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A Dwell Time Switching Approach to Channel Assignment for Stabilization of NCSs With Medium Access Constraint

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ABSTRACT This paper investigated the networked control of a collection of continuous-time linear time-invariant plants with medium access constraints. We aim to stabilize the plants by effectively assigning the wireless channels and properly designing feedback controllers. By modeling the plant as a switched system and using a time-scheduled Lyapunov function approach, computable sufficient conditions on the stabilization and schedulability requirements are proposed in the framework of dwell time. Then, based on the results, an interesting channel assignment policy is obtained and time-scheduled state feedback controller design schemes are given. The effectiveness of the results is demonstrated by numerical simulations.

INDEX TERMS Medium-access constraint, dwell time, switched systems, channel assignment policy.

I. INTRODUCTION

Networked control systems (NCSs) have received substantial attention due to their wide applications in transportation systems, chemical processes, and many manufacturing plants [1]–[3]. In NCSs, information and control signals are exchanged by shared communication network. Although the introduce of network brings some advantages such as reduced wiring and weight, lower cost, simple and easy installation and maintenance [4]–[6], it has raised fundamentally new challenges in communications. For example, when an actual NCS involves a large amount of sensors and actuators while the network cannot accommodate them at the same time. As a consequence, the measurement and control signals cannot be updated for a long time that may lead to performance degradation or stability loss. This limitation is called medium access constraint [7].

There are many practical applications that motivate this issue, such as the climate monitoring system, a predication system for oil spill, and the sensor network used for monitoring earthquake [7]. To help make the motivation of this

paper more comprehensive, consider a network of mobile sensing/actuating agents deployed to monitor/intervene an ongoing incident of oil pervasion in a given oceanic zone. The agents are controlled by a remote control station via a wireless communication network. At any time, all the sensing/actuating agents are required to send their measurement to the control station simultaneously, but owing to the medium access constraint of the wireless network, only a part of agents are allowed to receive control signals computed in the control station. If not effectively schedule the communication of agents, oil diffusion may happen due to some agents lose of control for a long time.

How to deal with the negative effects of medium access constraint on the stability of an NCS has received ever-increasing attention in recent years. Various modeling, analysis and channel scheduling policies have been proposed. For example, Zhang and Hristu-Varsakelis [8] discuss the stabilization of a NCS in which sensors and actuators of a plant exchange information with a remote controller via a shared communication medium. A codesign strategy of feedback controller and scheduling policy was presented, where the access to that medium is governed by a pair of static periodic communication sequences. Different from [8],

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a co-design scheme of Try Once Discard (TOD) dynamic scheduling strategy and controller is proposed for a class of NCSs with random time delay [9] and quantized control [10], which is albeit more robust than static ones, but require frequent monitoring of the channel for feedback information and complicated processing. Zhang *et al.* [11]–[13] proposed a new redundant channel policy to resolve the issue of packet dropouts and alleviate the effect of disconnection of partial channels. But the results cannot be applied to the medium access constraint directly because there are no redundant channels for use in this study. To reduce the network bandwidth usage, a hybrid scheduling strategy and dynamic robust H-infinity output feedback controller is proposed in [14]. The hybrid scheduling scheme integrates dead zone scheduling and TOD scheduling that can get stronger adaptability and flexibility than the single scheduling.

The phenomenon of medium access constraint also exists in large-scale systems with numerous subsystems sharing a common wireless network. For a large-scale system subject to medium access constraints, the procedure of control synthesis involves designing not only a controller, but also a network-access assignment policy for the plants at the same time. To address this problem, the switched system approach was introduced in [15]. By modeling each plant as a switched system, sufficient condition of the existence of scheduling policy and controller design has been obtained in the framework of average dwell time and attention rate. Then, based on the results, a scheduling and feedback control codesign procedure is proposed for the simultaneous stabilization of collection of networked linear time-invariant (LTI) systems with uncertain delays. Along this line of research within the periodic scheduling framework, the results were extended to the case with limited communication energy [16], and a more complicated codesign method of channel assignment, transmission power allocation and stabilizing control are derived. The presented methodology can guarantee a desired decay rate and a given energy consumption for each plant. By using the same average dwell time technique as in [15], [16], an alternative methodology for most regular binary sequences protocol and controller co-design is proposed in [17], [18], in which the transmission intervals vary with time. Much research is done on the codesign of the controller and scheduler of large-scale systems with medium access constraints. Unfortunately, in the above work with the average dwell time technique, the switching behaviors are always viewed to increase the value of the Lyapunov function. This viewpoint is too restrictive without considering the stabilization characteristic of switchings. On the other hand, in many actual applications, the stability analysis results in [15], [16] is trivial since it is not easy to check conditions for all switching instants, especially for non-period scheduling policy.

Here, we discuss the stability of large-scale systems in the framework of dwell time and aim to find a scheduling and control codesign algorithm such that the collection of plants can be simultaneously stabilized with the medium access constraint. By constructing a new time-varying Lyapunov

function of each plant, computable sufficient conditions for the (robust) global stabilization of a single plant are proposed within the framework of dwell-time. The dwell time is confined by a certain pair of upper and lower bounds, which restrict the channel accessing time and disconnecting time. Then, based on the dwell time conditions, an interesting co-design framework is derived for period channel assignment policy and switched controllers, which can guarantee the global stabilization of the whole set of plants with medium access constraints. The rest of this paper is divided into four sections. Section II introduces the definition of the problem setup of this paper, while Section III states our main results regarding the stabilization conditions of switched linear systems and schedulable conditions of the channel assignment policy. A numerical example is presented in Sections IV and V concludes this paper.

Notation: Throughout this paper, R^n and $R^{n \times m}$ represent the n -dimensional Euclidean space and the set of $n \times m$ real matrices, respectively. N represents the set of non-negative integers and N^+ represents the set of positive integers. Superscript “ T ” represents the transpose. For Hermitian matrices, and $Y = Y^T \in R^{n \times n}$, $X > Y$ means that matrix $X - Y$ is positive definite. $\|\cdot\|$ represents the Euclidean norm for a vector.

II. PROBLEM FORMULATION

Consider a NCS consisting of M plants. The dynamics of each plant is described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t), \quad i = 1, 2, \dots, M, \quad (1)$$

where $x_i(t) \in R^n$ and $u_i(t) \in R^m$ are the states and input of plant i , $A_i \in R^{n \times n}$ and $B_i \in R^{n \times m}$ are constant matrices.

Without loss of generality, suppose that M plants share r wireless channels at any time, where $r < M$. Denote binary-valued function $\delta_i(t) : R \rightarrow \{0, 1\}$, $i = 1, 2, \dots, M$, as the channel-access status of plant i at time t . When $\delta_i(t) = 1$, plant i is accessing the channel and the remote controller i can receive the measurement; otherwise, $\delta_i(t) = 0$, no measurement is transmitted and plant i falls into open-loop status. Due to medium access constraint, the following inequality holds true at any time t

$$\delta_1(t) + \delta_2(t) + \dots + \delta_M(t) \leq r.$$

The controller of plant i is a state feedback controller given by

$$u_i(t) = K_i(t) \hat{x}_i(t), \quad (2)$$

where $K_i(t) \in R^{m \times n}$ is the controller gain, $\hat{x}_i(t)$ is the state actually received by the controller, which, according to the above discussions, can be represented as

$$\hat{x}_i(t) = \delta_i(t) x_i(t). \quad (3)$$

As $\delta_i(t)$ is a binary function describing network access status, each plant is actually a switching system comprising the following two modes:

closed-loop mode

$$\dot{x}_i(t) = [A_i + B_i K_i(t)]x_i(t), \quad \delta_i(t) = 1, \quad t \in [t_{2j}, t_{2j+1}),$$

and open-loop mode

$$\dot{x}_i(t) = A_i x_i(t), \quad \delta_i(t) = 0, \quad t \in [t_{2j+1}, t_{2j+2}),$$

where $j \in N, t_0 = 0$.

In the following, we will give a definition and an assumption utilized in the posterior part of the paper.

Definition 1 [19]: System (1) is globally asymptotic stable (GAS) if for all $x_i(0)$ and all σ , there exists $\sigma_0 > 0$ such that

- 1) $\|x_i(0)\| \leq \sigma_0 \Leftrightarrow \|x_i(t)\| \leq \sigma$ for all $t \geq 0$;
- 2) $\|x_i(t)\| \rightarrow 0$ as $t \rightarrow \infty$.

Assumption 1: The following assumptions are made:

- 1) There exists a uniform lower-bound τ_i on the lengths of $[t_{2j}, t_{2j+1})$, i.e., $t_{2j+1} - t_{2j} \geq \tau_i$, for all $j \in N$.
- 2) There exists a uniform upper-bound Γ_i on the lengths of $[t_{2j+1}, t_{2j+2})$, i.e., $t_{2j+2} - t_{2j+1} \leq \Gamma_i$, for all $j \in N$.

Remark 1: Similar to [20], the dwell time proposed in this paper has upper and lower bounds. The meaning of the upper-bound Γ_i is that plant i cannot fall into the open-loop mode indefinitely whereas the lower bound τ_i ensures that plant i has enough time to fully recover from the open-loop. In the view of switched systems, the quantities τ_i and Γ_i are minimum dwell-time and maximal dwell-time respectively. Denote f_i as the attention rate of plant i . Then from Assumption 1 we know that $f_i \geq \tau_i/(\tau_i + \Gamma_i)$. Specifically, if all the plants have the same dynamics, then the minimal number of channels is given by $r_{\min} = \lceil M\tau/(\tau + \Gamma) \rceil$, where $\lceil \cdot \rceil$ denotes the upper integer bound.

III. MAIN RESULTS

In this section, sufficient conditions for GAS of the single plant are given first, which is followed by the controller design method. Then we extend the results to nonzero disturbance case with H_∞ robust performance. Finally, based on the results, schedulability requirements are given in the framework of dwell time and a period channel assignment policy is derived.

A. STABILITY ANALYSIS

The following result is concerned with the GAS of a single plant i .

Theorem 1: For given parameters $m_i, n_i \in N^+$, plant i is GAS if there exist matrices $P_{i,1,m}, P_{i,2,n} > 0$, $m = 0, 1, \dots, m_i - 1, n = 0, 1, \dots, n_i - 1$, such that

$$\Phi_{i,1,m} < 0, \quad \Phi_{i,2,m} < 0, \quad \Phi_{i,m_i} < 0, \quad (4)$$

$$\Psi_{i,1,n} < 0, \quad \Psi_{i,2,n} < 0, \quad (5)$$

$$P_{i,1,0} - P_{i,2,n_i} < 0, \quad (6)$$

$$P_{i,2,0} - P_{i,1,m_i} < 0, \quad (7)$$

where

$$\Phi_{i,1,m} = (P_{i,1,m+1} - P_{i,1,m})m_i/\tau_i + [A_i + B_i K_i(t)]^T P_{i,1,m} + P_{i,1,m}[A_i + B_i K_i(t)],$$

$$\Phi_{i,2,m} = (P_{i,1,m+1} - P_{i,1,m})m_i/\tau_i + [A_i + B_i K_i(t)]^T P_{i,1,m+1} + P_{i,1,m+1}[A_i + B_i K_i(t)],$$

$$\Phi_{i,m_i} = [A_i + B_i K_i(t)]^T P_{i,1,m_i} + P_{i,1,m_i}[A_i + B_i K_i(t)],$$

$$\Psi_{i,1,n} = (P_{i,2,n+1} - P_{i,2,n})n_i/\Gamma_i + A_i^T P_{i,2,n} + P_{i,2,n}A_i,$$

$$\Psi_{i,2,n} = (P_{i,2,n+1} - P_{i,2,n})n_i/\Gamma_i + A_i^T P_{i,2,n+1} + P_{i,2,n+1}A_i.$$

Proof: Choose a time-varying Lyapunov function as below

$$V_i(t) = \begin{cases} x_i^T(t)P_{i,1}(t)x_i(t), & t \in [t_{2j}, t_{2j+1}) \\ x_i^T(t)P_{i,2}(t)x_i(t), & t \in [t_{2j+1}, t_{2j+2}) \end{cases}, \quad (8)$$

where

$$P_{i,1}(t) = \begin{cases} (1 - \beta_1)P_{i,1,m} + \beta_1 P_{i,1,m+1}, & t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) \\ P_{i,1,m_i}, & t \in [t_{2j} + \alpha_{1,m_i}, t_{2j+1}) \end{cases},$$

$$\beta_1 = (t - t_{2j} - \alpha_{1,m})m_i/\tau_i, \quad \alpha_{1,m} = m\tau_i/m_i, \\ m = 0, 1, \dots, m_i - 1,$$

$$P_{i,2}(t) = (1 - \beta_2)P_{i,2,n} + \beta_2 P_{i,2,n+1}, \\ t \in [t_{2j+1} + \alpha_{2,n}, t_{2j+1} + \alpha_{2,n+1}), \\ \alpha_{2,n} = n\Gamma_i/n_i, \quad n = 0, 1, \dots, n_i - 1, \\ \beta_2 = (t - t_{2j+1} - \alpha_{2,n})n_i/\Gamma_i, \quad t_{2j+1} + \alpha_{2,n_i} = t_{2n+2}.$$

Then if $t \in [t_{2j}, t_{2j+1})$ and from (4), we have that

$$\dot{V}_i(t) = x_i^T(t)\dot{P}_{i,1}(t)x_i(t) + 2x_i^T(t)P_{i,1}(t)x_i(t) \\ = \begin{cases} x_i^T(t)[(1 - \beta_1)\Phi_{i,1,m} + \beta_1 \Phi_{i,2,m}]x_i(t), & t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) \\ x_i^T(t)\Phi_{i,m_i}x_i(t), & t \in [t_{2j} + \alpha_{1,m_i}, t_{2j+1}) \end{cases} \\ < 0$$

Similarly, if $t \in [t_{2j+1}, t_{2j+2})$, $\dot{V}_i(t) < 0$ can be obtained using (5).

If $t = t_{2j}$, then from (6), it arrives at $V_i(t_{2j}^+) = x_i^T(t)P_{i,1,0}x_i(t) < x_i^T(t)P_{i,2,n_i}x_i(t) = V_i(t_{2j}^-)$. Similarly, if $t = t_{2j+1}$, $V_i(t_{2j+1}^+) < V_i(t_{2j+1}^-)$ can be obtained by using (7). Then we can conclude that, $V_i(t)$ monotonically decreases in $[t_{2j}, t_{2j+1}), [t_{2j+1}, t_{2j+2})$ and at switching instants t_{2j}, t_{2j+1} , thus the GAS of plant i can be established by the standard Lyapunov theorem.

Remark 2: Theorem 1 provides us a sufficient condition ensuring GAS of switched linear system (1) composed of open-loop and closed-loop subsystems. Time-varying Lyapunov functions have been proven to be able to formulate nonconservative conditions for maximum and minimum dwell-time analysis. From Theorem 1 we know that plant i is GAS if it gains access to the channel for enough time τ_i and disconnects from the network for no more than time Γ_i .

B. CONTROLLER DESIGN

Theorem 2: For given parameters $m_i, n_i \in N^+$, plant i is globally asymptotic stabilized if there exist matrices $S_{i,1,m}, S_{i,2,n} > 0$ and $L_{i,1,m}, m = 0, 1, \dots, m_i - 1, n = 0, 1, \dots, n_i - 1$, such that

$$\bar{\Phi}_{i,1,m} < 0, \quad \bar{\Phi}_{i,2,m} < 0, \quad \bar{\Phi}_{i,m_i} < 0, \quad (9)$$

$$\bar{\Psi}_{i,1,n} < 0, \quad \bar{\Psi}_{i,2,n} < 0, \quad (10)$$

$$S_{i,1,0} - S_{i,2,n_i} > 0, \quad (11)$$

$$S_{i,2,0} - S_{i,1,m_i} > 0. \quad (12)$$

The controller gain is given by

$$K_i(t) = L_i(t)S_{i,1}^{-1}(t), \quad (13)$$

where

$$\begin{aligned} \bar{\Phi}_{i,1,m} &= A_i S_{i,1,m} + S_{i,1,m} A_i^T + B_i L_{i,m} + L_{i,m}^T B_i^T \\ &\quad - (S_{i,1,m+1} - S_{i,1,m}) m_i / \tau_i, \\ \bar{\Phi}_{i,2,m} &= A_i S_{i,1,m+1} + S_{i,1,m+1} A_i^T + B_i L_{i,m+1} + L_{i,m+1}^T B_i^T \\ &\quad - (S_{i,1,m+1} - S_{i,1,m}) m_i / \tau_i, \\ \bar{\Phi}_{i,m_i} &= A_i S_{i,1,m_i} + S_{i,1,m_i} A_i^T + B_i L_{i,m_i} + L_{i,m_i}^T B_i^T, \\ \bar{\Psi}_{i,1,n} &= A_i S_{i,2,n} + S_{i,2,n} A_i^T - (S_{i,2,n+1} - S_{i,2,n}) n_i / \Gamma_i, \\ \bar{\Psi}_{i,2,n} &= A_i S_{i,2,n+1} + S_{i,2,n+1} A_i^T - (S_{i,2,n+1} - S_{i,2,n}) n_i / \Gamma_i, \\ L_i(t) &= \begin{cases} (1 - \beta_1) L_{i,m} + \beta_1 L_{i,m+1}, \\ \quad t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) \end{cases}, \quad (14) \\ S_{i,1}(t) &= \begin{cases} (1 - \beta_1) S_{i,1,m} + \beta_1 S_{i,1,m+1}, \\ \quad t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) \end{cases}. \quad (15) \\ &\quad S_{i,1,m_i}, \quad t \in [t_{2j} + \alpha_{1,m_i}, t_{2j+1}) \end{aligned}$$

Proof: Define $S_{i,2}(t), t \in [t_{2j}, t_{2j+1})$ as

$$S_{i,2}(t) = (1 - \beta_2) S_{i,2,n} + \beta_2 S_{i,2,n+1}, \quad t \in [t_{2j+1}, t_{2j+2}),$$

where β_2 is defined in (8).

Since $S_{i,1,m} > 0, m = 0, 1, \dots, m_i - 1$ and $S_{i,2,n} > 0, n = 0, 1, \dots, n_i - 1$, we have $S_{i,1}(t) > 0$ and $S_{i,2}(t) > 0$. Define the Lyapunov function as below

$$V_i(t) = \begin{cases} x_i^T(t) S_{i,1}^{-1}(t) x_i(t), & t \in [t_{2j}, t_{2j+1}) \\ x_i^T(t) S_{i,2}^{-1}(t) x_i(t), & t \in [t_{2j+1}, t_{2j+2}) \end{cases}. \quad (16)$$

From the proof lines in Theorem 1, we have to prove

$$\dot{V}_i(t) < 0, \quad \forall t \in [t_{2j}, t_{2j+1}), \quad (17)$$

$$\dot{V}_i(t) < 0, \quad \forall t \in [t_{2j+1}, t_{2j+2}), \quad (18)$$

$$V_i(t_{2j}^+) < V_i(t_{2j}^-), \quad V_i(t_{2j+1}^+) < V_i(t_{2j+1}^-). \quad (19)$$

For $t \in [t_{2j}, t_{2j+1})$, (17) can be guaranteed by

$$[A_i + B_i K_i(t)]^T S_{i,1}^{-1}(t) + S_{i,1}^{-1}(t) [A_i + B_i K_i(t)] + \dot{S}_{i,1}^{-1}(t) < 0. \quad (20)$$

Using $\dot{S}_{i,1}^{-1}(t) = -S_{i,1}^{-1}(t) \dot{S}_{i,1}(t) S_{i,1}^{-1}(t)$, (20) is equivalent to

$$\begin{aligned} [A_i + B_i K_i(t)]^T S_{i,1}^{-1}(t) + S_{i,1}^{-1}(t) [A_i + B_i K_i(t)] \\ - S_{i,1}^{-1}(t) \dot{S}_{i,1}(t) S_{i,1}^{-1}(t) < 0 \end{aligned} \quad (21)$$

Multiplying both sides of (21) by $S_{i,1}(t)$, it arrives at

$$S_{i,1}(t) [A_i + B_i K_i(t)]^T + [A_i + B_i K_i(t)] S_{i,1}(t) - \dot{S}_{i,1}(t) < 0.$$

Then, by the similar guidelines in Theorem 1 and structure of \cdot and $L_i(t) = K_i(t) S_{i,1}(t)$, (17) can be established by (9).

Similarly for $t \in [t_{2j+1}, t_{2j+2})$, (18) can be ensured under condition (10), This completes the proof.

According to Theorem 2, for given τ_i , the maximal time Γ_i of tolerating the disconnecting from the network of plant i can be obtained by solving Problem 1.

Problem 1: $\Gamma_i = \max \tau$

s.t. (9), (11), (12)

$$A_i S_{i,2,n} + S_{i,2,n} A_i^T + (S_{i,2,n+1} - S_{i,2,n}) n_i / \tau < 0, ,$$

$$A_i S_{i,2,n+1} + S_{i,2,n+1} A_i^T + (S_{i,2,n+1} - S_{i,2,n}) n_i / \tau < 0.$$

C. ROBUST PERFORMANCE ANALYSIS

If there is a disturbance $w_i(t) \in L_2[0, \infty)$ in system (1). Then the dynamics of each plant is described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + C_i w_i(t), \quad i = 1, 2, \dots, M, \quad (22)$$

In this section, we are interested in establishing a design methodology in the sense that the above system (22) is robustly GAS with a H_∞ disturbance attenuation level $\gamma > 0$, namely, system (22) is GAS with $w_i(t) = 0$ and the state $x_i(t)$ under zero initial condition satisfies

$$\int_0^\infty x_i^T(t) x_i(t) dt \leq \gamma^2 \int_0^\infty w_i^T(t) w_i(t) dt$$

Theorem 3: For given parameters $m_i, n_i \in N^+$, if there exist matrices $P_{i,1,m}, P_{i,2,n} > 0, m = 0, 1, \dots, m_i - 1, n = 0, 1, \dots, n_i - 1$, such that

$$\begin{bmatrix} \Phi_{i,1,m} + I & P_{i,1,m} C_i \\ C_i^T P_{i,1,m} & -\gamma^2 I \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} \Phi_{i,2,m} + I & P_{i,2,m} C_i \\ C_i^T P_{i,2,m} & -\gamma^2 I \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} \Phi_{i,m_i} + I & P_{i,m_i} C_i \\ C_i^T P_{i,m_i} & -\gamma^2 I \end{bmatrix} < 0, \quad (24)$$

$$\begin{bmatrix} \Psi_{i,1,n} + I & P_{i,1,n} C_i \\ C_i^T P_{i,1,n} & -\gamma^2 I \end{bmatrix} < 0, \quad (25)$$

$$\begin{bmatrix} \Psi_{i,2,n} + I & P_{i,2,n} C_i \\ C_i^T P_{i,2,n} & -\gamma^2 I \end{bmatrix} < 0, \quad (25)$$

$$P_{i,1,0} - P_{i,2,n_i} < 0, \quad P_{i,2,0} - P_{i,1,m_i} < 0, \quad (26)$$

where the parameters $\Phi_{i,1,m}, \Phi_{i,2,m}, \Phi_{i,m_i}, \Psi_{i,1,n}, \Psi_{i,2,n}$ are defined in Theorem 1. Then, system (22) is robustly GAS with a H_∞ disturbance attenuation level $\gamma > 0$.

Proof: The GAS of system (22) with $w_i(t) = 0$ can be easily obtained from (23)-(26). Hereby, we mainly focused on the robust performance analysis. With the similar steps as in Theorem 1 and from (23)-(25), we can obtain that

$$x_i^T(t) x_i(t) - \gamma^2 w_i^T(t) w_i(t) + \dot{V}_i(t) < 0. \quad (27)$$

Now, integrate (27) from 0 to ∞ with respect to t yields

$$\int_0^\infty x_i^T(t) x_i(t) dt < \gamma^2 \int_0^\infty w_i^T(t) w_i(t) dt + V_i(0) - V_i(\infty)$$

TABLE 1. Computation on Γ_i with $d = 5$.

	Γ_1	Γ_2	Γ_3
$d = 5$	2.4s	1.5s	2.7s

TABLE 2. Computation on Γ_1 with different d .

$m_i = n_i$	1	2	5	8	10
Γ_1	0.2s	0.9s	2.4s	3.4s	4s

upper-bound Γ_i of each plant is guaranteed by the choice of scheduling period T . Then from Theorem 1 or Theorem 3 we can conclude that the above scheduling policy can guarantee the simultaneous stabilization of all the plants.

Remark 3: Theorem 3 gives a sufficient condition for the existence of period scheduling policy in the framework of dwell time. The obtained channel assignment policy generated from dwell time is easy for the implementation of time-varying switched controllers in Theorem 2.

IV. SIMULATION

A. STABILITY ANALYSIS WITH A NUMERICAL EXAMPLE

Consider three plants whose dynamics are given, respectively, by

$$\begin{aligned} \dot{x}_1(t) &= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} x_1(t) + \begin{bmatrix} 10 \\ 5 \end{bmatrix} u_1(t), \\ \dot{x}_2(t) &= \begin{bmatrix} 1.6 & 1 \\ 0 & -1.1 \end{bmatrix} x_2(t) + \begin{bmatrix} 8 \\ 0 \end{bmatrix} u_2(t), \\ \dot{x}_3(t) &= \begin{bmatrix} 0.5 & 1 \\ 1 & -1.5 \end{bmatrix} x_3(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_3(t). \end{aligned}$$

It is assumed that there are two wireless channels available, i.e., $r = 2$. Assuming $\tau_1 = \tau_2 = \tau_3 = 1s$ here, we try to design a stabilizing controller to maximize Γ_i , $i = 1, 2, 3$, i.e., maximizing the capability of tolerating the disconnecting from the network. Then, by Theorem 2 and letting $m_i = n_i = d$, the upper dwell time bound Γ_i , $i = 1, 2, 3$ is given in Table 1 with respect to $d = 5$. From Table 1 we can see that for the same d and τ_s , different plants corresponding to different Γ_i , i.e., different systems differ in the capability of tolerating the disconnecting from the network. The upper dwell time bound of plant 1 Γ_1 for different values d is given in Table 2. From Table 2 one can see that upper dwell time bound Γ_1 increases with d , i.e., the larger d is, the larger capability of tolerating the disconnecting from the network is. Thus we can say a less conservative result can be obtained by a larger d , but with a larger computational cost.

Let $d = 5$, then by Matlab software, the feedback controllers are given as below

$$K_i(t) = L_i(t)S_{i,1}^{-1}(t),$$

where

$$\begin{aligned} L_i(t) &= \begin{cases} (1 - \beta_1)L_{i,m} + \beta_1L_{i,m+1}, & t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) , \\ L_{i,m_i}, & t \in [t_{2j} + \alpha_{1,m_i}, t_{2j+1}) \end{cases} \\ \beta_1 &= 5(t - t_{2j}) - m, \quad \alpha_{1,m} = 0.2m, \quad m = 0, 1, \dots, 4, \\ S_{i,1}(t) &= \begin{cases} (1 - \beta_1)S_{i,1,m} + \beta_1S_{i,1,m+1}, & t \in [t_{2j} + \alpha_{1,m}, t_{2j} + \alpha_{1,m+1}) . \\ S_{i,1,m_i}, & t \in [t_{2j} + \alpha_{1,m_i}, t_{2j+1}) \end{cases} \end{aligned}$$

plant 1:

$$\begin{aligned} S_{1,1,0} &= \begin{bmatrix} 64.7053 & 5.9076 \\ 5.9076 & 19.0193 \end{bmatrix}, & L_{1,0} &= [-9.6066 & -10.5836] \\ S_{1,1,1} &= \begin{bmatrix} 53.9544 & 3.8779 \\ 3.8779 & 11.2310 \end{bmatrix}, & L_{1,1} &= [-8.7491 & -8.1890] \\ S_{1,1,2} &= \begin{bmatrix} 42.2572 & 2.8213 \\ 2.8213 & 4.6930 \end{bmatrix}, & L_{1,2} &= [-7.8492 & -5.7350], \\ S_{1,1,3} &= \begin{bmatrix} 29.3365 & 2.5546 \\ 2.5546 & 1.2390 \end{bmatrix}, & L_{1,3} &= [-6.8287 & -3.4300], \\ S_{1,1,4} &= \begin{bmatrix} 15.2018 & 1.6153 \\ 1.6153 & 0.7274 \end{bmatrix}, & L_{1,4} &= [-5.6813 & -1.4686], \\ S_{1,1,5} &= \begin{bmatrix} 0.2433 & -0.8690 \\ -0.8690 & 3.1423 \end{bmatrix}, & L_{1,5} &= [-4.1855 & 0.6188]. \end{aligned}$$

plant 2:

$$\begin{aligned} S_{2,1,0} &= \begin{bmatrix} 25.7133 & 0 \\ 0 & 10.9895 \end{bmatrix}, & L_{2,0} &= [-7.4831 & 0], \\ S_{2,1,1} &= \begin{bmatrix} 19.8866 & 0 \\ 0 & 8.6005 \end{bmatrix}, & L_{2,1} &= [-6.2588 & 0], \\ S_{2,1,2} &= \begin{bmatrix} 14.4927 & 0 \\ 0 & 7.0327 \end{bmatrix}, & L_{2,2} &= [-5.0406 & 0], \\ S_{2,1,3} &= \begin{bmatrix} 9.5678 & 0 \\ 0 & 6.0097 \end{bmatrix}, & L_{2,3} &= [-3.9415 & 0], \\ S_{2,1,4} &= \begin{bmatrix} 4.8522 & 0 \\ 0 & 5.3265 \end{bmatrix}, & L_{2,4} &= [-2.9851 & 0], \\ S_{2,1,5} &= \begin{bmatrix} 0.0043 & 0 \\ 0 & 4.8325 \end{bmatrix}, & L_{2,5} &= [-2.0355 & 0]. \end{aligned}$$

plant 3:

$$\begin{aligned} S_{3,1,0} &= \begin{bmatrix} 5.6233 & 1.4098 \\ 1.4098 & 2.6415 \end{bmatrix}, & L_{3,0} &= [-8.6322 & 0.3880], \\ S_{3,1,1} &= \begin{bmatrix} 4.2936 & 1.0786 \\ 1.0786 & 2.2104 \end{bmatrix}, & L_{3,1} &= [-7.3626 & 0.2661], \end{aligned}$$

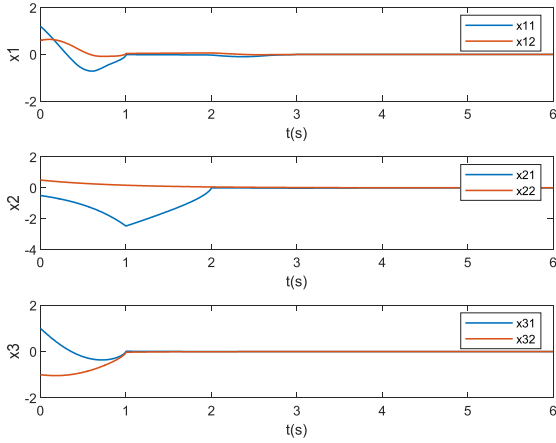


FIGURE 1. System states.

$$S_{3,1,2} = \begin{bmatrix} 3.1096 & 0.7297 \\ 0.7297 & 1.8390 \end{bmatrix}, \quad L_{3,2} = [-6.0747 \quad 0.1658],$$

$$S_{3,1,3} = \begin{bmatrix} 2.0644 & 0.3853 \\ 0.3853 & 1.5013 \end{bmatrix}, \quad L_{3,3} = [-4.9166 \quad 0.0536],$$

$$S_{3,1,4} = \begin{bmatrix} 1.1015 & 0.0335 \\ 0.0335 & 1.1726 \end{bmatrix}, \quad L_{3,4} = [-3.9519 \quad -0.0807],$$

$$S_{3,1,5} = \begin{bmatrix} 0.1443 & -0.3433 \\ -0.3433 & 0.8315 \end{bmatrix}, \quad L_{3,5} = [-3.0531 \quad -0.1936].$$

According to Theorem 5, there exists a period channel assignment policy stabilizes the three plants at the same time with $\Delta_1 = 1, \Delta_2 = 1, \Delta_3 = 1.6$. Let $T = 2s$, Then the channel assignment policy over a scheduling period is given as below

$$C_1 : \underbrace{P_1 \cdots P_1}_{1s} \quad \underbrace{P_2 \cdots P_2}_{1s}$$

$$C_2 : \underbrace{P_3 \cdots P_3}_{1.6s}$$

The states of the three plants are given in Figure 1, from which we can see that the above channel assignment policy can guarantee the stability of the three plants.

B. ROBUST ANALYSIS WITH VEHICLE PLATOON EXAMPLE

In this section, we will show how to apply the results to a platoon of vehicles. Consider a platoon of 11 vehicles running a horizontal road as in Figure 2, where 0 denotes the leading vehicle.

The dynamics of each vehicle i is written as [3]

$$\dot{z}_i(t) = A_i z_i(t) + B_i u_i(t),$$

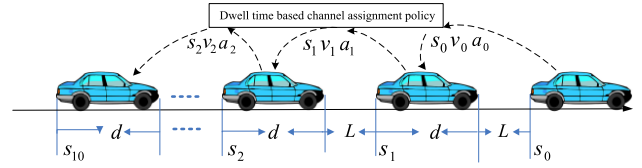


FIGURE 2. Platoon of vehicles.

where $A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\zeta_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\zeta_i \end{bmatrix},$

$$z_i(t) = \begin{bmatrix} s_i(t) \\ v_i(t) \\ a_i(t) \end{bmatrix}, s_i, v_i \text{ and } a_i \text{ denote the position, velocity}$$

and acceleration of the i -th following vehicle respectively, with $i = 1, 2, \dots, 10$, and $i = 0$ denoting the leading vehicle. ζ_i is the time constant of the lag in tracking any desired acceleration command, $u_i(t)$ is the control input.

The objective is to design a controller and channel scheduling policy for each following vehicle that can keep a constant distance with its preceding vehicle. The following vehicle exchanges information on preceding vehicle via wireless network. But due to the medium access constraint, at each time, only 9 following vehicles can gain access the channel and receive the information (i.e., inter-vehicle distance and the preceding vehicle’s velocity and acceleration) of preceding vehicle. Denote by $es_i(t) = s_{i-1} - s_i - d - L, ev_i(t) = v_{i-1} - v_i$ and $ea_i(t) = a_{i-1} - a_i$ the tracking error, velocity error and acceleration error, where $i = 1, 2, \dots, 10, L$ is the desired vehicle spacing and d is the vehicle length. Then the vehicle following error model can be written as

$$\dot{x}_i(t) = A_i x_i(t) - B_i u_i(t) + B_i w_i(t),$$

where $x_i(t) = [es_i(t) \quad ev_i(t) \quad ea_i(t)]^T, w(k) = u_{i-1}(k)$ is the disturbance.

Let $\zeta_i = 0.25, r = 3, m_i = n_i = 3, \tau_i = 0.9s$ and $\Gamma_i = 0.1s$, then from Theorem 4 and using Matlab LMI toolbox, the controller gain of each following vehicle is given by

$$K(t) = L(t)S_1^{-1}(t),$$

where

$$L(t) = \begin{cases} (1 - \beta_1)L_m + \beta_1 L_{m+1}, & t \in [t_{2j} + \alpha_m, t_{2j} + \alpha_{m+1}) \\ L_3, & t \in [t_{2j} + \alpha_3, t_{2j+1}), \end{cases}$$

$$S_1(t) = \begin{cases} (1 - \beta_1)S_{1,m} + \beta_1 S_{1,m+1}, & t \in [t_{2j} + \alpha_m, t_{2j} + \alpha_{m+1}) \\ S_{1,3}, & t \in [t_{2j} + \alpha_3, t_{2j+1}). \end{cases}$$

with $\beta_1 = 5(t - t_{2j}) - m, \alpha_m = 0.3m, m = 0, 1, 2,$

$$S_{1,0} = \begin{bmatrix} 0.5678 & -0.3916 & 0.0223 \\ -0.3916 & 0.6611 & -0.9403 \\ 0.0223 & -0.9403 & 21.8355 \end{bmatrix},$$

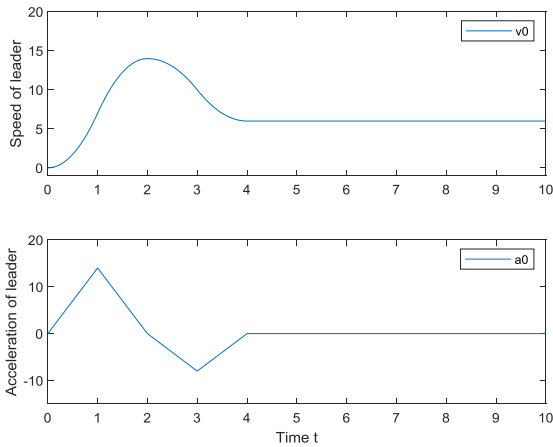


FIGURE 3. Profile of the leading vehicle.

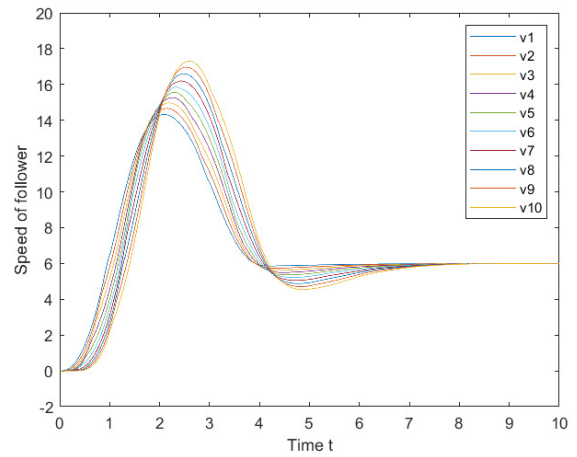


FIGURE 5. Speed of following vehicles.

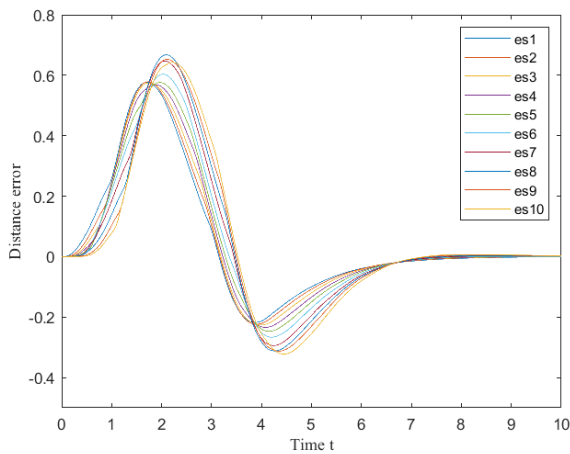


FIGURE 4. Distance errors.

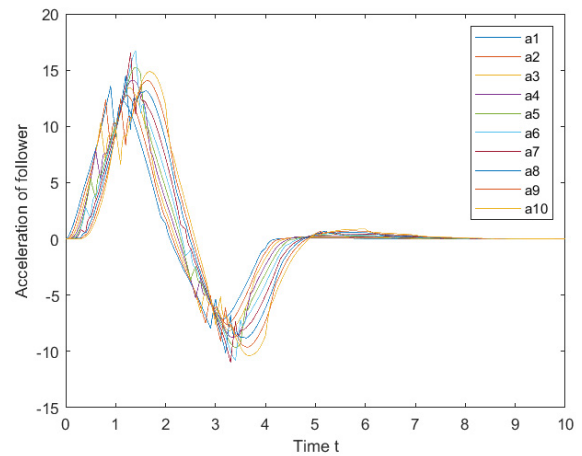


FIGURE 6. Acceleration of the following vehicles.

$$S_{1,1} = \begin{bmatrix} 0.5227 & -0.3645 & 0.0195 \\ -0.3645 & 0.5755 & -0.9358 \\ 0.0195 & -0.9358 & 8.2858 \end{bmatrix},$$

$$S_{1,2} = \begin{bmatrix} 0.4808 & -0.3379 & 0.0275 \\ -0.3379 & 0.5079 & -0.9021 \\ 0.0275 & -0.9021 & 8.0672 \end{bmatrix},$$

$$S_{1,3} = \begin{bmatrix} 0.4397 & -0.3034 & 0.0706 \\ -0.3034 & 0.4346 & -0.7774 \\ 0.0706 & -0.7774 & 3.0931 \end{bmatrix},$$

$$L_{1,0} = [-0.0548 \quad 1.4958 \quad 66.2026],$$

$$L_{1,1} = [-0.1280 \quad 0.9134 \quad 28.9724],$$

$$L_{1,2} = [-0.1397 \quad 0.9125 \quad 26.7082],$$

$$L_{1,3} = [-0.1613 \quad 0.7905 \quad 24.5844].$$

Suppose that the leading vehicle accelerates first and then decelerate. The profile of the leading vehicle’s velocity and acceleration is shown in Figure 3.

According to Theorem 5, there exists a period channel assignment policy that stabilizes the vehicle platoon systems with period $T = 1s$. In one scheduling period T , each following vehicle gain access the channel for $0.9s$ and then disconnected from the network for $0.1s$. The communication

scheduling policy for the 11 following vehicles is given as below.

$$C_1 : \underbrace{V_1 \cdots V_1}_{0.9s} \quad \underbrace{V_2 \cdots V_2}_{0.1s}$$

$$C_2 : \underbrace{V_2 \cdots V_2}_{0.8s} \quad \underbrace{V_3 \cdots V_3}_{0.2s}$$

$$C_3 : \underbrace{V_3 \cdots V_3}_{0.7s} \quad \underbrace{V_4 \cdots V_4}_{0.3s}$$

$$C_4 : \underbrace{V_4 \cdots V_4}_{0.6s} \quad \underbrace{V_5 \cdots V_5}_{0.4s}$$

$$C_5 : \underbrace{V_5 \cdots V_5}_{0.5s} \quad \underbrace{V_6 \cdots V_6}_{0.5s}$$

$$C_6 : \underbrace{V_6 \cdots V_6}_{0.4s} \quad \underbrace{V_7 \cdots V_7}_{0.6s}$$

$$C_7 : \underbrace{V_7 \cdots V_7}_{0.3s} \quad \underbrace{V_8 \cdots V_8}_{0.7s}$$

$$C_8 : \underbrace{V_8 \cdots V_8}_{0.2s} \quad \underbrace{V_9 \cdots V_9}_{0.8s}$$

$$C_9 : \underbrace{V_9 \cdots V_9}_{0.1s} \quad \underbrace{V_{10} \cdots V_{10}}_{0.9s}$$

All the following vehicles are controlled to follow the lead vehicle by the proposed controller and scheduling policy. The distances error, velocity and acceleration of the following vehicles are given in Figure 4, Figure 5 and Figure 6, respectively. From Figure 4 one can see that all the distances errors converge to zeros. Then we can conclude that the proposed controller and scheduling policy can drive the following vehicles to keep a desired spacing with their proceeding vehicles no matter whether the leading vehicle accelerates or decelerates.

V. CONCLUSION

This paper addressed a joint problem of control and channel allocation for simultaneous stabilization of a collection of LTI systems sharing a limited bandwidth communication network. By employing a time-scheduled Lyapunov function, sufficient conditions on the GAS stability of each plant are presented which relates to a pair of upper and lower bounds of dwell time. Correspondingly, based on this stability condition, a co-design methodology of channel scheduling and feedback control was presented. The periodic channel assignment policy proposed is easy for implementation, but it is worthwhile and interesting to extend the framework to non-periodic scheduling schemes in the future work. As channels may be stochastically assigned in practice, the results of this study can also be extended to the case of stochastically assigned channels.

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