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SPECIAL SECTION ON NEW WAVEFORM DESIGN AND AIR-INTERFACE FOR FUTURE HETEROGENEOUS NETWORK TOWARDS 5G

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# Optimal and Robust Interference Efficiency Maximization for Multicell Heterogeneous Networks

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**ABSTRACT** To improve energy efficiency and reduce the interference to macro users (MUs), this paper investigates the robust resource allocation (RA) problem for maximizing the interference efficiency of users (i.e., total rate/sum interference) in heterogeneous networks. Under perfect channel state information (CSI), the considered RA problem is formulated as a multivariate nonlinear programming problem with constraints on the maximum interference power of MUs, the minimum rate requirement of each femto user, and the maximum transmit power of the femto base station. The original fractional programming problem is converted into a convex optimization problem by using Dinkelbach's method, the logarithmic transformation method, and the successive convex approximation method, where the closed-form solution is obtained with the Lagrangian dual approach. Moreover, with imperfect CSI consideration, the original problem is reformulated as a robust RA problem that is transformed into a deterministic and convex optimization problem by using an inequality transformation approach. In addition, the computational complexity and the cost of robustness are also analyzed. The simulation results verify that the proposed algorithm has better interference efficiency and robustness by comparing with the existing algorithms.

**INDEX TERMS** Heterogeneous wireless networks, interference efficiency, robust resource allocation, successive convex approximation.

## I. INTRODUCTION

With the advent of next-generation Internet technologies and the rapid growth of the number of wireless terminal devices, the available spectrum resources are becoming less and less. The contradiction between the continued increase in the bandwidth requirements of users and the limited spectrum resources has become more apparent. How to improve spectrum utilization is an important direction for the development of 5G communication technologies [1]. Heterogeneous networks (HetNets) [2], [3] have become a research hotspot for next-generation communication technologies to improve spectrum resource utilization and reduce coverage holes by allowing small cells (microcell, picocell, femtocell) coexisting with macro cells via spectrum multiplex mode.

Comparing with HetNets, there are many blind spots for conventional homogeneous networks (e.g., single cellular networks), and the data rate requirements of users in hot spots are difficult to meet. In HetNets, macro cell networks (MCNs) usually provide wide-area coverage, however, they cannot provide reliable communication for the indoor environment due to shadowing effect. Additionally, by introducing small cells to the existing MCNs, it is very promising to improve data requirements and system throughput for indoor scenarios or local communication environments. However, both the cross-tier interference and the intra-tier interference are the bottlenecks of performance improvement. In order to realize resource sharing and network coexistence in such complex network scenarios, resource allocation (RA) has become a research hotspot in academia and industry because it can alleviate the mutual interference among users and improve network capacity [4], [5].

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The current research works on RA problems in HetNets have yielded many meaningful results. The overview can be divided into three categories: (1) interference management and coordination, such as reducing the interference between two types of overlapping networks; (2) maximum transmission rate/throughput, e.g., maximize the total rate or throughput; (3) maximum energy efficiency (EE), namely, maximize the ratio of the total data rate to the total power consumption [6]. In terms of interference management, Ahuja *et al.* in [7] proposed a distributed interference management strategy to eliminate the interference power among adjacent users in HetNets. Shifat *et al.* in [8] used game theory to study interference management and quality of service (QoS) supervising in HetNets. Elsherif *et al.* in [9] proposed an adaptive graph coloring method to achieve interference management and user's fairness. Although the above interference management methods can effectively alleviate and control the interference power among different users or networks, it cannot improve the overall network performance, such as rate maximization. From the aspect of overall performance improvement, Al-Zahrani and Yu [10] studied the power control algorithm for maximizing the sum rate of femto users (FUs). In order to analyze and solve the problem conveniently, they assumed that only one user is each femtocell network (FN), and channel state information (CSI) is assumed to be perfectly obtained. This system model is too ideal to meet the needs of actual network. In [11], Huang *et al.* investigated the fairness-based distributed RA algorithm for maximizing the total throughput of FUs. Although the above results can effectively improve the overall communication quality, they cannot further improve energy utilization. In order to improve transmission rate in unit energy, Zhang *et al.* in [12] used the gradient-assisted binary search algorithm to study the EE-based power allocation and backhaul bandwidth allocation in heterogeneous small cell networks. For multi-user cognitive HetNets, Xie *et al.* in [13] studied the problem of maximizing the FUs' EE. With the consideration of user's QoS constraint, based on the non-cooperative game model, Bacci *et al.* in [14] studied the power allocation problem for maximizing uplink EE of users in orthogonal frequency division multiple access (OFDMA) HetNets. For heterogeneous macro-pico networks, Li *et al.* in [15] studied the problem of EE maximization RA with user's priority.

Although the above works constrain the maximum interference power from FUs to each MU for protecting the QoS of MUs, it does not impose the effect to overall system performance. In this paper, we introduce a novel interference efficiency[1] (IE) that is defined as the ratio of total data rate of FUs to the overall cross-tier interference power of MUs to address this issue. In order to reduce the interference power to MUs as well as improve the data rate of FUs, we study the IE maximization RA problems in multiuser HetNets by joint optimizing user association and power allocation under the cases of perfect CSI and imperfect CSI. Our main contributions are listed:

- We build a hybrid RA optimization model with user association and power allocation where the constraints of minimum data rate requirement of each FU, the maximum transmit power of each femto BS (FBS), the maximum interference power of each MU and the user association are considered simultaneously. The original problem is a non-convex nonlinear programming (NCNP) problem, and it is not easy to obtain the optimal solutions.
- Under perfect CSI, the NCNP problem is transformed into a subtractive one by using the Dinkelbach's method. And the problem is converted into a convex one by using the successive convex approximation (SCA) method and the exponential transformation approach. Moreover, the analytical solutions are obtained using Lagrangian dual theory and subgradient methods. Computational complexity is also given.
- Under imperfect CSI, the NCNP problem (e.g., good channel parameters) is reformulated as a robust RA problem under bounded channel uncertainties. By using the worst-case approach and the Cauchy-Buniakowsky-Schwarz inequality, robust uncertain constraints and objective function are converted into the convex ones. Finally, the cost of robustness (i.e., the performance gap between the non-robust approach and the robust approach) is analyzed.
- Simulation results demonstrate the proposed algorithm has better IE and robustness by comparing with the traditional EE-based maximization RA approach and the rate-based maximization RA approach.

The rest of the paper is organized as follows. In Section II, we introduce the system model and formulate an IE-based maximization RA problem for multicell HetNets. Next, Section III presents the optimal user association and power allocation algorithm under perfect CSI to solve our formulated non-convex optimization problem. Then Section IV studied the robust RA problem under the spherical uncertainty sets of uncertain channel gains, and the cost of robustness is also presented. Section V gives several simulation results to show the advantage of the proposed algorithms. Finally, Section VI concludes the paper.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a two-tier multiuser HetNet with one MCN and $M$ FNs, as shown in Fig. 1. Each user and each BS are equipped with a single antenna. There are $N$ MUs and $K$ FUs defined as $\forall n \in \mathcal{N} = \{1, 2, \cdots, N\}$ and $\forall i, k \in \mathcal{K} = \{1, 2, \cdots, K\}$, respectively. Multiple FNs can access the licensed spectrum owned by MUs in an underlaying way. That is to say that

[1] Please note that such a metric was firstly introduced and defined as the number of bits transmitted per unit of interference energy imposed on the primary user receiver [16], but it cannot be directly used in our considered HetNets. The reason is as follows. In cognitive networks, there only exists the interference power on neighboring users, while in our considered network, the interference power is from not only the cross-tier interference power but also the intra-tier interference power.

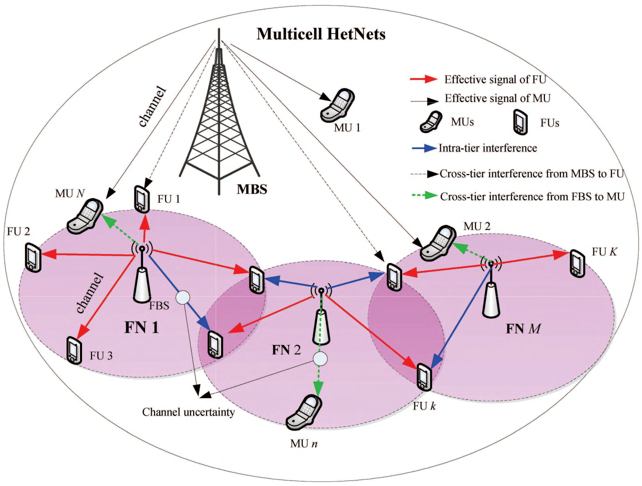


**FIGURE 1. Multicell underlay heterogeneous networks.**

FUs can adjust transmit power to satisfy their performance requirements while protecting the QoS of MUs. Without loss of generality, the normalized bandwidth is considered in this paper, i.e., $B = 1$ Hz. System symbols are presented in Table 1.

According to information theory, the signal-to-interference-plus-noise ratio (SINR) of FU $k$ in FN $m$ can be formulated as

$$\gamma_{k,m} = \frac{p_{k,m}h_{k,m}}{z_{k,m}}, \tag{1}$$

where $z_{k,m} = \sum_{i \neq k} p_{i,m}h_{i,m} + \sigma_{k,m}$ denotes the sum of interference and noise at the $k$-th FU in FN $m$. The first item denotes the intra-tier interference power from other FNs. $\sigma_{k,m} = \sigma^2 + \sum_n p_n g_{n,k,m}$ denotes the noise and interference from MUs (e.g., cross-tier interference).

Based on Shannon's theorem, the achievable data rate of FU $k$ in FN $m$ is

$$R_{k,m} = \log_2(1 + \gamma_{k,m}). \tag{2}$$

Because each FU can only access one FBS during every time slot, the user-association factor needs to satisfy the following constraint, given by

$$\sum_m s_{k,m} = 1, \quad s_{k,m} = \{0, 1\}, \ \forall k, m. \tag{3}$$

Moreover, each BS is impossible to provide unlimited transmission power due to the limitation of device capacity. The transmit power from the FBS to FUs should not exceed the maximum power threshold, i.e.,

$$\sum_k s_{k,m}p_{k,m} \leq p_m^{max}, \quad \forall m. \tag{4}$$

In order to guarantee the service quality of each FU, the allocated power to each FU should also satisfy the following minimum rate constraint, i.e.,

$$\sum_m s_{k,m}R_{k,m} \geq R_k^{min}, \quad \forall k. \tag{5}$$

**TABLE 1. Symbol notations.**

| Symbol | Meaning |
|---|---|
| $N$ | the number of MUs |
| $K$ | the number of FUs |
| $U$ | the total IE of FUs |
| $s_{k,m}$ | the user association factor |
| $M$ | the number of FNs |
| $\sigma^2$ | the background noise at the receiver |
| $R_{k,m}$ | the data rate of FU $k$ in FN $m$ |
| $h_{k,m}$ | the channel gain from FBS $m$ to FU $k$ |
| $p_n$ | the transmit power from MBS to MU $n$ |
| $p_{k,m}$ | the transmit power from FBS $m$ to FU $k$ |
| $R_k^{min}$ | the minimum rate requirement of FU $k$ |
| $\gamma_{k,m}$ | the received SINR at the $k$-th FU in FN $m$ |
| $z_{k,m}$ | sum of interference and noise at FU $k$ in FN $m$ |
| $I_n^{th}$ | the maximum interference threshold at MU $n$ |
| $g_{n,k,m}$ | the channel gain from MU $n$ to FU $k$ in FN $m$ |
| $g_{k,m,n}$ | the channel gain from FU $k$ in FN $m$ to MU $n$ |

Furthermore, to achieve spectrum sharing, the cross-tier interference from all FBSs to each MU receiver (MU-Rx) is constrained by

$$\sum_m \sum_k s_{k,m}p_{k,m}g_{k,m,n} \leq I_n^{th}, \quad \forall n. \tag{6}$$

In order to realize resource sharing in HCNs, it is necessary to satisfy the performance of MUs and simultaneously ensure the QoS of the FUs for optimizing system performance. In order to protect the QoS of MUs greatly, it is possible to improve the date rate of FNs as well as reduce the interference power to MUs as much as possible by designing the following IE criteria.

$$U = \frac{\sum_m \sum_k s_{k,m}R_{k,m}}{\sum_n \sum_m \sum_k s_{k,m}p_{k,m}g_{k,m,n}}, \tag{7}$$

The physical meaning of the function (7) can be explained that FUs can adjust their transmit power to achieve higher data rate and reduce more harmful interference to MUs as much as possible.

According to the constraints (3)-(6), the joint user association and power allocation problem for maximizing the IE of FUs can be formulated as

$$\begin{aligned}
\max_{s_{k,m},p_{k,m}} \ & U = \frac{\sum_m \sum_k s_{k,m}R_{k,m}}{\sum_n \sum_m \sum_k s_{k,m}p_{k,m}g_{k,m,n}} \\
s.t. \ C1: & \sum_k s_{k,m}p_{k,m} \leq p_m^{max}, \quad \forall m, \\
C2: & \sum_m \sum_k s_{k,m}p_{k,m}g_{k,m,n} \leq I_n^{th}, \quad \forall n, \\
C3: & \sum_m s_{k,m}R_{k,m} \geq R_k^{min}, \quad \forall k, \\
C4: & \sum_m s_{k,m} = 1, \quad s_{k,m} = \{0, 1\}, \ \forall k, m,
\end{aligned} \tag{8}$$

where the constraints $C2$ and $C3$ are used to guarantee the QoS requirements of MUs and FUs. Moreover, $C1$ and $C2$ constrain the upper bound of transmit power $p_{k,m}$.

*Remark*: Here we highlight the difference between the IE and the conventional EE. To evaluate the performance of problem (8), the conventional EE-based objective function is given as

$$\eta_E = \frac{\sum_m \sum_k s_{k,m} R_{k,m}}{\sum_m \sum_k s_{k,m} p_{k,m} + M P_c}. \tag{9}$$

where $P_c$ denotes the circuit power consumption of each FBS. Compared with two kinds of utility function in (8) and (9), it is shown that the efficiency of IE-based function is bigger than that of EE-based function under the assumption of the same power allocation strategy. The reason is that the EE-based function tries to improve the EE by reducing the total transmit power of FUs, but the IE-based function is to enhance the EE by limiting the total interference power to MUs. Due to the fact of fading channel, the channel gain $g_{k,m,n} \leq 1$ holds [19]. As a result, if the feasible region is same, the IE of (8) is better than that of the EE-based function in (9).

## III. OPTIMAL RA WITH PERFECT CSI

Problem (8) is a non-robust RA problem where there are no parameter uncertainties in the optimization problem. In this section, we will discuss the optimal RA under the assumption of perfect CSI. The design of robust optimization problem will be provided in the next section.

### A. TRANSFORMATION OF CONVEX PROBLEM

Due to the impact of the fractional objective function $U$ and the binary parameter $s_{k,m}$, the problem (8) is a non-convex problem which is difficult to directly obtain the optimal solutions. Furthermore, the computational complexity is very high if the exhaustive search approach is applied for achieving user association. When the number of FUs $K \to \infty$ holds, the designed strategy cannot be used in practical HetNets. Besides, if the user association is completed, namely, $s_{k,m}, \forall k, m$ is a fixed value, the above problem is still non-convex due to the coupled transmit power (i.e., $p_{k,m}$ and $p_{i,m}$) in the utility function.

In order to solve the issue, based on time-sharing scheme [18], the integer variable $s_{k,m}$ can be relaxed to be the range of [0,1] by using the transformation of $\tilde{p}_{k,m} = p_{k,m} s_{k,m}$, which has a zero-duality gap proved in [19]. Therefore, the original problem (8) can be equivalent to the following form, i.e.,

$$\begin{aligned} \max_{s_{k,m}, \tilde{p}_{k,m}} \quad & U = \frac{\sum_m \sum_k s_{k,m} \tilde{R}_{k,m}}{\sum_n \sum_m \sum_k \tilde{p}_{k,m} g_{k,m,n}} \\ s.t.\ & \tilde{C}1 : \sum_k \tilde{p}_{k,m} \leq p_m^{max}, \quad \forall m, \\ & \tilde{C}2 : \sum_m \sum_k \tilde{p}_{k,m} g_{k,m,n} \leq I_n^{th}, \quad \forall n, \\ & \tilde{C}3 : \sum_m s_{k,m} \tilde{R}_{k,m} \geq R_k^{min}, \quad \forall k, \\ & \tilde{C}4 : 0 \leq s_{k,m} \leq 1, \quad \forall k, m, \end{aligned} \tag{10}$$

where $\tilde{R}_{k,m} = \log_2(1 + \tilde{\gamma}_{k,m})$, $\tilde{\gamma}_{k,m} = \tilde{p}_{k,m} h_{k,m} / \tilde{z}_{k,m}$ and $\tilde{z}_{k,m} = s_{k,m} z_{k,m}$. According to Dinkelbach's method [20]–[22], the objective function in (10) can be rewritten as

$$f(U) = \sum_m \sum_k s_{k,m} \tilde{R}_{k,m} - U \sum_n \sum_m \sum_k \tilde{p}_{k,m} g_{k,m,n}. \tag{11}$$

From (11), it is clear that $f(U) < 0$ when $U \to +\infty$. Otherwise $f(U) \geq 0$. Thus the function $f(U)$ ia a decreasing function with respect to the variable $U$. Assume the optimal RA strategy is $(s_{k,m}^*, p_{k,m}^*)$, the maximum IE $U^*$ can be obtained by the following function

$$\begin{aligned} f(U^*) = & \sum_m \sum_k s_{k,m}^* \tilde{R}_{k,m}(\tilde{p}_{k,m}^*) \\ & - U^* \sum_n \sum_m \sum_k \tilde{p}_{k,m}^* g_{k,m,n} = 0 \end{aligned} \tag{12}$$

Thus the optimal IE is

$$U^* = \frac{\sum_m \sum_k s_{k,m}^* \tilde{R}_{k,m}(\tilde{p}_{k,m}^*)}{\sum_n \sum_m \sum_k \tilde{p}_{k,m}^* g_{k,m,n}}. \tag{13}$$

As a result, we obtain the following problem with the subtractive utility function, i.e.,

$$\begin{aligned} \max_{s_{k,m}, \tilde{p}_{k,m}} \quad & \sum_m \sum_k s_{k,m} \tilde{R}_{k,m} \\ & - U \sum_n \sum_m \sum_k \tilde{p}_{k,m} g_{k,m,n} \\ s.t.\ & \tilde{C}1 : \sum_k \tilde{p}_{k,m} \leq p_m^{max}, \quad \forall m, \\ & \tilde{C}2 : \sum_m \sum_k \tilde{p}_{k,m} g_{k,m,n} \leq I_n^{th}, \quad \forall n, \\ & \tilde{C}3 : \sum_m s_{k,m} \tilde{R}_{k,m} \geq R_k^{min}, \quad \forall k, \\ & \tilde{C}4 : 0 \leq s_{k,m} \leq 1, \quad \forall k, m. \end{aligned} \tag{14}$$

Problem (14) is still non-convex due to the form of $\tilde{R}_{k,m}$. With the help of SCA [23], the non-convex rate function can be substituted by the corresponding lower bound, i.e.,

$$\tilde{R}_{k,m} \geq a_{k,m} \log_2(\tilde{\gamma}_{k,m}) + b_{k,m}. \tag{15}$$

where $a_{k,m}$ and $b_{k,m}$ are auxiliary variables, i.e.,

$$a_{k,m} = \frac{\bar{\gamma}_{k,m}}{1 + \bar{\gamma}_{k,m}}, \tag{16}$$

$$b_{k,m} = \log_2(1 + \bar{\gamma}_{k,m}) - a_{k,m} \log_2(\bar{\gamma}_{k,m}), \tag{17}$$

For the first iteration, we can initialize $a_{k,m} = 1$, $b_{k,m} = 0$. And in the next iteration, they can be updated by (16) and (17). This initialized values are reasonable, because $\log_2(1 + \gamma_{k,m}) \approx \log_2 \gamma_{k,m}$ holds with the increasing SINR of each user. Additionally, the same initial value has been given in [23], [24]. The lower bound will become tight when $\tilde{\gamma}_{k,m} = \bar{\gamma}_{k,m}$. Additionally, since $\tilde{\gamma}_{k,m}$ is a non-convex function with respect to $p_{k,m}, \forall k, m$, the problem (14) can be transformed into the following optimization problem with the logarithmic transformation $\bar{p}_{k,m} = \ln(\tilde{p}_{k,m})$, i.e.,

$$\begin{aligned} \max_{s_{k,m}, \bar{p}_{k,m}} \quad & \sum_m \sum_k s_{k,m} \bar{R}_{k,m} \\ & - U \sum_n \sum_m \sum_k e^{\bar{p}_{k,m}} g_{k,m,n} \\ s.t.\ & \bar{C}1 : \sum_k e^{\bar{p}_{k,m}} \leq p_m^{max}, \quad \forall m, \end{aligned}$$

$$\begin{aligned}
&\bar{C}2: \sum\nolimits_m \sum\nolimits_k e^{\bar{p}_{k,m}} g_{k,m,n} \le I_n^{th}, \quad \forall n,\\
&\bar{C}3: \sum\nolimits_m s_{k,m} \bar{R}_{k,m} \ge R_k^{min}, \quad \forall k,\\
&\tilde{C}4: 0 \le s_{k,m} \le 1, \quad \forall k, m. \qquad (18)
\end{aligned}$$

where $\bar{R}_{k,m} = \frac{a_{k,m}}{\ln 2} \times \ln(\frac{e^{\bar{p}_{k,m}} h_{k,m}}{\sigma_{k,m}+\sum_{i\neq k} e^{\bar{p}_{i,m}} h_{i,m}}) + b_{k,m}$. Note that the log-sum-exp function is convex [25], therefore the problem (18) becomes a convex optimization problem. In order to obtain the analytical solutions of resource allocation problem, we will use the Lagrange dual approach and subgradient method to solve the problem (18).

### B. OPTIMAL RA ALGORITHM

For the optimization problem (18), it can be directly solved by using the Lagrange dual approach. The Lagrange function of (18) is written as

$$\begin{aligned}
&L\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&= \sum\nolimits_m \sum\nolimits_k s_{k,m}\bar{R}_{k,m} - U \sum\nolimits_n \sum\nolimits_m \sum\nolimits_k e^{\bar{p}_{k,m}} g_{k,m,n}\\
&\quad + \sum\nolimits_m \lambda_m (p_m^{max} - \sum\nolimits_k e^{\bar{p}_{k,m}}) + \sum\nolimits_m \sum\nolimits_k \alpha_{k,m}(1 - s_{k,m})\\
&\quad + \sum\nolimits_k \chi_k (\sum\nolimits_m s_{k,m}\bar{R}_{k,m} - R_k^{min})\\
&\quad + \sum\nolimits_n \beta_n (I_n^{th} - \sum\nolimits_m \sum\nolimits_k e^{\bar{p}_{k,m}} g_{k,m,n}), \qquad (19)
\end{aligned}$$

where $\lambda_m \ge 0$, $\beta_n \ge 0$, $\chi_k \ge 0$ and $\alpha_{k,m} \ge 0$ are the related Lagrange multipliers for the constraints in (18). The function can be rewritten as

$$\begin{aligned}
&L\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad = \sum\nolimits_m \sum\nolimits_k L_{k,m}\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad\quad + \sum\nolimits_m \lambda_m p_m^{max} + \sum\nolimits_n \beta_n I_n^{th} - \sum\nolimits_k \chi_k R_k^{min}\\
&\quad\quad + \sum\nolimits_m \sum\nolimits_k \alpha_{k,m}. \qquad (20)
\end{aligned}$$

where

$$\begin{aligned}
&L_{k,m}\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad = (1+\chi_k) s_{k,m}\bar{R}_{k,m} - \sum\nolimits_n (U+\beta_n) e^{\bar{p}_{k,m}} g_{k,m,n}\\
&\quad\quad - \lambda_m e^{\bar{p}_{k,m}} - \alpha_{k,m} s_{k,m}, \qquad (21)
\end{aligned}$$

The dual problem is

$$\begin{aligned}
&\min_{\lambda_m, \beta_n, \chi_k, \alpha_{k,m}} D\left(\lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad s.t.\ \lambda_m \ge 0, \quad \beta_n \ge 0,\ \chi_k \ge 0,\ \alpha_{k,m} \ge 0. \qquad (22)
\end{aligned}$$

where

$$D\left(\lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right) = \max_{s_{k,m}, \bar{p}_{k,m}} L\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right), \qquad (23)$$

According to (20)-(23), it can be well-known that the dual problem can be solved by the two-layer iteration approach, namely, the Dinkelbach method for the outer layer iteration (e.g., $U$) and the updating Lagrange multipliers as well as primal variables for the inner layer iteration (e.g., $p_{k,m}$, $\lambda_m$).

According to the Karush-Kuhn-Tucker condition [25], the optimal power allocation is

$$p_{k,m}^* = \frac{\tilde{p}_{k,m}}{s_{k,m}} = \frac{e^{\bar{p}_{k,m}}}{s_{k,m}} = \frac{(1+\chi_k) a_{k,m}/\ln 2}{\lambda_m + \sum_n (U+\beta_n) g_{k,m,n}}. \qquad (24)$$

In order to obtain user association factor $s_{k,m}$, the partial derivative with the variable $s_{k,m}$ is

$$\frac{\partial L_{k,m}(\cdot)}{\partial s_{k,m}} = \varphi_{k,m} - \alpha_{k,m} \begin{cases} < 0, & s_{k,m} = 0\\ = 0, & 0 < s_{k,m} < 1\\ > 0, & s_{k,m} = 1, \end{cases} \qquad (25)$$

where

$$\varphi_{k,m} = (1+\chi_k)\bar{R}_{k,m}(p_{k,m}^*) - \sum\nolimits_n (U+\beta_n) p_{k,m}^* g_{k,m,n} \varphi_{k,m}. \qquad (26)$$

Since $\alpha_{k,m}$ is the Lagrange multiplier for user association (BS selection), the optimal FBS $m^*$ is assigned to the $k$-th FU with the largest $\varphi_{k,m}$, namely

$$s_{k,m}^* = 1 \Big| m^* = \max_m \varphi_{k,m}, \quad \forall k. \qquad (27)$$

Additionally, the outer layer Lagrange multipliers can be updated by using the subgradient method, that is

$$\lambda_m(t+1) = [\lambda_m(t) - d_1 \times (p_m^{max} - \sum\nolimits_k \tilde{p}_{k,m})]^+, \qquad (28)$$

$$\beta_n(t+1) = [\beta_n(t) - d_2 \times (I_n^{th} - \sum\nolimits_m \sum\nolimits_k \tilde{p}_{k,m} g_{k,m,n})]^+, \qquad (29)$$

$$\chi_k(t+1) = [\chi_k(t) - d_3 \times (\sum\nolimits_m s_{k,m}\bar{R}_{k,m} - R_k^{min})]^+. \qquad (30)$$

where $t$ denotes the iteration index. $d_1$, $d_2$ and $d_3$ are non-negative step sizes, and $[x]^+ = \max(0, x)$. When the step sizes are suitably chose, the algorithm can converge to the equilibrium quickly.

### C. COMPUTATIONAL COMPLEXITY

Define $L$ as the maximum iteration number for outer layer loop and $T$ as the maximum iteration number for inner layer loop. According to (26) and (27), the calculation for every FBS on each FU needs $\mathcal{O}(KM)$ operations. The calculation of $\lambda_m$, $\beta_n$ and $\chi_k$ requires $\mathcal{O}(M)$, $\mathcal{O}(N)$ and $\mathcal{O}(K)$ operations, respectively. Therefore, the complexity of Lagrange multipliers is $\mathcal{O}(MNK)$. Because $T$ is a polynomial function of $\mathcal{O}(M^2NK^2T)$, the total complexity of the proposed algorithm is $\mathcal{O}(M^2K^2TLN)$. Furthermore, the value of $L$ can be very small when the step sizes are well chosen. The iterative IE-based RA algorithm is given in Algorithm 1.

## IV. ROBUST RESOURCE ALLOCATION WITH IMPERFECT CSI

Since macro cells and femtocells are two different kinds of networks, they have no obligation to provide related CSI to each other. There must be channel estimation errors. In this section, with the bounded channel uncertainties, we study the

**Algorithm 1** An Iterative IE-Based Maximization RA Algorithm

```
Initialize the maximum number of iterations L and the maximum tolerance ε; Initialized transmit power p_{k,m}(0), ∀k, m with equal power allocation strategy.
Initialize system parameters: maximum transmit power p_m^{max}, maximum interference power threshold I_n^{th}, and minimum data rate threshold R_k^{min}. Given the number of MUs and FUs as well as the number of FNs. Define a_{k,m}(0) and b_{k,m}.
Calculate the initialized IE U(0) by (13).
Set the iteration number l ← 0.
while | (Σ_m Σ_k s_{k,m} R̃_{k,m}(l)) / (Σ_n Σ_m Σ_k p̃_{k,m}(l) g_{k,m,n}) − U(l−1) | > ε or l ≤ L do
    Initialize the maximum number of the inner loop iterations T; Set the initialized iteration t ← 0; Initialize Lagrange multipliers λ_m(0), β_n(0), χ_k(0); The step sizes are 0 < d_e < 1, ∀e = {1, 2, 3}.
    while t ≤ T or ||F(t+1) − F(t)|| > ε (F = λ_m, β_n, χ_k) do
        for n = 1 to N do
            for k = 1 to K do
                for m = 1 to M do
                    Calculate the optimal power p_{k,m} by (24);
                    Update the Lagrange multipliers λ_m, β_n and χ_k by (28)-(30);
                    Update s_{k,m*} by using (26) and (27);
                    Update a_{k,m} and b_{k,m} by (16) and (17);
                end for
            end for
        end for
        t ← t + 1;
    end while
    l ← l + 1; U(t) = (Σ_m Σ_k s_{k,m} R̃_{k,m}(t−1)) / (Σ_n Σ_m Σ_k p̃_{k,m}(t−1) g_{k,m,n});
end while
```

robust transmission problem for achieving the IE maximization of FUs. According to the definition of protection function [27], the non-robust problem (10) can be reformulated as the robust optimization problem, given by

$$\begin{aligned}
&\max_{s_{k,m},\tilde{p}_{k,m}} \frac{\sum_m \sum_k s_{k,m}\tilde{R}_{k,m}(\bar{h}_{k,m}) + \sum_k \Delta_k^R}{\sum_n \sum_m \sum_k \tilde{p}_{k,m}\bar{g}_{k,m,n} + \sum_n \Delta_n^{viloate}}\\
&s.t.\ \tilde{C}1, \tilde{C}4,\\
&\quad \tilde{C}2: \sum_m \sum_k \tilde{p}_{k,m}\bar{g}_{k,m,n} + \Delta_n^{viloate} \le I_n^{th}, \quad \forall n,\\
&\quad \tilde{C}3: \sum_m s_{k,m}\tilde{R}_{k,m}(\bar{h}_{k,m}) + \Delta_k^R \ge R_k^{min}, \quad \forall k, \quad (31)
\end{aligned}$$

where the additive uncertainties [28] of channel gains can be formulated as

$$\begin{cases} h_{k,m} = \bar{h}_{k,m} + \Delta h_{k,m}, & \forall k, m, \\ g_{k,m,n} = \bar{g}_{k,m,n} + \Delta g_{k,m,n}, & \forall k, m, n, \end{cases} \quad (32)$$

where $\bar{h}_{k,m}$ and $\bar{g}_{k,m,n}$ are the estimated channel gains of FU's links and FU-to-MU's links, respectively. $\Delta h_{k,m}$ and $\Delta g_{k,m,n}$ are the corresponding channel estimation errors which are random variables. When $\Delta h_{k,m} < 0$, $\bar{h}_{k,m} > h_{k,m}$ indicates that the parameter $h_{k,m}$ is overestimated. Otherwise, the channel gain $h_{k,m}$ is underestimated.

Additionally, the protection functions $\Delta_k^R$ and $\Delta_n^{viloate}$ are used to avoid any outage and balance the optimality and the robustness, that is

$$\Delta_n^{viloate} = \max_{g_{k,m,n}\in R_g} \sum_m \sum_k (g_{k,m,n} - \bar{g}_{k,m,n})\tilde{p}_{k,m}. \quad (33)$$

$$\Delta_k^R = \min_{h_{k,m}\in R_h} \sum_m s_{k,m}\tilde{R}(h_{k,m} - \bar{h}_{k,m}). \quad (34)$$

where $R_g$ and $R_h$ are the uncertainty sets. Since the problem (31) becomes an infinite and non-convex problem due to the impact of uncertainty, it is challenging to solve. Combining with (23), based on the Lagrange dual approach, we have

$$\begin{aligned}
&D^{robust}\left(\lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad = \max_{s_{k,m},\tilde{p}_{k,m}} L^{robust}\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right). \quad (35)
\end{aligned}$$

where

$$\begin{aligned}
&L^{robust}\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\\
&\quad = L\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\Big|_{h_{k,m}=\bar{h}_{k,m}, g_{k,m,n}=\bar{g}_{k,m,n}}\\
&\quad\quad + \sum_k (1+\chi_k)\Delta_k^R - \sum_n (U+\beta_n)\Delta_n^{viloate}, \quad (36)
\end{aligned}$$

where the first part presents the Lagrange function of nominal optimization problem, namely, non-robust resource allocation problem. The second item denotes the perturbations of channel uncertainties. In order to get the analytical solution of problem (31), we define the following ball uncertainty sets, i.e.,

$$R_g = \left\{\boldsymbol{g}_n \,\middle|\, \left\|\boldsymbol{g}_n - \bar{\boldsymbol{g}}_n\right\| \le \varepsilon_n \right\}, \quad (37)$$

$$R_h = \left\{\boldsymbol{h}_k \,\middle|\, \left\|\boldsymbol{h}_k - \bar{\boldsymbol{h}}_k\right\| \le \upsilon_k \right\}, \quad (38)$$

where $\varepsilon_n$ and $\upsilon_k$ denotes the upper bound of uncertainty. $\boldsymbol{h}_k = [\frac{h_{1,m}}{h_{k,m}}, \frac{h_{2,m}}{h_{k,m}}, \cdots, \frac{h_{K,m}}{h_{k,m}}]^T$ and $\boldsymbol{g}_n = [g_{1,1,n}, g_{1,2,n}, \cdots, g_{M,1,n}, \cdots, g_{M,K,n}]^T$. As a result, the problem (36) can be decomposed into two subproblems, i.e.,

$$\begin{aligned}
&\max_{g_{k,m,n}\in R_g} \sum_n (U+\beta_n) \sum_m \sum_k (g_{k,m,n} - \bar{g}_{k,m,n})\tilde{p}_{k,m}\\
&s.t.\ R_g = \left\{\boldsymbol{g}_n \,\middle|\, \left\|\boldsymbol{g}_n - \bar{\boldsymbol{g}}_n\right\| \le \varepsilon_n \right\}. \quad (39)
\end{aligned}$$

$$\begin{aligned}
&\min_{h_{k,m}\in R_h} \sum_k (1+\chi_k) \sum_m s_{k,m}\tilde{R}_{k,m}(h_{k,m} - \bar{h}_{k,m})\\
&s.t.\ R_h = \left\{\boldsymbol{h}_k \,\middle|\, \left\|\boldsymbol{h}_k - \bar{\boldsymbol{h}}_k\right\| \le \upsilon_k \right\}. \quad (40)
\end{aligned}$$

In order to obtain the upper bound of problem (39), based on Cauchy-Buniakowsky-Schwarz inequality, we have

$$\begin{aligned}
\Delta_n^{viloate} &= \max_{g_{k,m,n}\in R_g} \sum_m \sum_k (g_{k,m,n} - \bar{g}_{k,m,n})\tilde{p}_{k,m}\\
&= \varepsilon_n \sqrt{\sum_m \sum_k \tilde{p}_{k,m}^2}. \quad (41)
\end{aligned}$$

Therefore, the uncertain part of $\tilde{C}2$ in problem (31) becomes a deterministic one. Since it is very difficult to deal with the square of $\tilde{p}_{k,m}$ for obtaining the analytical solution of transmit power, the constraint relaxation is used here. Practically speaking, the approach can guarantee no outage. And $\varepsilon_n$ can be interpreted as the overall channel uncertainties from FUs to each MU $n$ below the designed threshold. It provides strong protection for MUs.

Because the forward channel of FU $K$ can be obtained by the feedback channel, we can assume the channel estimation error is very small, such as $\Delta h_{k,m} \approx 0$. On the contrary, the channel gains on other intra-tier links cannot be exactly obtained since other neighboring users have no obligation to provide the related information via a cooperation way, e.g., $\Delta h_{i,m} \neq 0, \forall i \neq k$. Therefore, the data rate in (39) can rewritten as

$$\min_{h_k \in R_h} \tilde{R}_{k,m} = a_{k,m}\log_2\left(\frac{\min\limits_{h_{k,m}}(p_{k,m}h_{k,m})}{\max\limits_{h_{i,m}}(z_{k,m})}\right) + b_{k,m}$$
$$\Leftrightarrow a_{k,m}\log_2\left(\frac{p_{k,m}}{\max\limits_{h_{k,m}}(z_{k,m}/h_{k,m})}\right) + b_{k,m}. \quad (42)$$

Based on (38), the problem $\max\limits_{h_{k,m}}(z_{k,m}/h_{k,m})$ can be approximated as

$$\begin{aligned}&\frac{\sigma_{k,m}}{\bar{h}_{k,m}} + \sum\nolimits_{i\neq k} p_{i,m}\frac{\bar{h}_{i,m}}{\bar{h}_{k,m}} \\ &\qquad + \max_{h_{k,m}} \sum\nolimits_{i\neq k} p_{i,m}(h_{i,m} - \bar{h}_{i,m})/\bar{h}_{k,m} \\ &\quad \leq \frac{\sigma_{k,m}}{\bar{h}_{k,m}} + \sum\nolimits_{i\neq k} p_{i,m}\frac{\bar{h}_{i,m}}{\bar{h}_{k,m}} \\ &\qquad + \upsilon_k\sqrt{\sum\nolimits_{i\neq k} p_{i,m}^2}/\bar{h}_{k,m}. \end{aligned} \quad (43)$$

Moreover, because of $\sqrt{\sum_r x_r^2} \leq \sum_r \sqrt{x_r^2}$, substituting (41), (43) into (36), we have

$$\begin{aligned}&L^{robust}\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right) \\ &= L\left(s_{k,m}, p_{k,m}, \lambda_m, \beta_n, \chi_k, \alpha_{k,m}\right)\Big|_{h_{k,m}=\bar{h}_{k,m}, g_{k,m,n}=\bar{g}_{k,m,n}} \\ &\quad + \sum\nolimits_k \sum\nolimits_m (1+\chi_k)s_{k,m}R^r_{k,m} \\ &\quad - \sum\nolimits_n \sum\nolimits_m \sum\nolimits_k (U+\beta_n)\tilde{p}_{k,m}(\bar{g}_{k,m,n}+\varepsilon_n), \end{aligned} \quad (44)$$

where $R^r_{k,m} = a_{k,m}\log_2(\frac{p_{k,m}\bar{h}_{k,m}}{\sum_{i\neq k} p_{i,m}(\bar{h}_{i,m}+\upsilon_k)+\sigma_{k,m}}) + b_{k,m}$. Similar to Section III, based on the Karush-Kuhn-Tucker (KKT) condition and the subgradient updating approach [25], the analytical solution of the robust RA problem (31) is easily obtained through the partial derivative of (44), which is omitted here.

*The cost of robustness*: Define the performance gap as $L_{gap} = L^{robust}(\cdot) - L(\cdot)$, and set the initial IE $U = 0$, we also assume the optimal Lagrange multipliers $\chi_k^*$, $\beta_n$ and power allocation $p^*_{k,m}$. According to (44), the performance gap between the robust approach (e.g., channel perturbation is not zero) and the non-robust approach (e.g., channel perturbation is zero) is presented as

$$\begin{aligned}L_{gap} = &\sum\nolimits_k \sum\nolimits_m (1+\chi_k^*)s^*_{k,m}R^{gap}_{k,m} \\ &- \sum\nolimits_n \sum\nolimits_m \sum\nolimits_k \beta_n^* s^*_{k,m}p^*_{k,m}\varepsilon_n, \end{aligned} \quad (45)$$

where $R^{gap}_{k,m} = -\frac{\sum_{i\neq k} p^*_{i,m}\upsilon_k}{\ln 2\bar{z}_{k,m}}$ and $\bar{z}_{k,m} = \sum_{i\neq k} p^*_{i,m}(\bar{h}_{i,m} + \upsilon_k) + \sigma_{k,m}$. The proof is given in Appendix . Obviously, the Lagrange multiplier $\lambda_m$ for the constraint $\bar{C}1$ cannot influence the performance gap because there is no perturbed parameter in that condition. According to (45), it is analyzed that the sum IE under the optimal RA approach is better than that of robust approach due to $L_{gap} \leq 0$. That is to say that the robust approach can improve the robustness and reduce the outage probabilities by sacrificing certain optimality which is demonstrated in simulation parts of Section V.

## V. SIMULATION RESULTS

In this section, we provide simulation results to show the performance of our proposed algorithm by comparing with the existing algorithms. For simplicity, during simulations, the sum-rate maximization RA algorithm in [10] is defined as 'Max-rate approach'. The EE maximization RA algorithm in [13] is defined as 'Max-EE approach', the proposed IE-based approaches with and without perfect CSI are defined as 'Non-robust IE approach' and 'Robust IE approach', respectively. There are one MCN with two users and three FNs with six users. The transmission radii of MBS and FBS are 500m and 30m respectively. The pathloss model and channel fading model are determined by the standard of heterogeneous development in [30], and the path loss exponent is 3. The noise power density is $-174$dBm/Hz. The maximum transmit power from MBS to MU is 45dBm. The maximum transmit power of FBS is 30dBm. The minimum data requirement of each FU is $R_k^{min} = 1$ bit/s/Hz. The circuit power consumption is $Pc = 0.01$ mW.

Fig. 2 shows the convergence of the proposed algorithm with different FUs. From the figure, it is obvious that the total IE of FUs can quickly converge to the equilibrium point within 10 iterations. Furthermore, the total IE of FNs improves a lot with the increasing number of FUs. Due to the impact of the mutual interference among different FUs, the utility function (e.g., sum IE) cannot increase exponentially when the number of users doubles.

Fig. 3 shows the total IE of FUs versus the maximum interference power threshold under different approaches. As shown in the figure, the total IE of FUs decreases with the increasing interference threshold of MU since the available transmit power becomes bigger. Our proposed non-robust IE approach has the best performance in the IE of FUs, and the max-rate approach has the worst performance. The reason is that the later achieves the RA with consideration of energy consumption. Moreover, the IE performance under the max-EE approach is lower than that of our approaches because the

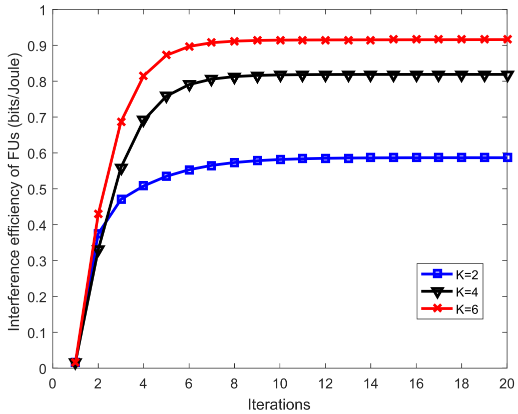


**FIGURE 2.** **Convergence performance of the optimal RA algorithm.**

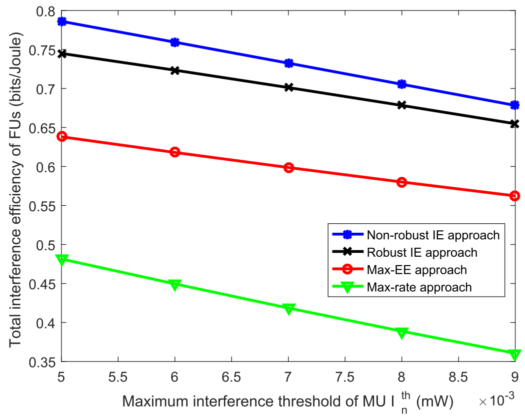


**FIGURE 3.** **Total IE of FUs versus maximum interference threshold $I_n^{th}$ under $\varepsilon_n = 0.02$, $\upsilon_k = 0.001$.**

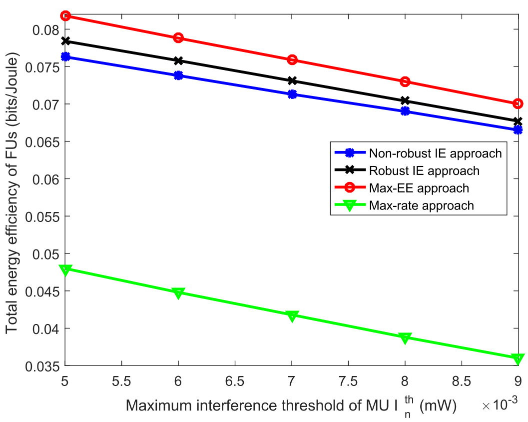


**FIGURE 4.** **Total EE of FUs versus maximum interference threshold $I_n^{th}$ under $\varepsilon_n = 0.02$, $\upsilon_k = 0.001$.**

former causes more interference power to the MUs. Furthermore, the total IE of the proposed robust IE approach is a little less than that of the non-robust approach.

Fig. 4 shows the total EE of FUs versus the maximum interference power threshold under different approaches. As presented in the figure, the total EE of the max-EE approach has the best performance, and the total EE of the max-rate approach has the lowest one. Compared with the max-rate approach, our proposed IE approach has better performance in the EE. The reason is that, the max-rate approach does not consider the total power consumption so that it improves the available transmit power as much as possible to pursuing the objective of rate maximization. The total EE under the robust IE approach is a little higher than that of the non-robust IE approach. Because the maximum available transmit power of the robust IE approach becomes more compact to overcome the effect of channel uncertainties, so that it cannot further improve the total IE of FUs.

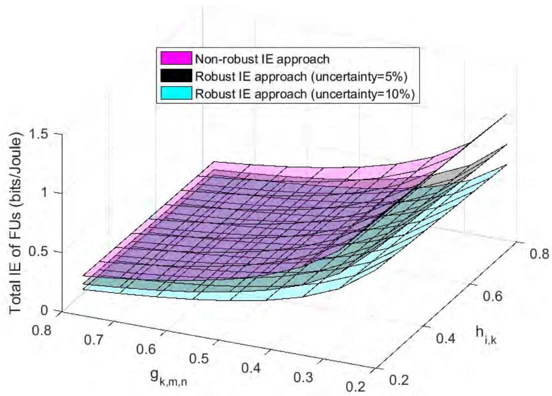


**FIGURE 5.** **Total EE of FUs versus channel gains under different uncertainties.**

Fig. 5 shows the total EE of FUs versus the channel gains under different uncertainties. Define two different levels of uncertainty, such as $\Delta h_{k,m} = 5\%\bar{h}_{k,m}$, $\Delta g_{k,m,n} = 5\%\bar{g}_{k,m,n}$ and $\Delta h_{k,m} = 10\%\bar{h}_{k,m}$, $\Delta g_{k,m,n} = 10\%\bar{g}_{k,m,n}$. From the figure, it is demonstrated that the total IE of both the non-robust IE approach and the robust IE approach increases with the increasing channel gain $h_{i,k}$. Because the available transmit power becomes bigger to improve the data rate of FUs according to (5). Moreover, with the increasing channel gain $g_{k,m,n}$, the total IE of FUs decreases due to the smaller maximum transmit power. The bigger $g_{k,m,n}$ means that the MU-Rx is very close to the FBS. In order to protect the MUs, the available transmit power under this case becomes small. Additionally, with the increasing uncertainty, the performance gap between the non-robust IE approach and the robust IE approach increases for giving more protection to the MUs ahead of time, which has been analyzed in Section IV.

Fig. 6 presents the actually received interference power at the MU-Rx versus the maximum transmit power of FBS. From the figure, it is clear that the received interference power of MU improves with the increasing maximum transmit power threshold of FBS which enlarges the available maximum transmission power. Therefore, there is more power resource to improve the transmission rate of users. Moreover, the interference power under the max-rate approach has the highest interference to the MUs due to users' rate maximization. The received interference power of MU under the robust IE approach is less than that of other approaches.

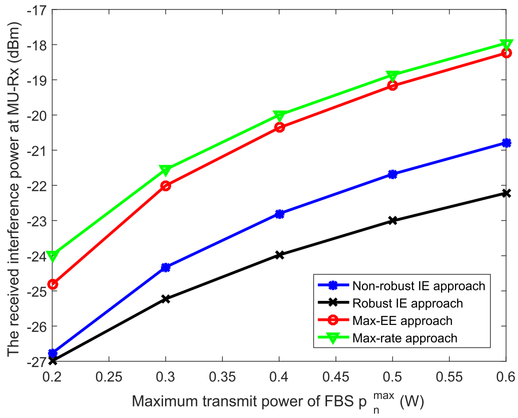


**FIGURE 6.** The received interference power at MU-Rx versus maximum transmit power of FBS.

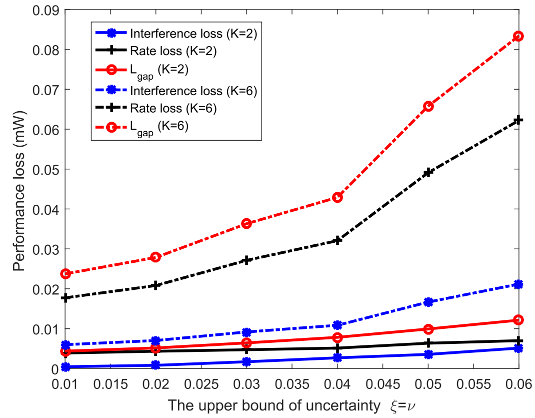


**FIGURE 8.** Performance loss versus the upper bound of channel uncertainties $\varepsilon = \upsilon, \forall n, k$.

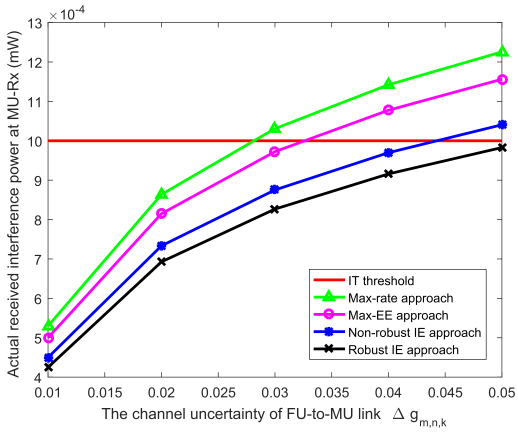


**FIGURE 7.** The received interference power at MU-Rx versus channel uncertainties of FU-to-MU link under $I_n^{th} = 0.001$mW, $\varepsilon_n = 0.05$.

The reason is that it limits the available transmission power to obtain good robustness under channel uncertainties so that it sacrifices some optimal EE.

In order to show the effect of channel uncertainties, Fig. 7 shows the received interference power at the MU-Rx versus channel uncertainties of FU-to-MU link. From the figure, it is well-known that the actual received interference at the MU-Rx increases with the bigger channel uncertainty. Because, for the fixed channel estimation value, the true channel gain becomes bigger with the increase of channel uncertainties. It also means the estimated channel gain deviates from the true value. The channel estimation error has a greater impact on system performance. Obviously, when the received interference power is above the interference temperature (IT) threshold, MUs will generate an outage probability. Compared with the other two approaches (e.g., max-rate approach, max-EE approach), our approaches have the lowest outage probability. The robust IE approach can protect the performance of MU completely. With the objective of maximizing the IE of FUs, our non-robust IE approach generates less interference than traditional approaches. It is well demonstrated that our approaches have better performance.

Fig. 8 shows the performance loss versus the upper bound of channel uncertainties. In order to verify the performance loss conveniently, taking negative operation on both sides in (45), we define the first item as the rate loss (the effect from channel uncertainties on FUs' links) and the second item as the interference loss (the effect from channel uncertainties on FU-to-MU links). From the figure, the performance loss (e.g., optimal-robust) increases with the increasing upper bound of channel uncertainties. Additionally, the performance loss under $K = 6$ is bigger than that under $K = 2$. Because more admitted users improve intra-tier interference and generate more effective interference among users. Moreover, under the higher performance loss, the optimal IE under the non-robust approach is better than that of the robust IE approach, which also verifies the correctness of the theoretical analysis of the cost of robustness.

## VI. CONCLUSION

In this paper, we studied the RA problem for maximizing the total IE of FUs in a two-tier downlink multi-cell HetNet with perfect CSI and imperfect CSI in an underlay spectrum sharing mode. Due to the complexity of the considered user association and power allocation problem, we transformed the fractional programming problem into a convex optimization form. Specifically, the time-sharing method was used to convert the integer programming problem into a continuous optimization problem. The Dinkelbach's method was used to transform the fractional objective function into a subtractive form. The SCA approach and the logarithmic transformation scheme were introduced to convert the problem into a convex one which was solved by using the Lagrange dual approach and subgradient updating methods. Moreover, to further improve the robustness of HetNets, we reformulated the original problem into a robust optimization problem under bounded channel uncertainties. Considering the spherical uncertainty sets, the Lagrange dual function was converted into a deterministic one with the help of the worst-case approach and the Cauchy-Buniakowsky-Schwarz inequality. Furthermore, the cost of robustness and computational complexity were also analyzed. Simulation results demonstrated that the proposed algorithm can improve the IE, the robust performance and reduce the cross-tier interference.

## APPENDIX
## PROOF OF (45)

According to (44), it is easy to obtain the gap of interference power, i.e.,

$$\begin{aligned} I_{gap} &= I^{robust}(g_{k,m,n}) - I^{non-robust}(\bar{g}_{k,m,n}) \\ &= \sum_n \sum_m \sum_k \beta_n^{robust} \tilde{p}_{k,n}^{robust}(\bar{g}_{k,m,n} + \varepsilon_k) \\ &\quad - \sum_n \sum_m \sum_k \beta_n^{non-robust} \tilde{p}_{k,n}^{non-robust} \bar{g}_{k,m,n}. \end{aligned} \tag{46}$$

According to the sensitivity analysis in [29], when the parameter uncertainty is assumed to be very small, the optimal value under the non-robust case and the robust case can be assumed to be equal, thus we have

$$I_{gap} = \sum_n \sum_m \sum_k \beta_n^* \tilde{p}_{k,n}^* \varepsilon_k. \tag{47}$$

Similarly, define $f_{k,m} = \sum_{i \neq k} p_{i,m} \bar{h}_{i,m} + \sigma_{k,m}$, the gap of data rate of each FU $k$ on the FN $m$ can be defined as

$$\begin{aligned} R_{k,m}^{gap} &= R_{k,m}^r(h_{k,m}) - \bar{R}_{k,m}(\bar{h}_{k,m}) \\ &= \log_2\left(\frac{p_{k,m}\bar{h}_{k,m}}{\sum_{i\neq k} p_{i,m}(\bar{h}_{i,m} + \upsilon_k) + \sigma_{k,m}}\right) \\ &\quad - \log_2\left(\frac{p_{k,m}\bar{h}_{k,m}}{\sum_{i\neq k} p_{i,m}\bar{h}_{i,m} + \sigma_{k,m}}\right) \\ &= \log_2(p_{k,m}\bar{h}_{k,m}) - \log_2(\sum_{i\neq k} p_{i,m}\upsilon_k + f_{k,m}) \\ &\quad - \log_2(p_{k,m}\bar{h}_{k,m}) + \log_2 f_{k,m} \\ &= \log_2 f_{k,m} - \log_2(\sum_{i\neq k} p_{i,m}\upsilon_k + f_{k,m}). \end{aligned} \tag{48}$$

Based on Lagrange mean value theorem, we have

$$\begin{aligned} \log_2(\sum_{i\neq k} p_{i,m}\upsilon_k + f_{k,m}) &= \log_2 f_{k,m} \\ &\quad + \frac{\sum_{i\neq k} p_{i,m}\upsilon_k}{\ln 2(\sum_{i\neq k} p_{i,m}\upsilon_k + f_{k,m})} + o. \end{aligned} \tag{49}$$

Combining (48) and (49), we have

$$R_{k,m}^{gap} = -\frac{\sum_{i\neq k} p_{i,m}^* \upsilon_k}{\ln 2 \bar{z}_{k,m}}, \tag{50}$$

where $\bar{z}_{k,m} = \sum_{i\neq k} p_{i,m}^*(\bar{h}_{i,m} + \upsilon_k) + \sigma_{k,m}$.

Bring (47), (50) into (44), we have

$$\begin{aligned} L_{gap} &= \sum_k \sum_m (1 + \chi_k^*) s_{k,m}^* R_{k,m}^{gap} \\ &\quad - \sum_n \sum_m \sum_k \beta_n^* s_{k,m}^* p_{k,m}^* \varepsilon_n, \end{aligned} \tag{51}$$

where $R_{k,m}^{gap} = -\frac{\sum_{i\neq k} p_{i,m}^* \upsilon_k}{\ln 2 \bar{z}_{k,m}}$ and $\bar{z}_{k,m} = \sum_{i\neq k} p_{i,m}^*(\bar{h}_{i,m} + \upsilon_k) + \sigma_{k,m}$. The proof is completed.

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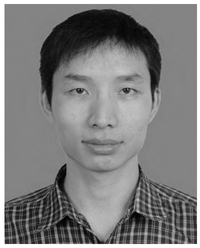

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