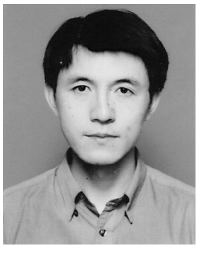

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