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Intuitionistic Evidence Sets

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ABSTRACT Dempster–Shafer evidence theory is efficient to deal with uncertain information. However, the traditional basic probability assignment (BPA) only considers the support degree of the focal elements. In this paper, in order to make decision-making processes more reasonable and flexible, intuitionstic evidence sets (IESs) are proposed. The essential part of IES is the intuitionstic basic probability assignment (IBPA). An IBPA can be regarded as a pair of ordered BPAs, the first BPA shows the support degrees and second BPA shows non-support degrees. Compared with the traditional BPA, the proposed IBPA considers both the support degree and non-support degree of focal elements. Compared with intuitionstic fuzzy set (IFS), the proposed IBPA assigns the support degree and non-support degree to both singletons and multiple subsets of the frame of discernment. The feasibility and the effectiveness of the proposed method are illustrated in an application of multi-criteria group decision making.

INDEX TERMS Dempster-Shafer evidence theory, intuitionstic evidence sets, intuitionstic fuzzy sets, combination rule, group decision making.

I. INTRODUCTION

Dempster-Shafer evidence theory widely used to deal with uncertain information [1], [2], such as information fusion [3]–[7], decision making [8]–[10], risk assignment [11]–[13], expert systems [14], medical diagnosis [15]–[20], environment management [21], target recognition and tracking [22], fault diagnosis [23]–[25] and pattern classification [26]–[29].

Nevertheless, the Dempster-Shafer evidence theory has some shortcomings. For example, the counterintuitive results may be obtained when the fused evidences are highly conflicted each other [30]. There are many methods are proposed to improve the D-S evidence theory. Some methods improve the Dempster combination rule, such as the new combination rule proposed by Yager [31], the rule of combination proposed by Smets [32], the combination operator proposed by Dubois and Prade [33], uncertainty modeling [34] and as so on [35], [36]. Some methods to correct the source of evidence, such as the discounting coefficients method [37], [38], average approach [39], modified average approach [40] etc. [41]–[43]. Other methods is to improve the way of the D-S evidence theory modeling uncertain information, such as interval-valued evidence theory [44], R numbers [45], [46] and D numbers [47]–[52].

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Besides the evidence theory, many other theories are developed to deal with uncertainty, such as fuzzy sets [53], rough sets [54], Z numbers [55], Intuitionistic fuzzy sets (IFSs) [56], etc. [57]–[60]. Intuitionistic fuzzy sets (IFSs) introduced by Atanassov [56] are the extension of traditional fuzzy sets, which consider three aspects of the degree of membership, the degree of non-membership and hesitancy. Therefore, they are more flexible and practical than the traditional fuzzy sets in dealing with uncertain information. IFSs have been applied in many fields, such as decision making [61]–[64], pattern recognition [61], [65]–[67], medical diagnosis [68], [69] and so on [70], [71]. Additionally, many extensions of IFSs have been developed, such as, interval-valued intuitionstic fuzzy sets (IVIFSs) [72], interval-valued intuitionstic hesitant fuzzy sets (IVIHFSs) [73], linguistic interval-valued Atanassov intuitionistic fuzzy set (LIVAIFS) [74], interval type-2 fuzzy sets(IT2FSs) [75], hesitant Pythagorean fuzzy sets [76], hesitant fuzzy linguistic term sets(HFLTSs) [77] and so on.

In this paper, another improvement on BPA, not the combination rule, of evidence theory is presented. The traditional basic probability assignment only considers the support degrees of hypothesis. To address this issue, an intuitionstic evidence set (IES) is proposed, inspired by the idea of IFSs. In IES, both the support degree and the non-support degree of the focal elements are considered. The essential part of

IES is the intuitionstic basic probability assignment (IBPA). An IBPA can be regarded as a pair of ordered BPAs, the first BPA shows the support degrees and second BPA shows the non-support degrees. The basic values of IFS are singletons, but the focal element of IBPA can be the multiple subset of the frame of discernment. The proposed IES provides a more flexible way to model uncertainty in decision making. It combines the features of Dempster-Shafer evidence theory and IFS, can handle more information.

This paper is organized as follows. The preliminaries, including Dempster-Shafer evidence theory and IFS, are briefly introduced in Section II. The proposed IES is detailed in Section III. In Section IV, the application on multi-criteria group decision making and comparison with other methods are used to illustrate the efficiency of the proposed method. Finally, this paper is concluded in Section V.

II. BACKGROUND

In this section, the background material of Dempster-Shafer evidence theory [1], [2] and IFSs [56] will be briefly introduced.

A. DEMPSTER-SHAFER EVIDENCE THEORY AND RELATED WORK

Dempster-Shafer theory was defined on a finite set of mutually exclusive elements. This finite set is called the frame of discernment denotes as Θ, its power set denotes as 2^{Θ}. Evidence theory allows belief to be assigned to not only the single subsets of the frame of discernment but also the multiple subsets. The evidence theory will be degenerated as the probability theory when the belief only to be assigned to the single subsets [1], [2].

Definition 1: Let the frame of discernment is $\Theta = \{h_1, h_2, \cdots, h_n\}$. The power set of Θ is 2^{Θ}, $2^{\Theta} = \{\emptyset, \{h_1\}, \cdots, \{h_n\}, \{h_1, h_2\}, \cdots, \{h_1, h_2, \cdots, h_i\}, \cdots, \Theta\}$. A basic probability assignment (BPA) function m is a mapping of 2^{Θ} to a interval $[0, 1]$, defined as [1], [2]:

$$m: \quad 2^{\Theta} \rightarrow [0, 1] \tag{1}$$

which satisfies the following two conditions:

$$m(\emptyset) = 0 \quad \sum_{A \in 2^{\Theta}} m(A) = 1 \tag{2}$$

where $\emptyset$ is an empty set, and A is any element of 2^{Θ}. The mass $m(A)$ shows the degree of the evidence support A.

Definition 2: For a BPA m on Θ, each element of 2^{Θ} such that $m(A) > 0$ is called a focal element of m [1], [2].

Definition 3: For a BPA m on Θ, the belief function Bel and the plausibility function Pl are defined, respectively, as [1], [2]

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{A \cap B \neq \emptyset} m(B) = 1 - \sum_{A \cap B = \emptyset} m(B) \tag{4}$$

The belief function is also called lower function, which can be interpreted as the belief of "A is true". The plausibility function be also called upper function, which can be interpreted as the belief of "A is not false". So the possibility of a subset A lies the interval, $Poss(A) \in [Bel(A), Pl(A)]$. It is obviously that $Bel(\emptyset) = Pl(\emptyset) = 0$, $Pl(\Theta) = Pl(\Theta) = 1$ and if $A \subseteq B \subseteq \Theta$ then $Pl(A) \leq Pl(B)$ and $Bel(A) \leq Bel(B)$.

Definition 4: Given two BPAs m_1 and m_2 on Θ, the Dempster's combination rule is used to fuse BPAs. The result BPA is denoted as $m_1 \otimes m_2$, is given by [1]:

$$\begin{cases} m_1 \otimes m_2(\emptyset) = 0 \\ m_1 \otimes m_2(A) = \dfrac{\sum\limits_{B \bigcap C = A} m_1(B) m_2(C)}{1 - K} \end{cases} \tag{5}$$

where $K = \sum\limits_{B \bigcap C = \emptyset} m_1(B) m_2(C)$.

The counterintuitive results may be obtained by Dempster's combination rule when the fused evidences are highly conflicted each other. In order to address this problem, an average method is proposed by Murphy [39]. This method average the masses and then calculate the combined masses by combining the average masses multiple times using classical Dempster's combination rule. However, the importance of evidence may not be equal. The weighted average method is proposed by Deng *et al.* based on Murphy's average method [78].

Definition 5: Given two BPAs m_1 and m_2 on Θ, the importance weights of two BPAs are w_1, w_2. A is any subset of Θ. The weighted average combination method is defined as [78]

$$m_1 \otimes m_2(A) = AVE(m_1, m_2) \otimes AVE(m_1, m_2)(A), \tag{6}$$

where $AVE(m_1, m_2)$ is a new BPA with

$$AVE(m_1, m_2)(A) = m_1(A) \times w_1 + m_2(A) \times w_2. \tag{7}$$

Definition 6: The pignistic probability of a BPA m on Θ is defined as [32], [79]

$$BetP(A) = \sum_{B \in 2^{\Theta}} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \quad \forall A \in 2^{\Theta} \tag{8}$$

where $|A|$ is the cardinality of A. The pignistic probability is a probability distribution on Θ with

$$p_k = BetP(h_k) = \sum_{h_k \in B} \frac{m(B)}{|B|} \tag{9}$$

Definition 7: Assume m is a BPA on Θ, the cost of a focal element A, $A \subseteq \Theta$, is defined as [80]

$$Cost(A) = \frac{n - |A|}{n - 1} \tag{10}$$

The cost is a decreasing function of the cardinality, the smaller the cardinality the most the cost. We can know $Cost(\Theta) = \frac{n-n}{n-1} = 0$, it is the least costly. $Cost(h_k) = \frac{n-1}{n-1} = 1$, it is the most costly.

Definition 8: Assume m is a BPA on Θ, the cost of m is defined as [80]

$$Cost(m) = \sum_{A \subseteq \Theta} Cost(A) m(A) \tag{11}$$

When m is a pure probability distribution, m is the most costly to use with $Cost(m) = 1$. When m with one focal element $m(\Theta) = 1$, m is the least costly with $Cost(m) = 0$. It can be seen that more imprecise the BPA the less costly, the more precise the focal elements the more costly.

Definition 9: Assume m is a BPA on Θ, the specificity of m is defined as [81]

$$Sp(m) = \sum_{A \subseteq \Theta, A \neq \emptyset} \frac{m(A)}{|A|} \tag{12}$$

When m with one focal element $m(\Theta) = 1$, the specificity of m has minimal value $Sp(m) = \frac{m(\Theta)}{|\Theta|} = \frac{1}{n}$. When m is a pure probability distribution, the specificity of m has maximal value $Sp(m) = \sum_{i=1}^{n} m(\{h_i\}) = 1$.

Recently, some researches about negation of BAP have been studied [82]. In addition, due to the efficiency of entropy function to model uncertainty [83], [84], the entropy function of BPA has been paid greatly attention [85], [86].

B. INTUITIONSTIC FUZZY SETS

Definition 10: Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe set, then an IFS in X is defined as [56]:

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in X\} \tag{13}$$

where $\mu_A(x) : X \to [0, 1]$ and $\upsilon_A(x) : X \to [0, 1]$ are the degree of membership and the degree of non-membership, respectively, such that

$$0 \leq \mu_A(x) + \upsilon_A(x) \leq 1 \tag{14}$$

The third parameter of IFS is the degree of hesitancy, $\pi_A(x)$:

$$\pi_A(x) = 1 - (\mu_A(x) + \upsilon_A(x)) \tag{15}$$

It is obviously that $0 \leq \pi_A(x) \leq 1, \forall x \in X$. When $\pi_A(x) = 0$, the IFS degenerates into the classical fuzzy set. The classical fuzzy set has the form $\{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in X\}$.

III. INTUITIONSTIC EVIDENCE SETS

The essential part of IES is the intuitionstic basic probability assignment (IBPA). Similar to the BPA in evidence theory, the IBPA is defined on a finite set of mutually exclusion elements, known as the frame of discernment.

Definition 11: Assume a frame of discernment $\Theta = \{h_1, h_2, \cdots, h_3\}$, its power set is $2^\Theta = \{\emptyset, \{h_1\}, \cdots, \{h_n\}, \{h_1, h_2\}, \cdots, \{h_1, h_2, \cdots, h_i\}, \cdots, \Theta\}$. An intuitionstic basic probability assignment on 2^Θ is defined as

$$m(A) = \langle m^+(A), m^-(A)\rangle \tag{16}$$

where $m^+(A) : 2^\Theta \to [0, 1]$ and $m^- : 2^\Theta \to [0, 1]$ are support degree and non-support degree, respectively. A is any element of 2^Θ. They must satisfy the following conditions:

(1) $m^+(\emptyset) = 0$ and $m^-(\emptyset) = 0$;
(2) $\sum_{A \in 2^\Theta} m^+(A) = 1$;
(3) $\forall A \neq \Theta, m^+(A) + m^-(A) \leq 1$.

Definition 12: For an IBPA m on Θ, each subset A of Θ such that $m^+(A) > 0$ or $m^-(A) > 0$ is called a focal element of m.

Definition 13: Let m be an IBPA on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A)\rangle$, and $A \in 2^\Theta$. The belief function and plausibility function of A also have two components, defined respectively as:

$$Bel(A) = \langle Bel^+(A), Bel^-(A)\rangle \tag{17}$$
$$Pl(A) = \langle Pl^+(A), Pl^-(A)\rangle \tag{18}$$

where

$$Bel^+(A) = \sum_{B \subseteq A} m^+(B) \tag{19}$$
$$Bel^-(A) = \sum_{B \subseteq A} m^-(B) \tag{20}$$
$$Pl^+(A) = \sum_{B \cap A \neq \emptyset} m^+(B) \tag{21}$$
$$Pl^-(A) = \sum_{B \cap A \neq \emptyset} m^-(B) \tag{22}$$

The possibility of a subset A also has two components $Poss(A) = \langle Poss^+(A), Poss^-(A)\rangle$, $Poss^+(A) \in [Bel^+(A), Pl^+(A)]$ and $Poss^-(A) \in [Bel^-(A), Pl^-(A)]$.

Definition 14: Given an IBPA m on Θ with intuitionstic probability masses $m(A) = \langle m^+(A), m^-(A)\rangle$, and $A \in 2^\Theta$. The pignistic probability of m is defined by:

$$BetP(A) = \langle BetP^+(A), BetP^-(A)\rangle, \quad \forall A \in 2^\Theta \tag{23}$$

where

$$BetP^+(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} \frac{m^+(B)}{1 - m^+(\emptyset)} \tag{24}$$
$$BetP^-(A) = \sum_{B \in 2^\Theta} \frac{|A \cap B|}{|B|} \frac{m^-(B)}{1 - m^-(\emptyset)} \tag{25}$$

Definition 15: Assume m_1 and m_2 are two IBPAs on Θ such that for each subset A they have $Bel_2^+(A) \leq Bel_1^+(A)$, $Bel_2^-(A) \leq Bel_1^-(A)$, $Pl_2^+(A) \geq Pl_1^+(A)$ and $Pl_2^-(A) \geq Pl_1^-(A)$, $[Bel_1^+(A), Pl_1^+(A)] \subseteq [Bel_2^+(A), Pl_2^+(A)]$ and $[Bel_1^-(A), Pl_1^-(A)] \subseteq [Bel_2^-(A), Pl_2^-(A)]$, we say that m_1 entail m_2, denoted $m_1 \subset m_2$.

If m is an IBPA on Θ, so for each subset A, $Poss^+(A) \in [Bel^+(A), Pl^+(A)]$ and $Poss^-(A) \in [Bel^-(A), Pl^-(A)]$. Assume $Bel^+(A) = a^+$, $Bel^-(A) = a^-$, $Pl^+(A) = b^+$ and $Pl^-(A) = b^-$, $Poss^+(A) \in [a^+, b^+]$ and $Poss^-(A) \in [a^-, b^-]$. If $c^+ \leq a^+, c^- \leq a^-, d^+ \geq b^+$ and $d^- \geq b^-$, then $Poss^+(A) \in [c^+, d^+]$ and $Poss^-(A) \in [c^-, d^-]$.

Definition 16: Let an intuitionstic probability mass of A, $\langle m^+(A), m^-(A)\rangle$, is an IBPA m on Θ. The pure support degree is defined as

$$PS(A) = BetP^+(A) - BetP^-(A) \tag{26}$$

For the problem of target recognition, the bigger the pure support degree, the more likely the target is A. For the problem of decision making, the higher the pure support degree, the alternative A is more in line with requirements.

Definition 17: Assume m is a IBPA on Θ, the cost of m is defined as

$$Cost(m) = \sum_{A \subseteq \Theta} Cost(A) m^{+}(A) + \sum_{A \subseteq \Theta} Cost(A) m^{-}(A) \quad (27)$$

When all focal elements of m are singleton, the cost of m is the most cost, $Cost(m) = 2$. When m with one focal element $m(\Theta) = \langle 1, 1 \rangle$, the cost of m is the least cost, $Cost(m) = 0$.

Definition 18: Assume m is an IBPA on Θ, the specificity of m is defined as

$$Sp(m) = \sum_{A \subseteq \Theta, A \neq \emptyset} \frac{m^{+}(A) + m^{-}(A)}{|A|} \quad (28)$$

If all focal elements of m is singleton, the specificity of m is maximal $Sp(m) = 2$. If m only has one element $m(\Theta) = \langle 1, 1 \rangle$, the specificity of m is minimal $Sp(m) = \frac{2}{n}$.

An IBPA can transform into a traditional BPA, but a traditional BPA cannot transform into an IBPA, because IBPA contains more information than BPA.

Definition 19: Assume an IBPA m on Θ with intuitionstic probability masses $m(A) = \langle m^{+}(A), m^{-}(A) \rangle$, and $A \in 2^{\Theta}$. $m*$ is a BPA transformed by m, m can be calculate by

$$m*(A) = \frac{m^{+}(A) - m^{-}(A)}{\sum_{B \subseteq \Theta} (m^{+}(B) - m^{-}(B))} \quad (29)$$

It is obviously that $\sum_{A \subseteq \Theta} m*(A) = 1$.

Actually, an IBPA can be regarded as two BPAs, one represents the support degree and the other expresses the non-support degree. This thinking is adopted in the combination of IBPA.

Definition 20: Given two IBPA m_1 and m_2 on Θ, the combination result is denoted as $m_1 \otimes m_2$, is given by:

$$m_1 \otimes m_2(A) = \langle m^{+}(A), m^{-}(A) \rangle \quad (30)$$

where

$$\begin{cases} m^{+}(\emptyset) = 0 \\ m^{+}(A) = \dfrac{\sum\limits_{B \bigcap C = A} m_1^{+}(B) m_2^{+}(C)}{1 - \sum\limits_{B \bigcap C = \emptyset} m_1^{+}(B) m_2^{+}(C)} \end{cases} \quad (31)$$

$$\begin{cases} m^{-}(\emptyset) = 0 \\ m^{-}(A) = \dfrac{\sum\limits_{B \bigcap C = A} m_1^{-}(B) m_2^{-}(C)}{1 - \sum\limits_{B \bigcap C = \emptyset} m_1^{-}(B) m_2^{-}(C)} \end{cases} \quad (32)$$

If $m^{+}(A) + m^{-}(A) > 1$, the result is not a IBPA, so we have to do some processing on the result to make it satisfies the third condition of IBPA. In this case, the degree of support remains unchanged and the degree of non-support reduced to $m^{+}(A) + m^{-}(A) = 1$, and the remaining degree of non-support is assigned to Θ.

Theorem 1: The combination result of two IBPAs also is an IBPA.

This theorem is obviously established.

Obviously, the problem of evidence conflict also exists in combination of IBPA, the weighted average combination method also can be used in IBPA.

Definition 21: Given two IBPA m_1 and m_2 on Θ, the weighted average combination result IBPA, $m_1 \otimes m_2(A) = \langle m^{+}(A), m^{-}(A) \rangle$, is given by:

$$\begin{aligned} m^{+}(A) &= AVE(m_1^{+}, m_2^{+}) \otimes AVE(m_1^{+}, m_2^{+})(A) \\ m^{-}(A) &= AVE(m_1^{-}, m_2^{-}) \otimes AVE(m_1^{-}, m_2^{-})(A) \end{aligned} \quad (33)$$

IV. APPLICATION IN MULTI-CRITERIA GROUP DECISION MAKING

A. ILLUSTRATION OF THE PROPOSED APPROACH

The real world is very complex since the fact in complex systems are interactional each other dynamically [87]–[89]. Multi-criteria decision making has importance application in engineering, many decision making approaches are proposed. For example, the interval-valued intuitionistic fuzzy MULTIMOORA method [62], a new group decision model based on grey-intuitionistic fuzzy-ELECTRE and VIKOR [90], signed distance-based consensus in multi-criteria group decision-making [91] and as so on [92]–[98]. Consider the problem of air-condition brands selection. Suppose there are four air-condition brands A_1, A_2, A_3 and A_4, the alternative set is $A = \{A_1, A_2, A_3, A_4\}$. In order to evaluate alternative air-condition brands, a decision group consists of three decision makers has been formed. The set of decision maker is $D = \{D_1, D_2, D_3\}$. Suppose three criteria C_1(quality), C_2(price), C_3(degree of satisfaction), C_4(function) are considered in the selection problem. The criteria set is $C = \{C_1, C_2, C_3, C_4\}$. Procedure for the selection problem is shown in Figure 1 and contains the following steps:

Step 1: Determine the weights of criteria.

Evaluations of all criteria by decision makers are shown in Table 1.

The fused result m_c of decision makers' opinions can be calculate by Eq. (30)-(32).

$$\begin{aligned} m_c(\{C_1\}) &= \langle 0.3810, 0.1250 \rangle, \\ m_c(\{C_2\}) &= \langle 0.3175, 0.1250 \rangle, \\ m_c(\{C_3\}) &= \langle 0.1586, 0.2500 \rangle, \\ m_c(\{C_4\}) &= \langle 0.1429, 0.5000 \rangle \end{aligned}$$

The weights of criteria $W_c = [w_{c,1}, w_{c,2}, w_{c,3}, w_{c,4}]$ can be obtained by:

$$w_{c,j} = \langle w_{c,j}^{+}, w_{c,j}^{-} \rangle = \langle BetP^{+}(C_j), BetP^{-}(C_j) \rangle,$$

where $w_{c,j}^{+} = BetP^{+}(C_j)$ and $w_{c,j}^{-} = BetP^{-}(C_j)$ and $j = 1, \cdots, 4$. So,

$$W_c = [\langle 0.3810, 0.1250 \rangle, \langle 0.3175, 0.1250 \rangle, \langle 0.1586, 0.2500 \rangle, \langle 0.1429, 0.5000 \rangle]$$

Step 2: Calculate the initial fused results IBPAs based on the weights of criteria.

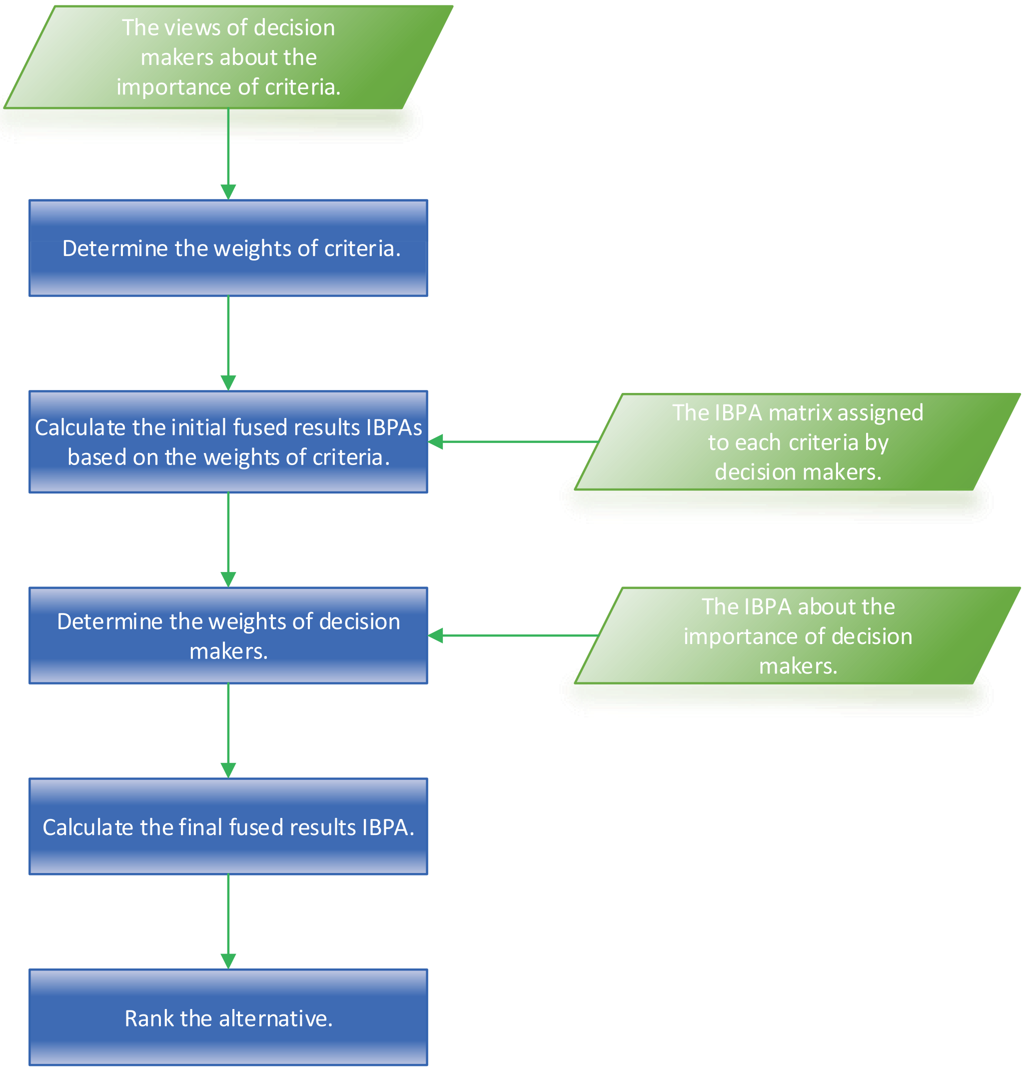

FIGURE 1. The procedure of proposed method.

TABLE 1. Opinions of all criteria by decision makers.

D_1		D_2		D_3	
$\{C_1\}$	$\langle 0.3, 0.1\rangle$	$\{C_1\}$	$\langle 0.2, 0.2\rangle$	$\{C_1\}$	$\langle 0.3, 0.1\rangle$
$\{C_2\}$	$\langle 0.2, 0.2\rangle$	$\{C_2\}$	$\langle 0.2, 0.2\rangle$	$\{C_2\}$	$\langle 0.3, 0.1\rangle$
$\{C_3\}$	$\langle 0.2, 0.3\rangle$	$\{C_3\}$	$\langle 0.2, 0.2\rangle$	$\{C_3\}$	$\langle 0.1, 0.2\rangle$
$\{C_4\}$	$\langle 0.2, 0.3\rangle$	$\{C_4\}$	$\langle 0.1, 0.3\rangle$	$\{C_4\}$	$\langle 0.1, 0.2\rangle$
$\{C_1, C_4\}$	$\langle 0.1, 0.1\rangle$	$\{C_1, C_2, C_3\}$	$\langle 0.1, 0.1\rangle$	$\{C_1, C_4\}$	$\langle 0.1, 0.2\rangle$
		$\{C_2, C_3, C_4\}$	$\langle 0.2, 0\rangle$	$\{C_2, C_3\}$	$\langle 0.1, 0\rangle$
				$\{C_2, C_3, C_4\}$	$\langle 0, 0.2\rangle$

The IBPA decision matrix M is

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{bmatrix}$$

where m_{jk} is an IBPA assigned to criteria C_j by decision maker D_k.

The IBPAs of decision matrix are shown in Table 2.

According to Eq. (33) the initial fused results $M^{in} = [m_1^{in}, m_2^{in}, m_3^{in}]$ can be calculated by

$$AVE(m_k^{in}) = \sum_{j=1}^{4} m_{jk} \times w_{cj}, \quad k = 1, 2, 3$$

$$m_k^{in} = AVE(m_k^{in}) \otimes AVE(m_k^{in}) \otimes AVE(m_k^{in}) \otimes AVE(m_k^{in}), \quad k = -1, 2, 3$$

TABLE 2. IBPAs of decision matrix.

Criteria	Subset of brands	Decision maker D_1	D_2	D_3
C_1	$\{A_1\}$	$\langle 0.22, 0.23\rangle$	$\langle 0.22, 0.22\rangle$	$\langle 0.21, 0.33\rangle$
	$\{A_2\}$	$\langle 0.18, 0.15\rangle$	$\langle 0.19, 0.12\rangle$	$\langle 0.15, 0.17\rangle$
	$\{A_3\}$	$\langle 0.27, 0.08\rangle$	$\langle 0.23, 0.11\rangle$	$\langle 0.25, 0.08\rangle$
	$\{A_4\}$	$\langle 0.18, 0.23\rangle$	$\langle 0.19, 0.22\rangle$	$\langle 0.21, 0.17\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.31\rangle$	$\langle 0, 0.33\rangle$	$\langle 0, 0.25\rangle$
	$\{A_1, A_3\}$	$\langle 0.15, 0\rangle$	$\langle 0.17, 0\rangle$	$\langle 0.18, 0\rangle$
C_2	$\{A_1\}$	$\langle 0.21, 0.23\rangle$	$\langle 0.23, 0.20\rangle$	$\langle 0.19, 0.15\rangle$
	$\{A_2\}$	$\langle 0.18, 0.18\rangle$	$\langle 0.20, 0.13\rangle$	$\langle 0.22, 0.23\rangle$
	$\{A_3\}$	$\langle 0.29, 0.06\rangle$	$\langle 0.23, 0.13\rangle$	$\langle 0.25, 0.08\rangle$
	$\{A_4\}$	$\langle 0.18, 0.24\rangle$	$\langle 0.17, 0.27\rangle$	$\langle 0.19, 0.23\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.29\rangle$	$\langle 0, 0.27\rangle$	$\langle 0, 0.31\rangle$
	$\{A_1, A_3\}$	$\langle 0.14, 0\rangle$	$\langle 0.17, 0\rangle$	$\langle 0.15, 0\rangle$
C_3	$\{A_1\}$	$\langle 0.21, 0.29\rangle$	$\langle 0.20, 0.30\rangle$	$\langle 0.24, 0.27\rangle$
	$\{A_2\}$	$\langle 0.18, 0.14\rangle$	$\langle 0.17, 0.20\rangle$	$\langle 0.18, 0.09\rangle$
	$\{A_3\}$	$\langle 0.22, 0.14\rangle$	$\langle 0.23, 0.10\rangle$	$\langle 0.20, 0.18\rangle$
	$\{A_4\}$	$\langle 0.21, 0.14\rangle$	$\langle 0.20, 0.20\rangle$	$\langle 0.20, 0.18\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.29\rangle$	$\langle 0, 0.20\rangle$	$\langle 0, 0.28\rangle$
	$\{A_1, A_3\}$	$\langle 0.18, 0\rangle$	$\langle 0.20, 0\rangle$	$\langle 0.18, 0\rangle$
C_4	$\{A_1\}$	$\langle 0.22, 0.25\rangle$	$\langle 0.22, 0.31\rangle$	$\langle 0.23, 0.20\rangle$
	$\{A_2\}$	$\langle 0.18, 0.17\rangle$	$\langle 0.15, 0.15\rangle$	$\langle 0.19, 0.13\rangle$
	$\{A_3\}$	$\langle 0.24, 0.08\rangle$	$\langle 0.25, 0.08\rangle$	$\langle 0.23, 0.20\rangle$
	$\{A_4\}$	$\langle 0.21, 0.17\rangle$	$\langle 0.19, 0.23\rangle$	$\langle 0.19, 0.20\rangle$
	$\{A_1, A_2\}$	$\langle 0, 0.33\rangle$	$\langle 0, 0.23\rangle$	$\langle 0, 0.27\rangle$
	$\{A_1, A_3\}$	$\langle 0.15, 0\rangle$	$\langle 0.19, 0\rangle$	$\langle 0.16, 0\rangle$

We can obtain

$$
\begin{aligned}
m_1^{in} : m_1^{in}(\{A_1\}) &= \langle 0.3534, 0.6466\rangle,\\
m_1^{in}(\{A_2\}) &= \langle 0.0211, 0.2804\rangle,\\
m_1^{in}(\{A_3\}) &= \langle 0.5893, 0.0005\rangle,\\
m_1^{in}(\{A_4\}) &= \langle 0.0256, 0.0070\rangle,\\
m_1^{in}(\{A_1, A_2\}) &= \langle 0, 0.0655\rangle,\\
m_1^{in}(\{A_1, A_3\}) &= \langle 0.0100, 0\rangle,\\
m_2^{in} : m_2^{in}(\{A_1\}) &= \langle 0.4385, 0.5615\rangle,\\
m_2^{in}(\{A_2\}) &= \langle 0.0211, 0.2164\rangle,\\
m_2^{in}(\{A_3\}) &= \langle 0.5007, 0.0008\rangle,\\
m_2^{in}(\{A_4\}) &= \langle 0.0215, 0.0266\rangle,\\
m_2^{in}(\{A_1, A_2\}) &= \langle 0, 0.0336\rangle,\\
m_2^{in}(\{A_1, A_3\}) &= \langle 0.0182, 0\rangle,\\
n_2^{in}(\Theta) &= \langle 0, 0.1609\rangle,\\
m_3^{in} : m_3^{in}(\{A_1\}) &= \langle 0.3973, 0.6027\rangle,\\
m_3^{in}(\{A_2\}) &= \langle 0.0223, 0.2605\rangle,\\
m_3^{in}(\{A_3\}) &= \langle 0.5330, 0.0083\rangle,\\
m_3^{in}(\{A_4\}) &= \langle 0.0316, 0.0162\rangle,\\
m_3^{in}(\{A_1, A_2\}) &= \langle 0, 0.0642\rangle,\\
m_3^{in}(\{A_1, A_3\}) &= \langle 0.0158, 0\rangle,\\
n_2^{in}(\Theta) &= \langle 0, 0.0481\rangle,
\end{aligned}
$$

Step 3: Determine the weights of decision makers.

The IBPA of importance of decision makers m_d is

$$
\begin{aligned}
m_d : m_d(\{D_1\}) &= \langle 0.4, 0.2\rangle\\
m_d(\{D_2\}) &= \langle 0.2, 0.35\rangle\\
m_d(\{D_3\}) &= \langle 0.3, 0.25\rangle\\
m_d(\{D_1, D_3\}) &= \langle 0.1, 0.1\rangle\\
m_d(\{D_2, D_3\}) &= \langle 0, 0.1\rangle
\end{aligned}
$$

The set of weights of decision makers, $W_d = [w_{d1}, w_{d2}, w_{d3}]$ can be calculated by

$$w_{dk} = \langle w_{dk}^+, w_{dk}^-\rangle = \langle BetP^+(D_k), BetP^-(D_k)\rangle, \quad k = 1, 2, 3$$

We get

$$W_d = [\langle 0.45, 0.25\rangle, \langle 0.2, 0.4\rangle, \langle 0.35, 0.35\rangle].$$

Step 4: Calculate the final fused results.

The final fused results IBPA m^{fi} is calculated by

$$AVE(m^{fi}) = \sum_{k=1}^{3} m_{k^{in}} \times w_{dk}$$

$$m^{fi} = AVE(m^{fi}) \otimes AVE(m^{fi}) \otimes AVE(m^{fi})$$

Hence

$$
\begin{aligned}
m^{fi} : m^{fi}(\{A_1\}) &= \langle 0.2606, 0.7394\rangle,\\
m^{fi}(\{A_2\}) &= \langle 0, 0.1196\rangle,\\
m^{fi}(\{A_3\}) &= \langle 0.7393, 0.0002\rangle,\\
m^{fi}(\{A_4\}) &= \langle 0.0001, 0.0010\rangle,\\
m^{fi}(\{A_1, A_2\}) &= \langle 0, 0.0042\rangle,\\
m^{fi}(\Theta) &= \langle 0, 0.1357\rangle,
\end{aligned}
$$

Step 5: Rank alternatives.

The pure support degree of each alternative in m^{fi} can be calculate by Eq. (23)-(26). Then rank the alternatives according to pure support degrees, and the larger the pure support degree, the more forward the alternative is.

$$
\begin{aligned}
PS(A_1) &= BetP^+(A_1) - BetP^-(A_1)\\
&= 0.2606 - 0.7754 = -0.5148\\
PS(A_2) &= BetP^+(A_2) - BetP^-(A_2)\\
&= 0 - 0.1556 = -0.1556\\
PS(A_3) &= BetP^+(A_3) - BetP^-(A_3)\\
&= 0.7393 - 0.0341 = 0.7052\\
PS(A_4) &= BetP^+(A_4) - BetP^-(A_4)\\
&= 0.0001 - 0.0349 = -0.0348
\end{aligned}
$$

The pure support degrees of alternatives were determined, and then four alternatives were ranked according to descending order of PS. Alternatives were ranked as $A_3 > A_4 > A_2 > A_1$. So A_3 should be selected among the four air-condition brands.

B. COMPARATIVE ANALYSIS AND DISCUSSION

If there is no non-support information in the problem of air-condition brands selection, in other words, the collected data is expressed as classical BPAs, the rank of air-condition brands is $A_3 > A_1 > A_4 > A_2$. Although the best alternative A_3 can be selected, the ranks of A_1, A_2 and A_4 under IBPA and BPA are totally different. Supprt degree of A_1 is bigger than A_1 and A_2, but the non-support degree of A_1 is also very large. If there is only non-support information in the problem of air-condition brands selection, the rank of air-condition

TABLE 3. Opinions of each criteria by decision makers expressed as IFSs.

	D_1	D_2	D_3
C_1	$\langle 0.75, 0.25\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.75, 0.25\rangle$
C_2	$\langle 0.5, 0.5\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.75, 0.25\rangle$
C_3	$\langle 0.4, 0.6\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.33, 0.67\rangle$
C_4	$\langle 0.4, 0.6\rangle$	$\langle 0.25, 0.75\rangle$	$\langle 0.33, 0.67\rangle$

TABLE 4. IFSs of decision matrix.

Criteria	brand	Decision maker D_1	D_2	D_3
C_1	A_1	$\langle 0.49, 0.51\rangle$	$\langle 0.50, 0.50\rangle$	$\langle 0.39, 0.61\rangle$
	A_2	$\langle 0.55, 0.45\rangle$	$\langle 0.61, 0.39\rangle$	$\langle 0.46, 0.54\rangle$
	A_3	$\langle 0.77, 0.23\rangle$	$\langle 0.67, 0.33\rangle$	$\langle 0.76, 0.24\rangle$
	A_4	$\langle 0.44, 0.56\rangle$	$\langle 0.46, 0.54\rangle$	$\langle 0.55, 0.45\rangle$
C_2	A_1	$\langle 0.47, 0.53\rangle$	$\langle 0.53, 0.47\rangle$	$\langle 0.55, 0.45\rangle$
	A_2	$\langle 0.50, 0.50\rangle$	$\langle 0.60, 0.40\rangle$	$\langle 0.49, 0.51\rangle$
	A_3	$\langle 0.83, 0.27\rangle$	$\langle 0.64, 0.36\rangle$	$\langle 0.76, 0.24\rangle$
	A_4	$\langle 0.42, 0.58\rangle$	$\langle 0.38, 0.62\rangle$	$\langle 0.45, 0.55\rangle$
C_3	A_1	$\langle 0.42, 0.58\rangle$	$\langle 0.40, 0.60\rangle$	$\langle 0.47, 0.53\rangle$
	A_2	$\langle 0.56, 0.44\rangle$	$\langle 0.46, 0.54\rangle$	$\langle 0.67, 0.33\rangle$
	A_3	$\langle 0.61, 0.39\rangle$	$\langle 0.76, 0.24\rangle$	$\langle 0.52, 0.48\rangle$
	A_4	$\langle 0.60, 0.40\rangle$	$\langle 0.50, 0.50\rangle$	$\langle 0.52, 0.48\rangle$
C_4	A_1	$\langle 0.46, 0.54\rangle$	$\langle 0.41, 0.59\rangle$	$\langle 0.53, 0.47\rangle$
	A_2	$\langle 0.51, 0.49\rangle$	$\langle 0.50, 0.50\rangle$	$\langle 0.59, 0.41\rangle$
	A_3	$\langle 0.75, 0.25\rangle$	$\langle 0.76, 0.24\rangle$	$\langle 0.53, 0.47\rangle$
	A_4	$\langle 0.55, 0.45\rangle$	$\langle 0.45, 0.55\rangle$	$\langle 0.49, 0.51\rangle$

brands is $A_3 = A_4 > A_2 > A_1$. The decision maker cannot select the best brand between A_3 and A_4. The IBPA considers both support and non-support information, can handle more information and help decision makers to make decision more accurately.

Compared with IFSs, the IES assigns the support degree and non-support degree to the power set of the frame of discernment, the IBPA can handle more uncertainty information. In the problem of air-condition brands selection, if we discard support degrees and non-support degrees of all multiple subsets and normalize the support degree and non-support degree of all single elements, then IBPAs become IFSs. Evaluations of each criteria by decision makers expressed as IFSs are shown in Table 3. The IFSs of decision matrix are shown in Table 4.

The IFS of importance of decision makers, d_k is

$$d_1 = \langle 0.67, 0.33\rangle$$
$$d_2 = \langle 0.36, 0.64\rangle$$
$$d_3 = \langle 0.54, 0.46\rangle$$

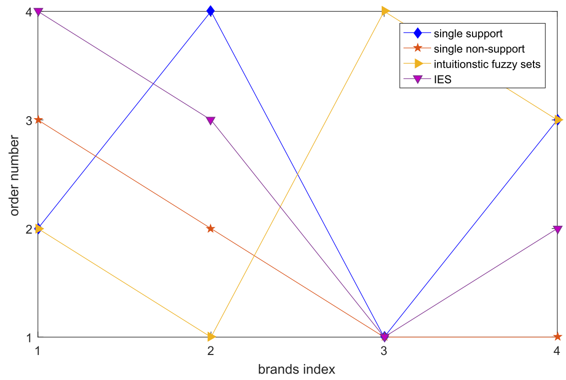

FIGURE 2. The ranking results of four methods.

According to the multi-criteria intuitionistic fuzzy group decision making method [99], the rank of air-condition brands under expressed as IFSs is $A_2 > A_1 > A_4 > A_3$. In this case, A_2 is the best selection. Four ranks of these methods are shown in Figure 2. It can be seems that the result of IES is more reasonable, because it combines the features of classical Dempster-Shafer evidence theory and IFSs.

C. SENSITIVITY ANALYSIS

Assume in the IBPA m_{jk}, when the support degree or non-support degree of alternative A_i assigned to criteria C_j by decision maker D_k changes, it only causes the support degree or non-support degree of another alternative A_l assigned to criteria C_j by decision maker D_k change. If the support degree, m^+_{ijk}, or non-support degree, m^-_{ijk}, of alternative A_i assigned to criteria C_j by decision maker D_k becomes $\hat{m}^+_{ijk}$ or $\hat{m}^-_{ijk}$, then the support degree, m^+_{ljk}, or non-support degree, m^-_{ljk}, of alternative A_l assigned to criteria C_j by decision maker D_k becomes $\hat{m}^+_{ljk}$ or $\hat{m}^-_{ljk}$, and $m^+_{ijk} + m^+_{ljk} = \hat{m}^+_{ijk} + \hat{m}^+_{ljk}$, $m^-_{ijk} + m^-_{ljk} = \hat{m}^-_{ijk} + \hat{m}^-_{ljk}$. The range of changes are $[0, m^+_{ijk} + m^+_{ljk}]$ and $[0, m^-_{ijk} + m^-_{ljk}]$ for non-support degree, respectively. We consider two alternatives A_2 and A_4 for all criteria and decision makers, and each change is 10% of $m^+_{ijk} + m^+_{ljk}$ or $m^-_{ijk} + m^-_{ljk}$. The results are shown in Table 5.

TABLE 5. The results of comparing alternatives A_2 and A_4 when the support and non-support degree change. Where $>$ indicates $A_2 > A_4$, $<$ indicates $A_2 < A_4$.

non-support degree of A_2 \ support degree of A_2	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0%	$<$	$<$	$>$	$>$	$>$	$>$	$>$	$>$	$>$	$>$	$>$
10%	$<$	$<$	$<$	$>$	$>$	$>$	$>$	$>$	$>$	$>$	$>$
20%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$>$	$>$	$>$
30%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$>$	$>$
40%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$>$
50%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$>$
60%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
70%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
80%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
90%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
100%	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$	$<$

The original sort is $A_2 < A_4$, when the non-support degree of A_2 is small enough and the support degree is large enough, $A_2 > A_4$. As can be seen from the table, the results of sorting are more sensitive to non-support degree. Because of both non-support degrees of A_2 and A_4 are larger than support degrees.

V. CONCLUSION

Though Dempster-Shafer evidence theory is widely used, it has many open issues. To make the classical BPA more flexible and reasonable to model the uncertain information, a new math model, named as intuitionstic evidence sets (IES), is presented. Compared with traditional BPA, the proposed IBPA considers both the support degree and non-support degree of focal elements. It combines the features of Dempster-Shafer evidence theory and IFS, can handle more information and more reasonable and flexible. Compared with IFSs, the proposed IBPA assigns the support degree and non-support degree to not only singletons but also multiple subsets of the frame of discernment. An application on group decision making under uncertain environment is used to illustrate the efficiency of the proposed IES.

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